Theoretical description of the forward-backward multiplicity correlations in windows separated in azimuth and rapidity

Vladimir Vechernin

St.-Petersburg State University

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Outline

- Connection of the forward-backward (FB) correlation coefficient b with two-particle correlation function C_2
- The b and C_2 in the model with independent identical emitters (strings)
- Fitting of the model parameters by FB multiplicity correlations between two small windows, separated in azimuth and rapidity
- Comparison of the FB correlation in large 2π windows with the model results
- Connection with the di-hadron correlation analysis
- Conclusions

Connection of the FB correlation coefficient with two-particle correlation function - 1

Traditionally one uses the following definition of the FB correlation coefficient:

$$b_{abs} \equiv \frac{\langle n_F n_B \rangle - \langle n_F \rangle \langle n_B \rangle}{D_{n_F}} \quad \text{where} \quad D_{n_F} = \langle n_F^2 \rangle - \langle n_F \rangle^2 \tag{1}$$

To avoid the trivial influence of absolute values of n_F and n_B on the correlation coefficient we go to the relative or scaled observables:

$$\nu_F = n_F / \langle n_F \rangle \quad \text{and} \quad \nu_B = n_B / \langle n_B \rangle \tag{2}$$

For these observables

$$b_{rel} = \frac{\langle \nu_F \nu_B \rangle - 1}{\langle \nu_F^2 \rangle - 1} = \frac{\langle n_F \rangle}{\langle n_B \rangle} \, b_{abs} \ . \tag{3}$$

Connection of the FB correlation coefficient with two-particle correlation function - 2

The two-particle correlation function C_2 is defined through the inclusive ρ_1 and double inclusive ρ_2 distributions:

$$C_2(\eta_F, \phi_F; \eta_B, \phi_B) = \frac{\rho_2(\eta_F, \phi_F; \eta_B, \phi_B)}{\rho_1(\eta_F, \phi_F)\rho_1(\eta_B, \phi_B)} - 1 \tag{4}$$

$$\rho_1(\eta, \phi) = \frac{d^2 N}{d\eta\, d\phi} \;, \qquad \rho_2(\eta_F, \phi_F; \eta_B, \phi_B) = \frac{d^4 N}{d\eta_F\, d\phi_F\, d\eta_B\, d\phi_B} \tag{5}$$

For a small window $\delta\eta\,\delta\phi$ around η, ϕ we have

$$\rho_1(\eta, \phi) \equiv \frac{\langle n \rangle}{\delta\eta\,\delta\phi} \;, \tag{6}$$

here $\langle n \rangle$ is the mean multiplicity in the acceptance $\delta\eta\,\delta\phi$.

Connection of the FB correlation coefficient with two-particle correlation function - 3

For two small windows: $\delta\eta_F\,\delta\phi_F$ around η_F, ϕ_F and $\delta\eta_B\,\delta\phi_B$ around η_B, ϕ_B we have

$$\rho_2(\eta_F,\phi_F;\eta_B,\phi_B) \equiv \frac{\langle n_F n_B\rangle}{\delta\eta_F\,\delta\phi_F\,\delta\eta_B\,\delta\phi_B}\ . \qquad (7)$$

The formulae (6) and (7) are the base for the experimental measurement of the one- and two-particle densities of charge particles ρ_1 and ρ_2, and hence of the two-particle correlation function C_2 (4),
for which by (6) and (7) we have:

$$C_2(\eta_F,\phi_F;\eta_B,\phi_B) \equiv \frac{\langle n_F n_B\rangle - \langle n_F\rangle\langle n_B\rangle}{\langle n_F\rangle\langle n_B\rangle}\ . \qquad (8)$$

where n_F and n_B are the event multiplicities in these two small windows.

Connection of the FB correlation coefficient with two-particle correlation function - 4

Comparing with traditional definition of the FB correlation coefficient (1), for small FB windows by (8) we have

$$b_{rel} = \frac{\langle n_F \rangle^2}{D_{n_F}} C_2(\eta_F, \phi_F; \eta_B, \phi_B) \tag{9}$$

Note that for small forward window:

$$D_{n_F} = \langle n_F \rangle [1 + \langle n_F \rangle C_2(\eta_F, \phi_F; \eta_F, \phi_F)] \ , \tag{10}$$

$$b_{rel} = \frac{\langle n_F \rangle}{1 + \langle n_F \rangle C_2(\eta_F, \phi_F; \eta_F, \phi_F)} C_2(\eta_F, \phi_F; \eta_B, \phi_B) \tag{11}$$

So by (11) we see that the traditional definition of the FB correlation coefficient in the case of small observation windows coincides with the standard definition of two-particle correlation function C_2 upto some common factor, which depends on the width of the forward window.

Connection of the FB correlation coefficient with two-particle correlation function - 5

One can go in C_2 to the variables:

$$\eta_{sep} = \eta_F - \eta_B \ , \qquad \eta_C = (\eta_F + \eta_B)/2 \tag{12}$$

$$\phi_{sep} = \phi_F - \phi_B \ , \qquad \phi_C = (\phi_F + \phi_B)/2 \tag{13}$$

and using the connection (8) or (11) check up *experimentally* the dependence of the two-particle correlation function C_2 on η_C for the different configurations and separations between FB observation windows. In the central rapidity region

$$C_2(\eta_F, \phi_F; \eta_B, \phi_B) = C_2(\eta_{sep}, \phi_{sep}) \tag{14}$$

and for small windows we have

$$D_{n_F} = \langle n_F \rangle [1 + \langle n_F \rangle C_2(0,0)] \ , \tag{15}$$

$$b_{rel} = \frac{\langle n_F \rangle}{1 + \langle n_F \rangle C_2(0,0)} C_2(\eta_{sep}, \phi_{sep}) \tag{16}$$

Connection of the FB correlation coefficient with two-particle correlation function - 6

For windows of arbitrary width in azimuth and rapidity, situated in the central rapidity region, a model-independent way, we obtain:

$$D_{n_F} = \langle n_F \rangle [1 + \langle n_F \rangle I_{FF}] \ , \tag{17}$$

$$b_{rel} = \frac{\langle n_F \rangle}{1 + \langle n_F \rangle I_{FF}} I_{FB}(\eta_{sep}, \phi_{sep}) \tag{18}$$

where

$$I_{FB}(\eta_{sep}, \phi_{sep}) = \frac{1}{\delta y_F \delta\varphi_F \delta y_B \delta\varphi_B} \int_{\delta y_F \delta\varphi_F} dy_1 d\varphi_1 \int_{\delta y_B \delta\varphi_B} dy_2 d\varphi_2 \, C_2(\eta_1 - \eta_2; \phi_1 - \phi_2) \tag{19}$$

$$I_{FF} = \frac{1}{(\delta y_F \delta\varphi_F)^2} \int_{\delta y_F \delta\varphi_F} dy_1 d\varphi_1 \int_{\delta y_F \delta\varphi_F} dy_2 d\varphi_2 \, C_2(\eta_1 - \eta_2; \phi_1 - \phi_2) \tag{20}$$

A.Capella, A.Krzywicki, Phys.Rev.D18, 4120 (1978).
C.Pruneau, S.Gavin, S.Voloshin, Phys.Rev.C66, 044904 (2002).

Model with independent identical emitters - 1

$$\rho_1^N(\eta) = N\lambda_1(\eta) \ , \tag{21}$$

$$\rho_2^N(\eta_F, \eta_B; \phi_{sep}) = N\lambda_2(\eta_F, \eta_B; \phi_{sep}) + N(N-1)\lambda_1(\eta_F)\lambda_1(\eta_B) \ . \tag{22}$$

Then after averaging over N the one- and two-particle densities of charge particles are given by

$$\rho_1(\eta) = \langle N\rangle\lambda_1(\eta) \ , \tag{23}$$

$$\rho_2(\eta_F, \eta_B; \phi_{sep}) = \langle N\rangle[\lambda_2(\eta_F, \eta_B; \phi_{sep}) - \lambda_1(\eta_F)\lambda_1(\eta_B)] + \langle N^2\rangle\lambda_1(\eta_F)\lambda_1(\eta_B) \tag{24}$$

and

$$\rho_2(\eta_F, \eta_B; \phi_{sep}) - \rho_1(\eta_F)\rho_1(\eta_B) = \tag{25}$$

$$= \langle N\rangle[(\lambda_2(\eta_F, \eta_B; \phi_{sep}) - \lambda_1(\eta_F)\lambda_1(\eta_B)] + D_N\lambda_1(\eta_F)\lambda_1(\eta_B) \ ,$$

where D_N is the event-by-event variance $D_N = \langle N^2\rangle - \langle N\rangle^2$ of the number of emitters.

M.A.Braun, C.Pajares, V.V., Phys.Lett.B493, 54 (2000).

Model with independent identical emitters - 2

Then we find

$$\boxed{C_2(\eta_F, \eta_B; \phi_F - \phi_B) = \frac{\Lambda(\eta_F,\eta_B;\phi_F-\phi_B)+\omega_N}{\langle N\rangle}} \ ,$$

where ω_N is the event-by-event scaled variance $\omega_N = D_N/\langle N\rangle$ of the number of emitters and

$$\Lambda(\eta_F, \eta_B; \phi_F - \phi_B) = \frac{\lambda_2(\eta_F, \eta_B; \phi_F - \phi_B)}{\lambda_1(\eta_F)\lambda_1(\eta_B)} - 1 \qquad (26)$$

is the two-particle correlation function for charged particles produced from a decay of a single emitter (string).

A.Capella, A.Krzywicki, Phys.Rev.D18, 4120 (1978).

Model with independent identical emitters - 3

In the central rapidity region, where each string contributes to the particle production in the whole rapidity region, one has the translation invariance in rapidity

$$\lambda_1(\eta) = \mu_0 = const \ , \qquad \Lambda(\eta_F, \eta_B; \phi_{sep}) = \Lambda(\eta_F - \eta_B; \phi_{sep}) \ , \tag{27}$$

then

$$\rho_1(\eta) = \langle N \rangle \mu_0 = const \ , \tag{28}$$

$$C_2(\eta_{sep}, \phi_{sep}) = \frac{\Lambda(\eta_{sep}, \phi_{sep}) + \omega_N}{\langle N \rangle} \ . \tag{29}$$

So we see that this common "pedestal" in $C_2(\eta_{sep}, \phi_{sep})$ is physically important. By (29) we see that from the height of the "pedestal" $(\omega_N/\langle N \rangle)$ one can obtain the important physical information on the magnitude of the fluctuation of the number of emitters N at different energies and centrality fixation.

FB correlation in the model - 1

FB multiplicity correlation strength b_{rel} in the case of the observation windows of arbitrary width in azimuth and rapidity, which are situated in the central rapidity region, are given by

$$b_{rel} = \frac{[\omega_N + J_{FB}(\eta_{sep}, \phi_{sep})]\mu_0\delta_F}{1 + [\omega_N + J_{FF}]\mu_0\delta_F} \ , \tag{30}$$

(the relative variables are using, V.V. arXiv:1210.7588, 1305.0857), where

$$J_{FB}(\eta_{sep}, \phi_{sep}) = \frac{1}{\delta\eta_F\delta\phi_F\delta\eta_B\delta\phi_B} \int_{\delta\eta_F\delta\phi_F} d\eta_1 d\phi_1 \int_{\delta\eta_B\delta\phi_B} d\eta_2 d\phi_2 \, \Lambda(\eta_1 - \eta_2; \phi_1 - \phi_2) \ , \tag{31}$$

$$J_{FF} = \frac{1}{(\delta\eta_F\delta\phi_F)^2} \int_{\delta\eta_F\delta\phi_F} d\eta_1 d\phi_1 \int_{\delta\eta_F\delta\phi_F} d\eta_2 d\phi_2 \, \Lambda(\eta_1 - \eta_2; \phi_1 - \phi_2) \ , \tag{32}$$

$\Lambda(\eta; \phi)$ is the pair correlation function for a single string,
$\omega_N = \frac{\langle N^2\rangle - \langle N\rangle^2}{\langle N\rangle}$ is the e-by-e scaled variance of the number of strings,
$\delta_F \equiv \delta\eta_F\delta\phi_F/2\pi$ is the acceptance of the forward window,
μ_0 is the average rapidity density of the charged particles from one string.

FB correlation in the model - 2

η_{sep}, ϕ_{sep} are the separations between the centers of FB windows in rapidity and azimuth.

$$b_{rel} = b^{LR} + b^{SR} \ , \tag{33}$$

$$b^{LR} = \frac{\omega_N \mu_0 \delta_F}{1 + [\omega_N + J_{FF}]\mu_0 \delta_F} \ , \tag{34}$$

$$b^{SR} = \frac{\mu_0 \delta_F}{1 + [\omega_N + J_{FF}]\mu_0 \delta_F} J_{FB}(\eta_{sep}, \phi_{sep}) \tag{35}$$

The Long-Range (LR) contribution arising due to e-by-e fluctuation in the number of emitters (strings).

The Short-Range (SR) contribution originating from the pair correlation function $\Lambda(\eta;\phi)$ of a single string.

Note that at $\Lambda(\eta,\phi) = 0$ we have $J_{FB} = J_{FF} = 0$ and $b^{SR}_{\Lambda=0} = 0$, but

$$b^{\Lambda=0}_{rel} = b^{LR}_{\Lambda=0} = \frac{\omega_N \mu_0 \delta_F}{1 + \omega_N \mu_0 \delta_F} \neq 0 \ .$$

M.A.Braun, R.S.Kolevatov, C.Pajares, V.V., Eur.Phys.J.C32, 535 (2004).

FB correlation in the model - 3

Note that by (31) and (32) for windows of small acceptances in rapidity and azimuth, when $\delta\eta \ll \eta_{cor}$ and $\delta\phi \ll \phi_{cor}$ (where η_{cor} and ϕ_{cor} are the characteristic correlation lengthes, defined by the behavior of the pair correlation function $\Lambda(\eta, \phi)$ (39) of a single string), we have

$$J_{FB}(\eta_{sep}, \phi_{sep}) \approx \Lambda(\eta_{sep}, \phi_{sep}) \ , \tag{36}$$

$$J_{FF} \approx \Lambda(0, 0) \tag{37}$$

and the formula (30) takes the simple form

$$b_{rel} \approx \frac{[\omega_N + \Lambda(\eta_{sep}, \phi_{sep})]\mu_0\delta_F}{1 + [\omega_N + \Lambda(0, 0)]\mu_0\delta_F} \ , \tag{38}$$

which *enables to fit the model parameters by experimental observation of the FB correlations between two small windows, varying the separation between these windows in azimuth and rapidity*.

The pair correlation function of a single string

The parametrization for the pair correlation function $\Lambda(\eta,\phi)$ of a single string (basing on the Schwinger mechanism of a string decay, V.V. arXiv:1210.7588):

$$\Lambda(\eta,\phi) = \Lambda_1 e^{-\frac{|\eta|}{\eta_1}} e^{-\frac{\phi^2}{\phi_1^2}} + \Lambda_2 \left(e^{-\frac{|\eta-\eta_0|}{\eta_2}} + e^{-\frac{|\eta+\eta_0|}{\eta_2}} \right) e^{-\frac{(|\phi|-\pi)^2}{\phi_2^2}} . \qquad (39)$$

This formula has the nearside peak, characterizing by parameters Λ_1, η_1 and ϕ_1, and the awayside ridge-like structure, characterizing by parameters Λ_2, η_2, η_0 and ϕ_2 (two wide overlapping hills shifted by $\pm\eta_0$ in rapidity, η_0 - the mean length of a string decay segment). We imply that in formula (39)

$$|\phi| \le \pi \ . \qquad (40)$$

If $|\phi| > \pi$, then we use the replacement $\phi \to \phi + 2\pi k$, so that (40) was fulfilled. With such completions the $\Lambda(\eta,\phi)$ meets the following properties

$$\Lambda(-\eta,\phi) = \Lambda(\eta,\phi) \ , \quad \Lambda(\eta;-\phi) = \Lambda(\eta,\phi) \ , \quad \Lambda(\eta,\phi+2\pi k) = \Lambda(\eta,\phi) \qquad (41)$$

Calculation of the integrals

The integrals (31) and (32) in the case of symmetric arbitrary windows $\delta\eta_F = \delta\eta_F \equiv \delta\eta$ and $\delta\phi_F = \delta\phi_F \equiv \delta\phi$ reduce to

$$J_{FB}(\eta_{sep}, \phi_{sep}) = (\delta\eta\delta\phi)^{-2} \int_{-\delta\eta}^{\delta\eta} d\eta \int_{-\delta\phi}^{\delta\phi} d\phi \, \Lambda(\eta + \eta_{sep}, \phi + \phi_{sep}) \, t_{\delta\eta}(\eta) \, t_{\delta\phi}(\phi) \, , \tag{42}$$

$$J_{FF} = I_{FB}(0,0) = (\delta\eta\delta\phi)^{-2} \int_{-\delta\eta}^{\delta\eta} d\eta \int_{-\delta\phi}^{\delta\phi} d\phi \, \Lambda(\eta, \phi) \, t_{\delta\eta}(\eta) \, t_{\delta\phi}(\phi) =$$

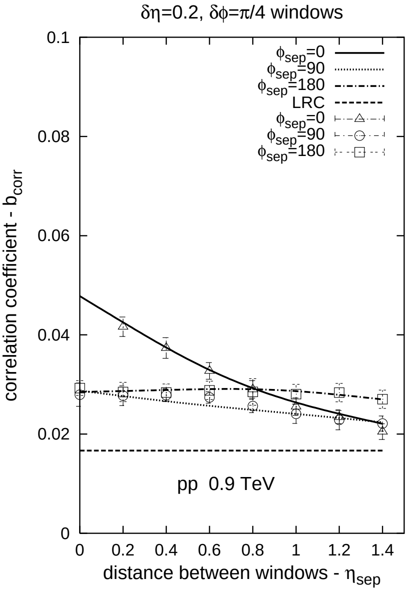
$$= 4(\delta\eta\delta\phi)^{-2} \int_0^{\delta\eta} d\eta \int_0^{\delta\phi} d\phi \, \Lambda(\eta; \phi) \, (\delta\eta - \eta) \, (\delta\phi - \phi) \, . \tag{43}$$

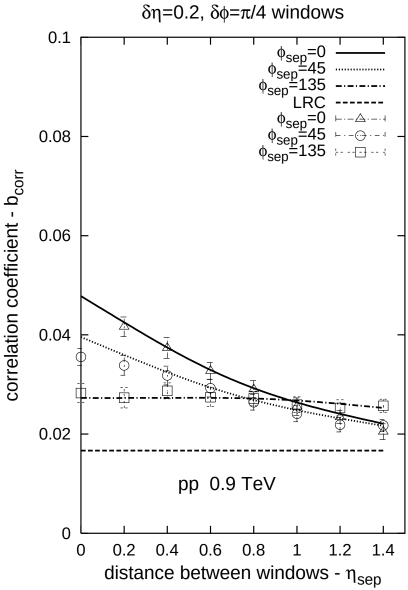
where $t_{\delta y}(y)$ is a "triangular" weight function arising at integration due to phase:

$$t_{\delta y}(y) = [\theta(-y)(\delta y + y) + \theta(y)(\delta y - y)] \, \theta(\delta y - |y|) \, . \tag{44}$$

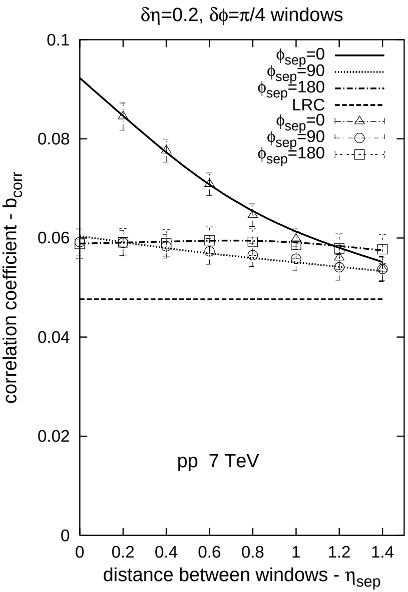
Then the η–ϕ factorization for near and away side contributions in the fit (39) for $\Lambda(\eta; \phi)$ enables to reduce the (42) and (43) to single integrals.

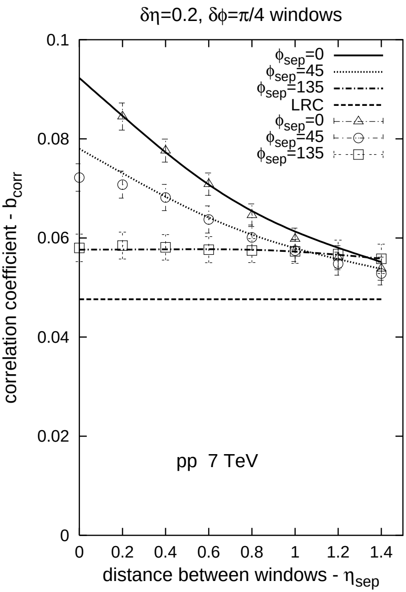
Fitting of model parameters by FBC in small windows - 1





Fitting of model parameters by FBC in small windows - 2





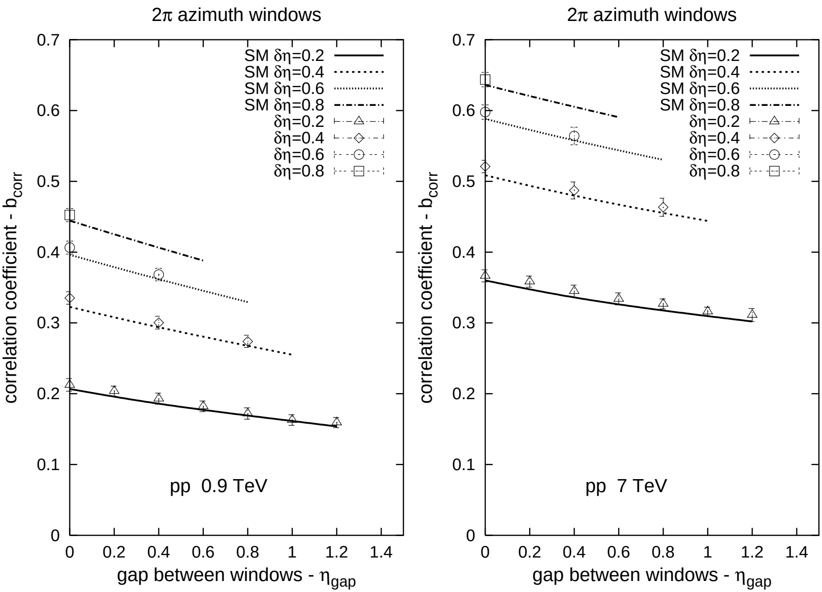
Fitting of model parameters by FBC in small windows - 3

Results of the fitting of the model parameters by FB correlations between two small windows, separated in azimuth and rapidity:

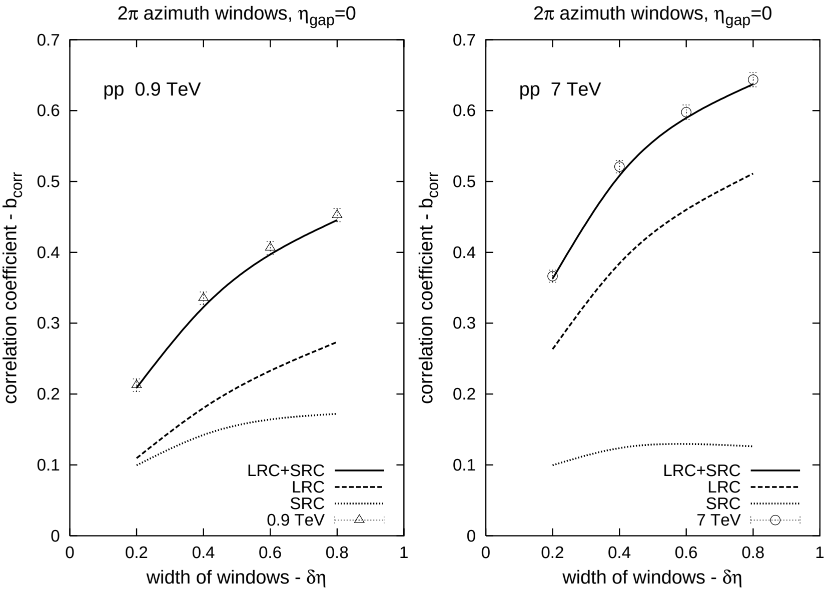
$\sqrt{s}$, TeV		0.9	7.0
LRC	$\mu_0\omega_N$	0.7	2.1
SRC	$\mu_0\Lambda_1$	1.5	2.3
	η_1	0.75	0.75
	ϕ_1	1.2	1.1
	$\mu_0\Lambda_2$	0.4	0.4
	η_2	2.0	2.0
	ϕ_2	1.7	1.7
	η_0	0.9	0.9

$\omega_N = \frac{\langle N^2\rangle - \langle N\rangle^2}{\langle N\rangle}$ is the e-by-e scaled variance of the number of strings,
μ_0 is the average rapidity density of the charged particles from one string,
$i=1$ corresponds to the nearside and $i=2$ to the awayside contributions,
η_0 is the mean length of a string decay segment.
I.Altsybeev, PhD Thesis, SPbSU, 2013.
G.Feofilov et al. (for ALICE Collaboration), PoS (Baldin ISHEPP XXI) 075, 2012.

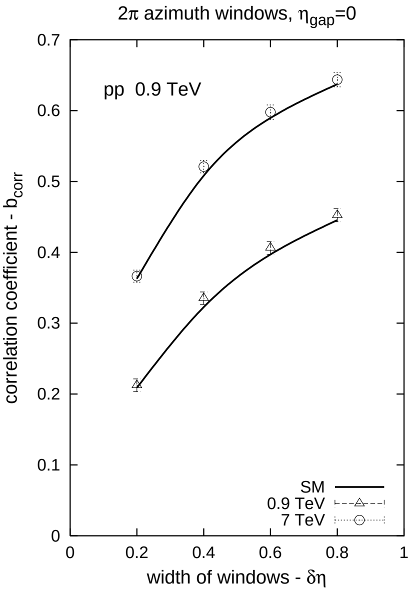
Comparison of FBC in large 2π windows with SM - 1



Comparison of FBC in large 2π windows with SM - 2



Comparison of FBC in large 2π windows with SM - 3



Connection with the di-hadron correlation analysis - 1

The di-hadron correlation function

$$C(\Delta y, \Delta\phi) \equiv S/B - 1 \tag{45}$$

takes into account all possible pair combinations of particles produced in given event in some ONE LARGE pseudorapidity window $\Delta y \in (-Y, Y)$, where

$$S = \frac{d^2 N}{d\Delta y \, d\Delta\phi} \tag{46}$$

and the B is the same but in the case of uncorrelated particle production. Experimentalists obtain the B by the event mixing procedure.
We can express the enumerator of (45) through the two-particle correlation function:

$$S(\Delta y, \Delta\phi) = \int_{-Y/2}^{Y/2} dy_1 dy_2 \, \rho_2(y_1, y_2; \Delta\phi) \, \delta(y_1 - y_2 - \Delta y) \tag{47}$$

Connection with the di-hadron correlation analysis - 2

In the central rapidity region, when the translation invariance takes place within the whole rapidity interval $(-Y/2, Y/2)$, we have

$$\rho_2(y_1, y_2;\ \Delta\phi) = \rho_2(y_1 - y_2;\ \Delta\phi)$$

and one can fulfill the integration in (47):

$$S(\Delta y, \Delta\phi) = \rho_2(\Delta y; \Delta\phi)\, t_Y(\Delta y) \qquad (48)$$

where the $t_Y(\Delta y)$ is a "triangular" weight function

$$t_{\delta\eta}(y) = [\theta(-y)(\delta\eta + y) + \theta(y)(\delta\eta - y)]\, \theta(\delta\eta - |y|)\ . \qquad (49)$$

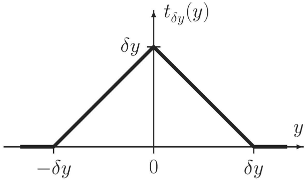


Figure: The "triangular" weight function arising due to phase space .

Connection with the di-hadron correlation analysis - 3

In the denominator of (45) we should replace the $\rho_2(y_1, y_2; \Delta\phi)$ by the product $\rho_1(y_1)\rho_1(y_2)$, which due to the translation invariance in rapidity reduces simply to ρ_0^2. Then

$$B(\Delta y, \Delta\phi) = \rho_0^2 \; t_Y(\Delta y) \; . \tag{50}$$

Substituting into (45) we get

$$C(\Delta y, \Delta\phi) = \frac{\rho_2(\Delta y; \Delta\phi)}{\rho_0^2} - 1 = C_2(\Delta y, \Delta\phi) \; , \tag{51}$$

We see that if the translation invariance in rapidity takes place within the whole interval $(-Y/2, Y/2)$, then the definition (45) for the di-hadron correlation function C leads to the standard two-particle correlation function C_2 (4) (see meanwhile the remarks below).

Comment on the normalization

By the formulae:

$$C_2(\eta_F,\phi_F;\eta_B,\phi_B)=\frac{\rho_2(\eta_F,\phi_F;\eta_B,\phi_B)}{\rho_1(\eta_F,\phi_F)\rho_1(\eta_B,\phi_B)}-1\ ,$$

$$\rho_1(\eta,\phi)\equiv\frac{\langle n\rangle}{\delta\eta\,\delta\phi}\ ,$$

$$\rho_2(\eta_F,\phi_F;\eta_B,\phi_B)\equiv\frac{\langle n_F n_B\rangle}{\delta\eta_F\,\delta\phi_F\,\delta\eta_B\,\delta\phi_B}$$

we see that calculating

$$C(\Delta\eta,\Delta\phi)\equiv\frac{S}{B}-1$$

we must to normalize S and B dividing by the number of events (N_{ev}), not by the number of triggers (N_{tr}), pairs (N_{par}) or $B(0,0)$. Otherwise the C will not coincide with C_2.

Conclusions

- The observation of multiplicity-multiplicity correlation with *two* small (in azimuth and rapidity) windows, enables to measure the two-particle correlation function C_2 in accordance with the standard definition even in the case of nonhomogeneous distributions (e.g. as in pA) without using the event mixing procedure.
- The model with strings as independent identical emitters well describes the FB multiplicity correlation in large 2π windows, when its parameters are fitted by the correlation in small windows.
- The relative contribution of the Long-Range Correlation (LRC), originating from e-by-e fluctuation in the number of emitters (strings), considerably increases with energy while the contribution of the Short-Range Correlations (SRC), originating from the pair correlation function of a single string remains practically the same.
- We must to apply the proper normalization to obtain C_2 by the di-hadron analysis.

Backup - 1

Backup slides - 1

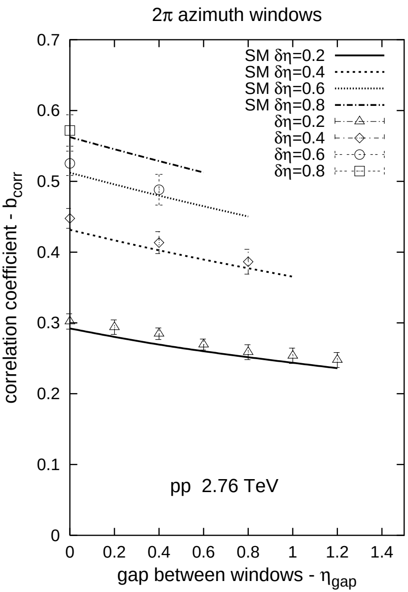
Fitting the model parameters at 2.76 TeV

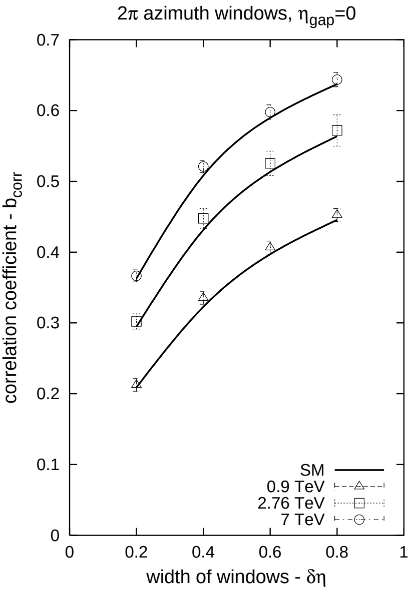
At present we have not the experimental value of FB correlation coefficient b_{rel} with small windows ($\delta\eta = 0.2$, $\delta\phi = \pi/4$) at the 2.76 TeV energy for fitting of the model parameters at this energy. So for a rude evaluation we take the mean value of the parameters at 0.9 and 7 TeV:

$\sqrt{s}$, TeV		0.9	2.76	7.0
LRC	$\mu_0\omega_N$	0.7	1.4	2.1
SRC	$\mu_0\Lambda_1$	1.5	1.9	2.3
	η_1	0.75	0.75	0.75
	ϕ_1	1.2	1.15	1.1
	$\mu_0\Lambda_2$	0.4	0.4	0.4
	η_2	2.0	2.0	2.0
	ϕ_2	1.7	1.7	1.7
	η_0	0.9	0.9	0.9

Then with these parameters we have:

Comparison of FBC in large 2π windows with SM





Backup slides - 2

Backup slides - 2

Comments on the event mixing - 1

In the framework of the model with strings as independent identical emitters we have for the enumerator and the denominator of (45):

$$S(\Delta y,\Delta\phi)=\rho_2(\Delta y;\Delta\phi)\,t_Y(\Delta y)=\langle\rho_2^N(\Delta y;\Delta\phi)\rangle\,t_Y(\Delta y)= \tag{52}$$

$$=[\langle N\rangle\Lambda(\Delta y,\Delta\phi)+\langle N^2\rangle]\mu_0^2\,t_Y(\Delta y)\;,$$

$$B(\Delta y,\Delta\phi)=\int_{-Y/2}^{Y/2}dy_1dy_2\,\rho_1(y_1)\rho_1(y_2)\,\delta(y_1-y_2-\Delta y)=$$

$$=\int_{-Y/2}^{Y/2}dy_1dy_2\,\langle\rho_1^N(y_1)\rangle\langle\rho_1^N(y_2)\rangle\,\delta(y_1-y_2-\Delta y)=$$

$$=\rho_0^2\;t_Y(\Delta y)=\langle N\rangle^2\mu_0^2\;t_Y(\Delta y)\;, \tag{53}$$

we have noted that $\lambda_1(y)=\mu_0$. Then by $C=S/B-1$ we get

$$C(\Delta y,\Delta\phi)=\frac{\omega_N+\Lambda(\Delta y,\Delta\phi)}{\langle N\rangle}=C_2(\Delta y,\Delta\phi)\;, \tag{54}$$

Comments on the event mixing - 2

But if instead of (53) one has

$$B(\Delta y, \Delta\phi) = \int_{-Y/2}^{Y/2} dy_1 dy_2 \, \langle \rho_1^N(y_1)\rho_1^N(y_2)\rangle \, \delta(y_1 - y_2 - \Delta y) = \langle N^2\rangle \mu_0^2 \, t_Y(\Delta y)$$

as it sometimes takes place in a di-hadron data analysis (or if some other artificial normalization conditions for the $B(\Delta y, \Delta\phi)$ are being used), then instead of (54) by $C = S/B - 1$ we get

$$C(\Delta y, \Delta\phi) = \frac{\langle N\rangle}{\langle N^2\rangle}\Lambda(\Delta y, \Delta\phi) \; , \tag{55}$$

which does not correspond to the standard two-particle correlation function $C_2(\Delta y, \Delta\phi)$, defined by (4). Compare (55) with (54) we see that in this case the resulting $C(\Delta y, \Delta\phi)$ does not have an additional contribution reflecting the event-by-event fluctuation in the number of emitters. It depends only on the pair correlation function of a single string $\Lambda(\Delta y, \Delta\phi)$ and, therefore, is equal to zero in the absence of the pair correlation from one string.