Relativistic corrections to the pair B_c mesons production in proton–proton collisions

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Pair charmonium production

e^+e^- annihilation:

- K. Abe et al. (Belle Collaboration), Phys. Rev. Lett. **89**, 142001 (2002)

$$\sigma[e^+e^- \to J/\psi + \eta_c] \times \mathcal{B}_{\geq 4} = 33^{+7}_{-6} \pm 9 \text{ fb}$$

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- E. Braaten and J. Lee, Phys. Rev. D **67**, 054007 (2003); **72**, 099901(E) (2005)
- K.Y. Liu, Z. G. He, and K. T. Chao, Phys. Lett. B **557**, 45 (2003)

$$\sigma[e^+e^- \to J/\psi + \eta_c] = 3.78 \pm 1.26 \text{ fb}$$
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New experiments:

- K. Abe et al. (Belle Collaboration), Phys. Rev. D **70**, 071102 (2004)
- B. Aubert et al. (BABAR Collaboration), Phys. Rev. D **72**, 031101 (2005)

$$\sigma[e^+e^- \to J/\psi + \eta_c] \times \mathcal{B}_{>2} = 25.6 \pm 2.8 \pm 3.4 \text{ fb}$$
$$\sigma[e^+e^- \to J/\psi + \eta_c] \times \mathcal{B}_{>2} = 17.6 \pm 2.8 \pm 2.1 \text{ fb}$$

Pair charmonium production

Improvement sources?

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- Y.-J. Zhang, Y.-J. Gao, K.-T. Chao, Phys. Rev. Lett. **96**, 092001 (2006)
- B. Gong and J.-X. Wang, Phys. Rev. D **77**, 054028 (2008)

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Found to be large in light–cone approach:

- A.E. Bondar, V.L. Chernyak, Phys. Lett. B **612**, 215 (2005)
- J.P. Ma and Z.G. Si, Phys. Rev. D **70**, 074007 (2004)
- V.V. Braguta, A. K. Likhoded, A.V. Luchinsky, Phys. Rev. D **72**, 074019 (2005)
- V.V. Braguta, Phys. Rev. D **79**, 074018 (2009)

Pair charmonium production

. . . as well as in potential models:

- D. Ebert, A.P. Martynenko, Phys. Rev. D **74**, 054008 (2006)
- D. Ebert, R.N. Faustov, V.O. Galkin, A.P. Martynenko, PLB **672**, 264 (2009)

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Relativistic corrections within NRQCD formalism:

- Z.-G. He, Y. Fan, K.-T. Chao, Phys. Rev. D **75**, 074011 (2007)
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- G.T. Bodwin, J. Lee, C. Yu, Phys. Rev. D **77**, 094018 (2008)

nonrelativistic result, (fb)	relativistic corrections	QED	NLO α_s (+QED)	correlations of relativistic & NLO α_s
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5.4	**2.9**	1.0	6.9	1.4

$$\sigma_{[Bodwin,Lee,Yu]} = \mathbf{17.6}^{+8.1}_{-6.7} \text{ fb}$$

Part of the NLO α_s effects is effectively accounted by the wave functions in light–cone and potential models:

- G.T. Bodwin, D. Kang, J. Lee, Phys. Rev. D **74**, 114028 (2006)

Relativistic corrections

Pair charmonium production (e^+e^- and pp):

- E.N. Elekina, A.P. Martynenko, Phys. Rev. D **81**, 054006 (2010)
- A.P. Martynenko, A.M. Trunin, Phys. Rev. D **86**, 094003 (2012); Phys. At. Nucl. **77**, 777 (2014)
- V.V. Braguta, A.K. Likhoded, A.V. Luchinsky, Phys. At. Nucl. **75**, 97 (2012)
- Y.-J. Li, G.-Z. Xu, K.-Y. Liu, Y.-J. Zhang, J. High Energy Phys. **1307**, 051 (2013)
- Y. Fan, J. Lee, C. Yu, Phys. Rev. D **87**, 094032 (2013)
- X.-H. Li, J.-X. Wang, arXiv:1301.0376 (2013)

Inclusive production:

- Y. Fan, Y.-Q. Ma, K.-T. Chao, Phys. Rev. D **79**, 114009 (2009)
- Z.-G. He, Y. Fan, K.-T. Chao, Phys. Rev. D **81**, 054036 (2010)
- Y. Jia, Phys. Rev. D **82**, 034017 (2010)
- G.-Z. Xu, Y.-J. Li, K.-Y. Liu, Y.-J. Zhang, Phys. Rev. D **86**, 094017 (2012); arXiv:1407.3783 (2014)

Pair B_c mesons production

In this work we consider the process $pp \to B_c + \bar{B}_c + X$:

- nonrelativistic cross sections
  [R. Li, Y.-J. Zhang, K.-T. Chao, Phys. Rev. D **80**, 014020 (2009)]

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  - non-zero bound energy effects

Quasipotential approach to relativistic quark model

Our calculation of relativistic corrections is based on the **quasipotential approach** (or the single–time formulation of the quantum field theory)

Bethe–Salpeter equation:

$$(\not{p}_1 - m_1)(\not{p}_2 - m_2)\psi_P(p) = i\int \frac{d^4q}{(2\pi)^4} K_{12}(p,q;P)\psi_P(q),$$

$\psi_P(x_1, x_2) = \langle 0|T\{\psi_1(x_1)\psi_2(x_2)\}|P\rangle$ — Bethe–Salpeter amplitude or wave function,

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Logunov–Tavkhelidze equation:

- A.A. Logunov, A.N. Tavkhelidze, Nuovo Cimento **29**, 380 (1963)
- V.G. Kadyshevsky, Nucl. Phys. **B** 6, 125 (1968)
- C. Itzykson, V.G. Kadyshevsky, I.T. Todorov, Phys. Rev. D **1**, 2823 (1970)
- R.N. Faustov, Teor. Mat. Fiz **3**, 240 (1970)

$$\left[M - \sqrt{\mathbf{p}^2 + m_1^2} - \sqrt{\mathbf{p}^2 + m_1^2}\right] \psi^{(+)}(\mathbf{p}) = \int \frac{d\mathbf{q}}{(2\pi)^3} V(\mathbf{p}, \mathbf{q}; M)\, \psi^{(+)}(\mathbf{q})$$

Quasipotential approach to relativistic quark model

Quasipotential equation in Schrödinger-like form:

- I.T. Todorov, Phys. Rev. D **3**, 2351 (1971)
- R.N. Faustov and A.P. Martynenko, Teor. Mat. Fiz **64**, 179 (1985)

$$\left[\frac{b^2(M)}{2\mu_R} - \frac{\mathbf{p}^2}{2\mu_R}\right]\psi^{(+)}(\mathbf{p}) = \int \frac{d\mathbf{q}}{(2\pi)^3} V(\mathbf{p},\mathbf{q};M)\,\psi^{(+)}(\mathbf{q}),$$

$$b^2(M) = \mathbf{p^2}\big|_{\text{on shell}} = \frac{1}{4M^2}\left[M^2-(m_1+m_2)^2\right]\left[M^2-(m_1-m_2)^2\right],$$

$\mu_R = \frac{1}{4M^3}\left[M^4-(m_1^2-m_2^2)^2\right]$ — relativistic reduced mass.

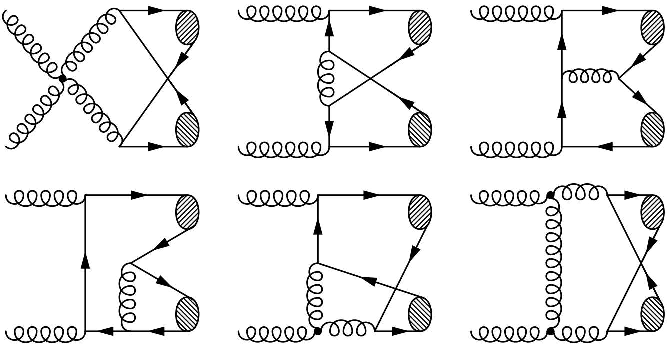
Quasipotential construction:

- R.N. Faustov, Fiz. El. Chast. Atom. Yad. **3**, 238 (1972)
- D. Ebert, V.O. Galkin, R.N. Faustov, Phys. Rev. D **57**, 5663 (1998)
- D. Ebert, R.N. Faustov, V.O. Galkin, Phys. Rev. D **72**, 034026 (2005)
- V.A. Matveev, V.I. Savrin, A.N. Sissakian, A.N. Tavkhelidze, Teor. Mat. Fiz **132**, 267 (2002)

31 LO α_s SPS gluon fusion diagrams

$$\mathcal{M}[gg \to B_c + \bar{B}_c] = \int \frac{d\mathbf{p}}{(2\pi)^3} \int \frac{d\mathbf{q}}{(2\pi)^3} \bar{\Psi}(p, P)\bar{\Psi}(q, Q) \otimes \mathcal{T}(p_1, p_2; q_1, q_2),$$

$$d\sigma[pp \to B_c + \bar{B}_c + X] = \int dx_1 dx_2\, f_{g/p}(x_1, \mu) f_{g/p}(x_2, \mu)\, d\sigma[gg \to B_c + \bar{B}_c]$$



Production amplitude

$$\mathcal{M}[gg \to B_c + \bar{B}_c](k_1, k_2, P, Q) = \frac{1}{9} M\pi^2\alpha_s^2 \int \frac{d\mathbf{p}}{(2\pi)^3} \int \frac{d\mathbf{q}}{(2\pi)^3} \mathrm{Tr}\, \mathfrak{M},$$

$$\mathfrak{M} = \bar{\Psi}_{P,p}\gamma_\beta \Gamma_1^{\beta\omega} \bar{\Psi}_{Q,q}\gamma_\omega + \bar{\Psi}_{P,p}\gamma_\beta \bar{\Psi}_{Q,q}\gamma_\omega \Gamma_2^{\beta\omega}$$
$$+\bar{\Psi}_{P,p}\hat{\varepsilon}_1 \frac{m_c - \hat{k}_1 + \hat{p}_1}{(k_1 - p_1)^2 - m_c^2}\gamma_\beta \bar{\Psi}_{Q,q}\Gamma_3^\beta + \bar{\Psi}_{P,p}\gamma_\beta \frac{m_c + \hat{k}_1 - \hat{q}_1}{(k_1 - q_1)^2 - m_c^2}\hat{\varepsilon}_1 \bar{\Psi}_{Q,q}\Gamma_4^\beta$$
$$+\bar{\Psi}_{P,p}\hat{\varepsilon}_2 \frac{m_c - \hat{k}_2 + \hat{p}_1}{(k_2 - p_1)^2 - m_c^2}\gamma_\beta \bar{\Psi}_{Q,q}\Gamma_5^\beta + \bar{\Psi}_{P,p}\gamma_\beta \frac{m_c + \hat{k}_2 - \hat{q}_1}{(k_2 - q_1)^2 - m_c^2}\hat{\varepsilon}_2 \bar{\Psi}_{Q,q}\Gamma_6^\beta,$$

$k_{1,2} = x_{1,2}\sqrt{S}/2\,(1, 0, 0, \pm 1)$ — the initial gluon four-momenta;
P, Q — the total four-momenta of outcoming B_c mesons;
$p = L_P(0, \mathbf{p})$, $q = L_Q(0, \mathbf{q})$ — the relative four-momenta of (anti)quarks.
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Vertex functions

$$
\begin{aligned}
\Gamma_1^{\beta\omega} = 18 D_\mu{}^\beta(p_1+q_1) D_\nu{}^\omega(p_2+q_2)\Big[2\varepsilon_1\varepsilon_2 g^{\mu\nu} - \varepsilon_1^\mu\varepsilon_2^\nu - \varepsilon_1^\nu\varepsilon_2^\mu \\
+ iD_{\kappa\lambda}(k_1-p_1-q_1)\mathfrak{E}_1^{\kappa\mu}(p_1+q_1)\mathfrak{E}_2^{\lambda\nu}(p_2+q_2) + iD_{\kappa\lambda}(k_1-p_2-q_2)\mathfrak{E}_1^{\lambda\nu}(p_2+q_2)\mathfrak{E}_2^{\kappa\mu}(p_1+q_1)\Big] \\
- i\varepsilon_1^\beta D_\mu{}^\omega(p_2+q_2)\frac{m_c-\hat{k}_1+\hat{p}_1}{(k_1-p_1)^2-m_c^2}\Big[\gamma^\mu\frac{m_c+\hat{k}_2-\hat{q}_1}{(k_2-q_1)^2-m_c^2}\hat{\varepsilon}_2 - 8\hat{\varepsilon}_2\frac{m_c-\hat{p}_2-\hat{q}_1-\hat{q}_2}{(p_2+q_1+q_2)^2-m_c^2}\gamma^\mu \\
- 9i\mathfrak{E}_2^{\nu\mu}(p_2+q_2)D_{\nu\rho}(k_1-p_1-q_1)\gamma^\rho\Big] - i\varepsilon_2^\beta D_\mu{}^\omega(p_2+q_2)\frac{m_c-\hat{k}_2+\hat{p}_1}{(k_2-p_1)^2-m_c^2} \\
\times\Big[\gamma^\mu\frac{m_c+\hat{k}_1-\hat{q}_1}{(k_1-q_1)^2-m_c^2}\hat{\varepsilon}_1 - 8\hat{\varepsilon}_1\frac{m_c-\hat{p}_2-\hat{q}_1-\hat{q}_2}{(p_2+q_1+q_2)^2-m_c^2}\gamma^\mu - 9i\mathfrak{E}_1^{\nu\mu}(p_2+q_2)D_{\nu\rho}(k_1-p_2-q_2)\gamma^\rho\Big] \\
+ 8iD^{\beta\omega}(p_2+q_2)\frac{m_c+\hat{p}_1+\hat{p}_2+\hat{q}_2}{(p_1+p_2+q_2)^2-m_c^2}\Big[\hat{\varepsilon}_1\frac{m_c+\hat{k}_2-\hat{q}_1}{(k_2-q_1)^2-m_c^2}\hat{\varepsilon}_2 + \hat{\varepsilon}_2\frac{m_c+\hat{k}_1-\hat{q}_1}{(k_1-q_1)^2-m_c^2}\hat{\varepsilon}_1\Big] \\
+ 9D_\nu{}^\omega(p_2+q_2)\Big[\mathfrak{E}_2^{\mu\nu}(p_2+q_2)D_\mu{}^\beta(k_1-p_1-q_1)\frac{m_c+\hat{k}_1-\hat{q}_1}{(k_1-q_1)^2-m_c^2}\hat{\varepsilon}_1 \\
+ \mathfrak{E}_1^{\mu\nu}(p_2+q_2)D_\mu{}^\beta(k_1-p_2-q_2)\frac{m_c+\hat{k}_2-\hat{q}_1}{(k_2-q_1)^2-m_c^2}\hat{\varepsilon}_2\Big]
\end{aligned}
\tag{10}
$$

Auxiliary functions:

$$
\begin{aligned}
\mathfrak{E}_{1,2}^{\mu\nu}(x) = g^{\mu\nu}(k_{1,2}-2x)\varepsilon_{1,2} + \varepsilon_{1,2}^\mu(2k_{1,2}^\nu - x^\nu) + \varepsilon_{1,2}^\nu(k_{1,2}^\mu + x^\mu), \\
\mathfrak{E}_{1,2}^{\mu}(x) = \varepsilon_{2,1}^\nu\mathfrak{E}_{1,2}^{\mu\nu}(x).
\end{aligned}
\tag{11}
$$

Transformation of relativistic wave functions

Quasipotential wave functions are calculated in the meson rest frame and then transformed to the reference frames moving with the four-momenta $P(Q)$:

$$\bar{\Psi}_{P,p} = \frac{\bar{\Psi}^0_{B_c}(\mathbf{p})}{\sqrt{\frac{\epsilon_c(p)}{m_c}\frac{\epsilon_c(p)+m_c}{2m_c}\frac{\epsilon_b(p)}{m_b}\frac{\epsilon_b(p)+m_b}{2m_b}}}\left[\frac{\hat{v}_1 - 1}{2} + \hat{v}_1 \frac{\mathbf{p}^2}{2m_b(\epsilon_b(p)+m_b)} - \frac{\hat{p}}{2m_b}\right]$$

$$\times \Sigma^P (1+\hat{v}_1)\left[\frac{\hat{v}_1 + 1}{2} + \hat{v}_1 \frac{\mathbf{p}^2}{2m_c(\epsilon_c(p)+m_c)} + \frac{\hat{p}}{2m_c}\right],$$

$$\bar{\Psi}_{Q,q} = \frac{\bar{\Psi}^0_{\bar{B}_c}(\mathbf{q})}{\sqrt{\frac{\epsilon_c(q)}{m_c}\frac{\epsilon_c(q)+m_c}{2m_c}\frac{\epsilon_b(q)}{m_b}\frac{\epsilon_b(q)+m_b}{2m_b}}}\left[\frac{\hat{v}_2 - 1}{2} + \hat{v}_2 \frac{\mathbf{q}^2}{2m_c(\epsilon_c(q)+m_c)} + \frac{\hat{q}}{2m_c}\right]$$

$$\times \Sigma^Q (1+\hat{v}_2)\left[\frac{\hat{v}_2 + 1}{2} + \hat{v}_2 \frac{\mathbf{q}^2}{2m_b(\epsilon_b(q)+m_b)} - \frac{\hat{q}}{2m_b}\right].$$

$v_1 = \frac{P}{M}$, $v_2 = \frac{Q}{M}$;
$\epsilon(p) = \sqrt{m^2 + \mathbf{p}^2}$;
$m_{b,c}$ — quark mass.

$\Sigma_{P,Q} = \gamma_5$ or $\varepsilon_{P,Q}$ — for pseudoscalar or vector B_c mesons, respectively

Expansion of quark and gluon propagators

$$
\begin{aligned}
\frac{1}{(p_{1,2}+q_{1,2})^2} &= \frac{1}{s\,\eta_{1,2}^2}\Big[1 \mp \frac{2(pQ+qP)}{s\,\eta_{1,2}} - \frac{p^2+2pq+q^2}{s\,\eta_{1,2}^2} + \dots\Big],\\
\frac{1}{(p_1+q_1+q_2)^2-m_b^2} &= \frac{1}{Z_1}\Big[1 - \frac{2pQ+p^2}{Z_1} + \frac{4(pQ)^2}{Z_1^2} + \dots\Big],\\
\frac{1}{(k_2-q_1)^2-m_c^2} &= \frac{1}{Z_2}\Big[1 + \frac{2k_2q-q^2}{Z_2} + \frac{4(k_2Q)^2}{Z_2^2} + \dots\Big],
\end{aligned}
\tag{12}
$$

where $s = x_1x_2S$ and $t = (P-k_1)^2 = (Q-k_2)^2$ — the Mandelstam variables for the gluonic subprocess $gg \to B_c + \bar{B}_c$.

Leading order denominators:

$$
Z_1 = s\,\eta_1 + \eta_2^2M^2 - m_b^2 \qquad Z_2 = t\,\eta_1 - \eta_1\eta_2M^2 - m_c^2 \tag{13}
$$

Neglecting bound state energy:

$$
s\,\eta_{1,2} \qquad s\,\eta_{1,2}^2 \qquad \eta_{1,2}(M^2-t) \qquad \eta_{1,2}(M^2-s-t) \tag{14}
$$

In the case of the most unfavourable values of the variables $x_{1,2}$ and t the expansion parameters in (12) can be roughly assessed as $4p^2/M^2$ and $4q^2/M^2$

Expansion of the amplitude

Example from $gg \to 2J/\psi$ amplitude:

$$\mathcal{M}_1^{ab} = \frac{32\alpha_s^2\delta^{ab}}{9\,m\,s^4}\int \frac{m+\epsilon(p)}{2\epsilon(p)}R(p)p^2dp\int \frac{m+\epsilon(q)}{2\epsilon(q)}R(q)q^2dq\Big\{3s^2\big[\varepsilon_1\cdot\varepsilon_2(s\,\varepsilon_P^*\cdot\varepsilon_Q^* -$$
$$-2\,\varepsilon_P^*\cdot Q\,\varepsilon_Q^*\cdot P) - 2\,\varepsilon_P^*\cdot\varepsilon_Q^*(\varepsilon_1\cdot P\,\varepsilon_2\cdot Q + \varepsilon_1\cdot Q\,\varepsilon_2\cdot P) + 2\,\varepsilon_P^*\cdot Q(\varepsilon_1\cdot P\,\varepsilon_2\cdot\varepsilon_Q^* +$$
$$+\varepsilon_1\cdot\varepsilon_Q^*\,\varepsilon_2\cdot P) - \varepsilon_1\cdot\varepsilon_P^* \times (s\,\varepsilon_2\cdot\varepsilon_Q^* - 2\,\varepsilon_2\cdot Q\,\varepsilon_Q^*\cdot P) - \varepsilon_2\cdot\varepsilon_P^*(s\,\varepsilon_1\cdot\varepsilon_Q^* -$$
$$-2\,\varepsilon_1\cdot Q\,\varepsilon_Q^*\cdot P)\big]\Big(3(\mathbf{1} - c_p - c_q - c_p^2 - c_q^2) + c_pc_q(67 + 3c_p + 3c_q) + 3c_p^2c_q^2\Big) + \ldots\Big\}$$

$$c_p = \frac{m-\epsilon(p)}{m+\epsilon(p)} \qquad c_q = \frac{m-\epsilon(q)}{m+\epsilon(q)}$$

Expansion of the amplitude

Example from $gg \to 2J/\psi$ amplitude:

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$$+\varepsilon_1\cdot\varepsilon_Q^*\,\varepsilon_2\cdot P) - \varepsilon_1\cdot\varepsilon_P^*\times(s\,\varepsilon_2\cdot\varepsilon_Q^* - 2\,\varepsilon_2\cdot Q\,\varepsilon_Q^*\cdot P) - \varepsilon_2\cdot\varepsilon_P^*(s\,\varepsilon_1\cdot\varepsilon_Q^* -$$

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$$c_p = \frac{m-\epsilon(p)}{m+\epsilon(p)} \qquad c_q = \frac{m-\epsilon(q)}{m+\epsilon(q)}$$

For B_c mesons — additional complication due to the unequal quark masses m_b and m_c, e.g.:

$$\frac{m+\epsilon(p)}{2\epsilon(p)} \to \sqrt{\frac{(\epsilon_c(p)+m_c)(\epsilon_b(p)+m_b)}{2\epsilon_c(p)\,2\epsilon_b(p)}} \tag{15}$$

The principal structure of the relativstic corrections to the amplitude remains the same

Effective relativistic Hamiltonian

$$H = H_0 + \Delta U_1 + \Delta U_2 + \Delta U_3,$$

$$H_0 = \sqrt{\mathbf{p}^2 + m_1^2} + \sqrt{\mathbf{p}^2 + m_2^2} - m_1 - m_2 - \frac{C_F \tilde{\alpha}_s}{r} + Ar + B,$$

$$\Delta U_1(r) = -\frac{C_F \alpha_s^2}{4\pi r}\left[2\beta_0 \ln(\mu r) + a_1 + 2\gamma_E \beta_0\right],$$

$$\Delta U_2(r) = -\frac{C_F \alpha_s}{2m_1 m_2 r}\left[\mathbf{p}^2 + \frac{\mathbf{r}(\mathbf{r}\mathbf{p})\mathbf{p}}{r^2}\right] + \frac{1}{2}\pi C_F \alpha_s \left(\frac{1}{m_1^2} + \frac{1}{m_2^2}\right)\delta(\mathbf{r}) -$$

$$-\frac{C_F \alpha_s}{2m_1 m_2}\left[\frac{\mathbf{S}^2}{r^3} - 3\frac{(\mathbf{S}\mathbf{r})^2}{r^5} - \frac{4\pi}{3}(2\mathbf{S}^2 - 3)\delta(\mathbf{r})\right] - C_A C_F \alpha_s^2 \frac{(m_1 + m_2)}{4 m_1 m_2 r^2},$$

$$\Delta U_3(r) = f_V\left[\frac{A}{4r}\left(\frac{1}{m_1^2} + \frac{1}{m_2^2}\right) + \frac{4A}{3m_1 m_2 r}\mathbf{S}_1\mathbf{S}_2 + \frac{A}{3m_1 m_2 r}\left(\frac{3}{r^2}(\mathbf{S}_1\mathbf{r})(\mathbf{S}_2\mathbf{r}) - \mathbf{S}_1\mathbf{S}_2\right)\right]. \quad (16)$$

'Rationalization' of the kinetic energy term:

$$T = \sqrt{\mathbf{p}^2 + m_1^2} + \sqrt{\mathbf{p}^2 + m_2^2} \approx 2 \times \frac{\mathbf{p}^2}{2\tilde{\mu}} + \frac{1}{2}\left(\frac{m_1^2}{\tilde{m}_1} + \frac{m_2^2}{\tilde{m}_2}\right),$$

$$\tilde{\mu} = \frac{2\tilde{m}_1 \tilde{m}_2}{\tilde{m}_1 + \tilde{m}_2}, \qquad \tilde{m}_{1,2} = \sqrt{\mathbf{p}_{\mathrm{eff}}^2 + m_{1,2}^2}. \quad (17)$$

Cross section

Nonrelativistic result:

$$d\sigma[gg \to B_c + \bar{B}_c](s,t) = \frac{\pi M^2 \alpha_s^4}{65\,536\, s^2} |R(0)|^4 F^{(1)}(s,t)$$

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$$-4(\omega_{01} + \omega_{10} - \omega_{11}) F^{(1)}(s,t) - 4 m_c^{-1} m_b^{-1} (m_c^2 \omega_{\frac{1}{2}\frac{3}{2}} + m_b^2 \omega_{\frac{3}{2}\frac{1}{2}}) F^{(1)}(s,t)$$
$$+6(\omega_{01} + \omega_{10})^2 F^{(1)}(s,t) + \omega_{\frac{1}{2}\frac{1}{2}} (1 - 3\omega_{01} - 3\omega_{10}) F^{(2)}(s,t) + \omega_{\frac{1}{2}\frac{1}{2}}^2 F^{(3)}(s,t) \Big]$$

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Relativistic generalization of $R(0) = \sqrt{\frac{2}{\pi}} \int R(p) p^2 \, dp$:

$$\tilde{R}(0) = \sqrt{\frac{2}{\pi}} \int_0^\infty \sqrt{\frac{(\epsilon_c(p) + m_c)(\epsilon_b(p) + m_b)}{2\epsilon_c(p)\, 2\epsilon_b(p)}} R(p) p^2 \, dp, \qquad (18)$$

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$$I_{nk} = \int_0^{m_c} p^2 R(p) \sqrt{\frac{(\epsilon_c(p) + m_c)(\epsilon_b(p) + m_b)}{2\epsilon_c(p)\, 2\epsilon_b(p)}} \left(\frac{\epsilon_c(p) - m_c}{\epsilon_c(p) + m_c} \right)^n \left(\frac{\epsilon_b(p) - m_b}{\epsilon_b(p) + m_b} \right)^k dp,$$
$$\omega_{nk} = \sqrt{\frac{2}{\pi}} \frac{I_{nk}}{\tilde{R}(0)}, \quad 0 < n + k \leq 2. \tag{19}$$

Numerical results

Energy $\sqrt{S}$	B_c mesons pair	CTEQ5L		CTEQ6L1	
		$\sigma_{\text{nonrel.}}$, nb	$\sigma_{\text{rel.}}$, nb	$\sigma_{\text{nonrel.}}$, nb	$\sigma_{\text{rel.}}$, nb
$\sqrt{S} = 7$ TeV	$B_c^* + \bar{B}_c^*$ $(S = 1)$	0.96	0.46	0.88	0.42
	$B_c + \bar{B}_c$ $(S = 0)$	0.57	0.62	0.52	0.56
$\sqrt{S} = 14$ TeV	$B_c^* + \bar{B}_c^*$ $(S = 1)$	2.1	1.0	1.8	0.88
	$B_c + \bar{B}_c$ $(S = 0)$	1.2	1.3	1.1	1.2

$$\sigma[pp \to 2J/\psi + X] = 23.0 \text{ (nonrel.)} \qquad 9.6 \text{ (rel.)} \quad \text{nb} \tag{20}$$

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Different sources of corrections:

- wave function — decrease (~ 3 times)
- amplitude exp — increase ($+30\%$ and $\times 2$)
- bound energy — increase ($+30 - 40\%$)

In total:

decrease (-50%) for B_c^*

some growth ($+10\%$) for pseudoscalar B_c

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As usual for this kind of calculations, the theoretical errors are expected to be as large as 50%

Thank you for attention!