XXII International Baldin Seminar on High Energy Physics Problems
Relativistic Nuclear Physics & Quantum Chromodynamics
September 15-20, 2014, Dubna, Russia

Self-similarity of Proton Spin

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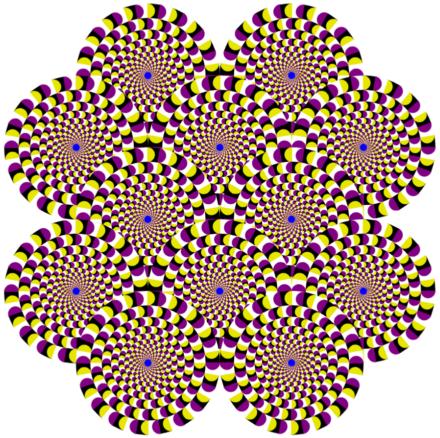


XXII International Baldin Seminar on High Energy Physics Problems
"Relativistic Nuclear Physics and Quantum Chromodynamics",
JINR, Dubna, Russia, September 15-20, 2014

Contents

- Introduction (motivation & goals)
- z-Scaling (principles, ideas, definitions,…)
- Self-similarity in unpolarized pp collisions
- Self-similarity in polarized pp collisions
- Spin-dependent fractal dimensions
- Spin-dependent constituent energy loss
- Conclusions

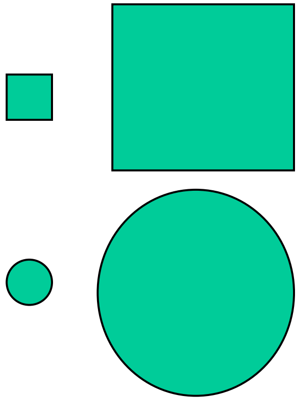
Similarity and self-similarity

1. Two geometrical objects are called similar if one is the result of a uniform scaling (enlarging or shrinking) of the other.

2. Object is called self-similar if it is composed of parts similar to it as a whole.

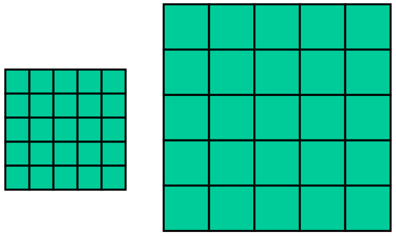
3. Object is called (self) similar fractal, if it consists of parts like him as a whole on any scale.

Similar objects



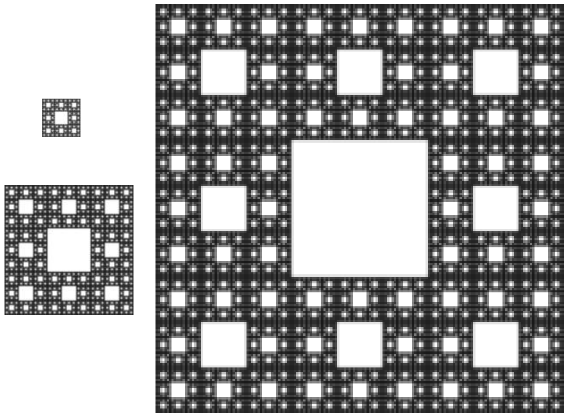
$D = 2, D_T = 2$

Self-similar object



$D = 2, D_T = 2$

Fractal

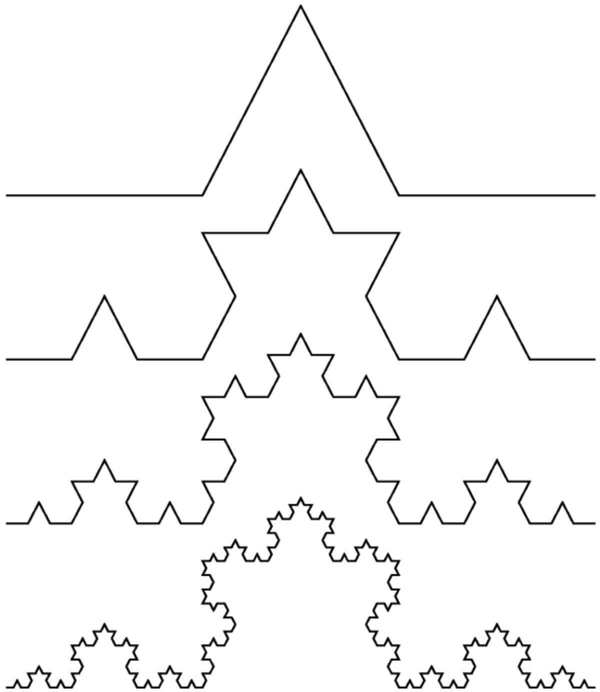


$D = \ln 8 / \ln 3 \approx 1.89, \quad D_T = 1$

Fractal Curve

$$p = 4, \quad q = 3$$

Swedish mathematician
Nils Fabian Helge von Koch



$$z = z_0 \cdot \varepsilon^{-\delta}$$

A curve length is a measure of fractal curve

$$z(\varepsilon)\big|_{\varepsilon^{-1} \to \infty} \to \infty$$

- $\varepsilon^{-1} = q^n$ resolution
- $D = \mathrm{Ln}(p)/\mathrm{Ln}(q)$ fractal dimension
- $D_T = 1$ topological dimension
- $\delta = D - D_T$ anomalous fractal dimension

z-Scaling - Universality & Saturation

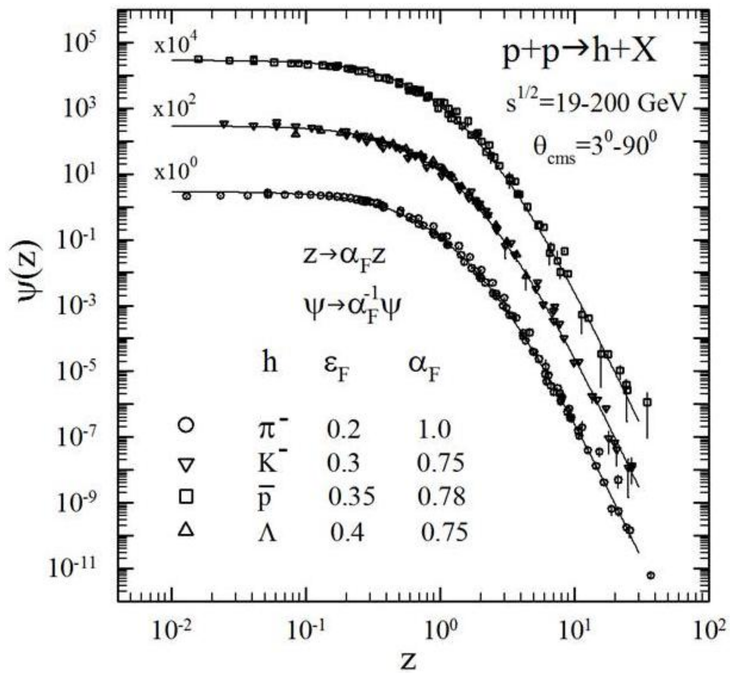
Inclusive cross sections of
π^-, K^-, $\bar{p}$, Λ
in pp collisions

FNAL:
PRD 75 (1979) 764

ISR:
NPB 100 (1975) 237
PLB 64 (1976) 111
NPB 116 (1976) 77
(low p_T)
NPB 56 (1973) 333
(small angles)

STAR:
PLB 616 (2005) 8
PLB 637 (2006) 161
PRC 75 (2007) 064901

Saturation at low z



Power law at high z

Energy scan of spectra
at U70, ISR, S$\bar{p}$pS, SPS, HERA, FNAL(fixed target), Tevatron, RHIC, LHC

MT & I.Zborovsky
T.Dedovich
Phys.Rev.D75,094008(2007)
Int.J.Mod.Phys.A24,1417(2009)
J. Phys.G: Nucl.Part.Phys. 37,085008(2010)
Int.J.Mod.Phys.A27,1250115(2012)
J.Mod.Phys.3,815(2012)

- Energy & angular independence
- Flavor independence (π, K, $\bar{p}$, Λ)
- Saturation for $z < 0.1$
- Power law $\Psi(z) \sim z^{-\beta}$ for high $z > 4$

Scaling – "collapse" of data points onto a single curve.
Universality classes – hadron species (ε_F, α_F).

Motivation & Goals

Development of z-scaling approach for description of processes with polarized particle production in inclusive reactions to understand the spin origin.

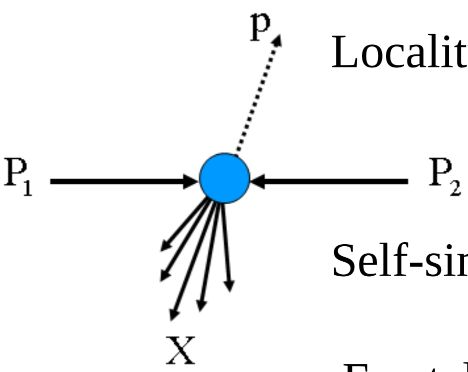
Analysis of double spin asymmetry of π meson production and transverse coefficient for Λ hyperon production in p+p collisions to determine spin-dependent fractal dimensions

The suggested approach can be used to study of

- Properties of sub-structure of the colliding objects, interactions of their constituents, and fragmentation process at small scales.
- Fractal properties of flavor (u,d,s,c,b,t)
- Fundamental principles (self-similarity, scale relativity, fractality, Lorentz invariance,…)
- Origin of mass, spin, charge,…, fractal topology of space-time,…

z-Scaling

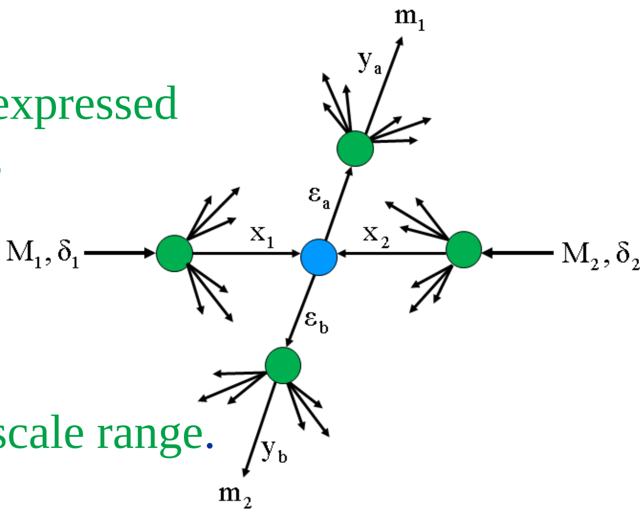
Principles: locality, self-similarity, fractality



Locality: collisions of hadrons and nuclei are expressed via interactions of their constituents (partons, quarks and gluons,...).

Self-similarity: interactions of the constituents are mutually similar.

Fractality: self-similarity is valid over a wide scale range.

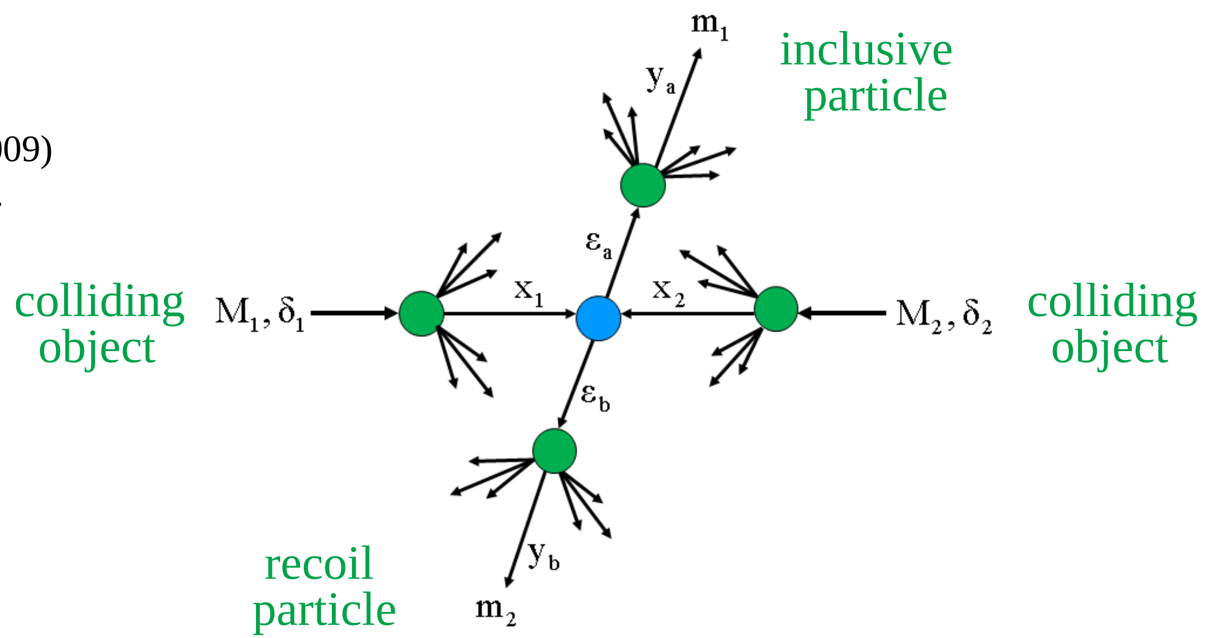


Hypothesis of z-scaling :

$s^{1/2}$, p_T, θ_{cms}	Inclusive particle distributions can be described in terms of constituent sub-processes and parameters characterizing bulk properties of the system.	x_1,x_2,y_a,y_b $\delta_1,\delta_2,\varepsilon_a,\varepsilon_b$,c
$Ed^3\sigma/dp^3$	Scaled inclusive cross section of particles depends in a self-similar way on a single scaling variable z.	$\Psi(z)$

Locality of hadron interactions

M.T. & I.Zborovský
Part.Nucl.Lett.312(2006)
PRD75,094008(2007)
Int.J.Mod.Phys.A24,1417(2009)
J.Phys.G: Nucl.Part.Phys. 37,085008(2010)



Constituent subprocess

$$(x_1 M_1) + (x_2 M_2) \Rightarrow (m_1/y_a) + (x_1 M_1 + x_2 M_2 + m_2/y_b)$$

Kinematical condition (4-momentum conservation law):

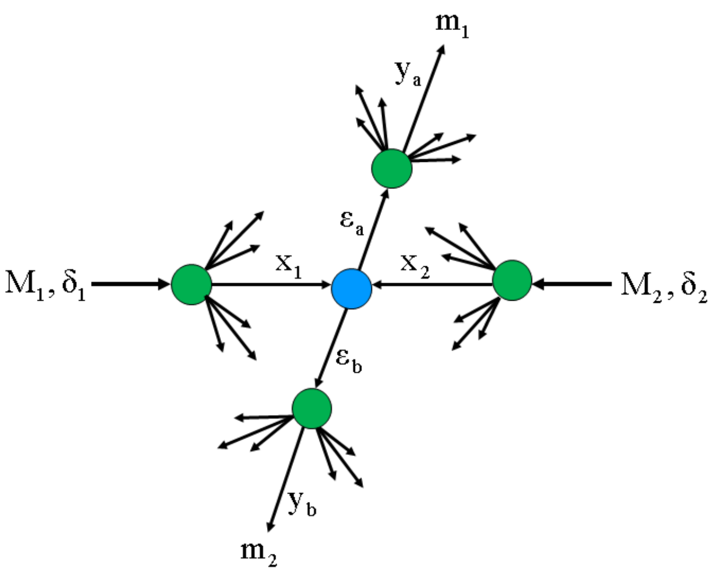
$$(x_1 P_1 + x_2 P_2 - p/y_a)^2 = M_X^2$$

Recoil mass: $M_X = x_1 M_1 + x_2 M_2 + m_2/y_b$

z as self-similarity parameter

$$z = z_0 \cdot \Omega^{-1}$$

$$z_0 = \frac{s_\perp^{1/2}}{(dN_{ch}/d\eta|_0)^c m}$$



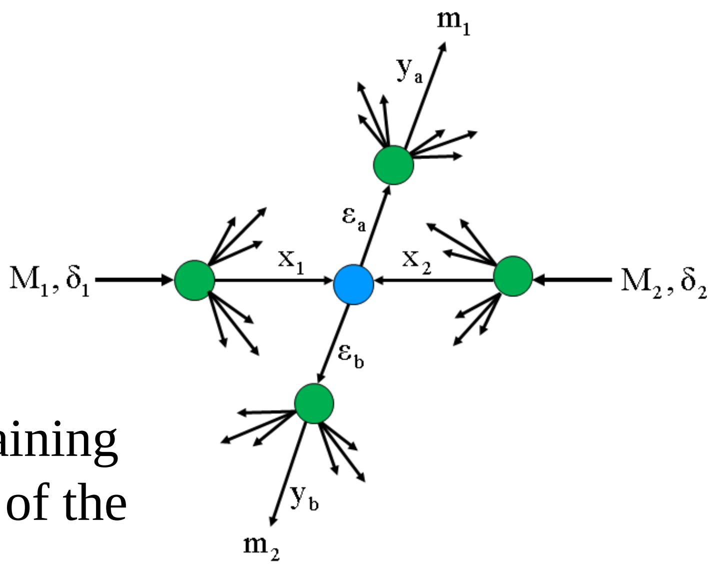
- Ω^{-1} is the minimal resolution at which a constituent subprocess can be singled out of the inclusive reaction
- $s_\perp^{1/2}$ is the transverse kinetic energy of the subprocess consumed on production of m_1 & m_2
- $dN_{ch}/d\eta|_0$ is the multiplicity density of charged particles at $\eta = 0$
- c is a parameter interpreted as a “specific heat” of created medium
- m is an arbitrary constant (fixed at the value of nucleon mass)

z as fractal measure

The fractality is reflected in definition of z

$$z = z_0 \cdot \Omega^{-1}$$

$$\Omega = (1-x_1)^{\delta_1}(1-x_2)^{\delta_2}(1-y_a)^{\varepsilon_a}(1-y_b)^{\varepsilon_b}$$



Ω is relative number of configurations containing a sub-process with fractions x_1, x_2, y_a, y_b of the corresponding 4-momenta

$\delta_1, \delta_2, \varepsilon_a, \varepsilon_b$ are parameters characterizing structure of the colliding objects and fragmentation process, respectively

$\Omega^{-1}(x_1, x_2, y_a, y_b)$ characterizes resolution at which a constituent sub-process can be singled out of the inclusive reaction

$$z(\Omega)|_{\Omega^{-1}\to\infty} \to \infty$$

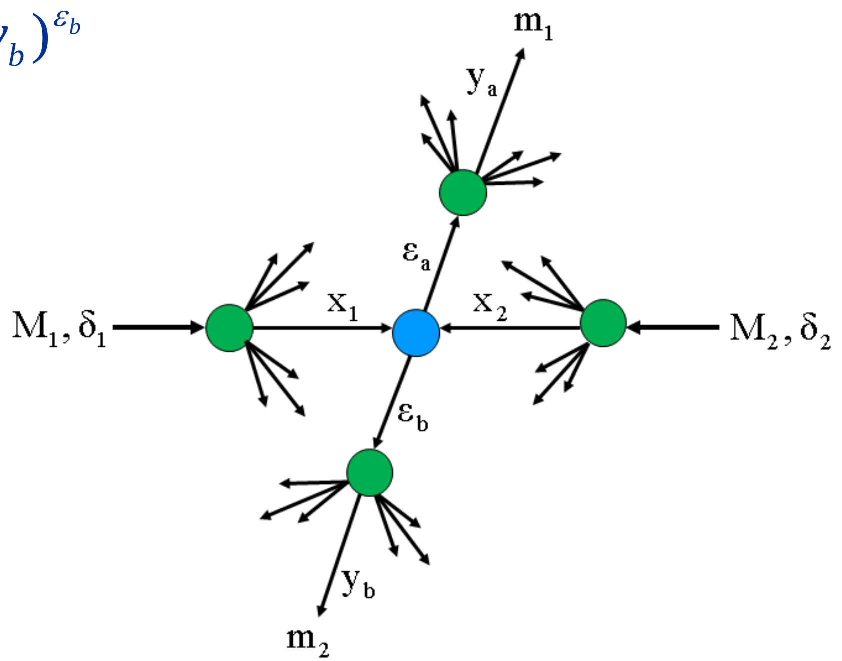
The fractal measure z diverges as the resolution Ω^{-1} increases.

Momentum fractions x_1, x_2, y_a, y_b

Principle of minimal resolution: The momentum fractions x_1, x_2 and y_a, y_b are determined in a way to minimize the resolution Ω^{-1} of the fractal measure z with respect to all constituent sub-processes taking into account 4-momentum conservation:

$$\Omega = (1-x_1)^{\delta_1}(1-x_2)^{\delta_2}(1-y_a)^{\varepsilon_a}(1-y_b)^{\varepsilon_b}$$

$$\begin{cases} \partial\Omega/\partial x_1 |_{y_a=y_a(x_1,x_2,y_b)} = 0 \\ \partial\Omega/\partial x_2 |_{y_a=y_a(x_1,x_2,y_b)} = 0 \\ \partial\Omega/\partial y_b |_{y_a=y_a(x_1,x_2,y_b)} = 0 \end{cases}$$



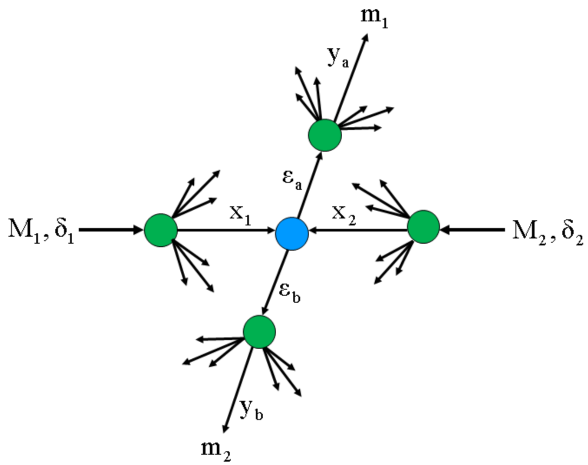
Momentum conservation law

$$(x_1P_1+x_2P_2-p/y_a)^2 = M_X^2$$

Recoil mass

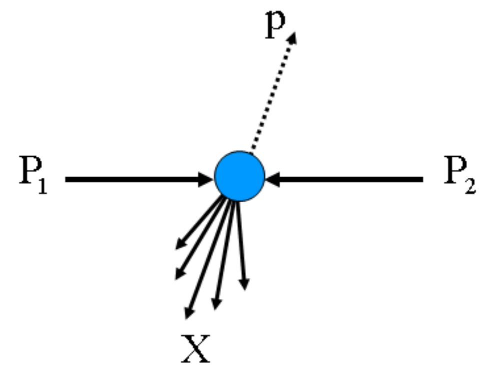
$$M_X = x_1M_1+x_2M_2+m_2/y_b$$

Scaling function $\Psi(z)$



$$\int_0^\infty \Psi(z)dz = 1$$

$$z \to \alpha_F z, \quad \Psi \to \alpha_F^{-1} \Psi$$



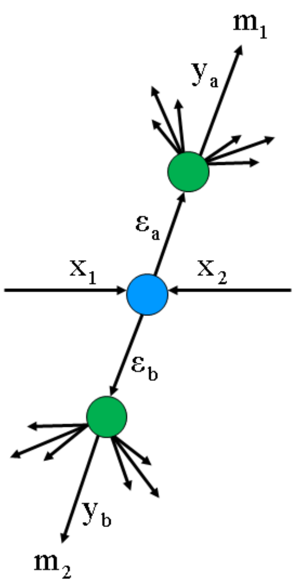
$$\Psi(z) = \frac{\pi}{(dN/d\eta)\cdot\sigma_{inel}} \cdot J^{-1} \cdot E\frac{d^3\sigma}{dp^3} \quad \Longleftrightarrow \quad \int E\frac{d^3\sigma}{dp^3} dy d^2p_\perp = \sigma_{inel}\cdot N$$

- σ_{in} - inelastic cross section
- N - average multiplicity of the corresponding hadron species
- $dN/d\eta$ - pseudorapidity multiplicity density at angle θ (η)
- $J(z,\eta;p_T^2,y)$ - Jacobian
- $Ed^3\sigma/dp^3$ - inclusive cross section

The scaling function $\Psi(z)$ is probability density to produce an inclusive particle with the corresponding z.

Transverse kinetic energy $\sqrt{s_\perp}$

$$s_\perp^{1/2} = \underbrace{y_1(s_\lambda^{1/2} - M_1\lambda_1 - M_2\lambda_2) - m_1}_{\text{energy consumed for the inclusive particle } m_1} + \underbrace{y_2(s_\chi^{1/2} - M_1\chi_1 - M_2\chi_2) - m_2}_{\text{energy consumed for the recoil particle } m_2}$$



Fraction decomposition: $x_{1,2} = \lambda_{1,2} + \chi_{1,2}$

$$\lambda_{1,2} = \kappa_{1,2}/y_1 + \nu_{1,2}/y_2$$

$$\chi_{1,2} = (\mu_{1,2}^2 + \omega_{1,2}^2)^{1/2} \mp \omega_{1,2}$$

$$\omega_{1,2} = \mu_{1,2} U, \quad U = \frac{\alpha - 1}{2\sqrt{\alpha}}\xi, \quad \alpha = \frac{\delta_2}{\delta_1}$$

$$\xi^2 = (\lambda_1\lambda_2 + \lambda_0)/[(1-\lambda_1)(1-\lambda_2)]$$

$$\kappa_{1,2} = \frac{(P_{2,1}p)}{(P_2P_1)}, \quad \nu_{1,2} = \frac{M_{2,1}m_2}{(P_2P_1)}$$

$$\mu_{1,2}^2 = \alpha^{\pm1}(\lambda_1\lambda_2 + \lambda_0)\frac{1 - \lambda_{1,2}}{1 - \lambda_{2,1}}$$

$$\lambda_0 = \bar{\nu}_0/y_2^2 - \nu_0/y_1^2$$

$$\bar{\nu}_0 = \frac{0.5m_2^2}{(P_1P_2)}, \quad \nu_0 = \frac{0.5m_1^2}{(P_1P_2)}$$

$$s_\lambda = (\lambda_1 P_1 + \lambda_2 P_2)^2 \qquad s_\chi = (\chi_1 P_1 + \chi_2 P_2)^2$$

The scaling variable z and scaling function Ψ(z) are expressed via Lorentz invariants.

Properties of $\Psi(z)$ in unpolarized $p\bar{p}$ & pp collisions

- Energy independence of $\Psi(z)$ ($s^{1/2} > 20$ GeV)
- Angular independence of $\Psi(z)$ ($\theta_{cms}=3^0$-90^0)
- Multiplicity independence of $\Psi(z)$ ($dN_{ch}/d\eta=1.5$-26)
- Power law, $\Psi(z) \sim z^{-\beta}$, at high z ($z > 4$)
- Flavor independence of $\Psi(z)$ ($\pi, K, \varphi, \Lambda, .., D, J/\psi, B, \Upsilon, \ldots$, top)
- Saturation of $\Psi(z)$ at low z ($z < 0.1$)

These properties reflect self-similarity, locality, and fractality
of the hadron interaction at a constituent level.
It concerns the structure of the colliding objects, interactions
of their constituents, and fragmentation process.

M.T. & I.Zborovsky
Phys.At.Nucl. 70,1294(2007)
Phys.Rev. D75,094008(2007)
Int.J.Mod.Phys. A24,1417(2009)
J. Phys.G: Nucl.Part.Phys. 37,085008(2010)
Int.J.Mod.Phys. A27,1250115(2012)
…………………

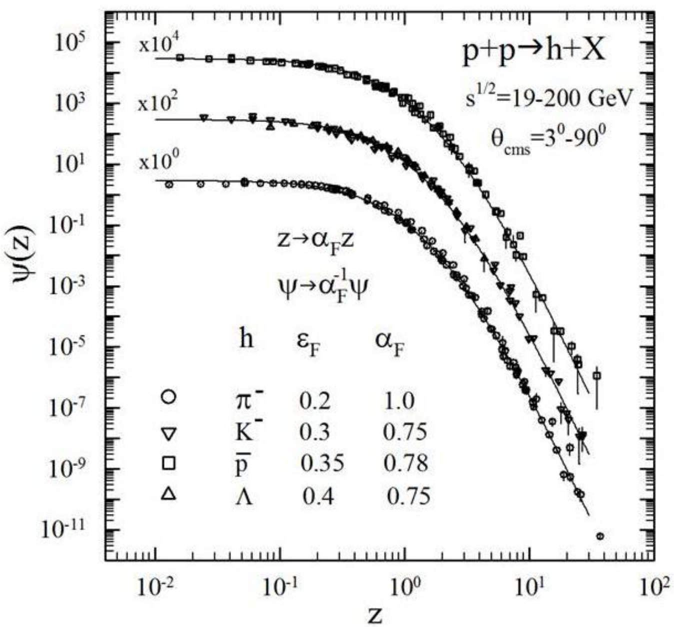
Flavor independence of $\Psi(z)$ at RHIC

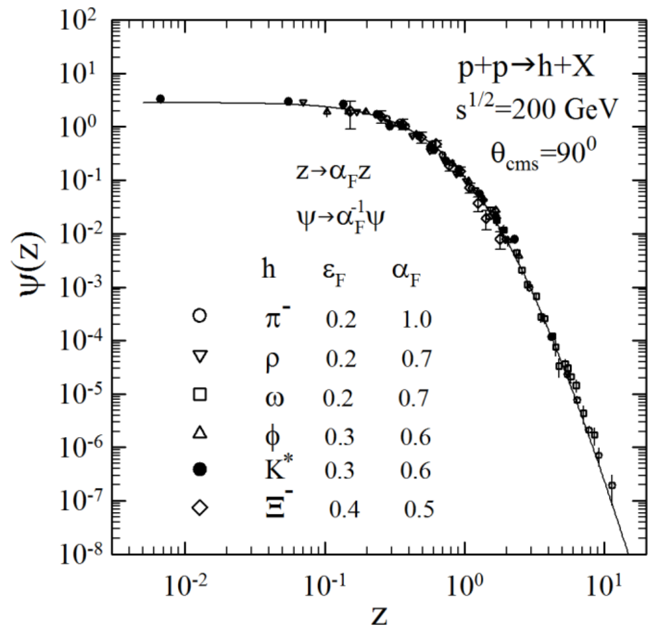
M.T. & I.Zborovský
Int.J.Mod.Phys.
A24,1417(2009)

π^-, ρ, ω, ϕ, K^*, Λ, Ξ, J/ψ, D, B, Υ

STAR:
PRL 92 (2004) 092301
PLB 612 (2005) 181
PRC 71 (2005) 064902
PRC 75 (2007) 064901

PHENIX:
PRC 75 (2007) 051902





- Energy independence
- Angular independence
- Flavor independence
- Saturation for $z < 0.01$

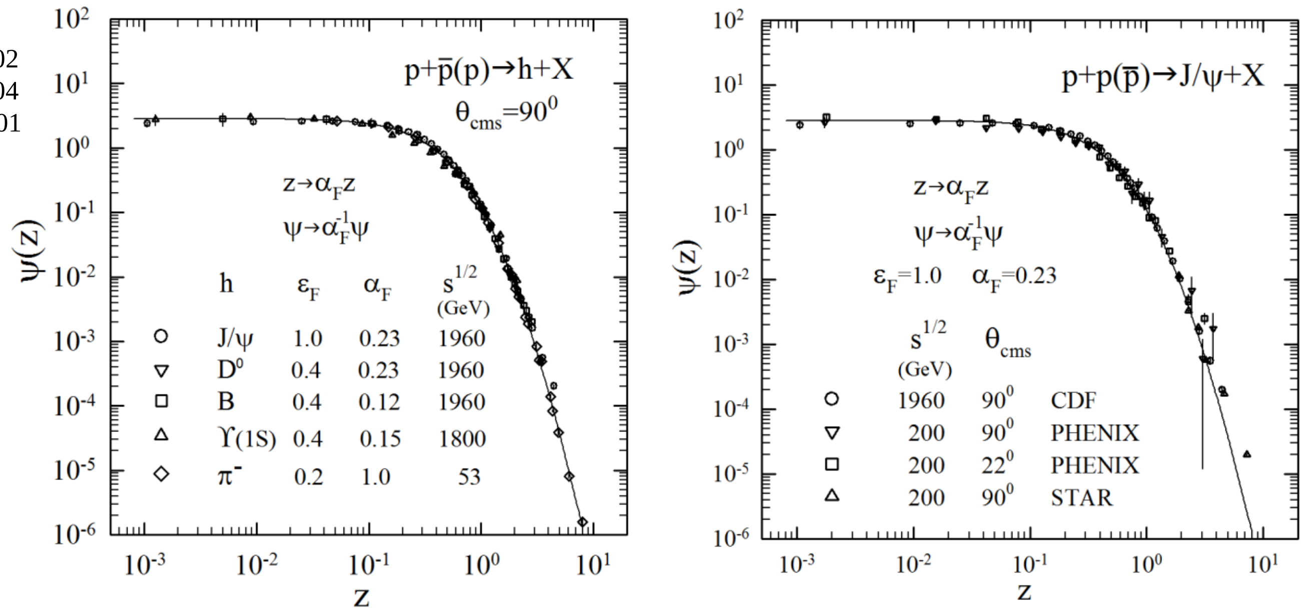
- Power law $\Psi(z) \sim z^{-\beta}$ at large z
- ε_F, α_F independent of p_T, $s^{1/2}$

Self-similarity of particle formation with various flavor content.

Flavor independence of $\Psi(z)$ at Tevatron

π^-, ρ, ω, ϕ, K^*, Λ, Ξ, J/ψ, D, B, Υ

CDF:
PRL 88 (2002) 161802
PRL 91 (2003) 241804
PRD 71 (2005) 032001

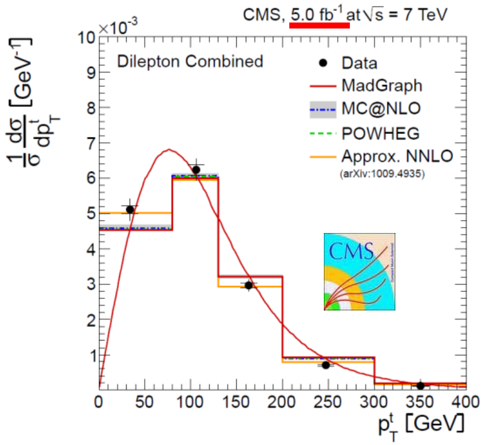


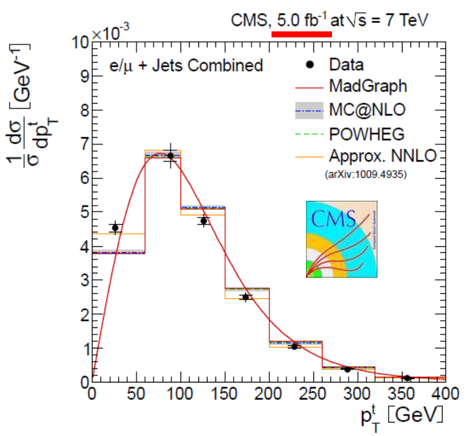
CDF:
PRD 71 (2005) 032001
PHENIX:
PRL 98 (2007) 232002
STAR:
Z.Tang
J.Phys.G35:104135, 2008

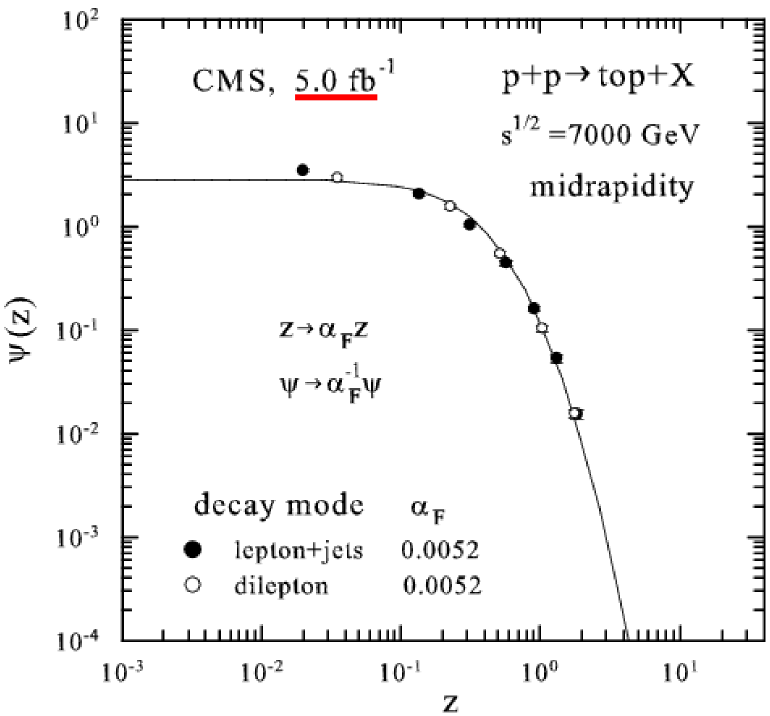
- Energy & angular independence
- Saturation of $\Psi(z)$ for $z < 0.1$
- Flavor independence of $\Psi(z)$
- Extra large $\varepsilon_F=1$ for J/ψ

Self-similarity of top quark production at LHC

Differential production cross sections
as a function of the transverse momentum
of the top quarks p_T







- Flavor independence of $\Psi(z)$
- Saturation of $\Psi(z)$ for or $z < 0.1$
- Fractal dimensions $\delta = 0.5$, $\varepsilon_{top} = 0$
- "Specific heat" $c = 0.25$

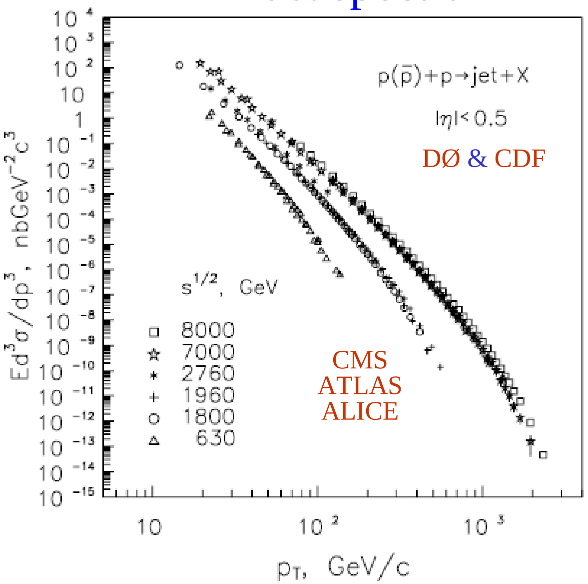
CMS data confirm self-similarity of top quark production in pp

CMS Collaboration,
CMS-PAS-TOP-11-013

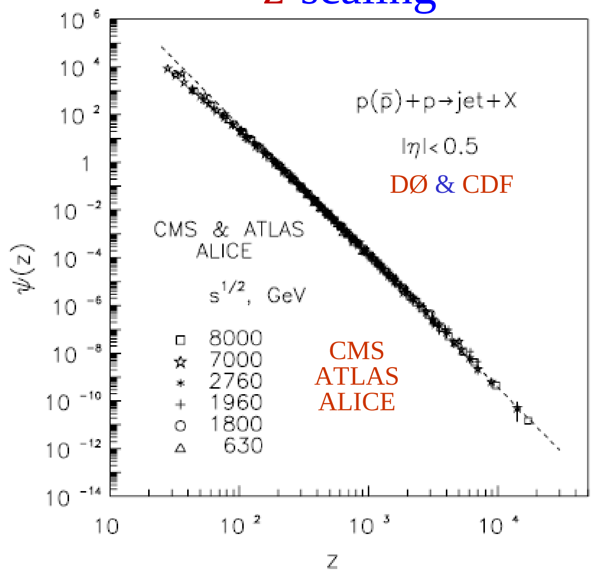
Self-similarity of jet production over a wide scale range

Highest energy - $\sqrt{s} = 8$ TeV, highest momentum - $p_T \approx 2.4$ TeV/c ,
smallest scale ~ $8 \cdot 10^{-5}$ fm

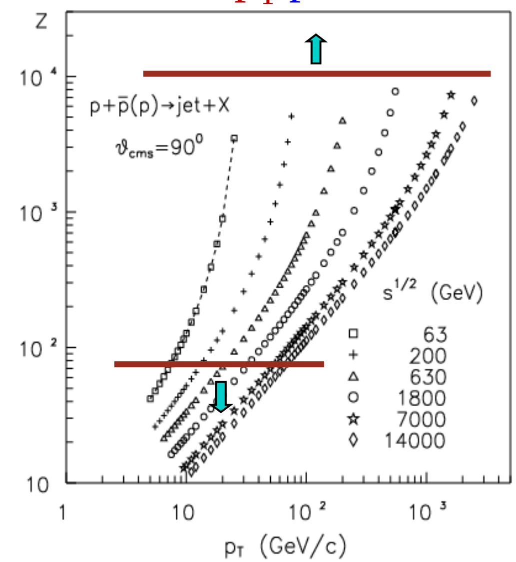
Jet spectra



z-scaling



z-p_T plot



New test of z-scaling at LHC

Structural phenomena ⟺ constituent substructure,…
Collective phenomena ⟺ multiple interactions, phase transitions,…
Self-similarity at small scales ⟺ fractal topology of momentum space,…

Search for new phenomena at LHC

Data on inclusive spectra obtained
at U70, ISR, SPS, S$\bar{p}$pS, RHIC, Tevatron, LHC
are consistent with z-scaling
for unpolarized processes

What about z-scaling hypothesis
for processes with polarized protons ?

F.Lehar

Self-similarity of spin structure

Hypothesis of z-scaling for processes with polarized particles

$s^{1/2}$, p_T, θ_{cms}
spin

Inclusive spin-dependent particle distributions can be described in terms of constituent sub-processes and parameters characterizing bulk properties of the system.

spin-dependent fractions
x_1, x_2, y_a, y_b

spin-dependent dimensions
$\delta_1, \delta_2, \varepsilon_a, \varepsilon_b$

spin-dependent cross section
$Ed^3\sigma/dp^3$

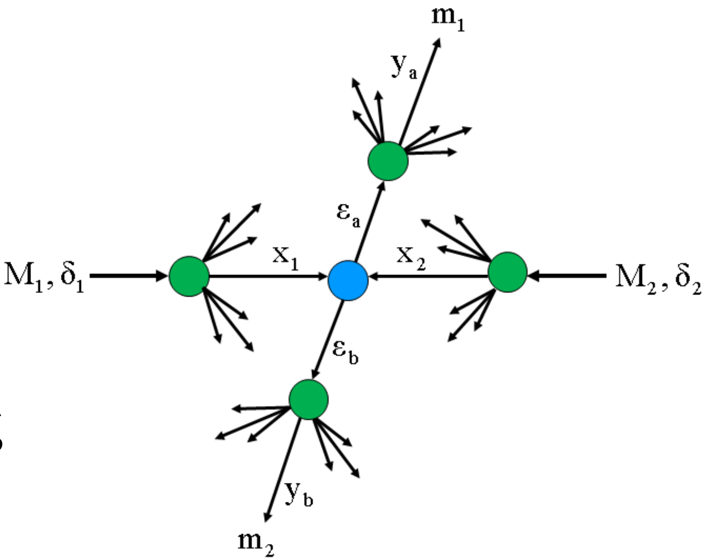
Scaled spin-dependent inclusive cross section of particle production depends in a self-similar way on a single spin-dependent scaling variable z.

spin-dependent
$\Psi(z)$

Universality of the shape of spin-dependent function $\Psi(z)$

z as spin-dependent fractal measure

Fractality is reflected in the definition of z



$$z = z_0 \cdot \Omega^{-1}$$

$$\Omega = (1-x_1)^{\delta_1}(1-x_2)^{\delta_2}(1-y_a)^{\varepsilon_a}(1-y_b)^{\varepsilon_b}$$

Ω is relative number of configurations containing a sub-process with spin-dependent fractions x_1, x_2, y_a, y_b of the corresponding 4-momenta

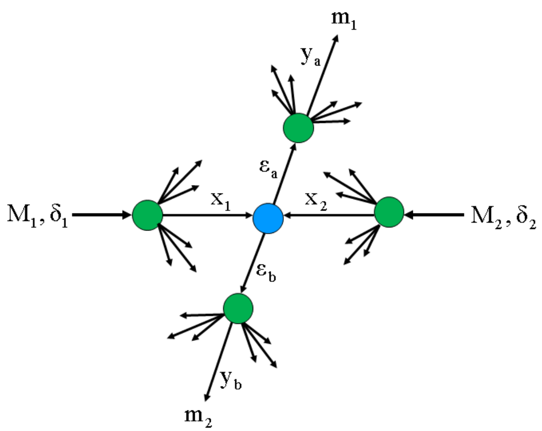
δ_1, δ_2, ε_a, ε_b are parameters characterizing spin-dependent structure of the colliding objects and fragmentation process, respectively

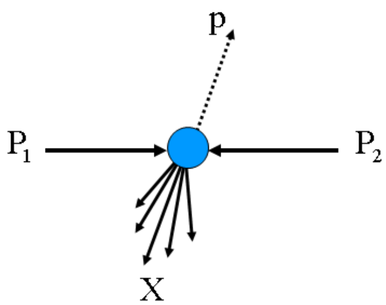
$\Omega^{-1}(x_1, x_2, y_a, y_b)$ characterizes spin-dependent resolution at which a constituent sub-process can be singled out of the inclusive reaction

$$z(\Omega)|_{\Omega^{-1}\to\infty} \to \infty$$

Spin-dependent fractal measure z diverges as the resolution Ω^{-1} increases.

Spin-dependent function $\Psi(z)$





spin-dependent
z & Ψ

$$\Psi(z) = \frac{\pi}{(dN/d\eta)\cdot\sigma_{inel}} \cdot J^{-1} \cdot E\frac{d^3\sigma}{dp^3}$$

spin-dependent
cross section σ

- σ_{in} - total inelastic cross section
- N - average multiplicity of the corresponding hadron species
- $dN/d\eta$ - pseudorapidity multiplicity density at angle θ (η)
- $J(z,\eta;p_T^2,y)$ - spin-dependent Jacobian
- $Ed^3\sigma/dp^3$ - spin-dependent inclusive cross section

$Ed^3\sigma/dp^3 \equiv \sigma$

spin-independent: σ , Ψ , z

spin-dependent: σ_{+-}, Ψ_{+-} , z_{+-}

Self-similarity in processes with polarized protons

New hypothesis:

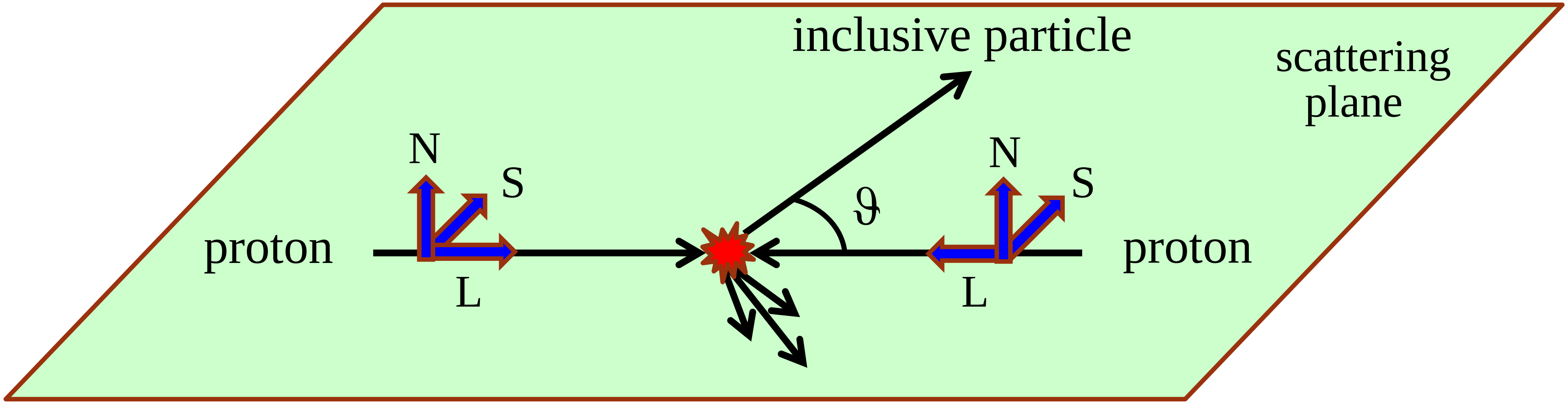
- Self-similarity of spin structure
- Fractality of proton spin
- Spin-dependent fractal dimensions

L, N, S represents the unit vectors along the spin directions of initial particles

L is along the incident momentum

N is along the normal to the scattering plane

S is along N×L



Double spin asymmetry

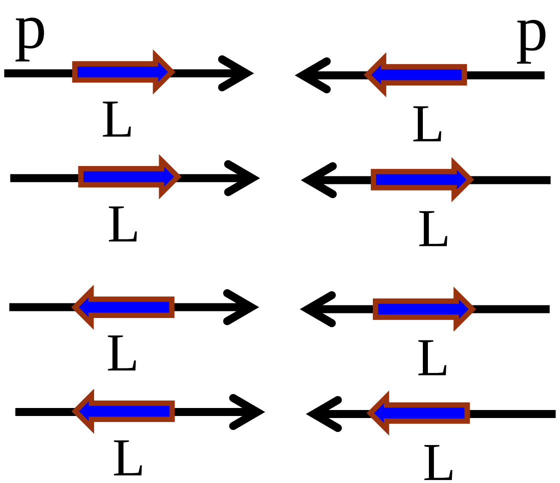
$$p^{\uparrow} + p^{\downarrow} \to h + X \qquad p^{\rightarrow} + p^{\leftarrow} \to h + X \qquad p^{\rightarrow} + p^{\uparrow} \to h + X$$

A_{NN} $\quad$ A_{LL} $\quad$ A_{LN}

Double spin asymmetry of pion production in pp

$$\vec{p} + \vec{p} \to \pi + X$$

$$A_{LL} = \frac{\sigma_{++} + \sigma_{--} - \sigma_{+-} - \sigma_{-+}}{\sigma_{++} + \sigma_{--} + \sigma_{+-} + \sigma_{-+}}$$



$\equiv \sigma_{++}$

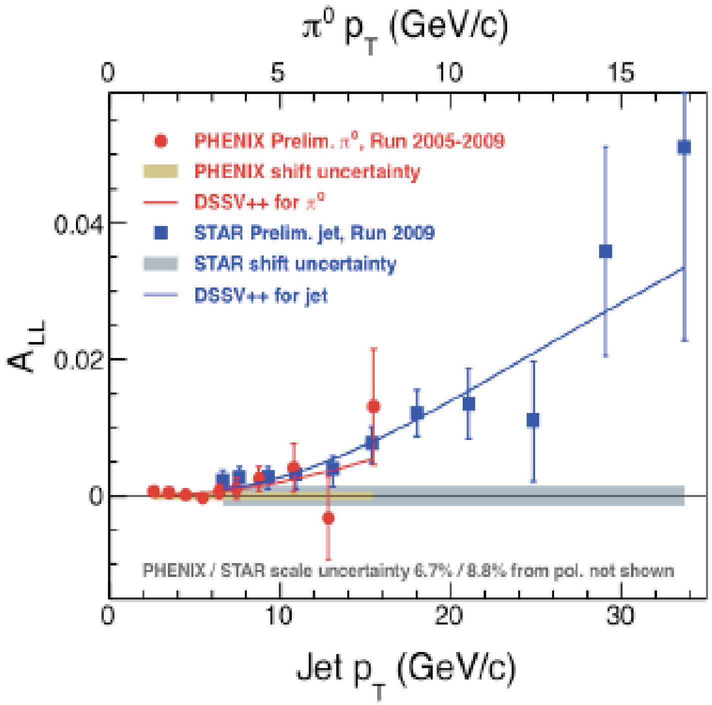
$\equiv \sigma_{+-}$

$\equiv \sigma_{--}$

$\equiv \sigma_{-+}$

$$\sigma_{++} + \sigma_{--} + \sigma_{+-} + \sigma_{-+} = 4\sigma_{00}$$

STAR & PHENIX at RHIC



PHENIX Collaborartion
Adare A .et al. hep-ex: 1402.6296
RHIC SPIN Collaboration
Arschenauer E.C. et al.
nucl-ex:1304.0079
STAR Collaboration,
Xu Q. , DSPIN2013, Dubna, Russia,
8-12 October, 2013

Self-similarity of spin-dependent function $\Psi(z)$

$$\vec{p} + \vec{p} \to \pi + X$$

$$\Psi_{++} = \Psi(z_{++}),\ \Psi_{+-} = \Psi(z_{+-}),\ \Psi_{00} = \Psi(z_{00})$$

$$\Omega = (1-x_1)^{\delta_1}(1-x_2)^{\delta_2}(1-y_a)^{\varepsilon_F}(1-y_b)^{\varepsilon_F}$$

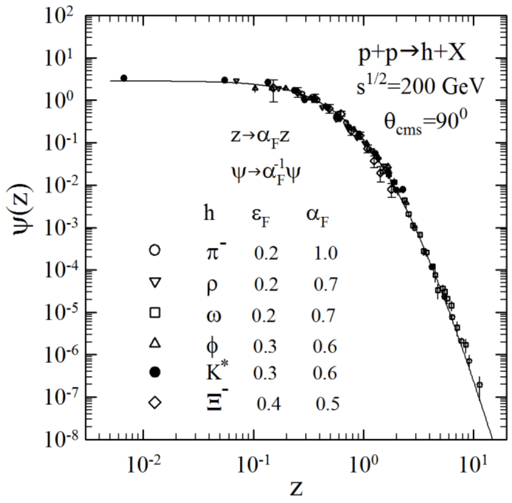
$$\Omega_{0000} =: \{\delta, \delta, \varepsilon_F, \varepsilon_F\}$$

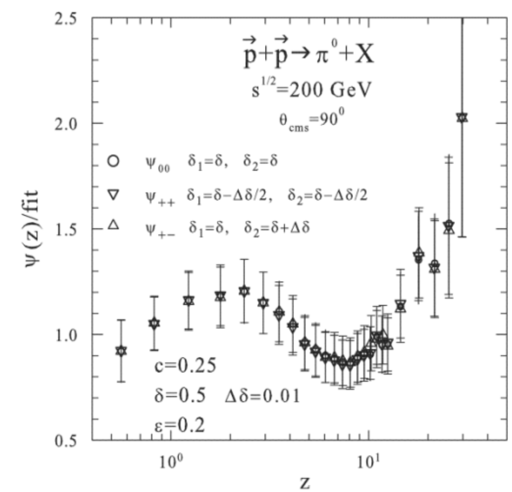
$$\Omega_{++00} =: \{\delta - \Delta\delta/2, \delta - \Delta\delta/2, \varepsilon_F, \varepsilon_F\}$$

$$\Omega_{--00} =: \{\delta - \Delta\delta/2, \delta - \Delta\delta/2, \varepsilon_F, \varepsilon_F\}$$

$$\Omega_{-+00} =: \{\delta + \Delta\delta, \delta, \varepsilon_F, \varepsilon_F\}$$

$$\Omega_{+-00} =: \{\delta, \delta + \Delta\delta, \varepsilon_F, \varepsilon_F\}$$

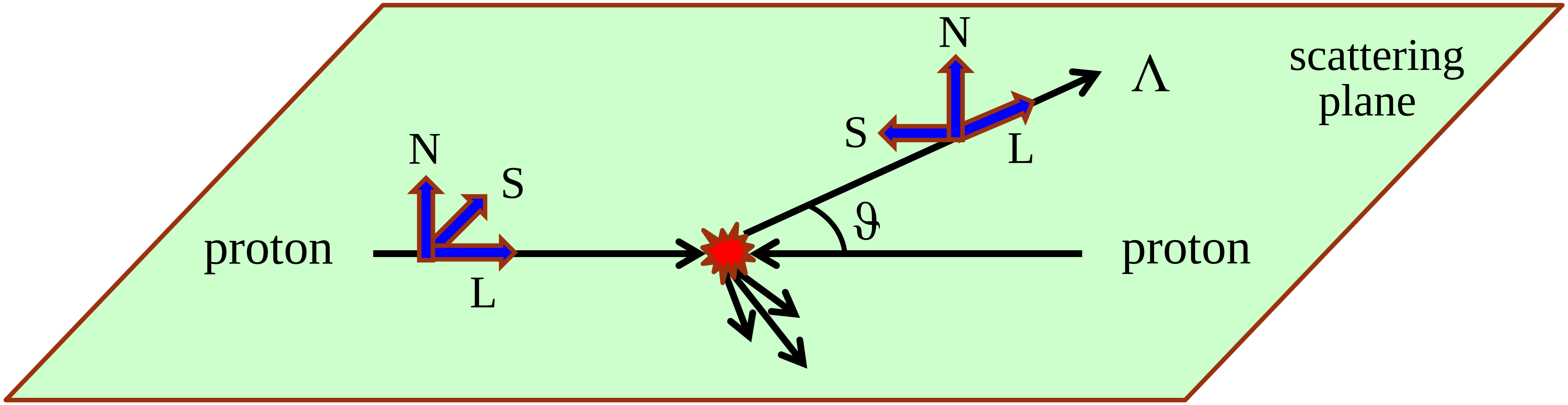




Additivity of fractal dimension: δ, $\Delta\delta$

Self-similarity in processes with polarized particles

- Self-similarity of spin structure
- Fractality of proton spin
- Spin-dependent fractal dimensions
- Self-similarity of spin-dependent fragmentation



Spin transfer coefficient

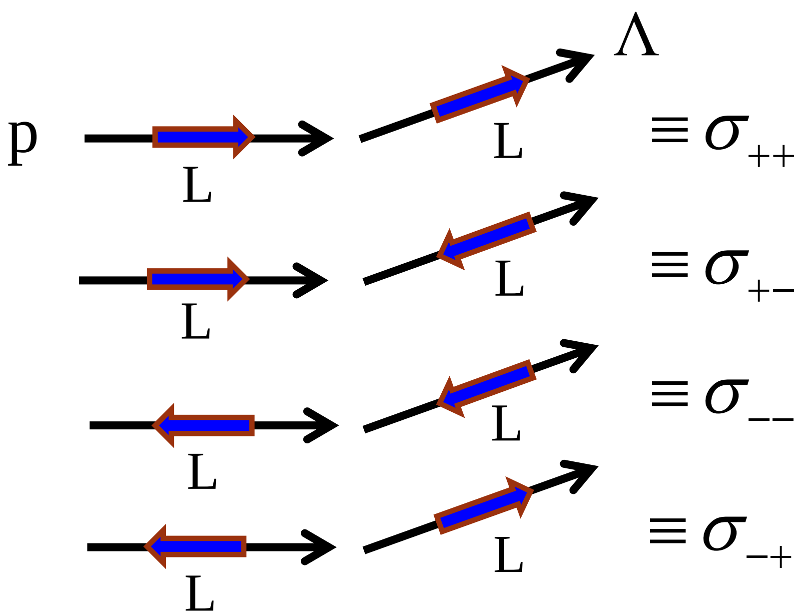
$$p^{\uparrow} + p \to \Lambda^{\uparrow} + X \qquad p^{\rightarrow} + p \to \Lambda^{\rightarrow} + X \qquad p^{\rightarrow} + p \to \Lambda^{\uparrow} + X$$

D_{NN} $\qquad$ D_{LL} $\qquad$ D_{LN}

Longitudinal spin transfer coefficient in p+p→Λ+X

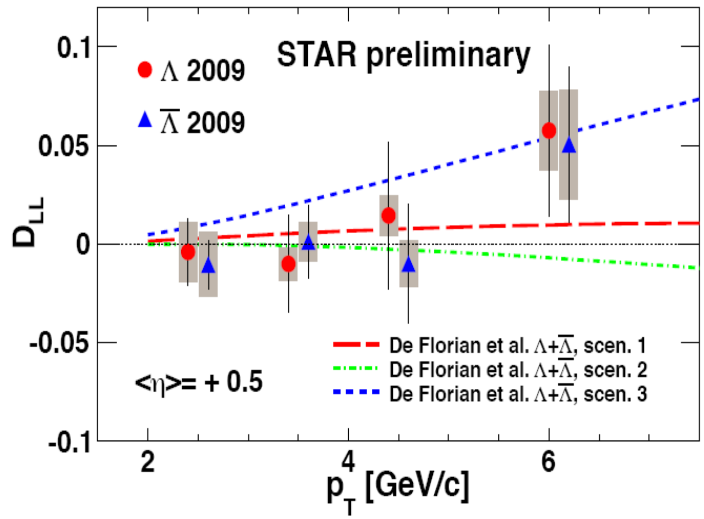
$$\vec{p} + p \to \vec{\Lambda} + X$$

$$D_{LL} = \frac{\sigma_{++} + \sigma_{--} - \sigma_{+-} - \sigma_{-+}}{\sigma_{++} + \sigma_{--} + \sigma_{+-} + \sigma_{-+}}$$



$$\sigma_{++} + \sigma_{--} + \sigma_{+-} + \sigma_{-+} = 4\sigma_{00}$$

STAR at RHIC



Xu Q. STAR Collaboration,
DSPIN2013, Dubna, Russia,
8-12 October, 2013

Self-similarity of spin-dependent function $\Psi(z)$

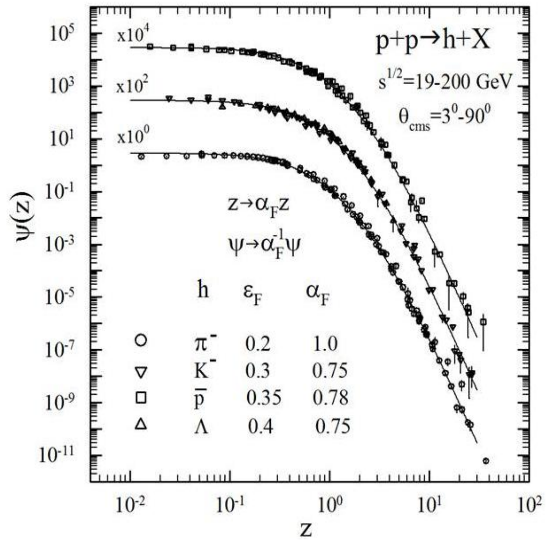
$$\vec{p} + p \to \vec{\Lambda} + X$$

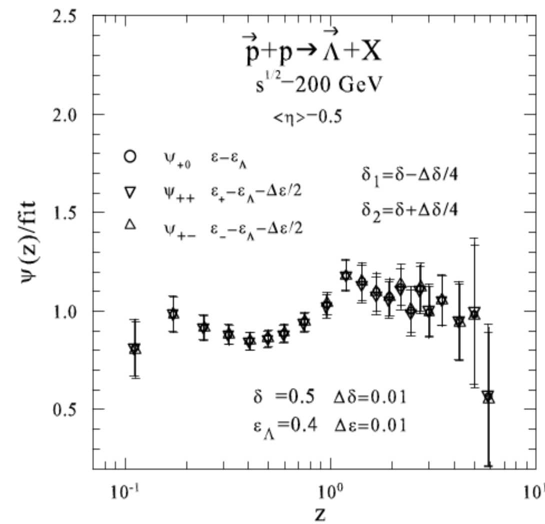
$$\Psi_{++} = \Psi(z_{++}),\ \Psi_{+-} = \Psi(z_{+-}),\ \Psi_{00} = \Psi(z_{00})$$

$$\Omega_{0000} =: \{\delta, \delta, \varepsilon_F, \varepsilon_F\}$$

$$\Omega_{+0+0} =: \{\delta - \Delta\delta/4, \delta + \Delta\delta/4, \varepsilon_F - \Delta\varepsilon_F/2, \varepsilon_F\}$$

$$\Omega_{+0-0} =: \{\delta - \Delta\delta/4, \delta + \Delta\delta/4, \varepsilon_F + \Delta\varepsilon_F/2, \varepsilon_F\}$$



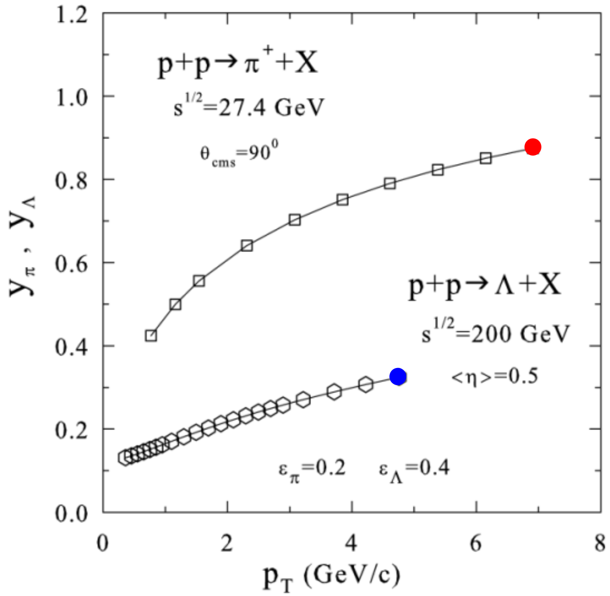


Additivity of fractal dimension: δ, $\Delta\delta$

Additivity of fragmentation fractal dimensions: ε_F, $\Delta\varepsilon_F$

Spin-dependent energy losses

Energy loss ~ $(1-y_a)$

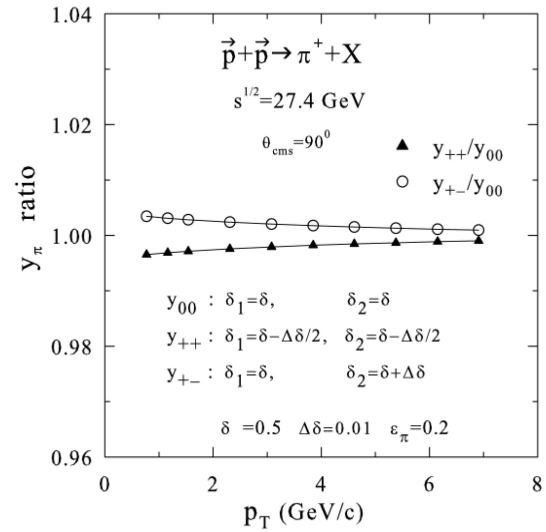


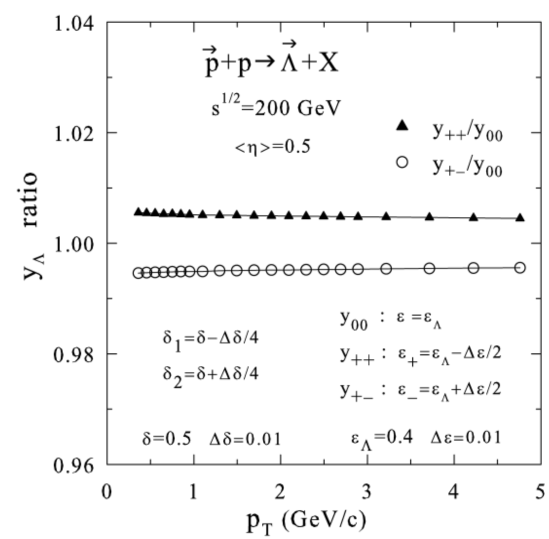
$p_T \approx 7$ GeV/c

π
10% energy loss
$q \approx 7.8$ GeV/c

$p_T \approx 5$ GeV/c

Λ
70% energy loss
$q \approx 16.7$ GeV/c





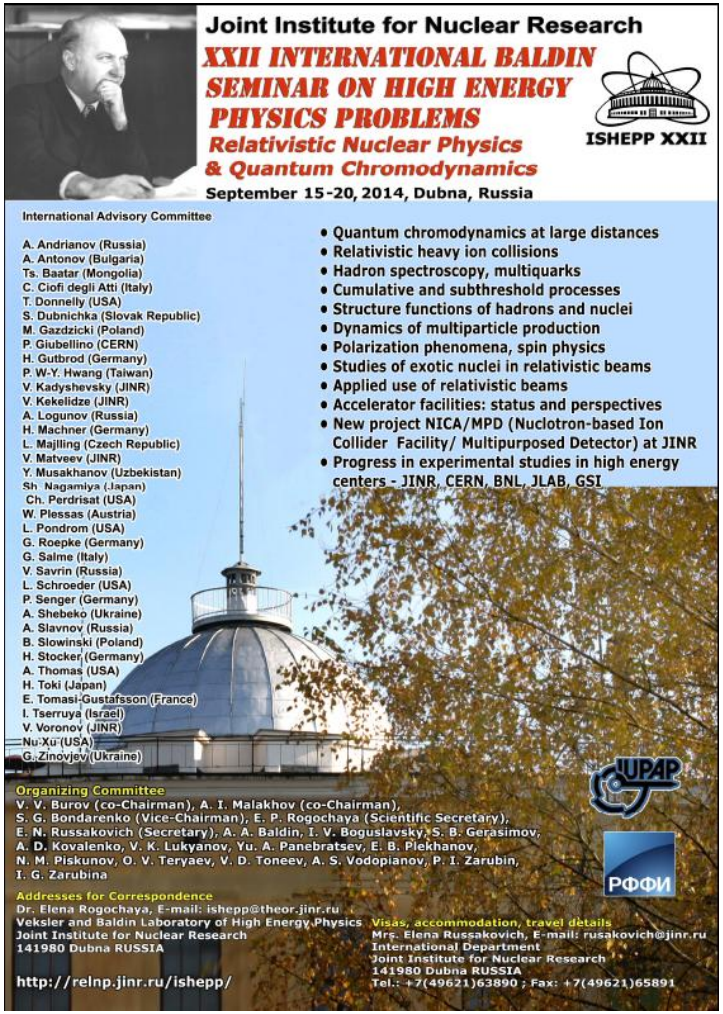
- Energy losses smear the features of hadron structure
- More higher p_T the smaller energy losses

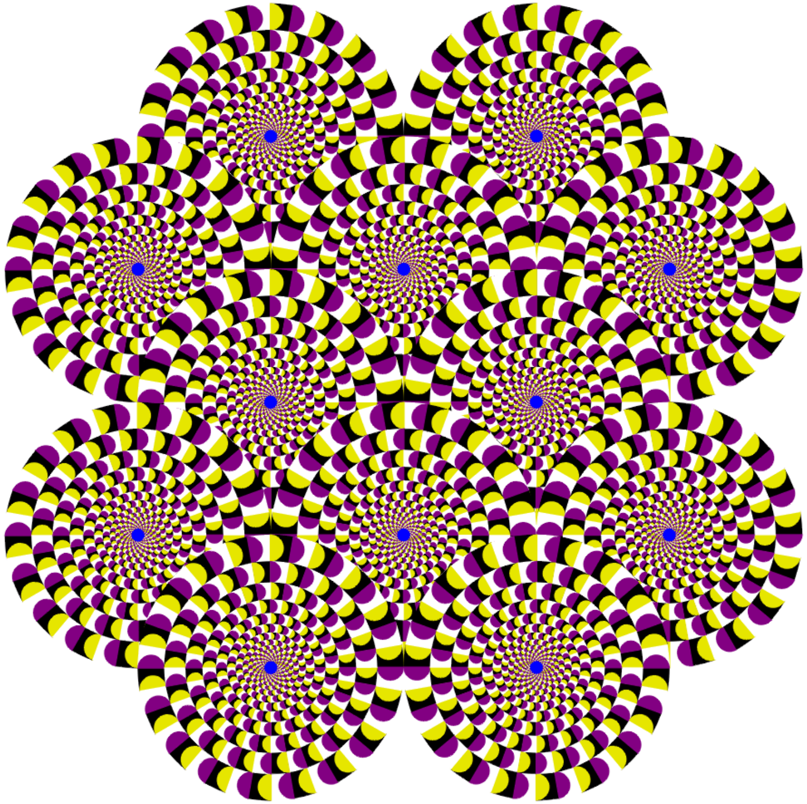
Conclusions

- A hypothesis of self-similarity of proton spin was formulated.
- Method of data analysis based on z-scaling was suggested for description of processes with polarized protons.
- Results of analysis of longitudinal double spin asymmetry A_{LL} of π production and longitudinal spin transfer coefficient D_{LL} of Λ production in pp collisions in z-scaling approach were presented.
- Spin-dependent fractal dimensions of proton structure and fragmentation to Λ hyperon were found.
- Spin-dependent constituent energy losses were estimated.

The investigation is motivated by expectations that particle production in pp collisions over energy range $\sqrt{s} = 10\text{–}30$ GeV is suitable for obtaining new information on fractal properties of proton spin.

Such experiments are planned to be carried out at the future SPD NICA facility in Dubna.





Thank you for your attention !

Back-up slides

Contribution to Letter of Intent

"Spin Physics Experiments at NICA-SPD
with polarized proton and deuteron beams" in Dubna

Measurements: Inclusive cross sections and asymmetries
of particles production in p-p collisions with polarized protons

Kinematic region: $\sqrt{s}$=10-30 GeV, high p_T , central rapidity range
Particles: π,.., J/ψ, Λ,…, γ, $l^+ l^-$,…

New characteristics of hadron production:
Spin-dependent fractal dimensions
Spin-dependent energy losses

New properties of spin origin:
Self-similarity of spin structure
Fractality of proton spin

Pion spectra in p_T and z presentation

MT & Zborovskỳ
Phys.Rev. D75
094008(2007)

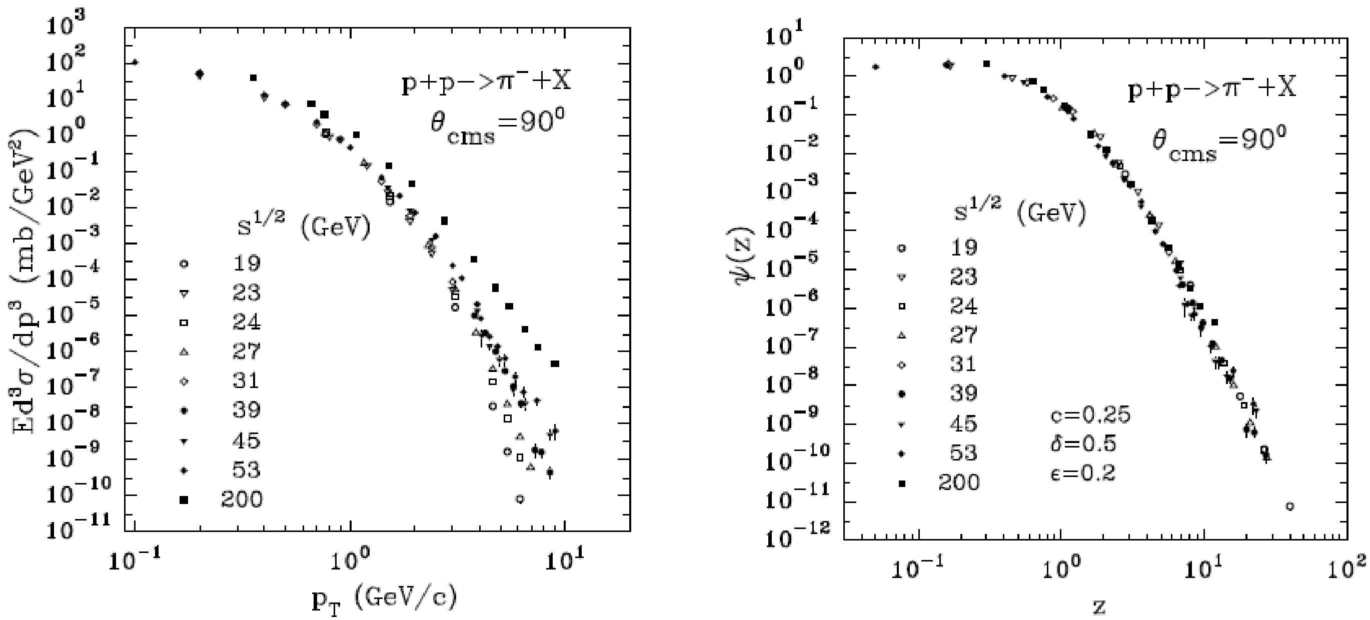


FIG. 3. (a) Transverse momentum spectra of the π^--mesons produced in *pp* collisions at $\sqrt{s} = 19 - 200$ GeV. Experimental data are taken from Refs. [22,23,25,27]. (b) The corresponding scaling function $\psi(z)$.

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