Bogoliubov Laboratory of Theoretical Physics

ISHEPP XXII September 15-20, 2014

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Cumulative photon emission off electron in intensive ultra-short electromagnetic fields

[Quantum processes in intensive short electromagnetic fields]

Hunt form mulpti-photon events

in collaboration with

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arXiv:1408.1040 [hep-ph]
Phys. Rev. A 87, 042106 (2013)
Phys. Rev. Lett. 108, 240406 (2012)
Phys. Lett. B, 715, 246 (2012)
Phys. Rev D 83, 053008 (2011)
Phys. Rev. ST Accel. Beams 12, 111301 (2009)

Motivation:

In QED a motion of charge particle in EM field is described by Lagrangian

$$\mathcal{L}^{\rm em} = -i\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi - M_e\bar{\psi}\psi - e\bar{\psi}\,\gamma\cdot A\,\psi - \frac{1}{4}F_{\mu}F^{\mu\nu}$$

In case of interaction of an electron with single photon(s) $|A^{s.p.}| \sim |\varepsilon_{\lambda}| = 1$

this theory has a small parameter $\alpha = \frac{e^2}{4\pi} = \frac{1}{137}$ *in other words, EM field does not disturb (modify) the plane wave solution* ψ_e^{PW} *of Dirac equation for electron in EM field*

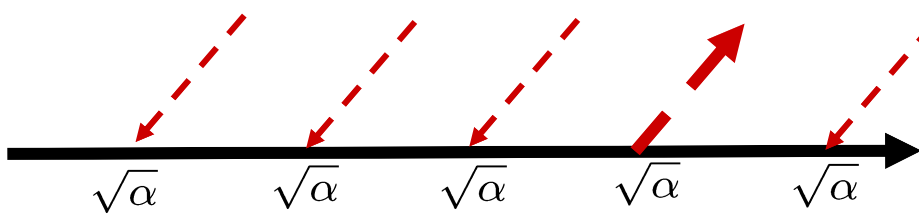
$$(i\not\nabla - eA - m)\psi_p = 0 \quad \rightarrow \quad (i\not\nabla - m)\psi_p^{PW} = 0$$

$$\psi_p(x) \simeq \psi_p^{PW}(x) = \left(u_p/\sqrt{2E_p}\right) \mathrm{e}^{-ip\cdot x},$$

Perturbation technique + Feynman rules → ***"Linear Electrodynamics"***
(conventionally)

Multi-photon processes with N photons are treated as the multi-step events and suppresses as α^N

$T_{fi} =$



Motivation (continuation)

*When an electron is moving in a strong background EM field with **LARGE** |A|*

$$|A| \gg |A^{s.p.}$$

$$(i\not\nabla - eA - m)\psi_p \neq (i\not\nabla - m)\psi_p^{PW}$$

$$\psi_e(x) \neq \psi_e^{PW}(x)$$

EM interaction becomes essentially non-linear → ***"Nonlinear Electrodynamics"***

Multi-photon events are not suppressed : charge particle may interact with N photons simultaneously.

Wanted: theory must describe "elementary" quantum processes in strong and "short" EM pulses

It has its own basic interest

May be used as an input for transport approaches in investigation of plasma preperties in strong EM field …

Laser pulse may be consider as a source of strong background field

$$|\vec{E}|^2 \sim |\dot{\vec{A}}|^2 \sim I$$

EM field *Pulse intensity*

$$|\vec{E}|(\frac{\mathrm{V}}{\mathrm{cm}}) \simeq 19.4\sqrt{I(\frac{\mathrm{W}}{\mathrm{cm}^2})}$$

- *UK [***VULCAN, HiPER** *(@Central Laser Facility (CLF))]*
- *EC [***Extreme Laser Infrastructure** *(ELI)]*
- *France [***APPOLON** *(@Institute de Lumiere Extreme (ILE))]*
- ⋮
- *US [* **TPL** *(Texas),(***BELLA***) Berkley, …..]*
- ⋮
- *Japan (ILE (GEKKO-LFEX ..) Spring-8 …)*
- *Russia (***PEARL** *(Nizhnij Nivgorod))*

A.Di Piazza, C.Muller, ***et al.*,**
Rev. Mod. Phys. 84, 1177 (2012)
G. Mourou, T. Tajima, S. V. Bulanov
Rev. Mod. Phys. 78, 309 (2006)

Laser Intensities $I \sim 10^{22} - 10^{25}\ \mathrm{W/cm^2}$

Pulse duration $\tau = 5,\ 10,\ 20, \ldots\ \mathrm{fs}$, $1\ \mathrm{fs} = 10^{-15}\ \mathrm{sec}$

τ_0 *(one oscillation)* = $3 \ldots 8\ \mathrm{fs}$

optical laser

Outline

- ***Introduction:***

Volkov solution of Dirac equation in Strong EM fields and its applications for infinitely long pulse $N_{\text{oscillations}} >>> 1$

Ritus *et al.*, '64~' 70

- ***Compton scattering in short EM pulses*** $N_{\text{oscillations}} = 2 \sim 10$

- ***Compton scattering in sub-cycle EM pulses*** $N_{\text{oscillations}} < 1$

- ***Quantum processes at sub-cycle pulses***

- ***Summry***

Electron in a strong electromagnetic field

D.M. Volkov, Z. Phys. 94, 250 (1935)

Über eine Klasse von Lösungen der Diracschen Gleichung.

1. Der Fall eines sinusoidalen Feldes. — 2. Lösung für den Fall, daß das äußere Feld aus polarisierten, in einer bestimmten Richtung fortschreitenden Wellen besteht, die ein *abzählbares* Spektrum nach Frequenz und Anfangsphasen haben.

A class of solutions of the Dirac equation

LL v.4, §40 (BLP)

$$(i\not\nabla - e\not A + m)(i\not\nabla - e\not A - m)\psi_p = 0,$$

Second order Dirac equation

Solution for special case $A = A(\phi)$ *with* $\phi = k \cdot x = \omega t - kz$ → *plane wave*

$$\psi_p = \left[1 + \frac{e(\gamma \cdot k)(\gamma \cdot A)}{2(k \cdot p)}\right] \frac{u_p}{\sqrt{2E_p}} e^{-ip\cdot x} \cdot e^{iS(\phi)}$$

← *phase factor*

spinor modification

$$S(\phi) = -\int_0^{kx} \left[\frac{e(p \cdot A)}{(k \cdot p)} - \frac{e^2 A^2}{2(k \cdot p)}\right] d\phi'$$

when $\vec{A} \to 0$

$$\psi_p \to \psi_p^{PW} = \frac{u_p}{\sqrt{2E_p}} e^{-ip\cdot x}$$

Dirac solution for free electron

Infinite plane wave:

$$A = (0, \vec{A}), \quad \vec{A} = a_x \cos kx + a_y \sin kx$$

S
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$$S(\phi) = -\int_0^{kx} \left[\frac{e(p\cdot A)}{(k\cdot p)} - \frac{e^2A^2}{2(k\cdot p)}\right] d\phi' \to -\frac{e^2a^2\cdot(kx)}{2(k\cdot p)} - \int_0^{k\cdot x} \frac{e(p\cdot A)}{(k\cdot p)} d\phi'$$

$$A^2 \to -a^2 = -a_x^2 = -a_y^2$$

$$\psi_p = [\ldots.]\cdot e^{iS'(\phi)} \frac{u_p}{\sqrt{2E_p}} e^{-i(p+\frac{e^2a^2}{2(k\cdot p)}k)\cdot x} = [\ldots.]\cdot e^{iS'(\phi)} \frac{u_p}{\sqrt{2E_p}} e^{iqx}$$

with

$$S'(\phi) = -\int_0^{kx} \frac{e(p\cdot A)}{(k\cdot p)} d\phi'$$

$$q^\mu \equiv p^\mu + \frac{e^2a^2}{2(k\cdot p)} k^\mu = p^\mu + \frac{\xi^2 m_e^2}{2(k\cdot p)} k^\mu$$

quasi-momentum

$$\xi^2 = \frac{e^2a^2}{M_e^2} = \frac{e^2E^2}{M_e^2\omega^2}$$

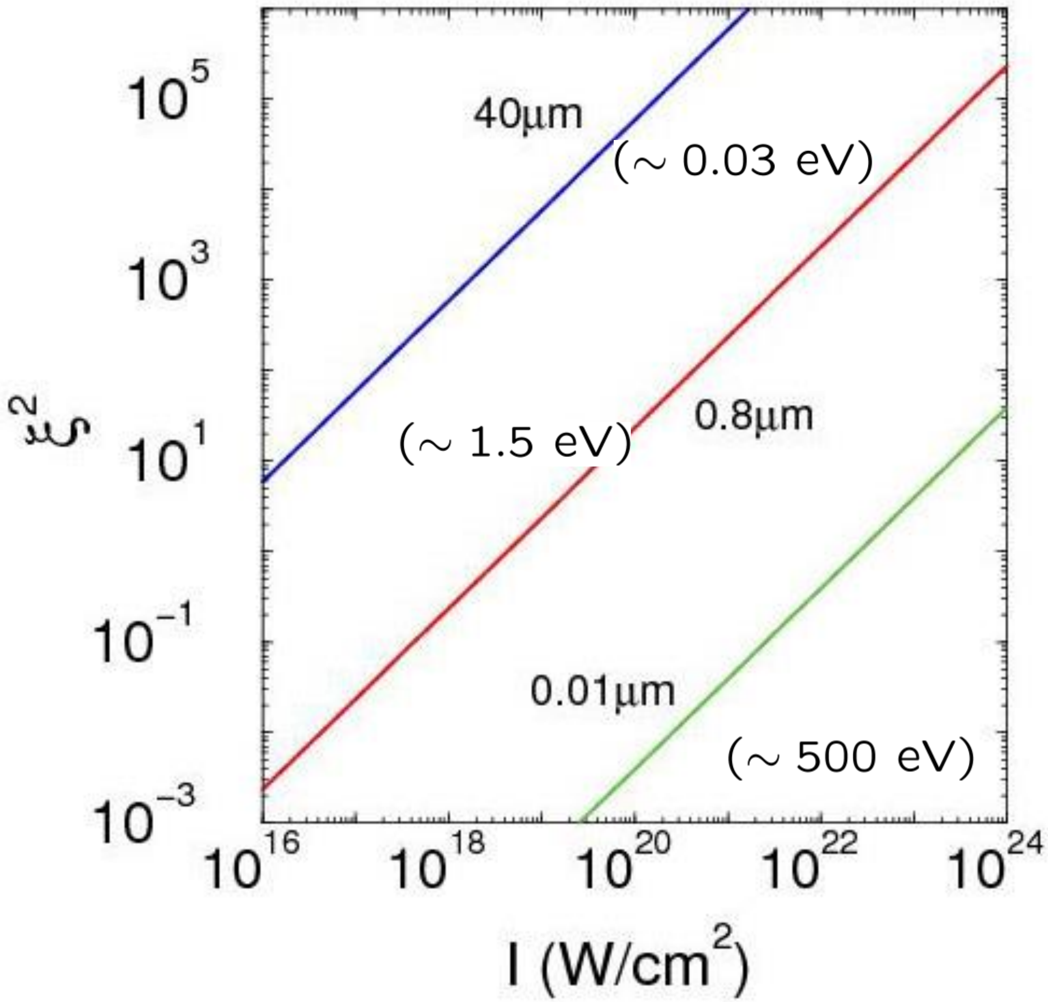
reduced field intensity

$$q^2 = M_*^2 \equiv M_e^2\left(1 + \frac{e^2a^2}{M_e^2}\right) = M_e^2\left(1+\xi^2\right) > M_e^2$$

effective mass

"quasi-momentum" and effective mass define energy-momentum conservation in processes with electrons (in infinite pulse !)

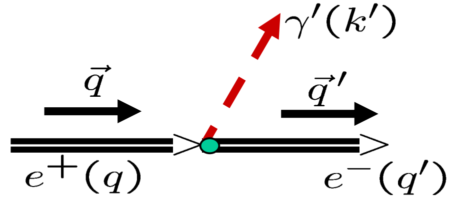
Dependance of reduced field strengh ξ^2 on laser pulse intensity I at different wavelength λ



$$\xi^2 = \frac{4\pi\alpha(\hbar c)^3}{M_e^2 c^4 \omega^2} \frac{I}{c}$$

$$\xi^2 = \frac{0.56}{[\omega(\text{eV})]^2} 10^{-18} I[\text{W/cm}^2]$$

Compton scattering for infinitely long pulse



$$T_{fi} = -ie\int \bar{\psi}_{e^-}(\gamma\cdot\varepsilon'(\gamma'))\psi_{e^+}\mathrm{e}^{-ik'\cdot x}\frac{d^4x}{\sqrt{2\omega'}} \;,$$

$$\frac{\bar{u}_{p'}}{\sqrt{2q'_0}}\mathrm{e}^{iq'\cdot x}\left[1+\frac{e(\gamma\cdot A)(\gamma\cdot k)}{2(k\cdot p')}\right]\mathrm{e}^{+i\int\limits_0^{k\cdot x}\frac{e(p'\cdot A)}{(k\cdot p')}d\phi'} \qquad \left[1+\frac{e(\gamma\cdot k)(\gamma\cdot A)}{2(k\cdot p)}\right]\mathrm{e}^{-i\int\limits_0^{kx}\frac{e(p\cdot A)}{(k\cdot p)}d\phi'}\frac{v_p}{\sqrt{2q_0}}\mathrm{e}^{iq\cdot x}$$

$$T_{fi} = \frac{-ie}{\sqrt{2q_0 2q'_0 2\omega'}}\int M_{fi}(kx)\mathrm{e}^{-i(k'-q+q')x}d^4x \qquad \neq (2\pi)^4\delta^4(k-k'+q-q')\cdot M$$

$$M_{fi}(kx) = \sum_{n=-\infty}^{\infty}\mathrm{e}^{-in\,k\cdot x}M_{fi}(n)$$

Fourier series
for functions defined in all space

$$T_{fi} = \frac{-ie}{\sqrt{2q_0 2q'_0 2\omega'}}\sum_{n=n_{\min}}^{\infty} M_{fi}(n)\,(2\pi)^4\,\delta^4(nk+q-k'-q')$$

$$M_{fi}(\phi) = \bar{u}_{p'}\left[\widehat{M}^{(0)}+\widehat{M}^{(1)}\cos\phi+\widehat{M}^{(2)}\sin\phi\right]v_p\mathrm{e}^{-iz\sin(\phi-\phi_0)},\quad \phi = k\cdot x,$$

$$M_{fi}(n) = \bar{u}_{p'}\left[\widehat{M}^{(0)}B_n^{(0)}+\widehat{M}^{(1)}B^{(1)}+\widehat{M}^{(2)}B^{(2)}\right]v_p$$

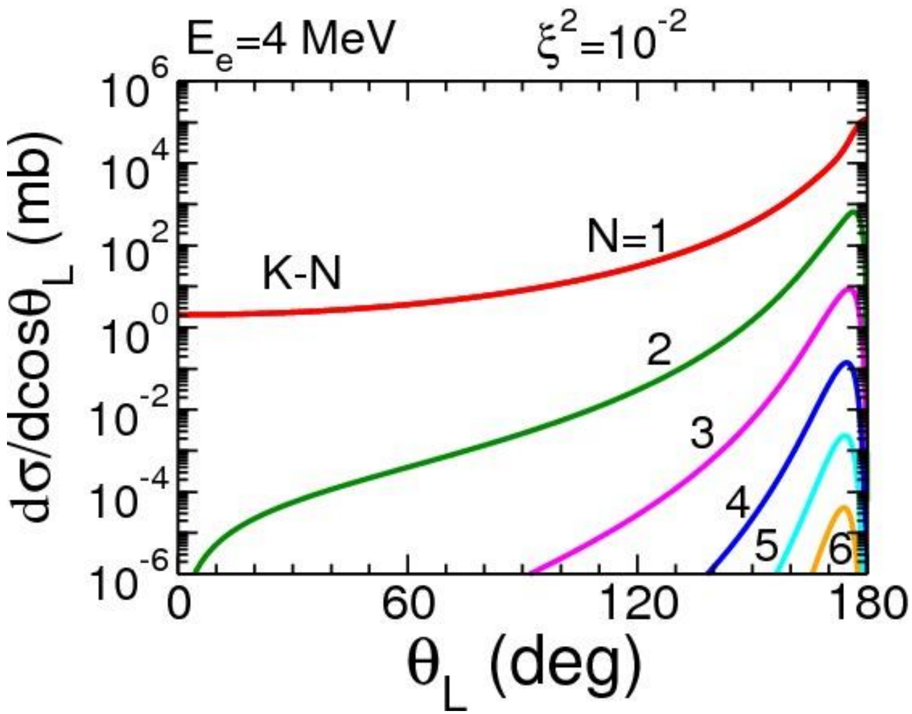
$$\widehat{M}^{(0)} = \not\varepsilon' - \frac{e^2|a|^2(\varepsilon'\cdot k)}{2(k\cdot p)(k\cdot p')}, \qquad \widehat{M}^{(1,2)} = -e\frac{\not\varepsilon'\not k\not a_{1,2}}{2(k\cdot p)}+e\frac{\not a_{1,2}\not k\not\varepsilon'}{2(k\cdot p)}$$

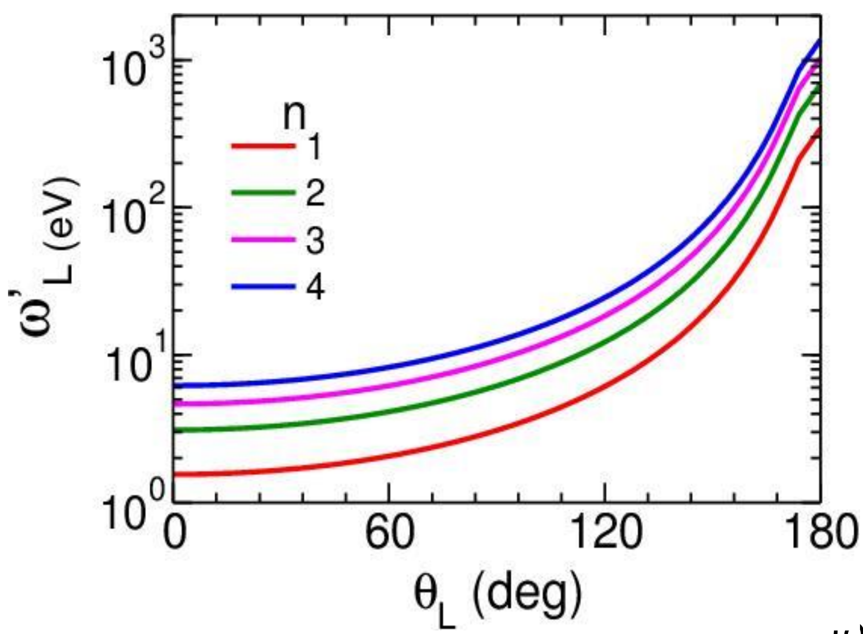
Differential cross section (infinite pulse)

$$\frac{d\sigma}{d\cos\theta_L} = \frac{4\pi\alpha^2}{\omega_L(s - M_e^2)(E_L + p_L)\,\xi^2} \sum_{n=1}^{\infty} \frac{{\omega'_L}^2}{n}\, w^{(n)}$$

$$w^{(n)} = -2J_n^2(z) + \xi^2(1 + \frac{u^2}{2(1+u})) [J_{n-1}^2(z) + J_{n+1}^2(z) - 2J_n^2(z)]$$

$$u = \frac{k\cdot k'}{k\cdot p'}, \qquad z^2 = \frac{4n^2\xi^2}{1+\xi^2}\,\frac{u}{u_n}(1 - \frac{u}{u_n}), \qquad u_n = 2n\,\frac{p\cdot k}{M_e^2(1+\xi^2)},$$





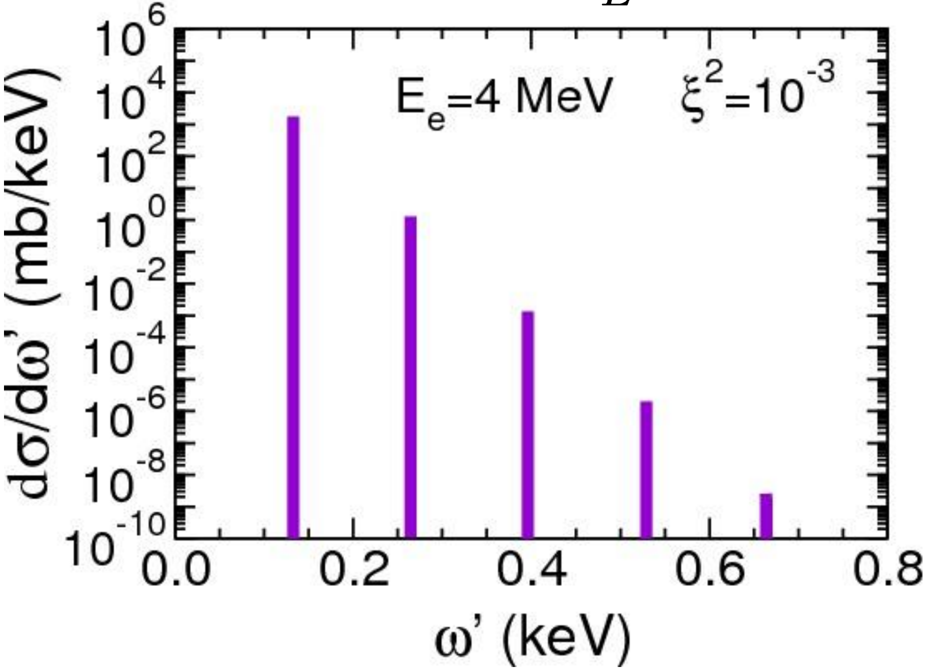
Differential cross section at fixed angle (inf. pulse)

$$\frac{d\sigma}{d\omega'_L} = \frac{4\pi\alpha^2 M_e^2}{(s - M_e^2)\,\xi^2} \sum_{n=1}^{\infty} \frac{1}{p_L - n\omega_L}\, w^{(n)}$$

$$w^{(n)} = -2J_n^2(z) + \xi^2(1 + \frac{u^2}{2(1+u}))[J_{n-1}^2(z) + J_{n+1}^2(z) - 2J_n^2(z)]$$

$$u = \frac{k\cdot k'}{k\cdot p'}, \qquad z^2 = \frac{4n^2\xi^2}{1+\xi^2}\,\frac{u}{u_n}(1 - \frac{u}{u_n}), \quad u_n = 2n\,\frac{p\cdot k}{M_e^2(1+\xi^2)},$$

$$\theta_L = 170^o$$



Discrete spectrum

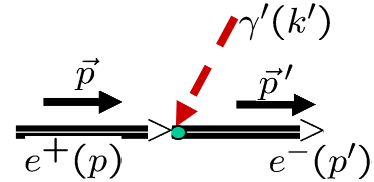
Compton scattering process in a short e.m. pulse

Short circularly polarized pulse:

$$A = (0, \vec{A}_\gamma) \quad \vec{A}_\gamma = f(kx)\,[\vec{a}_x \cos(kx) + \vec{a}_y \sin(kx)]\,, \qquad |\vec{a}_x| = |\vec{a}_y| = a \qquad \textit{and} \qquad \vec{a}_x \vec{a}_y = 0$$

where $f(kx)$ *is the pulse "envelope" function*

$$T_{fi} = -ie \int \bar{\psi}_{e^-} (\gamma \cdot \varepsilon'(\gamma')) \psi_{e^-} \mathrm{e}^{-ik' \cdot x} \frac{d^4x}{\sqrt{2\omega'}}\,,$$



$$\frac{\bar{u}_{p'}}{\sqrt{2p'_0}} \mathrm{e}^{ip' \cdot x} \left[1 + \frac{e(\gamma \cdot A)(\gamma \cdot k)}{2(k \cdot p')} \right] \mathrm{e}^{\,i \int_0^{k \cdot x} \left[\frac{e(p' \cdot A)}{(k \cdot p')} - \frac{e^2 A^2}{(2k \cdot p')} \right] d\phi'}$$

$$\left[1 + \frac{e(\gamma \cdot k)(\gamma \cdot A)}{2(k \cdot p)} \right] \mathrm{e}^{-i \int_0^{kx} \left[\frac{e(p \cdot A)}{(k \cdot p)} + \frac{e^2 A^2}{(2k \cdot p)} \right] d\phi'} \frac{u_p}{\sqrt{2p_0}} \mathrm{e}^{-ip \cdot x}$$

$$T_{fi} = \frac{-ie}{\sqrt{2q_0 2q'_0 2\omega'}} \int M_{fi}(kx) \mathrm{e}^{-i(p-k'-p')x} d^4x$$

$$M_{fi}(kx) = \int_{-\infty}^{\infty} dl \mathrm{e}^{-il\,k \cdot x}\, M_{fi}(l)$$

Fourier integral
for functions limited in configuration space

$$T_{fi} = \frac{-ie}{\sqrt{2p_0 2p'_0 2\omega'}} \int_{\zeta}^{\infty} dl\, M_{fi}(l) (2\pi)^4 \delta^4(lk + p - k' - p'),$$

Comparison between infinite and finite pulses

$$M_{fi}(n) = \sum_{i=0}^{2} M^{(i)} B_n^{(i)} \ ,$$

$$B_n^{(i)} = B\left[J_{n\pm1},\ J_n\right]$$

$$J_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} d\phi\, \mathrm{e}^{il\phi - iz\sin(\phi-\phi_e)}$$

$$\phi = k\cdot x$$

$$W = \sum_n [\text{one} - \text{dimentional integral}]$$

$$M_{fi}(l) = \sum_{i=0}^{3} M^{(i)} C^{(i)}(l) \ ,$$

$$C^{(i)}(l) = C\left[\mathcal{Y}(l\pm1),\ (\mathcal{Y}(l)\right]$$

$$\mathcal{Y}(l) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\phi\, \mathrm{e}^{il\phi - i\mathcal{P}(\phi)}\, f(\phi)$$

with

$$\mathcal{P}(\phi) = z\int_{-\infty}^{\phi} d\phi'\cos(\phi'-\phi_e) f(\phi') - \xi^2\zeta u \int_{-\infty}^{\phi} d\phi'\, f^2(\phi') \ .$$

$$W = [\text{five} - \text{dimentional integral}]$$

Pulse envelope functions

One parameter functions

$$f_{\mathrm{hs}}(\phi) = \frac{1}{\cosh\frac{\phi}{\Delta}}\,, \qquad \Delta = 2\pi N$$

hyperbolic secant (hs)

Two parameter function

$$f_{\mathrm{sF}}(\phi) = \frac{\cosh\frac{\Delta}{b} + 1}{\cosh\frac{\Delta}{b} + \cosh\frac{\phi}{b}}$$

$$\Delta = 2\pi N$$

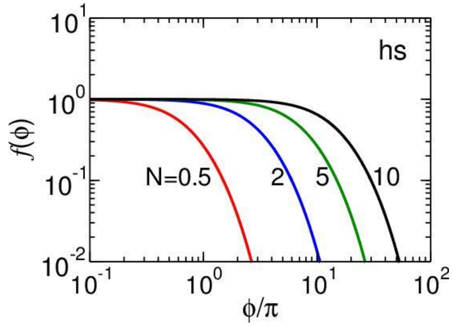
$$\frac{b}{\Delta}$$

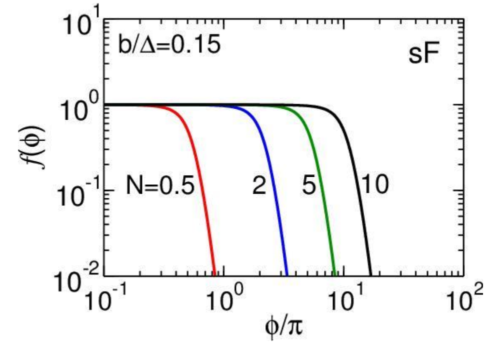
summarized Fermi (sF) *Lukyanov, 70'*

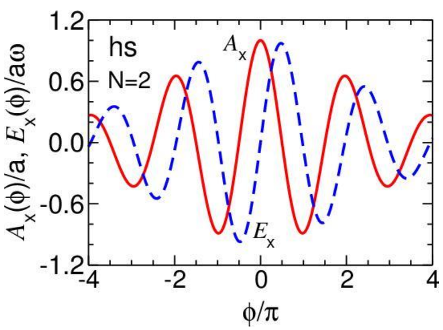
"ramping/decreasing" time

$$\frac{b}{\Delta} \to 0$$

rectangular shape

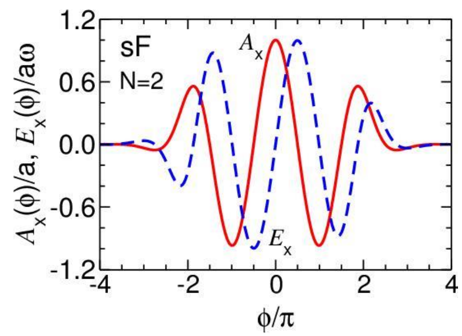


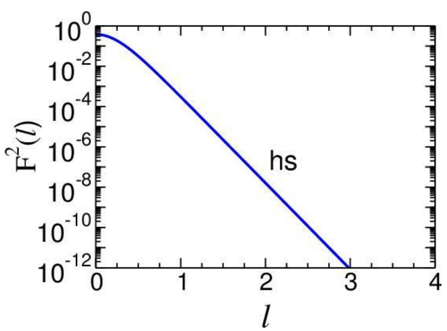




$$A_x(\phi)/a = f(\phi)\cos\phi$$

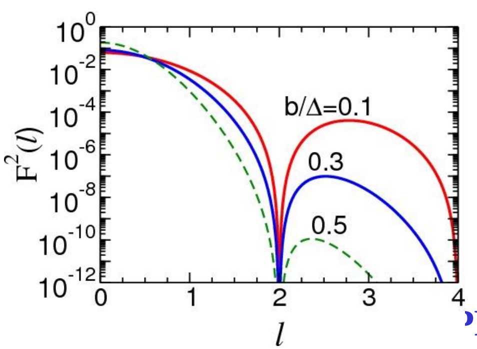
$$E_x = -\frac{\partial A_x}{\partial t}/a\omega$$





Fourier transforms

$$F(l) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\phi\, e^{il\phi}\, f(\phi);$$

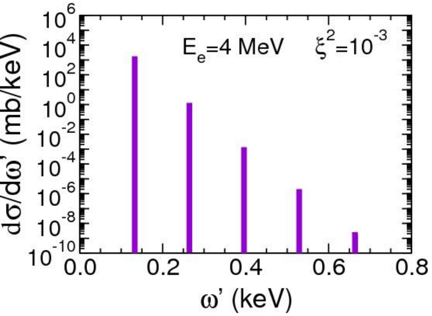


Differential cross section (short pulse)

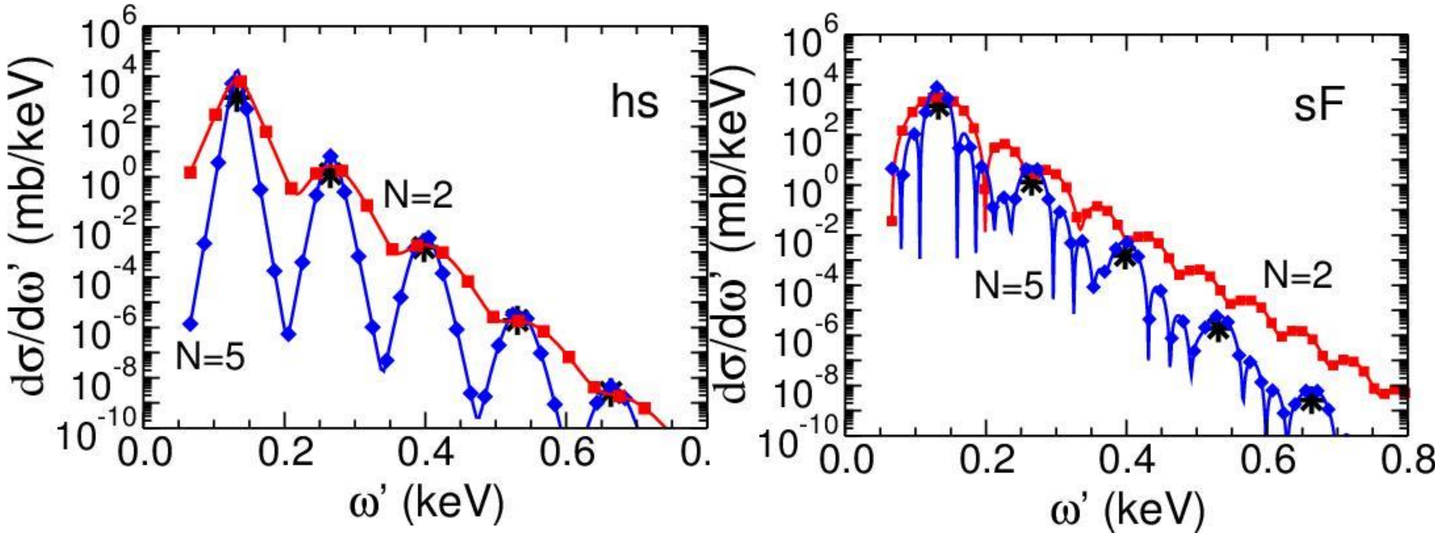
$$\frac{d\sigma}{d\omega' d\phi_{e'}} = \frac{2\alpha^2 M_e^2}{\xi^2(s - M_e^2)} \int\limits_{l_0<1}^{\infty} dl \frac{1}{p_e - l\omega} w_l$$

$$w_n = -2\tilde{Y}_l^2(z) + \xi^2(1 + \frac{u^2}{2(1+u}))[Y_{l-1}^2(z) + Y_{l+1}^2(z) - 2\mathrm{Re}\,\tilde{Y}_l(z)X_l^*(z)]$$

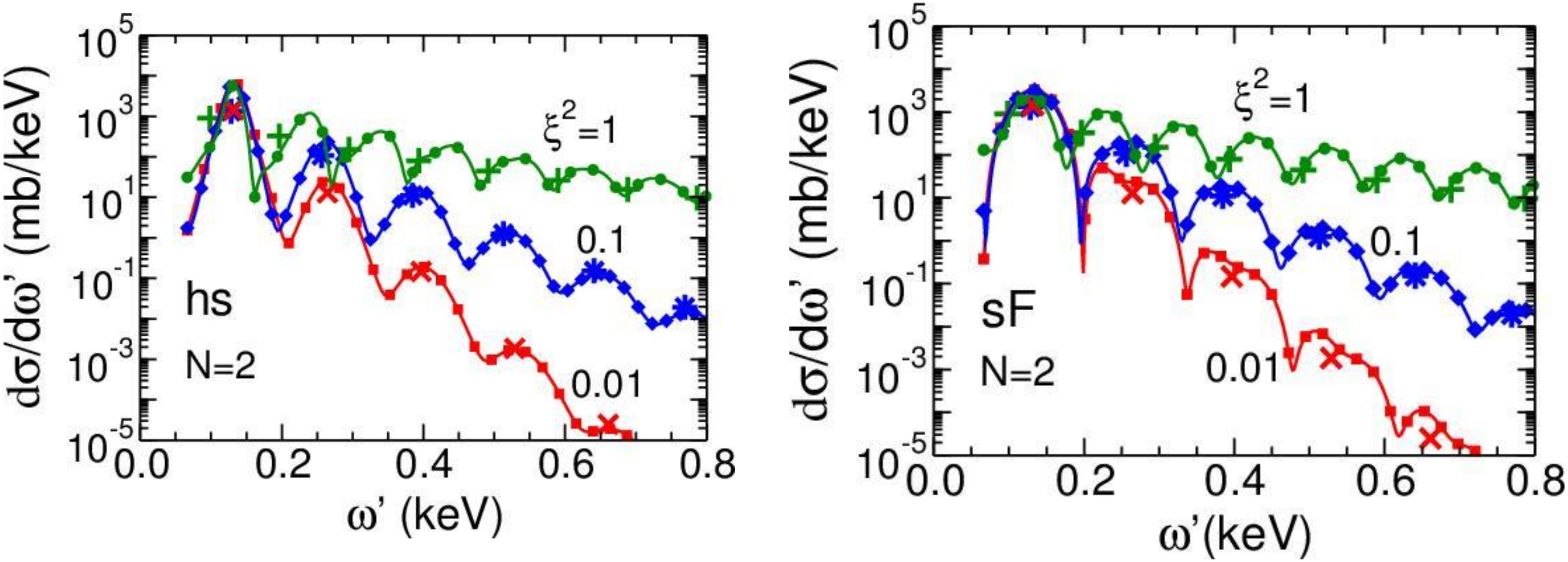
$$u = \frac{k\cdot k'}{k\cdot p'}, \qquad z^2 = 4l^2\xi^2\,\frac{u}{u_l}(1 - \frac{u}{u_l}), \quad u_l = 2l\,\frac{p\cdot k}{M_e^2}$$



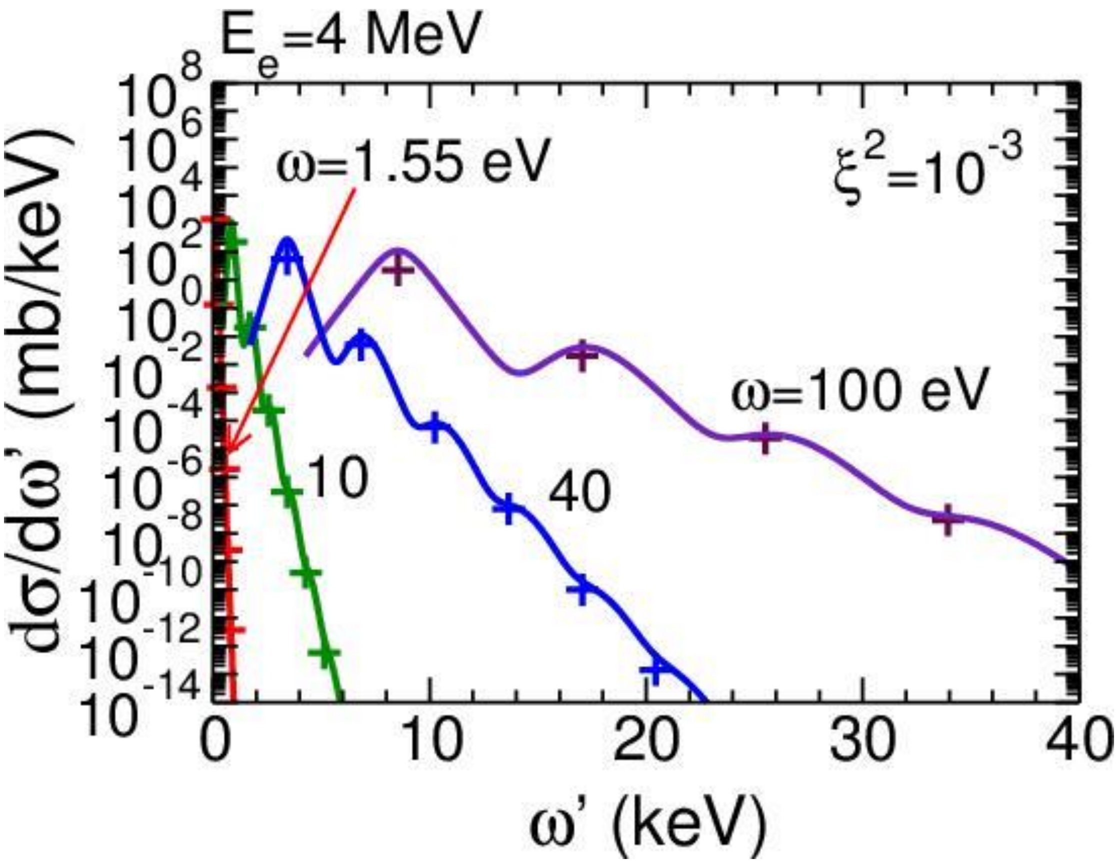
$$\xi^2 = 10^{-3}$$



Dependence on field intensity (short pulse)



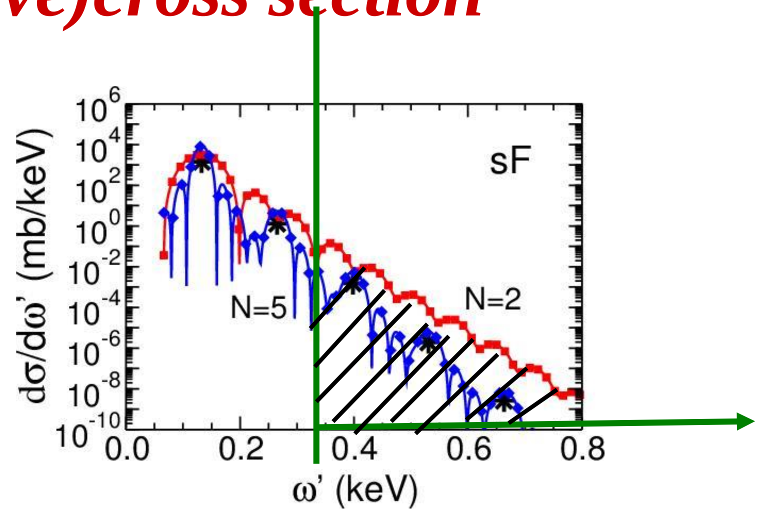
Dependence on the laser frequency



$$\omega'(l) \simeq \frac{l\omega(E+p)}{E + p\cos\theta' + l\omega(1-\cos\theta')}$$

Partly-integrated (cumulative)cross section

$$\tilde{\sigma}(\omega') = \int_{\omega'}^{\infty} d\bar{\omega}\, \frac{d\sigma}{d\bar{\omega}}$$

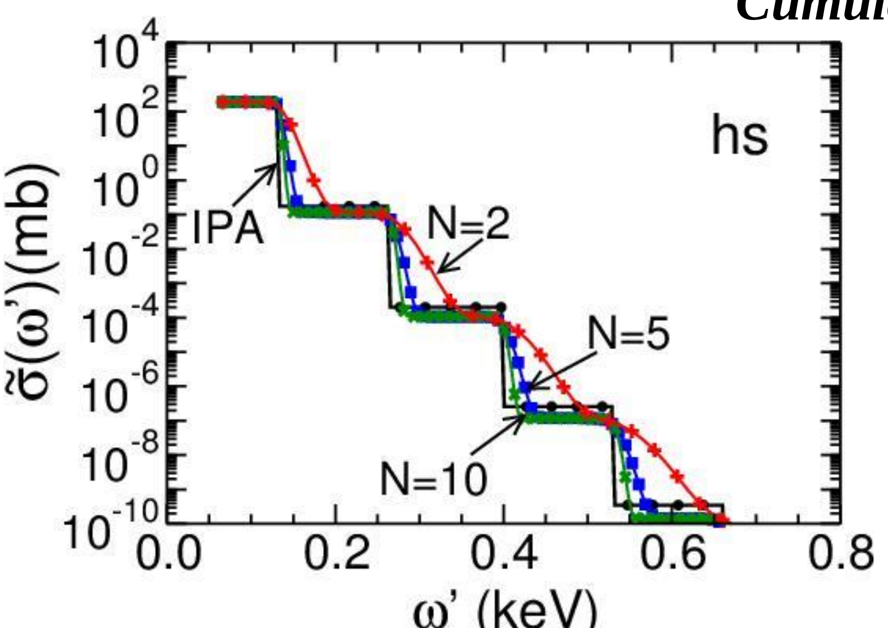


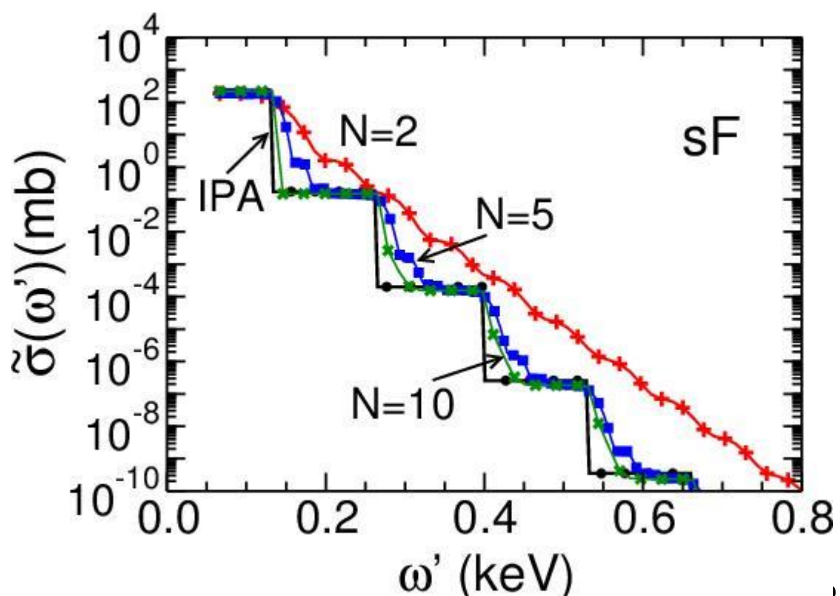
$$\kappa = \frac{\omega'}{\omega'(l=1)} > 1$$

sub-threshold parameter

$$\omega'(l) \simeq \frac{l\omega(E+p)}{E + p\cos\theta' + l\omega(1-\cos\theta')}$$

"Cumulative" cross section





Ultra-short pulse

$$\tilde{\sigma}(\omega') = \int_{\omega'}^{\infty} d\bar{\omega}\, \frac{d\sigma}{d\bar{\omega}} = \int_{l(\omega')}^{\infty} dl \frac{d\bar{\omega}}{dl}\frac{d\sigma}{d\bar{\omega}} \ ;$$

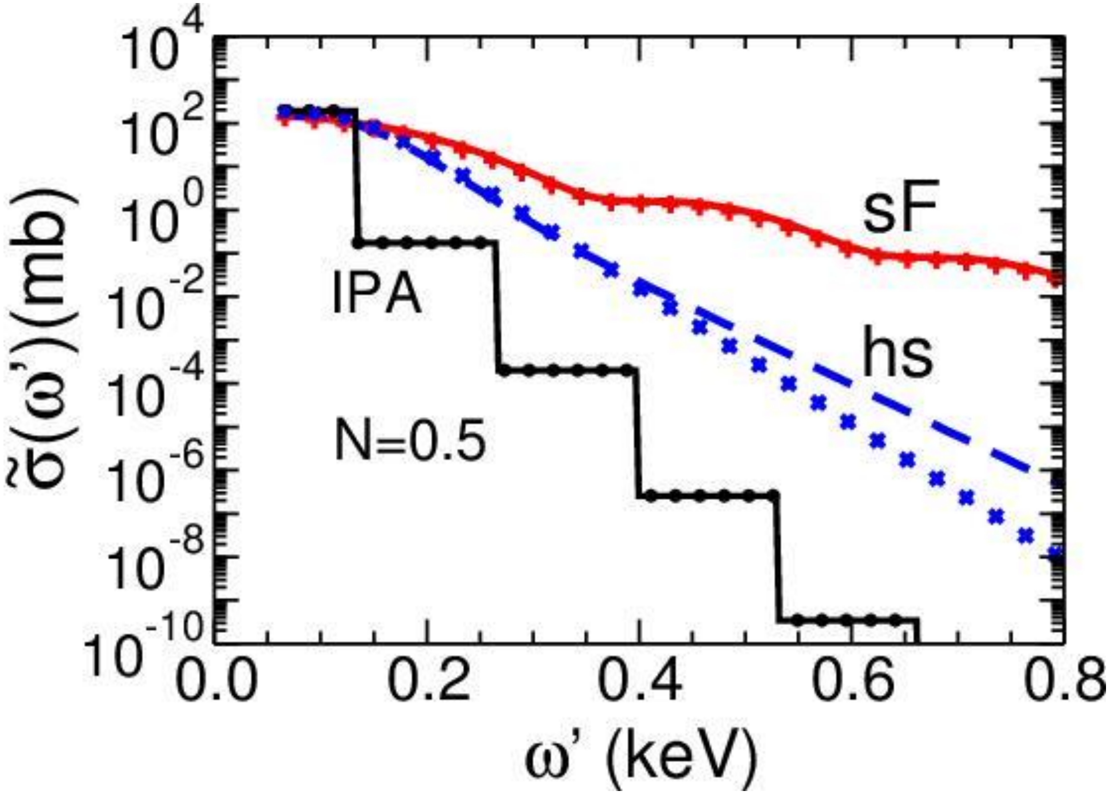
$$C^{(i)}(l) = C\left[\mathcal{Y}(l \pm 1),\, (\mathcal{Y}(l)\right]$$

$$\mathcal{Y}(l) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\phi\, \mathrm{e}^{il\phi - i\mathcal{P}(\phi)}\, f(\phi)$$

$$\mathcal{P}(\phi) \ll 1 \qquad \mathcal{Y}^2(l) \simeq F^2(l)$$

$$l \simeq \kappa = \frac{\bar{\omega}'}{\omega'(l=1)}$$

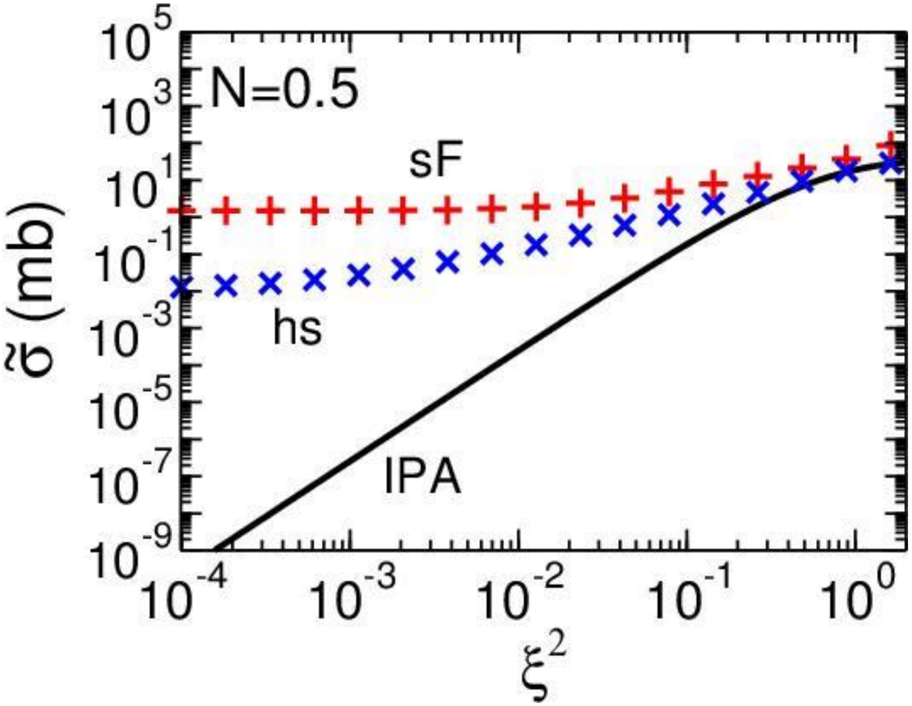
$$d\sigma(\bar{\omega}(l)) = d\sigma(F^2(l-1)) \ ;$$

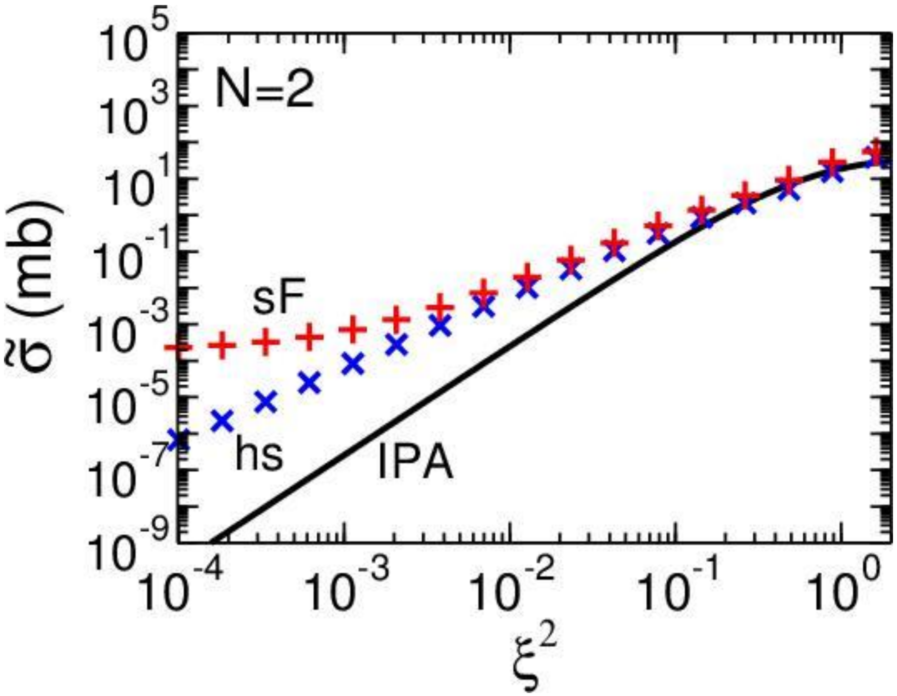


N=0.5

Dependence on the field intensity for sub-cycle/short pulse

$$\kappa = \omega'/\omega'_1 = 3$$



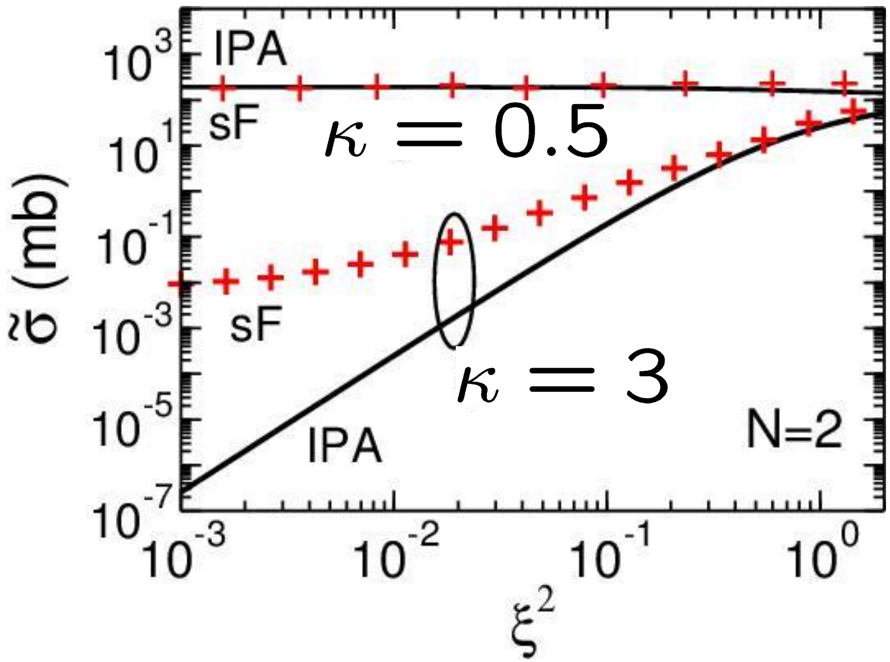


Sub-threshold effect is only at $\kappa > 1$ *!!!*

when

$$\kappa = \omega'/\omega'_1 = 0.5 < 1$$

$$\tilde{\sigma} \simeq \sigma_{\rm tot} = \text{constant}$$

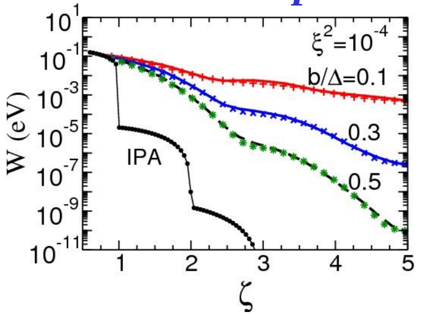


Summary

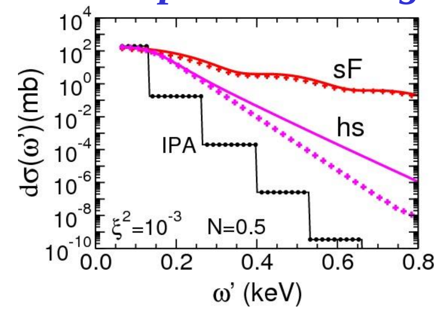
- *Elaborated a method for calculating the quantum processes in a short and intensive EM fields (laser pulses) in a wide range of pulse intensities for different shapes and duration.*

- *The short pulses "generates" high momentum components which produce a great amplifier effect for the multi-photon, sub-threshold events. Enhancement may reach many orders of magnitude depending on the field intensity and the beam shape, (relative to the infinite pulse)!*

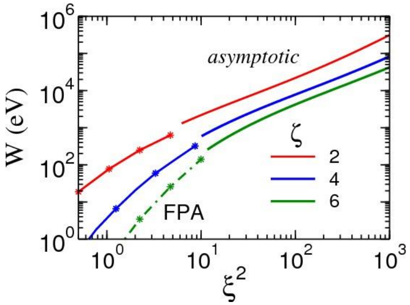
Breit-Wheeler process

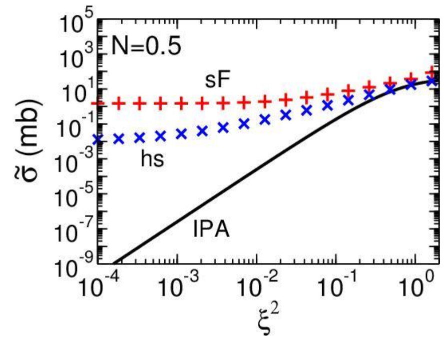


Compton scattering



- *Clarified the range of applicability of IPA*





- *Analyzed some subtle effects in particle production off a laser beam (anisotropies and asymmetries) , $\bar{\nu}\nu$ production etc.*

- *Developed a number of simple, asymptotic expressions for small* $\xi^2 \ll 1$

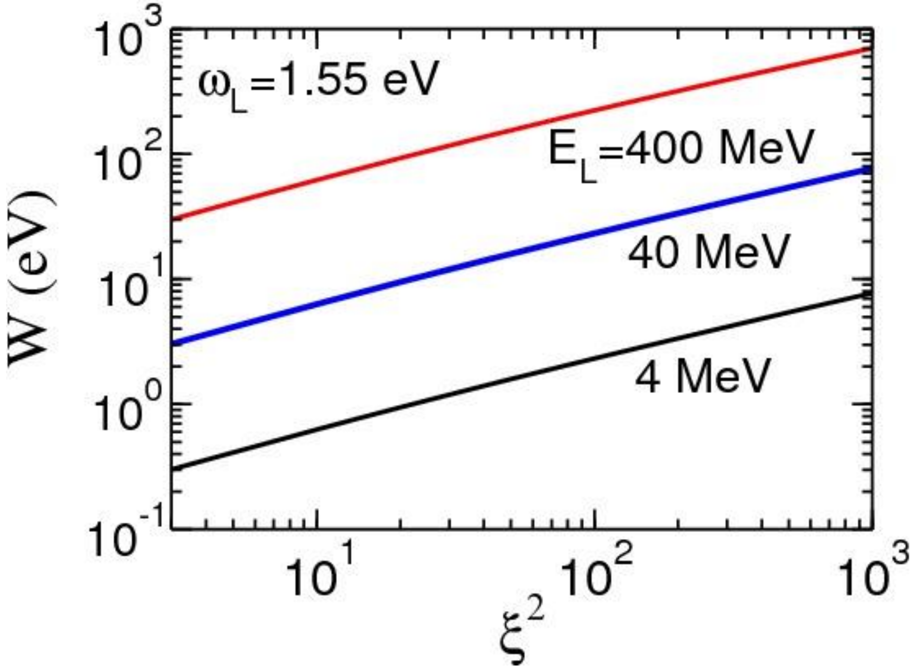
THE END

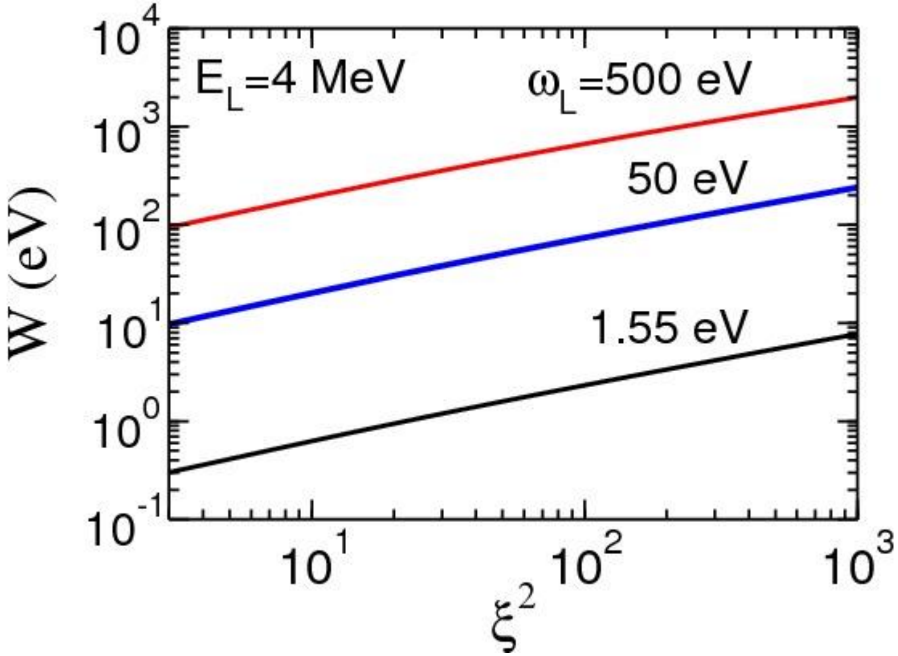
Thank you very much for attention !

Photon production at very large EM field intensity (rate)

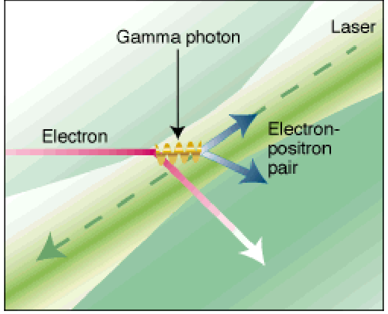
$$W \sim \left(\omega \frac{s - M_e^2}{M_e^2} \xi\right)^{\frac{2}{3}} = \left(\frac{4\omega E_e}{M_e^2} \xi\right)^{\frac{2}{3}}$$

$$\sigma \sim \frac{1}{\xi^2} W \sim \xi^{-\frac{4}{3}}$$





SLAC (E-144) experiment ***D. Burke et al.,PRL 79 (1997)***



$$\gamma' + L \to e^+ e^-$$ ***Generalized Breit-Wheeler process***

BW process (kinematics)

$$s_{thr}(\gamma'\gamma) = (k+k')^2 = 4\omega\omega' = 4M_e^2 \qquad \omega'_{thr} = \frac{M_e^2}{\omega} \simeq \frac{0.26\cdot 10^{12}(eV^2)}{2.35\,eV\,(SLAC)} \simeq 111 \text{ GeV}$$

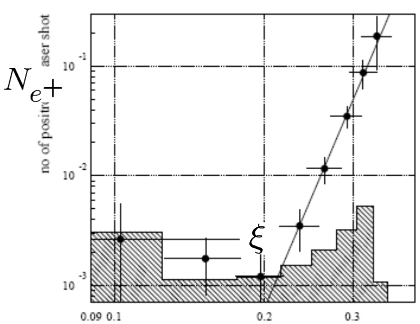
$$\omega'_{Bremsst} \simeq 29 \text{ GeV } (SLAC) \rightarrow \zeta = \frac{\omega'_{thr}}{\omega'_{Bremmst}} \simeq 3.83$$ ***with*** $0.1 < \xi < 0.36$

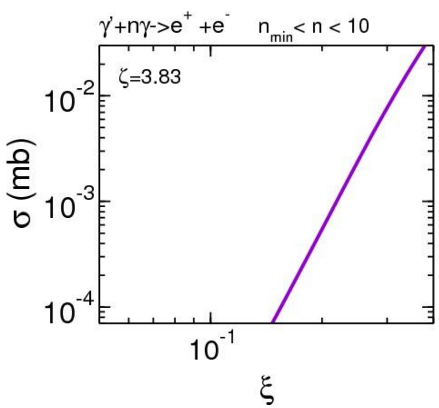
$$\gamma' + n\gamma \to e^+e^- \quad \to n_{\min} = I(\zeta(1+\xi^2)) + 1 = 5$$

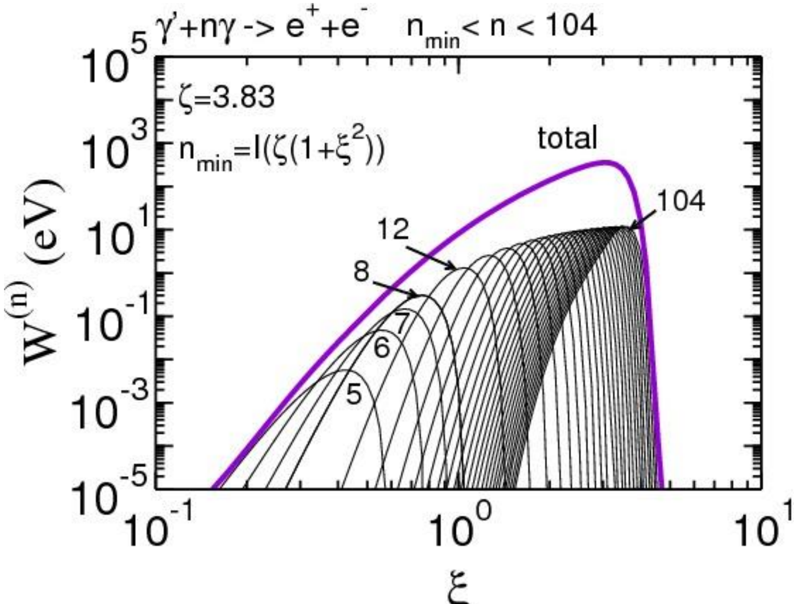
essentially multi-photon process

$$\xi^2 = 0.56(\omega(\text{eV}))^{-2}10^{-18}\text{I}/(\text{W/cm}^2) \;\; (\omega = 2.35 \text{ eV})$$

$$I \sim 2\times 10^{18} \text{ W/cm}^2 \rightarrow \xi^2 \sim 0.1$$







Lasers planned for the Extreme Light Infrastructure*					
Country	Facility focus	Power (PW)	Pulse energy (J)	Pulse width (fs)	Rep rate (Hz)
Romania	Nuclear physics	10 (×2)	200	20	0.1
Hungary	Attosecond physics	1	5	5	1000
		20	400	20	0.1
Czech Republic	Secondary beam radiation, high-energy particles	1	10	10	10
		5	50	10	10
		10 (×2)	200	20	0.1
To be determined	High intensity	10 beams of 10–20 PW each, phased and combined to create total power of 100–200 PW			
*Laser parameters still subject to change.					

FIG. 1 (color online). Summary of the four pillars of ELI. A power value of 10(×2) PW indicates the availability of two laser systems each with 10-PW power. From Feder, 2010.