Lorentz boosts in interacting systems

Eugene Stefanovich

Dubna. 17 September, 2014

OUTLINE

1. Wigner-Dirac theory of relativistic interactions
2. Currie-Jordan-Sudarshan “no interaction” theorem
3. Example: 2-particle interacting system
4. Example: μ-τ oscillating neutrino pair
5. Example: fast moving unstable particle
6. Conclusions

Basics of relativistic quantum theory

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quantum mechanics + principle of relativity =
Unitary representation of the Poincare group in the Hilbert space of the system

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space translations	Total momentum	**P**
space rotations	Total angular momentum	**J**
boosts		**K**
	Center-of-mass position	$\mathbf{R}=-c^2/2(\mathbf{K}H^{-1}+H^{-1}\mathbf{K})$ + spin terms

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Commutation relations of the Poincare Lie algebra:

$$[\mathbf{P},H]=0, \quad [J_i,P_j]=i\hbar\varepsilon_{ijk}P_k, \quad [K_i,P_j]=-(i\hbar/c^2)H\delta_{ij}, \quad [\mathbf{K},H]=-i\hbar\mathbf{P},$$
$$[R_i,P_j]= i\hbar\delta_{ij}, \; ....$$

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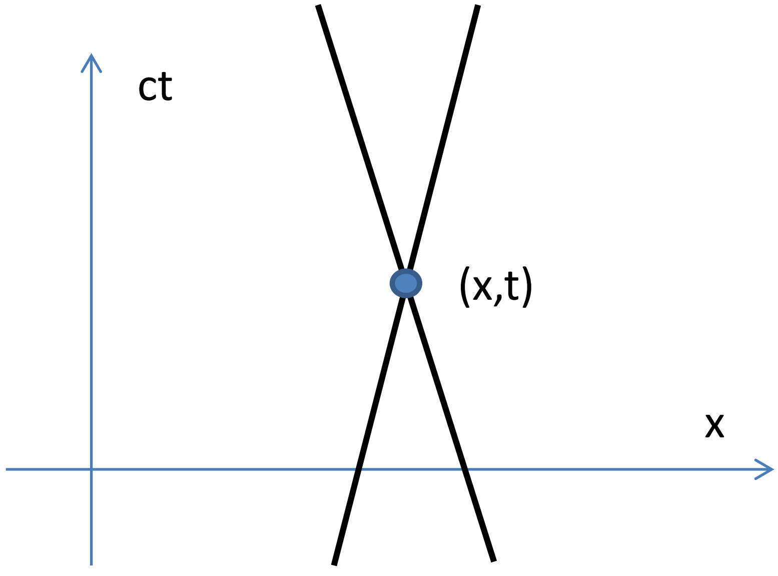

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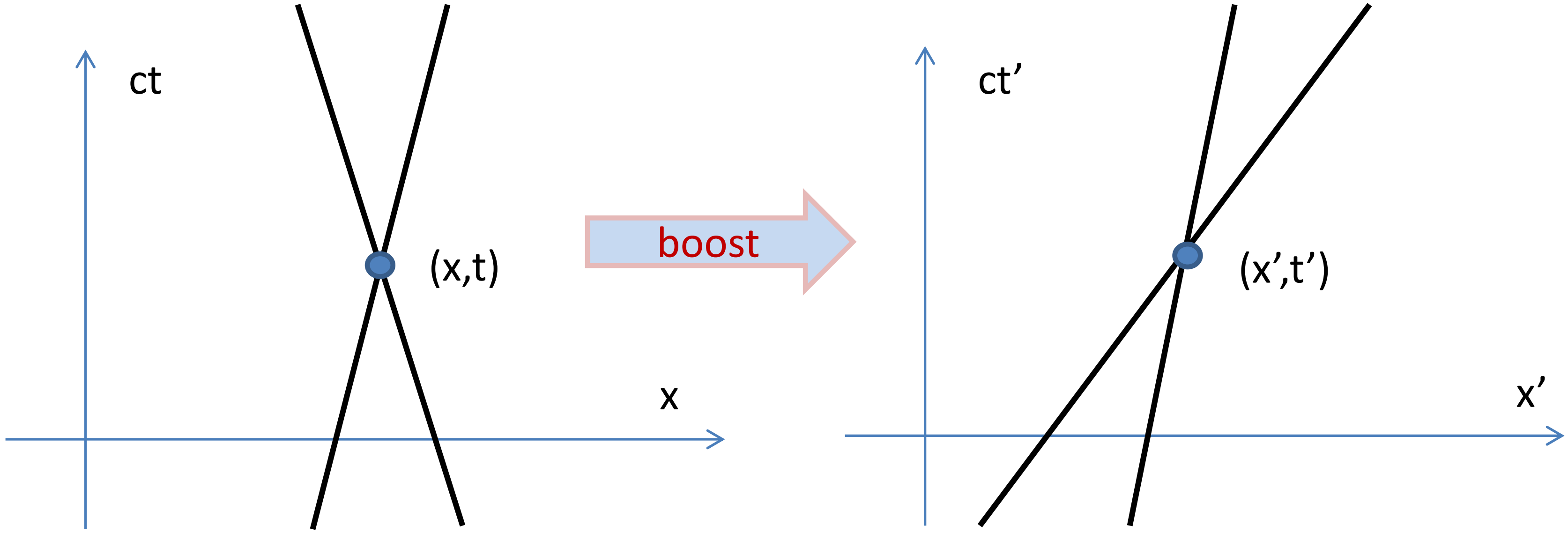

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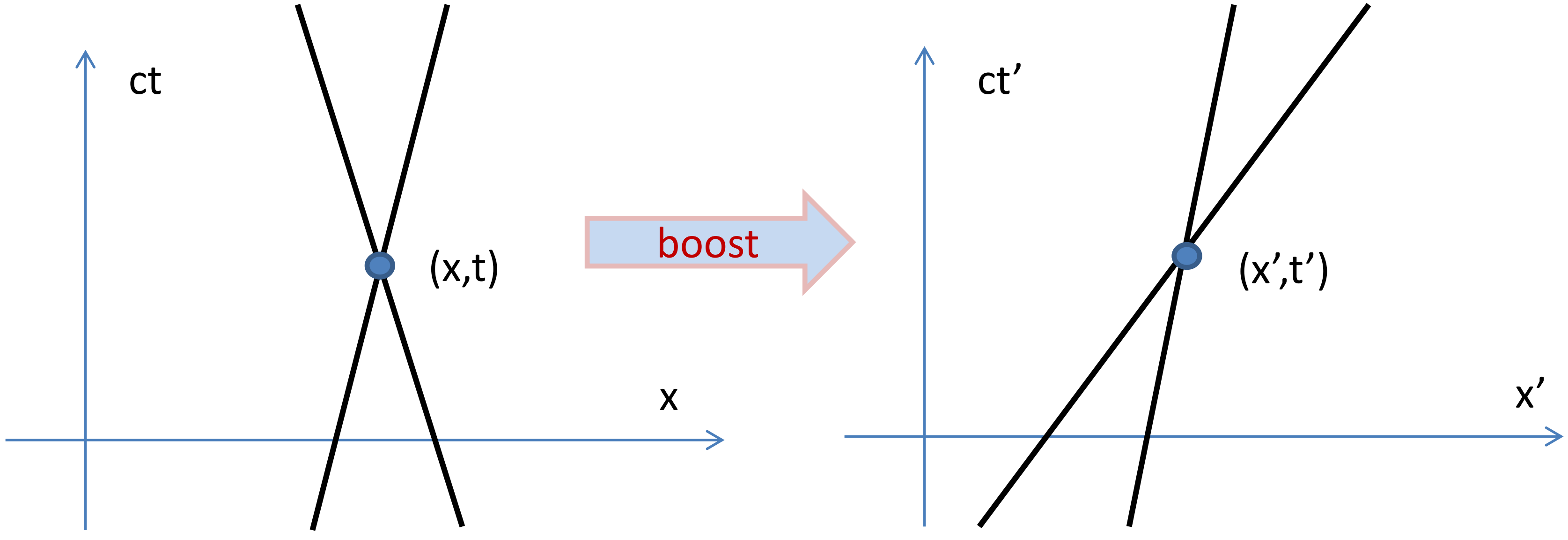

Lorentz transformations for times and positions of events

$$x' = x \cosh\theta - ct \sinh\theta$$
$$t' = -(x/c) \sinh\theta + t \cosh\theta$$

REVIEWS OF MODERN PHYSICS VOLUME 21, NUMBER 3 JULY, 1949

Forms of Relativistic Dynamics

P. A. M. Dirac
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1. Generators of the Poincare Lie algebra should be modified in the presence of interactions (Dirac's instant form)

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2. Interactions V and **Z** must be chosen so that all Poincare commutators are preserved, e.g.,

$$[K_{i0} + Z_i, P_j] = -(i\hbar/c^2)(H_0 + V)\delta_{ij}$$

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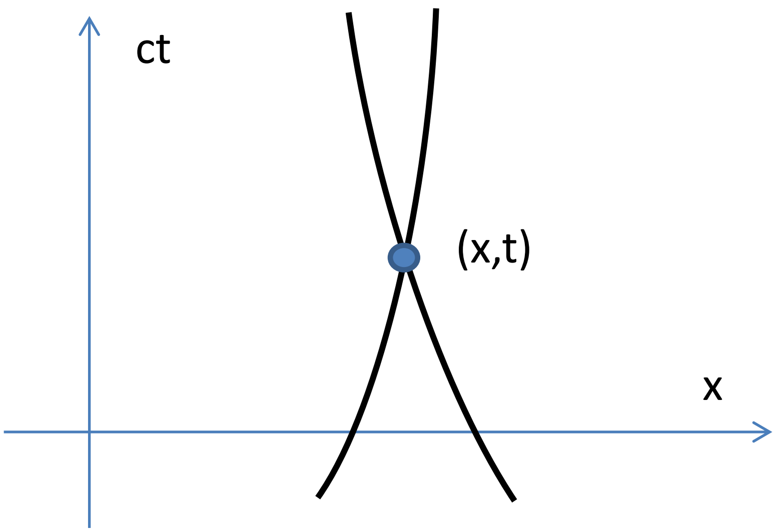

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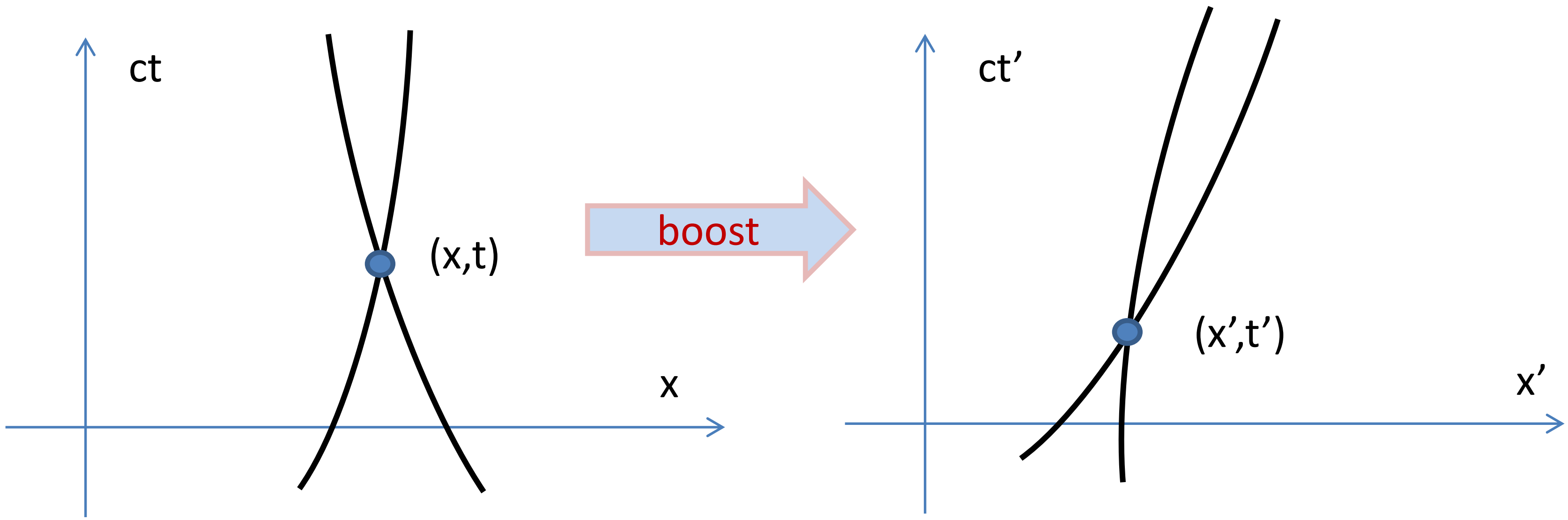

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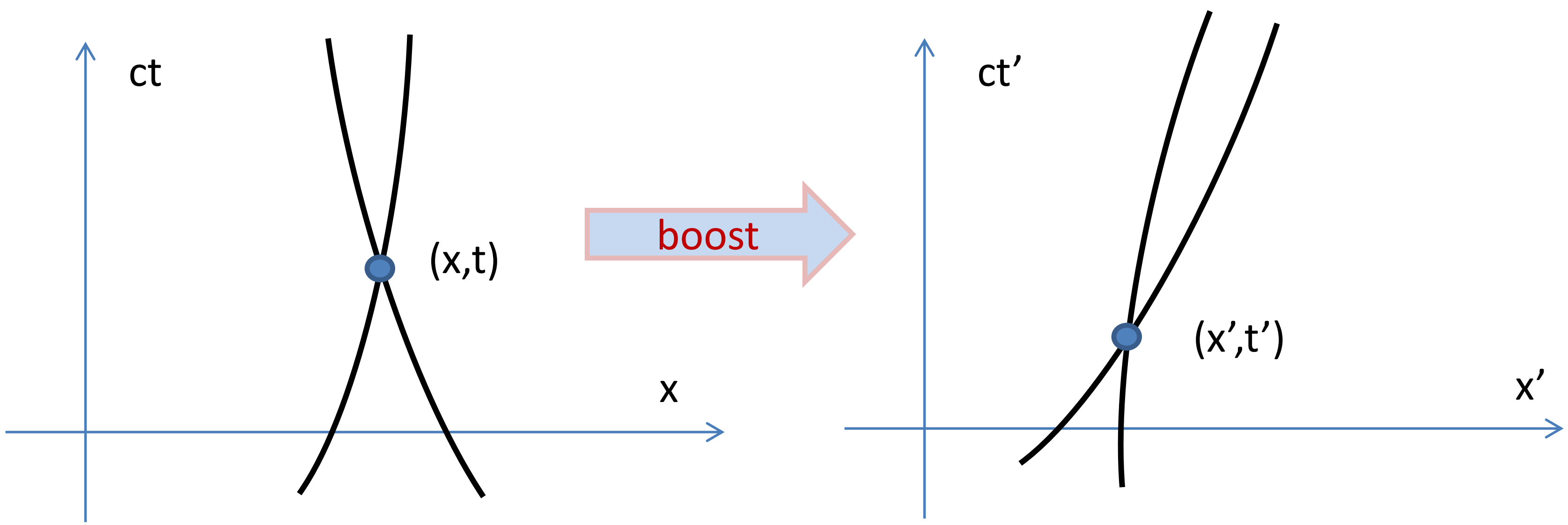

Lorentz transformations are **not** valid

$$x' \neq x \cosh\theta - ct \sinh\theta$$
$$t' \neq -(x/c) \sinh\theta + t \cosh\theta$$

Relativistic Invariance and Hamiltonian Theories of Interacting Particles*

D. G. CURRIE†, T. F. JORDAN, AND E. C. G. SUDARSHAN
Department of Physics and Astronomy, University of Rochester, Rochester, New York

REVIEWS OF MODERN PHYSICS VOLUME 35, NUMBER 2 APRIL 1963

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"...the combined assumptions of Lorentz symmetry and Lorentz transformation of particle positions rule out any interaction."

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"...the combined assumptions of Lorentz symmetry and Lorentz transformation of particle positions rule out any interaction."

How to resolve this paradox?

- reject Lorentz (Poincare) symmetry? No.
- reject Hamiltonian dynamics? No.
- reject existence of particle positions? No.
- reject existence of particles? No.
- **reject Lorentz transformations for positions (and momenta) of interacting particles? Yes!**

Two particles with a fully relativistic interaction in 1D

Shields, Morris, Ware, Su, Stefanovich, Grobe, Phys.Rev. A **82** (2010) 052116

Potential: $\mathbf{U(r) = U_0/[r^2+a^2]^{1/2}}$ introduced via Bakamjian-Thomas prescription

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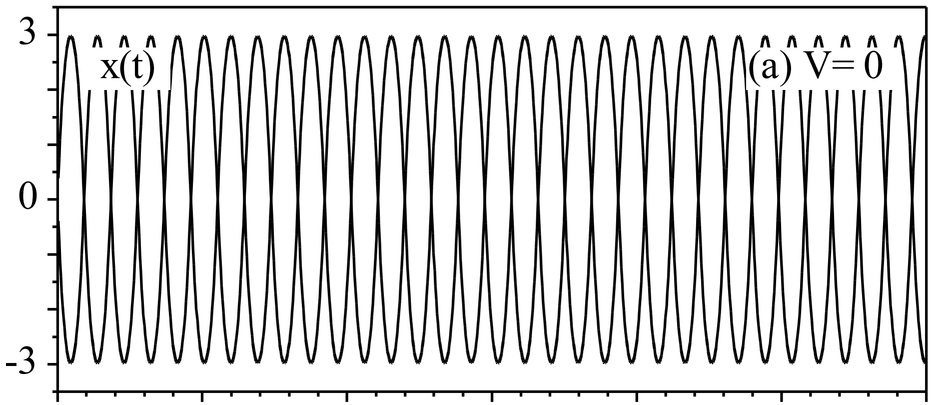

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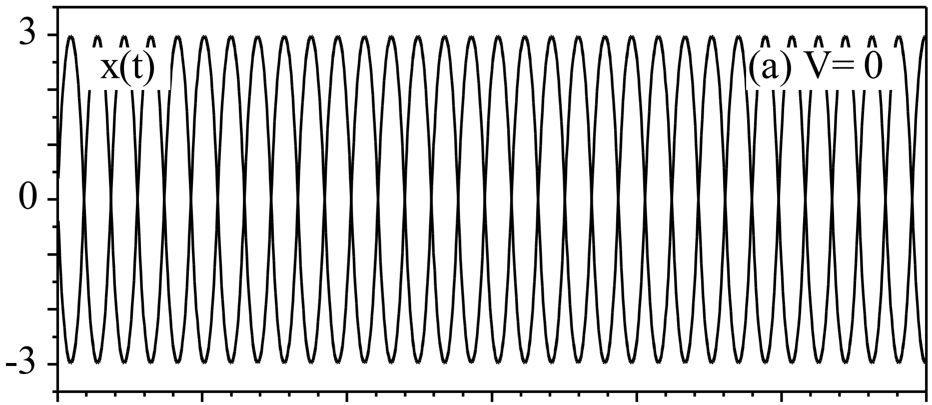

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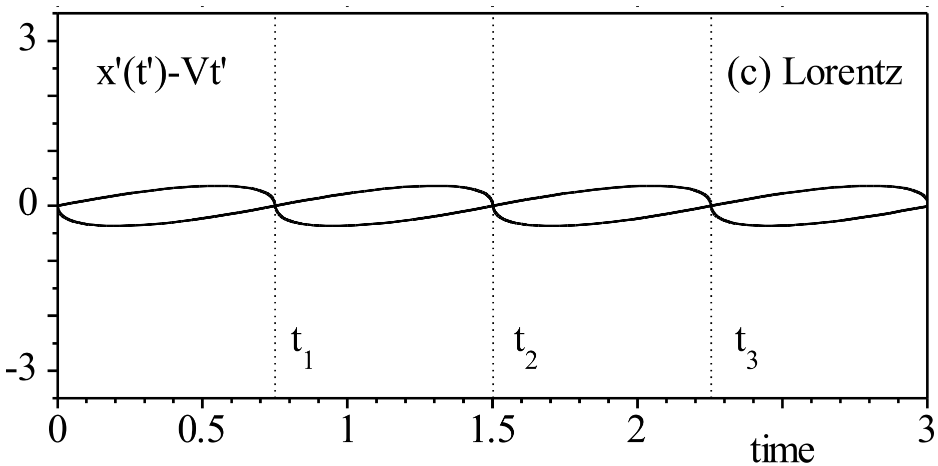

moving frame, Lorentz transformation

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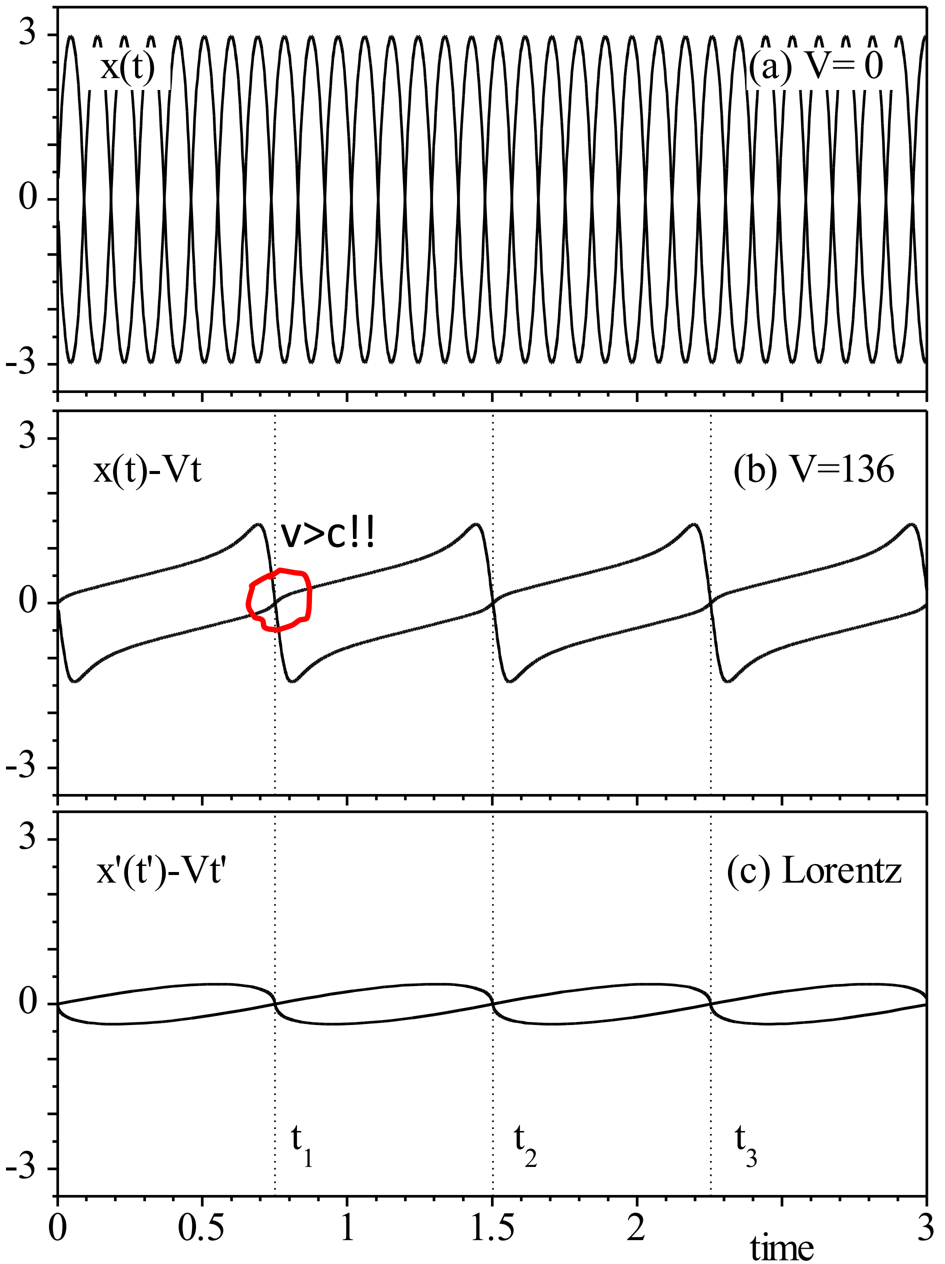

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Principle of causality: the cause **precedes** the effect in all reference frames

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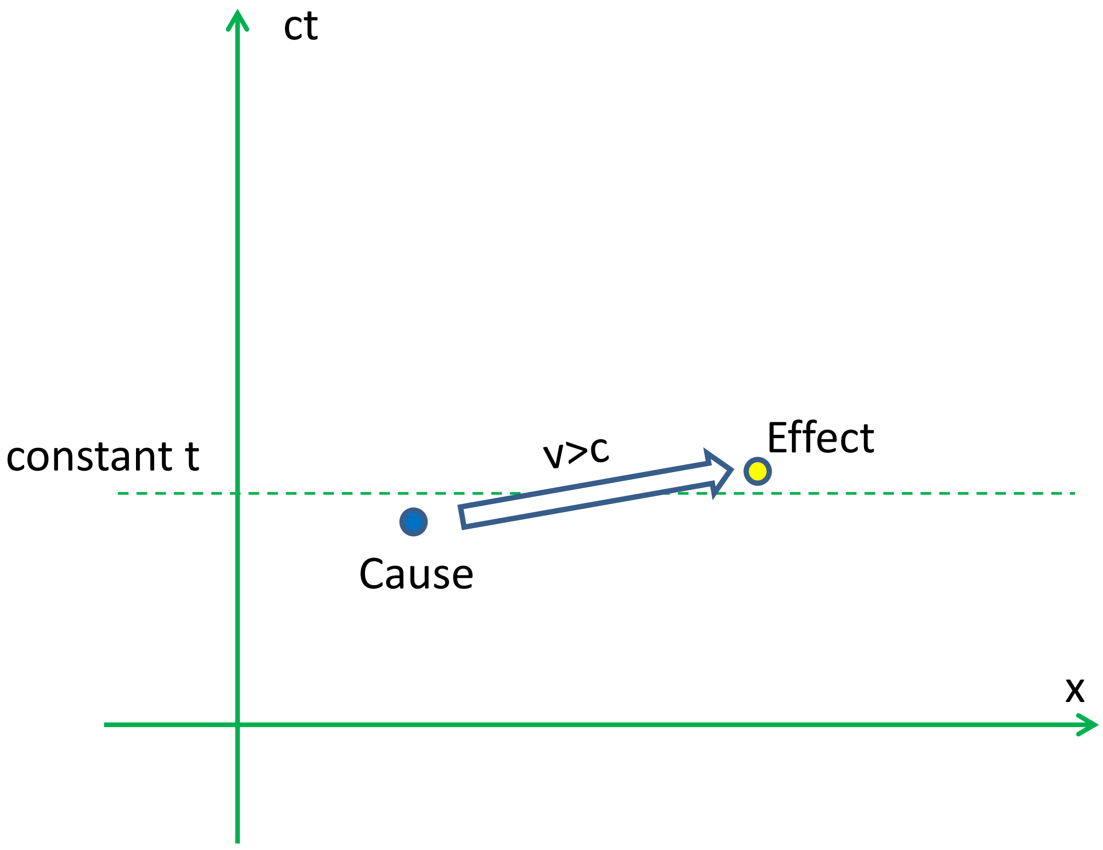

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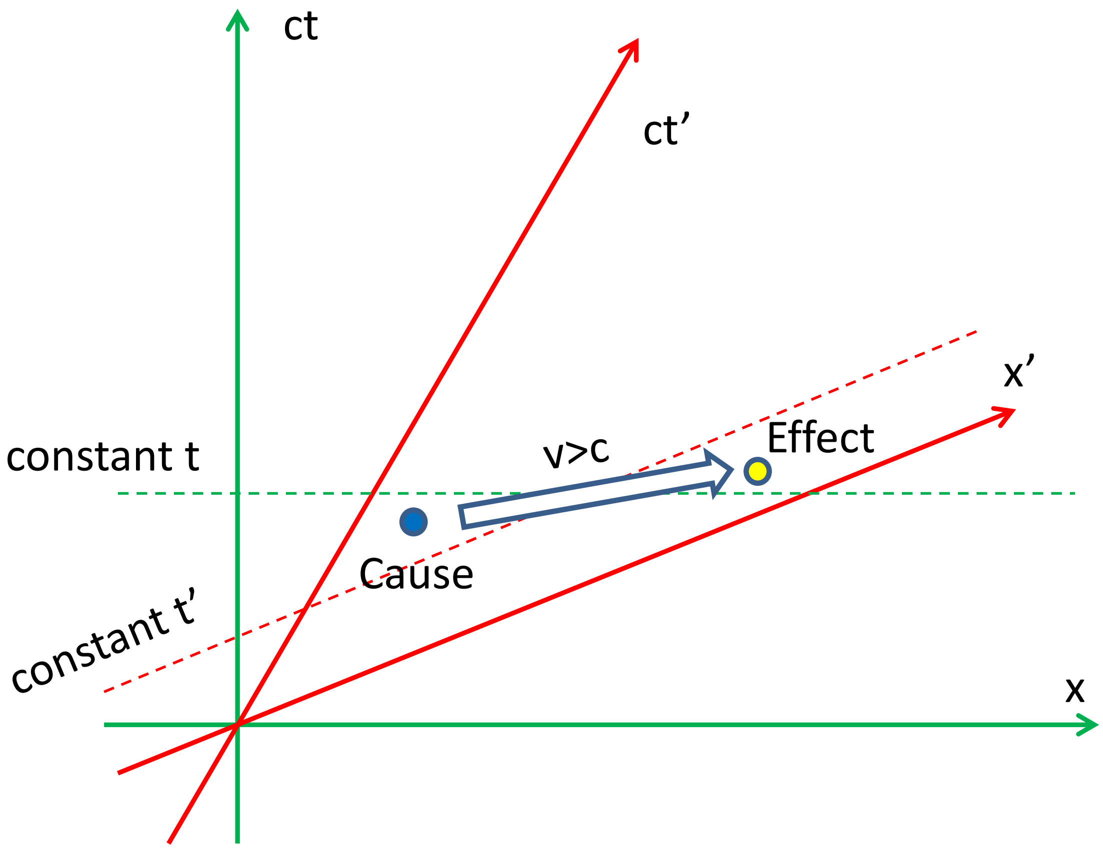

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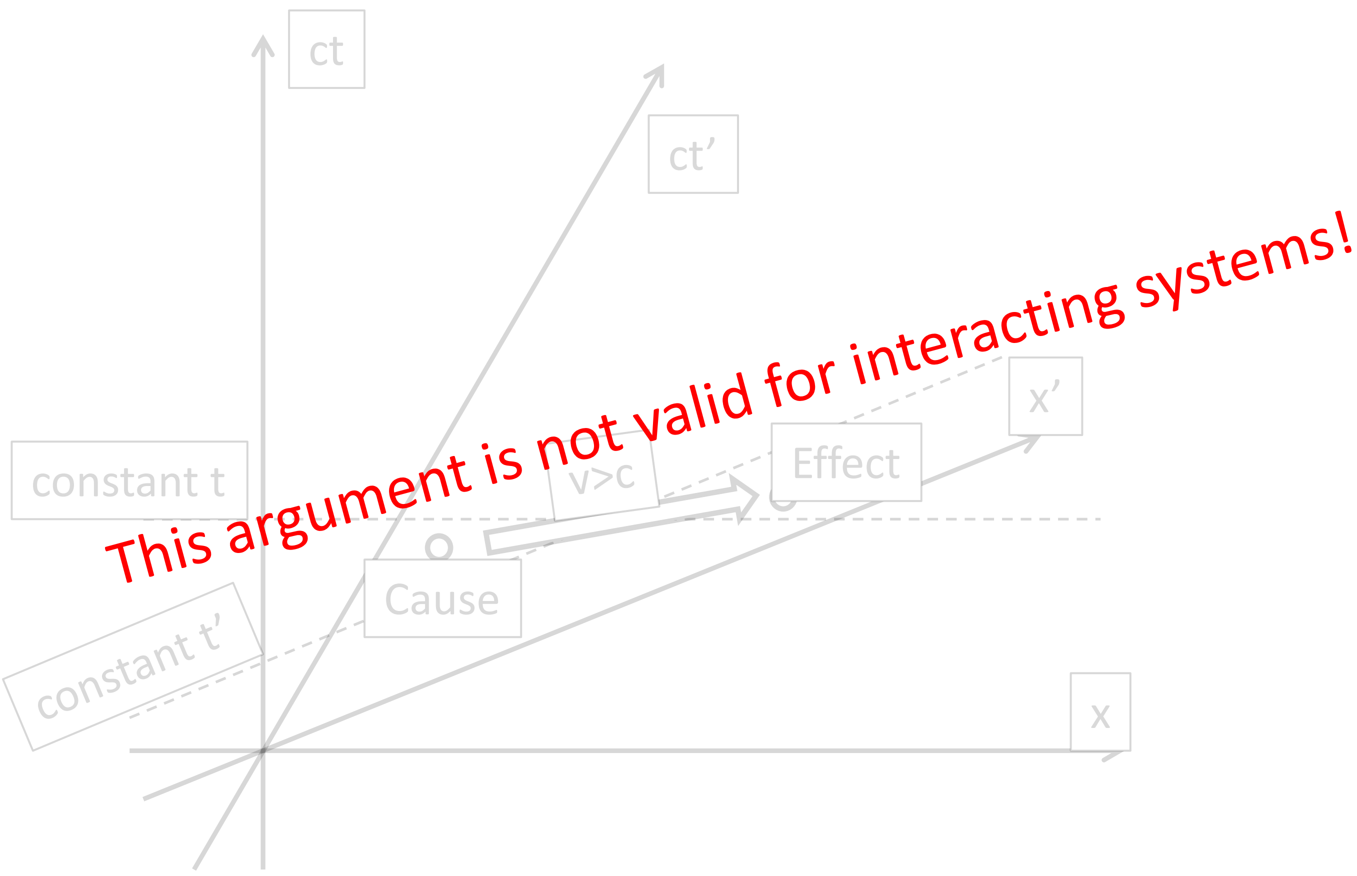

A fully relativistic model of neutrino propagation and oscillation

Stefanovich 2012

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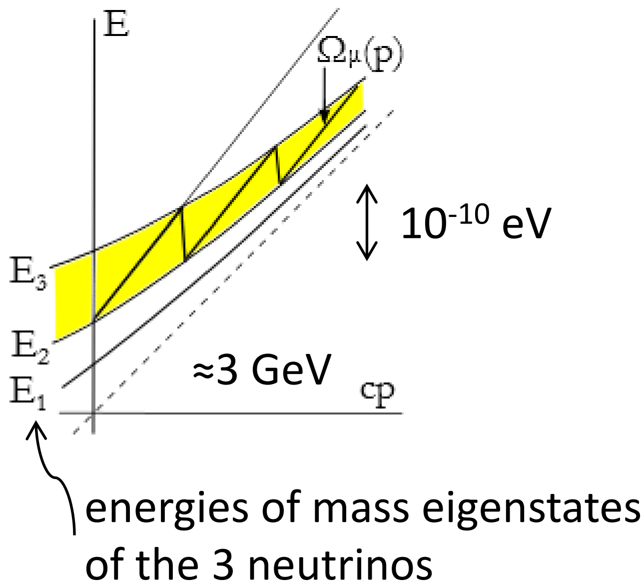

For a general relativistic Hamiltonian in the Hilbert space of the ν_μ-ν_τ system, the diagonal element $\Omega_\mu(p)$ is subject to only one condition:

$E_2(p) < \Omega_\mu(p) < E_3(p)$

$$H = \begin{bmatrix} \Omega_\mu(p) & f(p) \\ f^*(p) & \Omega_\tau(p) \end{bmatrix}$$

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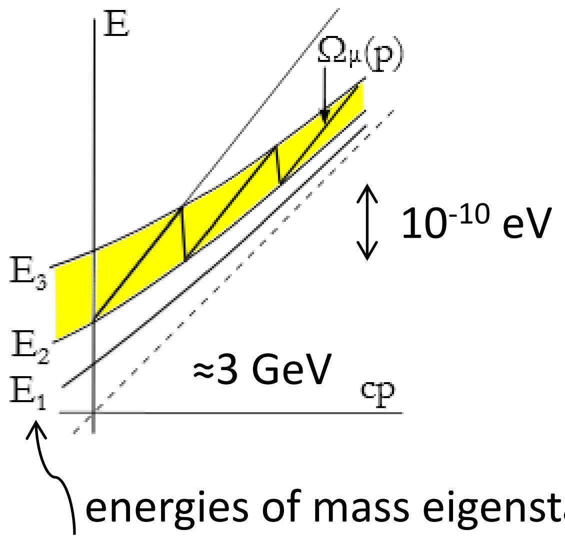

Trajectory of the μ-neutrino component

$\mathbf{L(t) = ct + \Delta L(E,t)}$

$\mathbf{\Delta L(E,t) = (d\Omega_\mu(p)/dp - c)\, g(E,t)}$

g(E,t) is an oscillating function of **t**

assumption: $\mathbf{d\Omega_\mu(p)/dp = 1.0000237c}$

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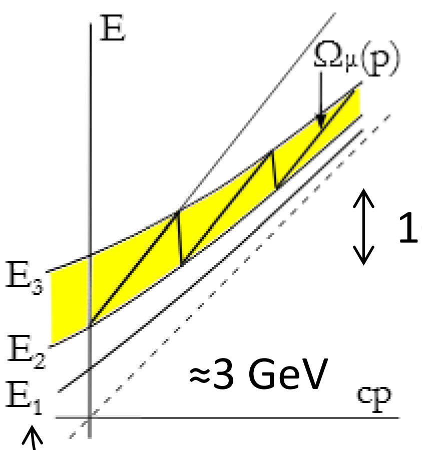
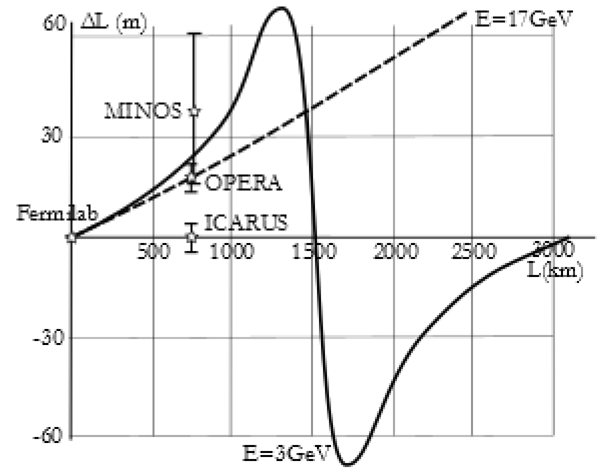


energies of mass eigenstates of the 3 neutrinos



Trajectory of the μ-neutrino component

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at rest	$\omega(0,t) \approx \exp(-\Gamma t)$
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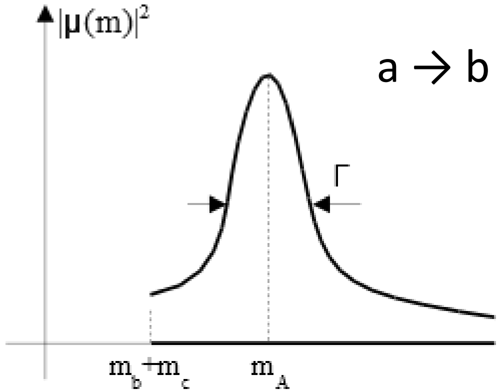

$a \rightarrow b + c$

Properties of the imaginary unstable particle a:
mass m_a = 1000 MeV/c^2
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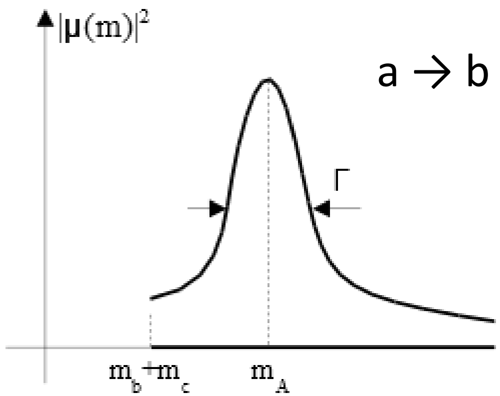

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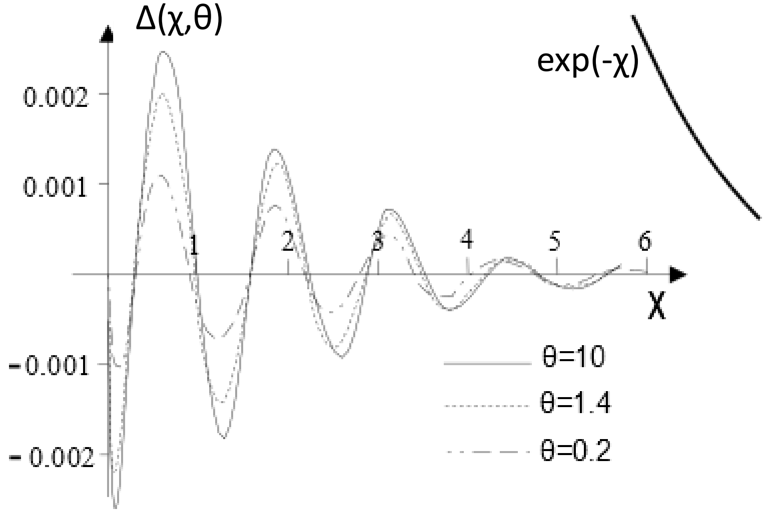

Corrections to Einstein's time dilation formula

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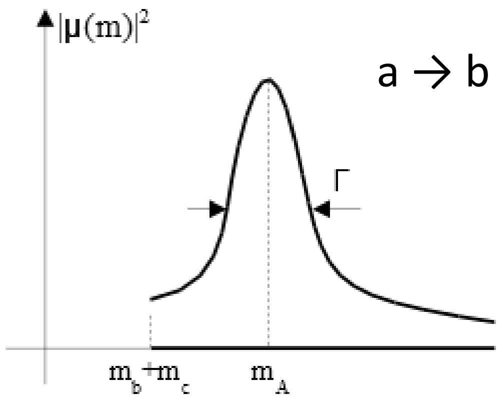

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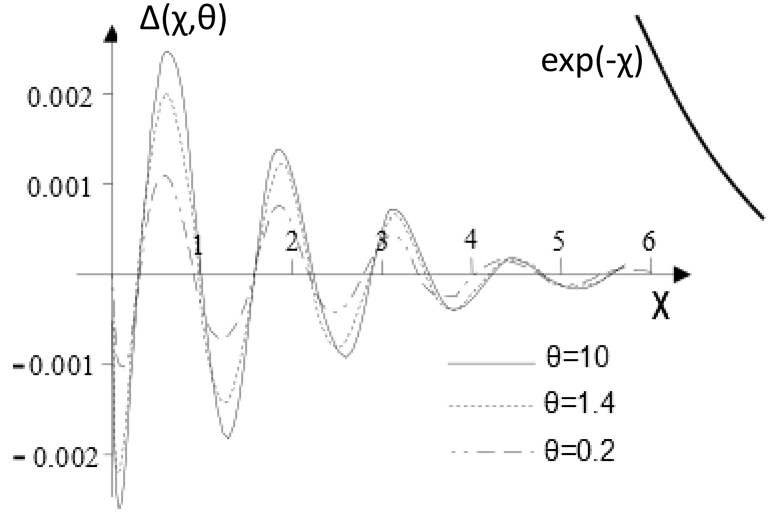

Corrections to Einstein's time dilation formula

The expected effect for muon (≈ 10^{-18}) is too small to be observed. ☹

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3. See, however, arXiv:1211.2913!