"It's hard to imagine that the substance is divisible again and again infinitely, but no less difficult to imagine, that such divisibility with necessity have the end" . Kant's antinomy

LIMITATION OF THE CLASSICAL MECHANICS AND WAYS IT'S EXPANSION

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INTRODUCTION

- All physical theories are built under some restrictions. Our report on how to expand the classical mechanics, when you delete some of its limitations.
- In according to the formalism of classical mechanics, the dynamics of systems are reversible. However, in the nature all processes are irreversible. It is one of the great contradiction between physics and reality, because the classical mechanics, anyway, forms the foundation for all branches of physics.
- The classical mechanics was constructed on the basis of the bodies model in the form of the material points (MP). Only for these idealization the Newton's laws is valid. Basing on the Newton's motion equation for MP, the Lagrange and Hamilton formalisms for the systems dynamics were created. But all bodies in the nature are the non-equilibrium systems which in motion in the space. Their dynamics are determined by the interaction of the system parts between each other and its interaction with the space. So there are two type of symmetry which the systems dynamics are determined: the internal bodies symmetry and the space symmetry. **We will call it as "a Principle of dualism of the symmetry" (PDS).** Thus only for the systems but not for structureless bodies, the PDS can be take into account. But only using the PDS, to construct the dissipative mechanics is possible.

This report is about as the classical mechanics expands based on the PDS and by replacing of MP on the structured particles (SP) where **SP is equilibrium system from a great number of potentially interacting MP.**

In this report the next questions will discussed:

- How SP mechanics can be constructed based on PDS and the Newton laws for MP;
- What are the properties of SP dynamics;
- Why and how irreversibility was lost in classical mechanics;
- How the concept of entropy appears in the classical mechanics due to the PDS;
- What is the nature of the break-symmetry of the time for SP mechanics;
- How the thermodynamics, statistical physics and kinetics based on the SP mechanics can be constructed.
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The SP mechanics will constructed in the frame of the next restrictions:

- 1) SP is a system consisting from a great number of potentially interacting MP.
- 2) All SP are considered to be equilibrium during motion.
- 3) Each MP is connected with a certain SP.
- In connection with PDS the motion equation of SP is determined by the energy which should be divided on two parts: the motion energy and the internal energy. The motion energy of the SP is determined by the velocity and coordinate of the center of mass (CM). The internal energy is determined by the motions of each MP into SP in relative to the CM.
- There are two independent groups of variables which determine the SP motion in connection with PDS. Each of these group is a different groups of the symmetry. Coordinates and velocities of the MP relative to the CM of SP are micro-variables. The coordinates and velocity of the SP are macro-variables.

The Newton's motion equation for one MP

$$E = T + U = const. \qquad dE/dt = 0. \;\; \vec{v}(m\dot{\vec{v}} + \partial U/\partial r) = 0.$$

$$m\dot{\vec{v}} = -\partial U/\partial r$$

The Newton's motion equation for one MP from SP

$$T = \sum_{i=1}^{N} mv_i^2/2; \; U_N(r_{ij}) = \sum_{i=1}^{N-1}\sum_{j=i+1}^{N} U_{ij}(r_{ij}); \quad F_i^{env}(r_i) = \partial U^{env}/\partial r_i \;;$$

$$\sum_{i=1}^{N} v_i(m\dot{v}_i + \sum_{j\neq i}^{N} F_{ij} + F_i^{env}) = 0; \quad m\dot{v}_i = -\sum_{j\neq i}^{N} F_{ij} - F_i^{env}$$

How to obtain the motion equation for SP

$$\sum_{i=1}^{N} v_i^2/2 = NV^2/2 + [\sum_{i=1}^{N-1}\sum_{j=i+1}^{N} v_{ij}^2]/(2N);$$

$$M_N = Nm; v_i = V_N + \tilde{v}_i; \sum_{i=1}^{N} \tilde{v}_i = 0; v_{ij} = v_i - v_j = \tilde{v}_i - \tilde{v}_j; \dot{R}_N = V_N = \sum_{i=1}^{N} v_i / N;$$

$$T_N = [M_N V_N^2 + m/N \sum_{i=1}^{N-1}\sum_{j=i+1}^{N} v_{ij}^2]/2 = M_N V_N^2/2 + \sum_{i=1}^{N} m\tilde{v}_i^2/2;$$

$$where \rightarrow r_i, v_i - micro\,\mathrm{var}\,iables; R_N, V_N - macro\,\mathrm{var}\,iables.$$

$$E_N = E_N^{tr} + E_N^{ins},$$

$$where \rightarrow E_N^{ins} = T_N^{ins} + U_N^{\mathrm{int}}, E_N^{tr} = T_N^{tr} + U^{env}$$

The law of energy conservation for SP is:

The sum of the internal energy and the motion energy remains constant.

This energy laws connected with the theorem according which the sum of the velocity vectors in square is equal the sum of the square of the sum of the MP velocities vector and sum of the square of the MP velocities vector difference.

Differentiating the SP energy with respect to the time, we obtain the equation of the energy exchange between different types of energy (between two groups of symmetry).

[Journal of Physics. 2005; New Adv. in Phys. V. 2, No 2, 2008].

$$V_N M_N \dot{V}_N + \dot{E}_N^{ins} = -V_N F^{env} - \Phi^{env} \qquad (1)$$

$$\dot{E}_N^{ins} = \sum_{i=1}^{N} \tilde{v}_i (m\dot{\tilde{v}}_i + F(\tilde{r}_i)_i); \quad \Phi^{env} = \sum_{i=1}^{N} \tilde{v}_i\, F_i^{env}(R_N, \tilde{r}_i); \quad F^{env} = \sum_{i=1}^{N} F_i^{env}(R_N, \tilde{r}_i);$$

Let us take a case: $R >> \tilde{r}_i \forall i;\ F^{env} = F^{env}(R + \tilde{r}_i); \quad F_i^{env} \approx F_i^{env}\Big|_R + (\tilde{r}_i \cdot \nabla) F_i^{env}\Big|_R$

$$\sum_{i=1}^{N} \tilde{v}_i = \sum_{i=1}^{N} \tilde{r}_i = 0; \qquad \sum_{i=1}^{N} F_{i0}^{env} = N F_{i0}^{env} = F_0^{env}(R);$$

Then we will have:

$$V_N M_N \dot{V}_N + \dot{E}_N^{ins} \approx -V_N F_0^{env}(R) - \sum_{i=1}^{N} (\tilde{r}_i \cdot \nabla) F_i^{env}\Big|_R \tilde{v}_i \qquad (2)$$

Thus the change of the internal energy will exist when the characteristic scale of inhomogeneities of the external field of forces is commensurable with the SP scale.

From the energy flows equation we come to the SP motion equation taking into account that the work of external forces goes to the change both the energy of motion and the internal energy.

Motion equation of the SP

$$M_N \dot{V}_N = -F^{env} - \alpha_N V_N, \ldots (3)$$

where $\alpha_N = (\Phi^{env} + \dot{E}_N^{ins}) / V_N^2$.

The coefficient α_N is determined the internal energy changes.

The area of application of the SP motion equation is determined by the next inequality

$$F_0^{inv} \square \sqrt{\frac{mkT}{2}} \left(\frac{R_F}{r_{sp}} \right)$$

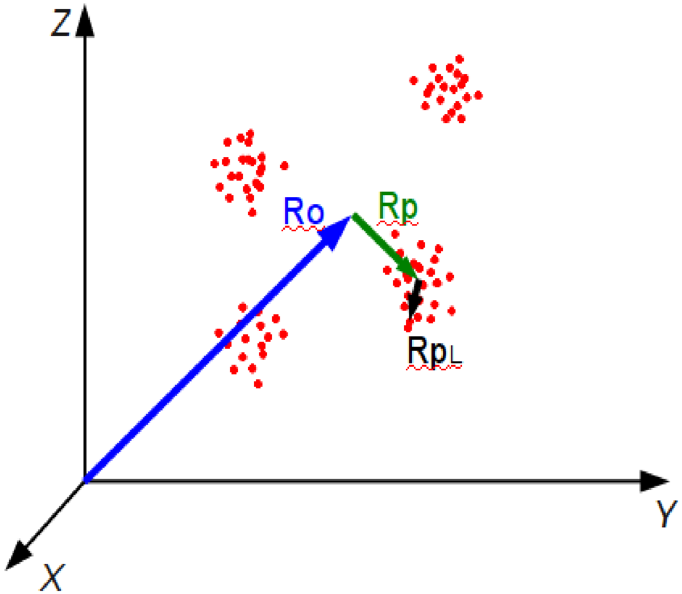
If this inequality have not a place, the SP is non-equilibrium and can be submitted as a set of equilibrium subsystems with the relative velocities. In this case we must use motion equation for each of these equilibrium subsystems taking into account the energy of their interaction.

- **There are basic differences of SP dynamics from dynamics of MP**
- The MP motion is determined by the point in the space. But SP have a scale. Therefore the SP motion can depend on from the gradient of the external forces.
- The MP motion is determined by the potential external forces. But SP motion is determined by the potential external forces and its gradients. These leads to the change of two types of energy: the motion energy and its internal energy.

All bodies in general are non-equilibrium systems.

The non-equilibrium systems in the local thermodynamic equilibrium approach can be submitted as a set of SP. Therefore for the description its dynamics it is necessary to used the equation for interacting SP.

Ro* –** coordinates of the center of mass of the nonequilibrium system, ***Rp – coordinates of SP relative to the center of mass ***Ro***, ***RpL*** – coordinates of MP relative center of mass of SP- ***Rp***.



$$E = M_N V_N^2/2 + \sum_{p=1}^{K} M_p V_p^2/2 + \sum_{p=1}^{K}\{\sum_{l=1}^{Lp} m\, v_{pl}^2/2\} + \sum_{P=1}^{K} U_P + \sum_{P_I=1}^{K-1}\sum_{P_J=1}^{K} U_{P_I,P_J} + U_N^{env} \quad (4)$$

We can understand the dynamics properties of non-equilibrium system on the example for two interacting SP

[*Soms. Journal of Phys. 23, 2005*].

$$V_L M_L \dot{V}_L + \dot{E}_L^{ins} = -\Phi_L - V_L \Psi \quad (5)$$

$$V_K M_K \dot{V}_K + \dot{E}_K^{ins} = \Phi_K + V_K \Psi \quad (6)$$

$$\dot{E}_L^{ins} = \sum_{i_L=1}^{L-1} \sum_{j_L=i_L+1}^{L} v_{i_L j_L} [m\dot{v}_{i_L j_L} / L + F_{i_L j_L}]; \; \dot{E}_K^{ins} = \sum_{i_K=1}^{K-1} \sum_{j_K=i_K+1}^{K} v_{i_K j_K} [m\dot{v}_{i_K j_K} / K + F_{i_K j_K}]$$

$$F_{i_L}^{K}(R_K, r_{i_L}) = \sum_{j_K=1}^{K} F_{i_L j_k}; \; F_{j_K}^{L}(R_L, r_{i_K}) = \sum_{i_L=1}^{L} F_{i_L j_k}; \quad R_K = (1/K)\sum_{i_K=1}^{K} r_{i_K}; R_L = (1/L)\sum_{i_L=1}^{L} r_{i_L};$$

$$\Psi = \sum_{i_L=1}^{L} F_{i_L}^{K}; \; \Phi_L = \sum_{i_L=1}^{L} \tilde{v}_{i_L} F_{i_L}^{K}; \; \Phi_K = \sum_{i_K}^{K} \tilde{v}_{j_K} F_{j_K}^{L}; \; V_L = \dot{R}_L; \; V_K = \dot{R}_K; Lm = M_L; \; Km = M_K.$$

The forces which transform the SP motion energy into their internal energy are nonlinear.

Motion equations for two SP

$$M_L \dot{\vec{V}}_L = -\vec{\Psi} - \alpha_L \vec{V}_L \ (7a)$$

$$M_K \dot{\vec{V}}_K = \vec{\Psi} + \alpha_K \vec{V}_K (7b)$$

$$\alpha_L = (\dot{E}_L^{ins} + \Phi_L)/V_L^2; \quad \alpha_K = (\Phi_K - \dot{E}_K^{ins})/V_K^2; \quad \dot{E}_L^{ins} = \sum_{i_L=1}^{L} \tilde{v}_{i_L} (m\dot{\tilde{v}}_{i_L} + F_{i_L});$$

$\dot{E}_K^{ins} = \sum_{i_K=1}^{K} \tilde{v}_{i_K} (m\dot{\tilde{v}}_{i_K} + F_{i_K})$. α_L, α_K - friction coefficients.

These equations determine the analytical form of the friction coefficients from the Newton’s laws for interacting SP.

The irreversibility in the dynamics of the non-equilibrium system is due to impossibility transition of an internal energy into the energy of SP relative motion. It is due to the law of conservation of momentum according to which the momentum of SP can’t changes by the work of the internal forces.

The Lagrange, Hamilton and Liouville equations for SP

- The Lagrange, Hamilton and Liouville equations which convenient for analyses of the nonequilibrium system can be derived based on the SP motion equation and D'Alembert equation by the ordinary way. The Liouville equation has a form [*Soms. IJBC, 2004, V 14, N11*]:

$$df/dt = -f\sum_{L=1}^{R} \partial F_L / \partial V_L \quad (8)$$

Here f is a distribution function for a set of SP, F_L is collective forces acting on the SP, V_L is the velocity of the L-SP. The right-hand side of eq.(8) is deterministic.

S-phase space for SP-systems.

The motion of the SP is defined by coordinates and velocities of its CM. Therefore the state of the non-equilibrium system from a set of SP will be defined in the phase space which consists from 6R-1 coordinates and momentums of SP, where R is the number of SP. We will call this space as S-phase space in order to distinguish it from ordinary phase space for MP. The S-phase space is compressible. It caused by the transformation of the energy of relative motions of SP into their internal energy.

The geometry of SP dynamics

(Somsikov V.M. arXiv:1006.3158v1 [physics.class-ph] 16 June. 2010)

Interval for MP

$$d\overline{s}^2 = 2T_N dt^2 = \sum_{i=1}^{N} \breve{v}_i^2 dt^2 = \sum_{i=1}^{N} (d\breve{x}_i^2 + d\breve{y}_i^2 + d\breve{z}_i^2)$$

$d\overline{s}$ -is a linear element or interval, $\breve{x}_i = \sqrt{m_i}\, x_i, \breve{y}_i = \sqrt{m_i}\, y_i, \breve{z}_i = \sqrt{m_i}\, z_i$

Interval for SP

$$d\overline{s}^2 = (2T^{tr} + 2T^{ins})dt^2 = ds_{tr}^2 + ds_{ins}^2 = N\breve{V}_0^2 dt^2 + (\sum_{i=1}^{N-1}\sum_{j=i+1}^{N} \breve{v}_{ij}^2)dt^2 / N$$

$$\breve{V}_0 = (\sum_{i=1}^{N} \breve{v}_i)/N, \breve{v}_{ij} = \breve{v}_i - \breve{v}_j$$

The geometry of SP dynamics is defined by the two type of symmetry:

1. Space symmetry,

2. Internal symmetry of SP.

How irreversibility was lost in classical mechanics

A Lagrange equation is obtained from the principle of d'Alembert.

$$\sum_{l=1}^{R}\left[F_i - \dot{p}_i\right]\delta r_l = 0 \ldots (9)$$

Here F_i -is active force which acted on i-th MP of the system; $\dot{p}_i$-is inertial force from i-th MP; δr_i -is a virtual displacement for i-th MP; $i = 1, 2 \ldots R$-is a number of MP in a system.

To obtain Lagrange equation it is necessary go to the independent generalized variables. After completing the necessary transformations we will have:

$$\delta w = \sum_{l=1}^{R}\left\{\left[\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_l}\right) - \frac{\partial T}{\partial q_l}\right] - Q_l\right\}\delta q_l = 0 \ldots (10)$$

Here t is time; T - is a kinetic energy for all MP; q_l are the general independents variables; δq_l - are the virtual displacements; Q_l -are the external forces.

In order to obtain the canonical Lagrange equation, the hypothesis about holonomicity constraints is used. This hypothesis implies that each term in the sum of eq.(10) is independent and equal to zero. **But the abovementioned condition is valid only when the external forces do not have non-linear terms which are dependent from the several variables. Only in this case we will have :**

$$\left[\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_l} \right) - \frac{\partial T}{\partial q_l} \right] - Q_l = 0........(11)$$

If we have the next condition ,

$$Q_l = -\sum_i \nabla_i V \frac{\partial r_i}{\partial q_l}.....................(12)$$

then we obtain:

$$\left[\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_l} \right) - \frac{\partial L}{\partial q_l} \right] = 0...where...L = T - V...(13)$$

where '*L*' is a Lagrange function.

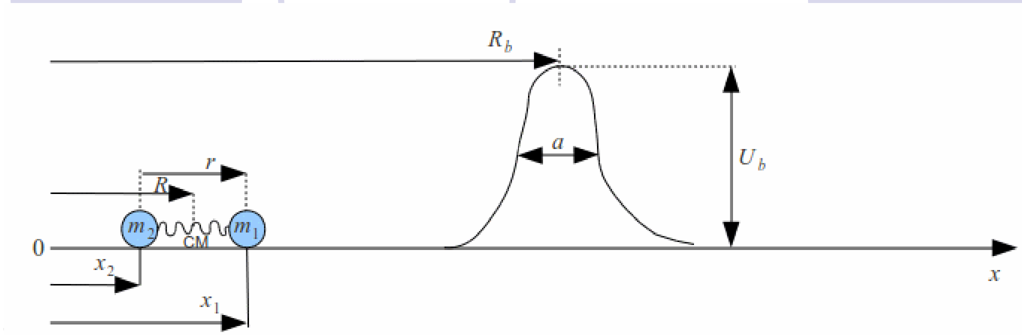
It is a Lagrange equation.

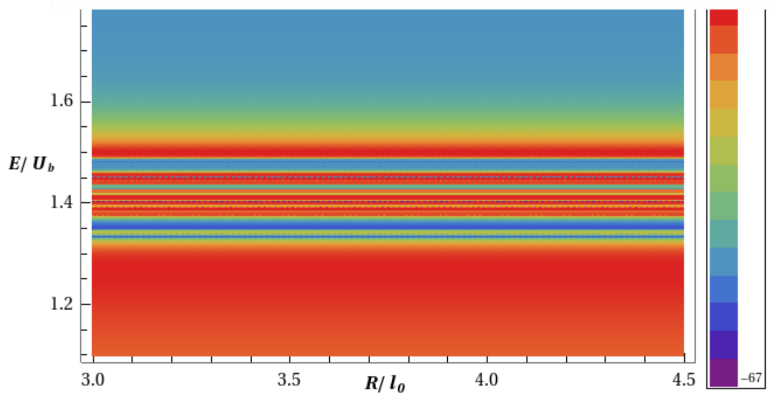
The requirement of holonomicity constraints is equivalent of the absence of the non-linear terms in the motion equation SP. But these terms determines the change of the internal energy. Indeed, the interdependence of the motion energy and internal energy can only be ensured by the nonlinear terms. Thus in the presence of nonlinearity at least two terms in the eq. (10) are different from zero. In this case, the requirement holonomicity of constraints is not performed.

These statements can be verified by the example of the motion of the oscillator through a potential barrier.

Numerical experiment for check of the holinomicity constraint

Somsikov, Denisnya. Peculiarities of passage of an oscillator through a potential barrier. Russ. Phys. Journ. Sept.2013, V. 56, **4;**
Somsikov,. Mokhnatkin, Non-Linear Forces and Irreversibility Problem in Classical Mechanics, *Journ. of Modern Phys.*, V.5 No.1, 2014;





$$M\ddot{R}(t) = F(1) + F(2),$$

$$\mu\ddot{r}(t) = F_1^i(r) + \frac{\mu}{m}\left[F(1) - F(2)\right]$$

If to consider the oscillator (OS) motion from the standpoint of the PDS, then we find that the OS passage through the barrier is possible even when the motion energy less than the barrier height but only if the total energy more than the height of the barrier. This passage have a place at the appropriate phase when the internal energy is converted into the motion energy due to the nonlinear transformation.

Thus, the hypothesis of holonomic constraints preclude nonlinear terms which responsible for the conversion between the energy of motion and internal energy of the OS, and the effect of transition OS through barrier is lost.

Restrictions of the Lagrange formalism

- The hypothesis about holonomicity of constraints limits the scope of formalism of classical mechanics by the reversible, non-dissipative processes which have a place only for systems close to equilibrium. In the practice this problem is overcome by adding empirical terms into the right-hand side of the Lagrange equation. These terms can be find in according with the non-holonomic constraints by using the method of Lagrange multipliers. But there are two problems.
- 1. The analytic form of the nonholonomic constraints are not always easy to find (for example, in the case of systems interaction).
- 2. The formalisms of the classical mechanics are used for the building of the foundations of the theory of statistical physics and kinetics. In this case it is very difficult to trace when and how to take into account the restriction of formalism of classical mechanics and to what consequences they may lead.

Similarities and differences between probabilistic irreversibility for Hamiltonian systems and deterministic irreversibility for SP

- In the contemporary explanation, the irreversibility in the Hamiltonian systems is appeared due to the fluctuations existence. But the presence of fluctuation is a sufficient condition for the irreversibility, but not necessary.
- The explanation of the deterministic irreversibility of SP has appeared in connection with the PDS. This explanation connected with that that the internal energy can change only due to the non-linear transformation of the SP motion energy into internal energy. Such transformation is possible in the presence of inhomogeneities of the external field of forces. This is due to the appearing of the interdependence of the variables between two groups of systems symmetry which in these forces.
- The nonlinear terms in the SP motion equations leads to breakdown of the invariance each of the two types of energy. The sum of these energies is invariant.

The SP mechanics, thermodynamics, stat.physics and kinetics

(*New Advances in Physics, Vol. 2, No 2, Sept. 2008, pp. 125-140*)

In the frame of the thermodynamics the work of the forces acting on the system is divided on two parts. The first part is connected with the reversible work. The change of the motion energy of system as a whole can be put in conformity for this part. The second part of the work go on the heating. The change of the internal energy of system corresponds with this part.

Dynamical Entropy

$$dS^d = dE^{\text{int}} / E^{\text{int}} = \sum_{L=1}^{R} \left\{ N_L \sum_{k=1}^{N_L} [\int \sum_s F_{ks}^L v_k dt] / E_L \right\} \rightarrow dQ / Q \Big|_{N\rightarrow\infty} \quad (14)$$

E_L -entrance energy for L - SP; R -is a number of SP; v_k -is velocity of k-element;
F_{ks}^L - is a force acting on k element of SP from the side of S - entrance element.

This entropy is a deterministic. It is characterizing relative change of the systems internal energy. Such entropy definition convenient for different number of MP in the systems and can be as positive as negative. Only for a great number of MP (for SP) it is corresponds to the empirical Clausius and Boltzmann definitions of entropy.

How statistics is appearing in SP mechanics

$$V_N = (\sum_{i=1}^{N} v_i) / N; \quad \sum_{i=1}^{N} \tilde{v}_i = 0; v_i = V_N + \tilde{v}_i; T_N = M_N V_N^2 / 2; \alpha_N = (\Phi^{env} + \dot{E}_N^{ins}) / V_N^2.$$

Сомсиков В.М., Андреев А.Б. Особенности перехода к термодинамическому описанию от динамического описания структурированных частиц. ПЭОС .т. 1. 2014 .

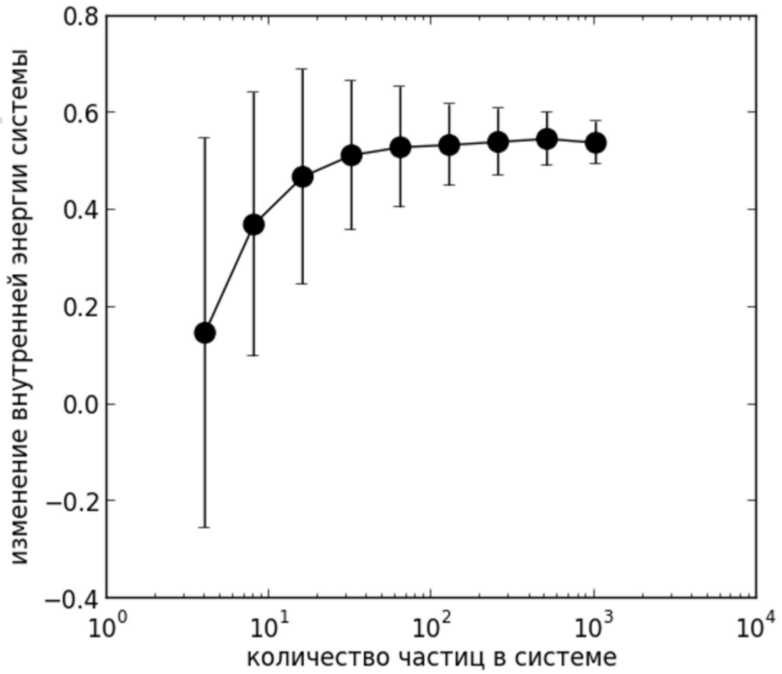


Fig. 3. Dependence of the relative value of the internal energy from the number of MP.

Conclusion

- Because all bodies have a structure their dynamics are determined by the PDS.
- In accordance with PDS the bodies motion are determined by the internal energy and the motion energy. Their motion invariant is a sum of these two energies. The dualism of the energy leads to the dualism of the forces. The SP motion equation follows from dual expression of energy.
- The change of the internal energy is possible when the characteristic scale of inhomogeneity of the external forces commensurate with the characteristic scale of SP. This change is determined by the nonlinear terms. The presence of the nonlinear terms leads to the break-symmetry of the time.
- The dissipative processes of the motion energy are going due to transformation of its into the SP internal energy. This defines a mechanism of equilibration and irreversibility.
- The concept of dynamical entropy as the ratio of change of the internal energies to the volume of the internal energy is appeared. In connection with the number of MP it can be as positive as negative. For SP the dynamical entropy is similar to the Clausius and Boltzmann entropy.

- The unacceptability of the canonical formalisms to describe the dissipative processes is connected with the facts that they are built without taking into account the PDS and by using the hypothesis of holonomic constraints. This hypothesis excludes the non-linear terms which are responsible to the transformation of the motion energy of the bodies into their internal energy.
- Because the dissipation is impossible for the structureless bodies, the formation of structures is impossible without dissipative processes. From here followed that the infinite divisibility of substance has a place. Thus, the Newton's laws, hierarchy of the fundamental forces leads to that the substance is a hierarchy of systems. Thus the problem of many-body dynamics, particle physics, etc. should be solved basing on the PDS.
- All branches of physics have emerged on the basis of the canonical formalism of classical mechanics. But they are unacceptable to the description of nonequilibrium processes of creation and annihilation of the systems. Therefore, there are a lot of difficult questions about how to change some branches of physics in connection with the need to use the PDS.

A little philosophy. Dualism of the world is in agreement with the Newton's laws!

If the evolution of systems is governed by the universal laws and we have the infinite divisibility of matter than we must have the fractality of the matter (see Figure, in which the author allowed himself to make some amendments to the traditional sign yin and yang, and will display the fractal self-similarity of the world).

The process of understanding the world like a settling of the fog in the mountains. When the fog in our understanding settles, the new heights of knowledge and the ridges connecting these heights begin to show .

Thank you very match for attention!