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# Polarazed nonsinglet $\Delta q_3(x)$ and nonsinglet fragmentation function $D_{u_v}^{\pi^+}(z)$ in the analytic approach to QCD

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# Outline

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- Overview of theoretical framework:
  APT/FAPT
  method (Jacobi polynomial expansion)
- QCD analysis of nonsinglet combination $x\Delta q_3(x,Q^2)$
- QCD analysis of nonsinglet fragmentation function $D_{u_V}^{\pi^+}(z,Q^2)$
- Summary

Polarized inclusive deep inelastic lepton-nucleon (DIS) scattering

$$\ell + p \to \ell' + X$$

Non-Singlet (NS) combination of polarized quark densities

$$x\Delta q_3(x,Q^2) = [x\,\Delta u(x,Q^2) - x\,\Delta\bar{u}(x,Q^2)] - $$
$$-[x\,\Delta d(x,Q^2) - x\,\Delta\bar{d}(x,Q^2)]$$

Semi-inclusive lepton-nucleon process (SIDIS)

$$\ell + p \to \ell' + h + X$$

$$D_{u_V}^{\pi^+}(z) \equiv D_u^{\pi^+}(z) - D_{\bar{u}}^{\pi^+}(z)$$

NS valence quark FF — the difference of pion fragmentation functions (FFs)

In our consideration we use results of QCD analysis performed by Leader-Sidorov-Stamenov (LSS) [PRD' 10] ($\Delta q_3$) for polarized PDFs and results of the analysis LSS'14 [hep-ph:1406.4678] for pions multiplicities data.

# Introduction

**We continue application of the analytic approach in QCD, called the analytic perturbation theory (APT)** [Shirkov-Solovtsov, Phys. Rev .Lett. 79 (1997)] **to the QCD analysis of the DIS data.**

**We study how the APT approach works in comparison with the ordinary perturbative QCD.**

This work could be consider as continuation of our previous analysis of the $F_3(x,Q^2)$ structure function [Sidorov-Solovtsova, hep-ph:1407.6858], where considering a combined set of the $F_3$ -data, we extracted values of the scale parameter $\Lambda_{QCD}$ , the parameters of the form of the $xF_3$, and compared the difference of the results of the PT and APT analysis with the corridor of experimental uncertainties.

The APT/(Fractional)APT has been applied to DIS in a set of works:

- ✓ G.Cvetic, A.Y. Illarionov, B.A. Kniehl, A.V.Kotikov, Phys. Lett. B 679 (2009) 350
- ✓ R.S. Pasechnik, D.V. Shirkov, O.V.Teryaev, O.P.Solovtsova, V.L.Khandramai, Phys. Rev. D81 (2010) 016010; Phys. Lett. B 706 (2012) 340
- ✓ A.V. Kotikov, V.G. Krivokhizhin, B.G. Shaikhatdenov, Phys. Atom. Nucl. 75 (2012) 507
- ✓ C. Ayala, S. Mikhailov, G. Cvetic, Calculation of Nucleon Structure Function in APT (in preparation) [This work was initiated by A. Bakulev]

**APT = PT + RG + $Q^2$ -analyticity**

*Well-known features of APT:*

D.V. Shirkov, I.L.Solovtsov,
Phys. Rev. Lett. 79 (1997) 1209

- analytic coupling

$$\mathcal{A}\left(Q^2\right) \equiv \alpha_{APT}(Q^2) = \frac{1}{\pi}\int_0^{+\infty} \frac{\rho(\sigma)\,d\sigma}{\sigma + Q^2} \quad (Kallen - Lehman\ representation)$$

$$[\alpha_{PT}(Q^2)]^n \Rightarrow \mathcal{A}_n\left(Q^2\right) = \frac{1}{\pi}\int_0^{+\infty} \frac{\rho_n(\sigma)\,d\sigma}{\sigma + Q^2}$$

- free from unphysical singularities and without additional parameters

$$\rho_n(\sigma) = \mathrm{Im}\left(\left[\alpha_{PT}(-\sigma - i\varepsilon)\right]^n\right)$$

- infrared stable point which is independent of the scale parameter $\Lambda_{QCD}$
- APT -> PT at large $Q^2$

$$\alpha_{APT}(0) = 1/\beta_0$$

Generalization: non-integer (fractional) power

*Fractional APT (FAPT)*

$$\left[\alpha_{PT}(Q^2)\right]^{\nu} \Rightarrow \mathcal{A}_{\nu}\left(Q^2\right) = \frac{1}{\pi}\int_0^{+\infty} \frac{\rho_{\nu}(\sigma)\,d\sigma}{\sigma + Q^2} \quad ,$$

$$\rho_{\nu}(\sigma) = \mathrm{Im}\left(\left[\alpha_{PT}(-\sigma)\right]^{\nu}\right)$$

A.P. Bakulev, S.V. Mikhailov, N. Stefanis,
Phys. Rev. D 72 (2005) 074014 (2005);
75 (2007) 056005
A.P. Bakulev, Phys. Part. Nucl. 40 (2009) 715-756.

FAPT: $Q^2$ – evolution SF

**In the description of $Q^2$ -evolution of the structure function (SF) moments the generalized powers (anomalous dimensions) for the running coupling appear. In the leading order (LO) the nonsinglet moments evolve as**

$$M_N(Q^2) = \frac{[\alpha_S(Q^2)]^{\nu}}{[\alpha_S(Q_0^2)]^{\nu}} M_N(Q_0^2)\ , \quad \nu(N) \equiv \gamma_{NS}^{(0)}(N)/2\beta_0$$

$$M_N(Q^2) = \int_0^1 x^{N-1} F(x, Q^2)\, dx$$

nonsinglet one-loop anomalous dimensions

In the framework of the APT this expression transforms as follows:

$$\mathcal{M}_N^{APT}(Q^2) = \frac{\mathcal{A}_\nu(Q^2)}{\mathcal{A}_\nu(Q_0^2)} \mathcal{M}_N^{APT}(Q_0^2)$$

**FAPT** ➔ $$\bar{\mathcal{A}}_\nu^{LO}(Q^2) = \left[\bar{a}_{\mathrm{PT}}^{LO}(Q^2)\right]^\nu - \frac{\mathrm{Li}_\delta(t)}{\Gamma(\nu)}\,, \qquad \bar{\mathcal{A}}_\nu = \beta_0 \mathcal{A}_\nu/(4\pi)$$

$$\mathrm{Li}_\delta(t) = \sum_{k=1}^{\infty} \frac{t^k}{k^\delta}\,, \quad t = \frac{\Lambda^2}{Q^2}\,, \quad \delta = 1 - \nu$$

$\mathrm{Li}_\delta$ is the polylogarithm function

$$\alpha_{\mathrm{APT}}^{LO}(Q^2) = \alpha_{\mathrm{PT}}^{LO}(Q^2) + \frac{4\pi}{\beta_0} \frac{\Lambda^2}{\Lambda^2 - Q^2} \quad (\nu = 1)$$

*additional term* $-\dfrac{4\pi}{\beta_0}\dfrac{\Lambda^2}{Q^2}$

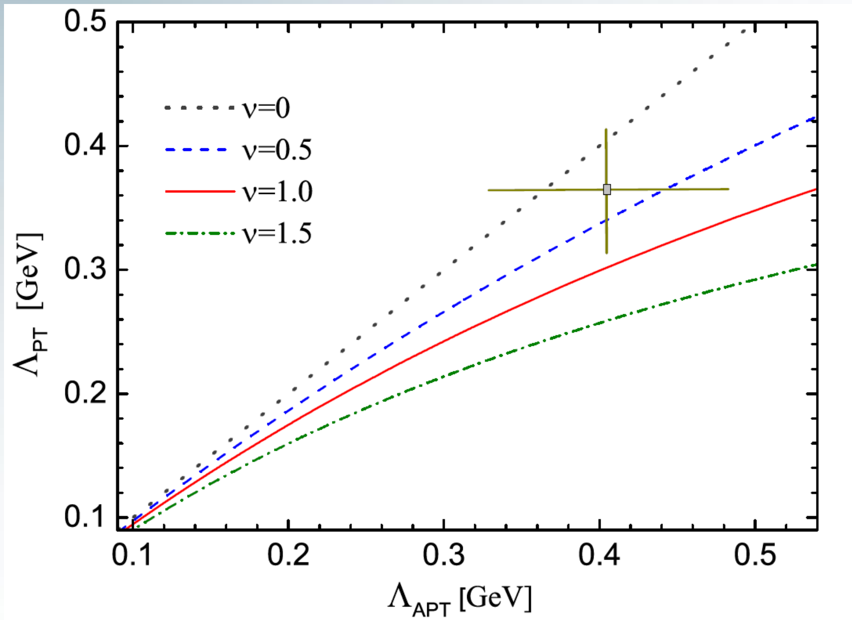
# Comparison of PT and APT couplings

✓ *At fixed $\Lambda$=250 MeV* $\alpha^{\nu}_{PT}(Q^2,\Lambda)$ *and* $\mathcal{A}_{\nu}(Q^2,\Lambda)$

| $Q^2$, $GeV^2$ | Method | ν=0.5 | ν =1 | ν =1.5 | ν =2 |
|---|---|---|---|---|---|
| 1 | APT<br>PT | 0.692<br>0.738 | 0.443<br>0.544 | 0.258<br>0.439 | 0.134<br>0.296 |
| 2 | APT<br>PT | 0.637<br>0.660 | 0.386<br>0.435 | 0.219<br>0.287 | 0.114<br>0.189 |

✓ *Connection between $\Lambda$ coming from the condition*

$$\alpha^{\nu}_{PT}(Q_0^2, \Lambda_{PT}) = \mathcal{A}_{\nu}(Q_0^2, \Lambda_{APT})$$




The point corresponds to extracted $\Lambda$-values (with errors) from the data on the structure function $xF_3$.

# Theoretical framework: $Q^2$–evolution and Jacobi polynomial expansion

In our analysis we follow the well-known approach based on the Jacobi polynomial expansion method. This method of solution of the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution equation was proposed by Parisi and Sourlas [Nucl. Phys. B 151 (1979) 432] and developed in JINR [Krivokhizhin, Kurlovich, Sanadze, Savin, Sidorov, Z.Phys. C36 (1987)]. The main formula of this method allows an approximate reconstruction of the nonsinglet combination through a finite number of Mellin moments

**PT**

$$x\Delta q_3^{N\max}(x,Q^2) = x^\alpha (1-x)^\beta \sum_{n=0}^{N\max} \Theta_n^{\alpha,\beta}(x) \sum_{j=0}^{n} c_j^{(n)}(\alpha,\beta) M_{j+2}(Q^2)$$

The $Q^2$–evolution

$$M_N(Q^2) = \frac{[\alpha_S(Q^2)]^{\nu(N)}}{[\alpha_S(Q_0^2)]^{\nu(N)}} M_N(Q_0^2) \ , \quad \nu(N) \equiv \gamma_{NS}^{(0)}(N)/2\beta_0$$

where $\Theta_n^{\alpha,\beta}$ are the Jacobi polynomials and $\alpha$ are $\beta$ their parameters fixed by the condition of the requirement of the minimization of the errors of the reconstruction.

**APT**

$$x\Delta q_3^{N\max}(x,Q^2) = x^\alpha (1-x)^\beta \sum_{n=0}^{N\max} \Theta_n^{\alpha,\beta}(x) \sum_{j=0}^{n} c_j^{(n)}(\alpha,\beta) \mathcal{M}_{j+2}(Q^2)$$

$$\mathcal{M}_N\left(Q^2\right) = \frac{\mathcal{A}_{\nu(N)}\left(Q^2\right)}{\mathcal{A}_{\nu(N)}\left(Q_0^2\right)} \mathcal{M}_N\left(Q_0^2\right) \qquad (*)$$

$$\mathcal{M}_N(Q_0^2) = \int_0^1 x^{N-1} \Delta q_3(x,Q_0^2) dz = \int_0^1 x^{N-2} A(Q_0^2) x^\alpha (1-x)^\beta (1+\gamma x) dx, \qquad N = 2,3...$$

# Results of fits

$$x\Delta q_3(x,Q_0^{\ 2}) = Ax^{\alpha}(1-x)^{\beta}(1+\gamma x)$$

A, α, β, γ, Λ - free parameters

We generate 'fake' data based on the results of the phenomenological analysis of polarized DIS data presented by Leader-Sidorov-Stamenov [Phys.Rev. D82 (2010) 114018].

The parameters are determined from the fit to these 'fake' data.

The kinematical region: $0.005 < x < 0.7,$ $Q_0^{\ 2} = 2\ GeV^2$

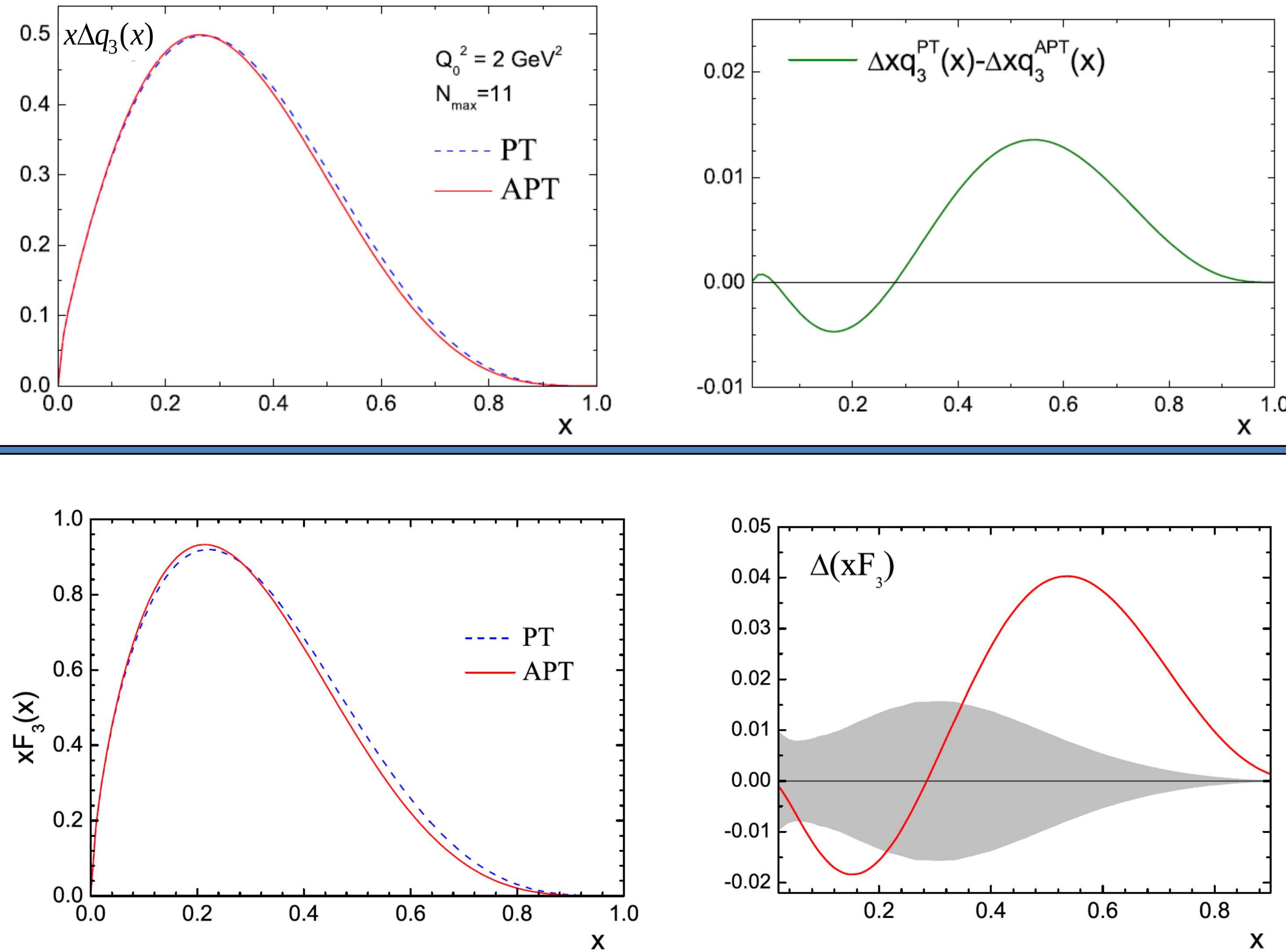
$1\ GeV^2 < Q^2 < 65\ GeV^2\ ,\ W^2 > 4\ GeV^2$ $N_{max} = 11$

| PT | | APT | |
|---|---|---|---|
| A | 0.807 +- 0.091 | A | 0.684 +- 0.052 |
| α | 0.536+- 0.024 | α | 0.505+- 0.016 |
| β | 3.43 +- 0.023 | β | 3.56+- 0.020 |
| γ | 9.89 +- 1.12 | γ | 12.55 +- 0.87 |

| PT | | APT | |
|---|---|---|---|
| $\Lambda\ (MeV)$ | $\chi^2$/D.o.f | $\Lambda\ (MeV)$ | $\chi^2$/D.o.f |
| 256 +- 11 | 33.3/260 | 280 +- 15 | 32.9/260 |

Note, $\Lambda_{APT} > \Lambda_{PT}$

# The x-shape $x\Delta q_3(x)$




Note, at x > 0.3 x $\Delta q_3^{PT}$ > x $\Delta q_3^{APT}$ ($xF_3^{PT}$ > $xF_3^{APT}$)

$$\Delta(xF_3) = xF_3^{PT}(Q^2) - xF_3^{APT}(Q_0^2)$$

# NS pion FF

The 'fake' data for NS pion FFs $D_{u_v}^{\pi^+}(z,Q^2)$ are constructed based on results of QCD analysis of pion multiplicities data [LSS'14 (to be published in PRD)].

The kinematical region: $0.2 < z < 0.7$ and $1.25\ \text{GeV}^2 < Q^2 < 10\ \text{GeV}^2$

$$zD_{u_v}^{\pi^+ N_{\max}}(z,Q^2) = z^{\alpha}(1-z)^{\beta}\sum_{n=0}^{N\max}\Theta_n^{\alpha,\beta}(z)\sum_{j=0}^{n}c_j^{(n)}(\alpha,\beta)\mathcal{M}_{j+2}(Q^2)$$

$Q^2$–evolution (*)

$$\mathcal{M}_N(Q_0^2) = \int_0^1 z^{N-1}D_{u_v}^{\pi^+}(z,Q_0^2)dz = \int_0^1 z^{N-2}A(Q_0^2)z^{\alpha}(1-z)^{\beta}(1+\gamma z)dz$$

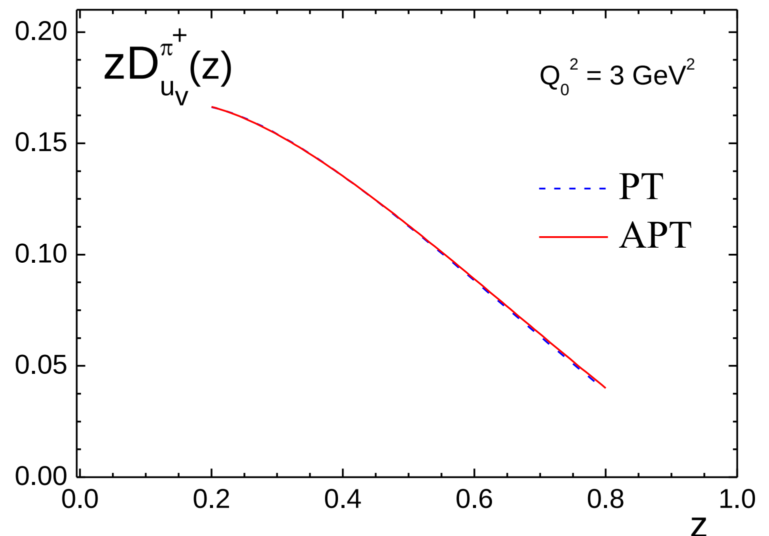
The parameters $A$, $\alpha$, $\beta$, $\gamma$, $\Lambda$ are determined from the fit to 'fake' data.

## Results of fits

$\Lambda_{APT} = 307 \pm 25$ MeV

$\Lambda_{PT} = 231 \pm 12$ MeV

$\Lambda_{APT} > \Lambda_{PT}$




z-shape

# Summary

We have demonstrated that APT $Q^2$-evolution could be apply for the analysis of 'fake' data for the polarized NS combination $x\Delta q_3$ and NS fragmentation function $D_{u_V}^{\pi^+}$.

We obtained values of scale parameter $\Lambda_{\mathrm{APT}}$ and $\Lambda_{\mathrm{PT}}$ .

We found that $\Lambda_{\mathrm{APT}} > \Lambda_{\mathrm{PT}}$ takes place both in $x\Delta q_3$ and $D_{u_V}^{\pi^+}$ cases.

The same inequality was found for the $xF_3$ data analysis.

For the future we have a wide field for work: the highest orders and various structural functions for the analysis.

***Thanks for your attention !***