# WKB – TYPE APPROXIMATIONS IN THE THEORY OF VACUUM PARTICLE CREATION IN STRONG FIELDS

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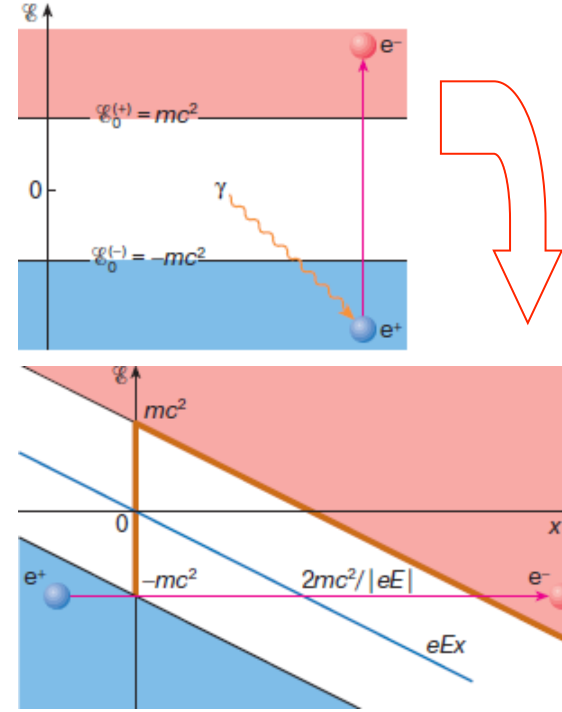
# HISTORICAL SKETCH

The first prediction belongs to F. Sauter (1931) and W. Heisenberg and H. Euler (1936) on the basis of the barrier formula for a constant electric field with strength $E_0$

$$w \sim \exp\left(-E_c / E_0\right)$$




J. Schwinger (1951) have got the exact result

$$w = \frac{\alpha^2}{\pi^2} E_0 \sum_{n=1}^{\infty} n^{-2} \exp\left(-\frac{n\pi E_c}{E_0}\right)$$

for the fermions $E_c = m^2 / |e| \sim 10^{16}$ V/cm

Invention of lasers leads to idea to use the strong laser fields for verification of the "Schwinger effect". At the present time there are realistic projects for the future experiments on the superpower laser facilities. For example

G. Gregory, D.B. Blaschke, P.P. Rajeev, H. Chen, R.J. Clarke, T. Huffman, A.V. Prozorkevich, C.D. Roberts, G. Röpke, S.M. Schmidt, S.A. Smolyansky, S. Wilks, R. Bingham //*J. High Energy Density Physics*, **6**, 166 (2010)

Success of the SHE prediction based on the tunnel mechanism in the case of a constant electric field excites a hope to use the similar tunnel approach for the "laser" electric fields with frequency $\omega << \nu_c = 2\pi / m$.

This brought to appearance of the classical work of E. Brezin and C. Itzykson (1970) and to series of the V.S. Popov works based on the different variants of the WKB approximation. On the other hand, doubt in validity of the WKB – type approaches in the case of the fast alternated fields was expressed in press (e.g., L.A. Khalfin (1986)).

**IDEA** is to verify correctness of the WKB results for the fast "laser" fields on the basis of a more strict approach.

## BASIC KINETIC EQUATION (KE)

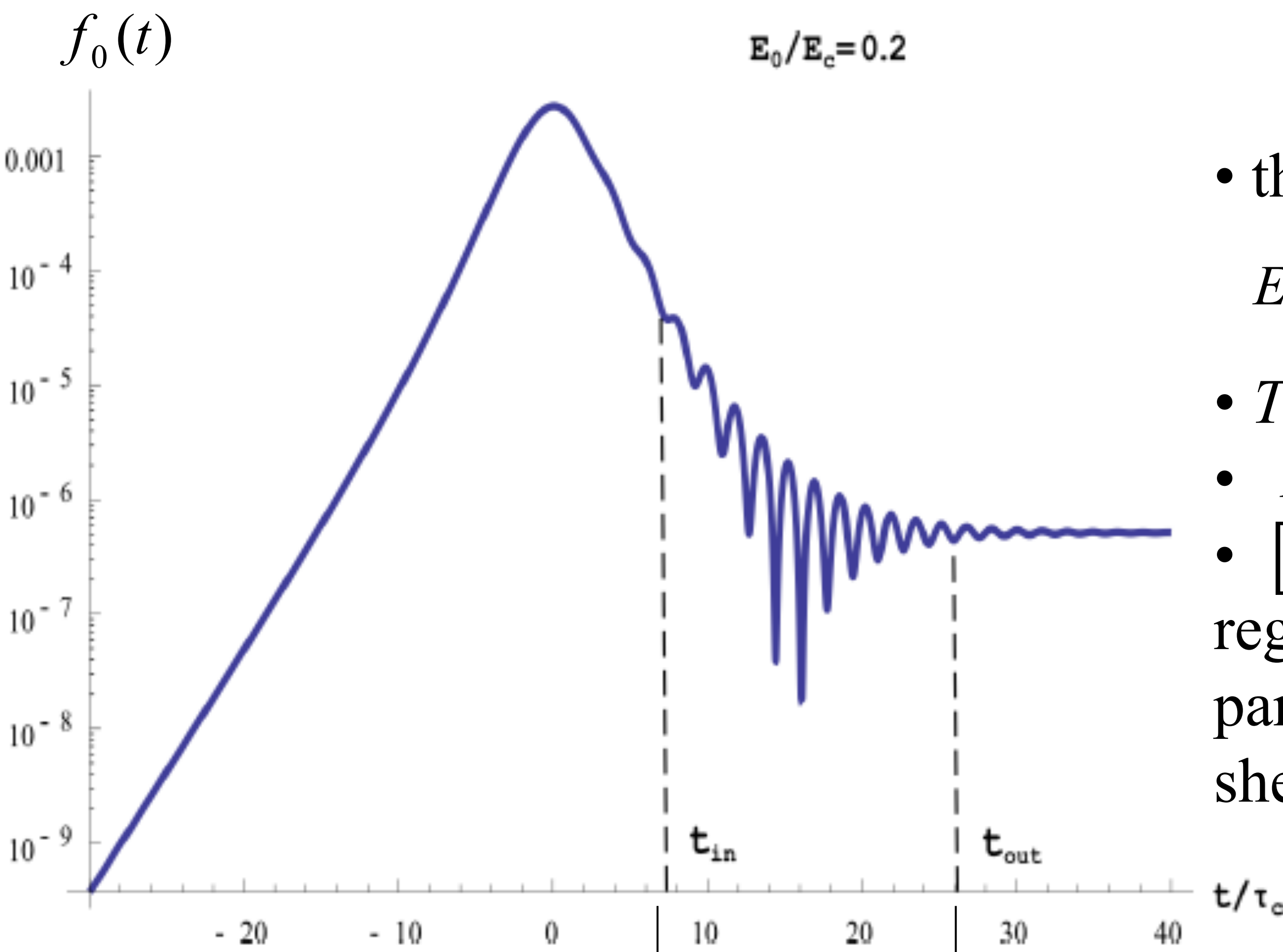
will used bellow in the capacity of such general approach (S.M. Schmidt, D.B. Blaschke, G. Röpke, S.A. Smolyansky, A.V. Prozorkevich, and V.D. Toneev (1998)). This KE is a strong consequence of the Dirac equations of motion for the case of the linear polarized electric field $A^{\mu}(t)\left(0,0,0,A^{3}=A(t)\right)$

$$\dot{f}(\vec{p},t)=\frac{1}{2}\lambda(\vec{p},t)\int_{t_0}^{t}dt'\lambda(\vec{p},t')\left(1-2f(\vec{p},t')\right)\cos\theta(t,t')$$

$$\lambda=\frac{eE\varepsilon_{\perp}}{\varepsilon^{2}},\ \ \varepsilon=\sqrt{\varepsilon_{\perp}^{2}+\vec{P}^{2}},\ \ \vec{P}=\vec{p}_{\|}-e\vec{A},\ \ \varepsilon_{\perp}=\sqrt{m^{2}+\vec{p}_{\perp}^{2}},\ \ \theta(t,t')=2\int_{t'}^{t}d\tau\varepsilon(\vec{p},\tau)$$

At the present time this KE and its different modifications are used in the strong field QED, in theory of the heavy ion collision, and cosmology.

Let us make a couple examples of using this KE




- the Sauter pulse

$$E(t) = E_0 \cosh^{-2}(t/T)$$

- $T = 0.02$ nm
- $p_\perp = p_\parallel = 0$
- $[t_{in}, t_{out}]$ - transient region between quasi-particle and mass-shell stage

quasiparticle EPP | transient region | residual EPP

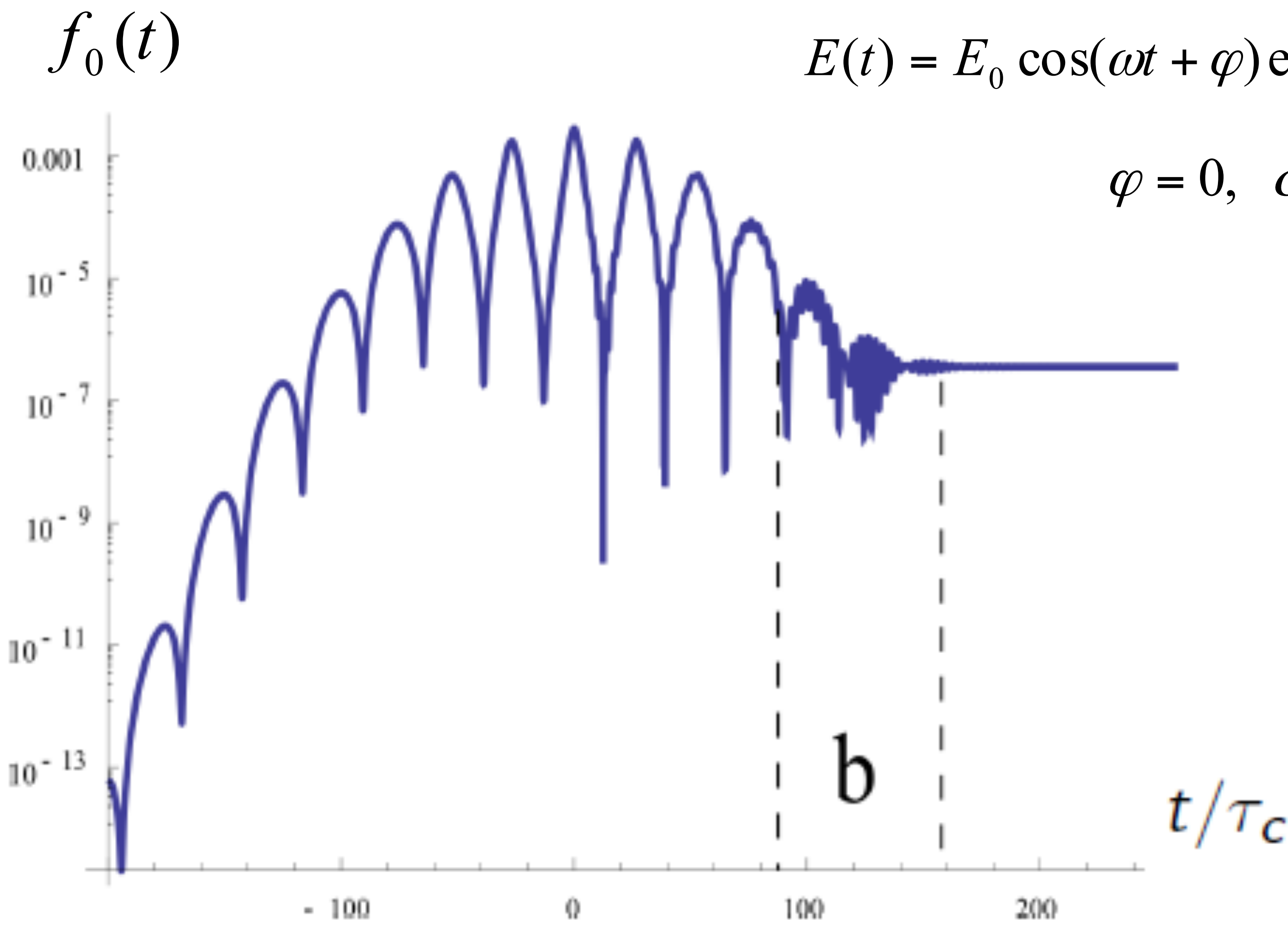
The region "**b**" corresponds to the transient process

$$E(t) = E_0 \cos(\omega t + \varphi) \exp\left(-t^2 / 2\tau^2\right)$$

$$\varphi = 0, \quad \sigma = \omega\tau = 0.5$$

$$p_\perp = p_\parallel = 0$$

$$E_0 = 0.2 E_c$$

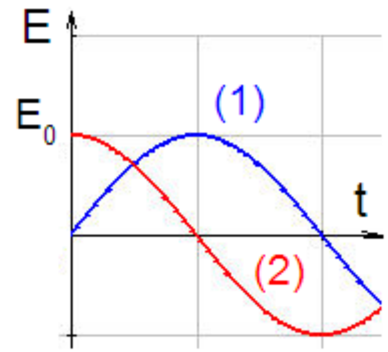



Results of the WKB approximation correspond to the out-state (residual EPP).

**COMPARISON OF THE RESULTS** of the kinetic theory (strict results) with the WKB – type approximations.

The simplest model of the periodical field is used with different initial conditions at $t_0 = 0$.

1) $E(t) = E_0 \sin \omega t, \; A(t) = (E_0 / \omega) \cos \omega t,$ (Brezin, Itzykson; Mocken et al)




2) $E(t) = E_0 \cos \omega t, \; A(t) = -(E_0 / \omega) \sin \omega t.$ (Popov)

The both field models belong to class of the non-adiabatic fields with different types of sharp switching on. Now it is well known that such difference leads to different results (F. Hebenstreit, et al (2009)). We compare these results of the WKB analysis with the kinetic approach.

In such field models there are set of the discrete out-states at $t_n = nT/2$. For small $n \sim 1$ the linear accumulation effects is observed, $f(t_n) > f(t_{n-1})$.

Then it can introduce the EPP production rate $w_n = n(t_n)/(nT/2)$

where $n(t_n) = 2\int \frac{d^3 p}{(2\pi)^3} f(\vec{p}, t_n)$ is the pair density of EPP.

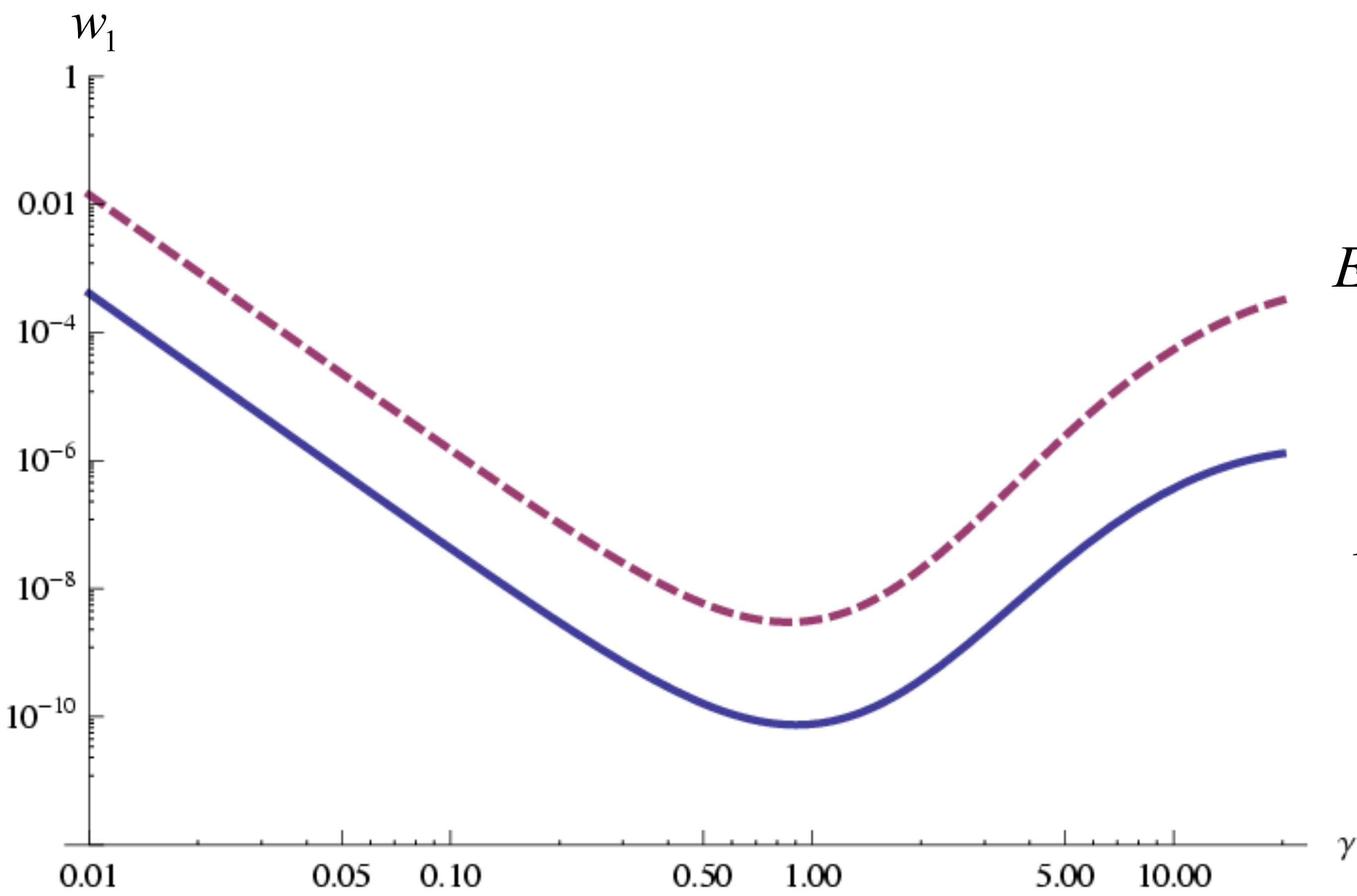
Adiabacity parameter of Keldysh as the $\gamma = \frac{E_c \omega}{E_0 m}$

dimensionless characteristic of an external field

$\gamma << 1$ - tunnel mechanism

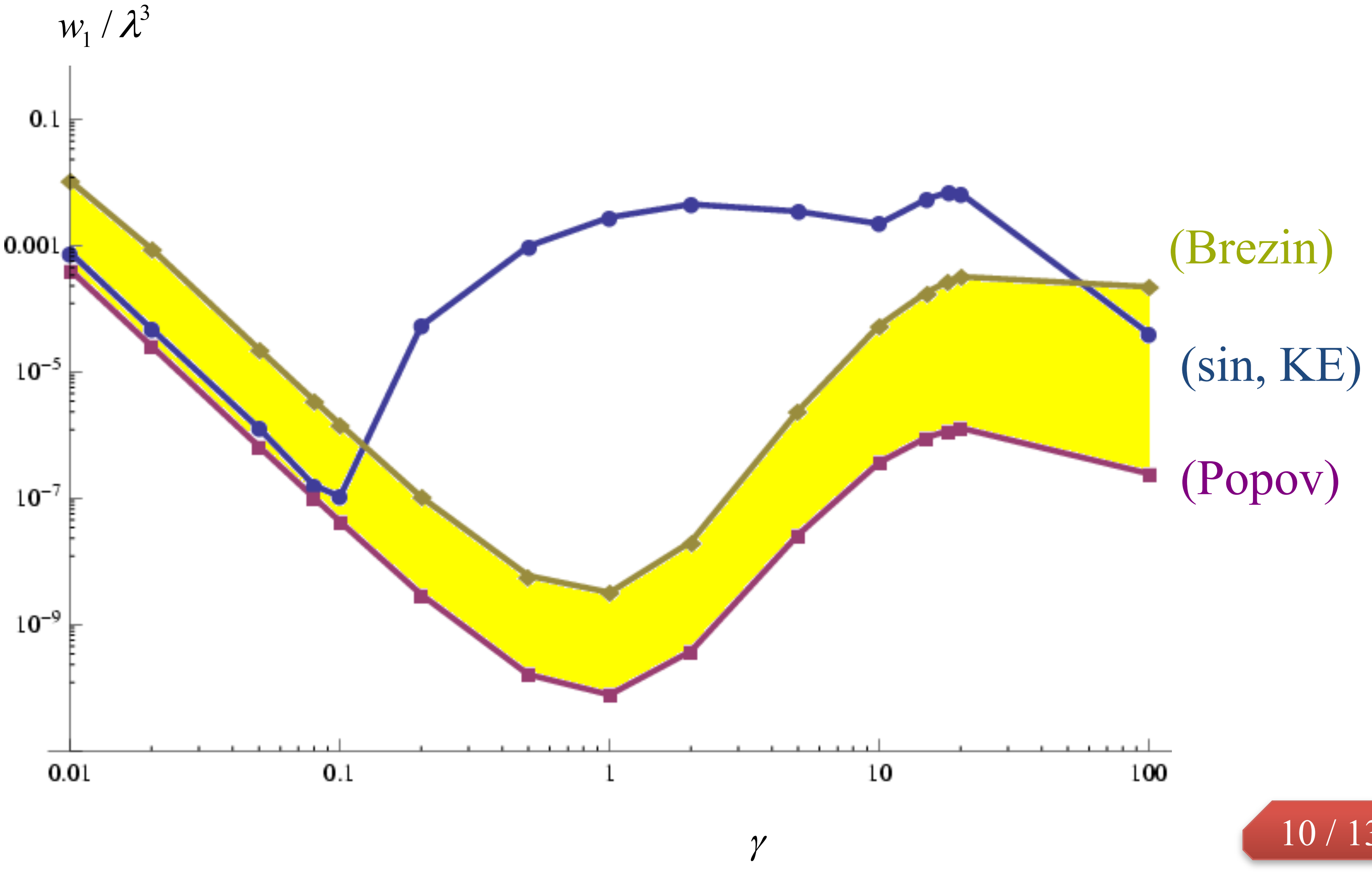
$\gamma >> 1$ - multi-photon mechanism

# COMPARISON OF THE BI AND POPOV RESULTS




$w_{BI} > w_P$ that contradict to the rule of fermionic dominance.

# COMPARISON WITH THE KINETIC APPROACH



**LOW DENSITY APPROXIMATION** (S. Schmidt, D. Blaschke, G. Röpke, A.V. Prozorkevich, S.A. Smolyansky, and V.D. Toneev (1999))

$$f(\vec{p},t) = \left|J(\vec{p},t)\right|^2, \quad J(\vec{p},t) = \frac{1}{2}\int_{t_0}^{t} dt'\,\lambda(\vec{p},t')\exp 2i\theta(\vec{p},t,t').$$

For a periodical field with an initial phase $\varphi$

$$E(t) = E_0 \sin\xi(t), \quad \xi(t) = \omega t + \varphi$$

we obtain the quadrature formula as function $\gamma$

$$\tilde{\varepsilon}_\perp = \varepsilon_\perp / m, \qquad \tilde{p}_\parallel = p_\parallel / m.$$

$$J(\vec{p},t) = \frac{\tilde{\varepsilon}_\perp}{2\gamma}\int_{\varphi}^{\xi(t)} dx \frac{\sin x \exp 2i\theta(\vec{p},t,t')}{\tilde{\varepsilon}_\perp^2 + \left(\tilde{p}_\parallel - 1/\gamma\cos x\right)^2}, \quad \theta(\vec{p},x) = 2\frac{m}{\omega}\int_{\varphi}^{x} dx' \sqrt{\tilde{\varepsilon}_\perp^2 + \left(\tilde{p}_\parallel - 1/\gamma\cos x'\right)^2}$$

## SUMMARY

For the class of periodical fields the fulfilled analysis had showed :

1) Inaccuracy of the WKB-approximation owing to difficulties with single-valuedness fixation of the pre-exponentional factor; in other words, dependence on selection of the method (saddle point approximation, imaginary time method, instanton approach etc).
2) Difficulties with limiting process to the case of on constant electric field (the Schwinger limit), that correspond to $\gamma \to 0$ .
3) WKB approximation is effective in the region of action of the tunnel mechanism only $\gamma << 1$ ;
4) The sharp growth of the pair production rate has been found in the nonperturbative region $\gamma \sim 1$. Apparently, this effects is stipulated by multi-photon resonance. Action of this mechanism relaxes by motion in the few photon region $\gamma >> 1$ .

## What is truth ?

1) The non-adiabatic periodical field models leads to ambiguous results.
   Thus, such kinds fields are non-physical by application to the nonlinear problems of kinetic theory.
2) We must move up to the adiabatic class of the “laser” fields. It’s our next problem.

**THANKS YOU VERY MUCH FOR YOUR ATTENTION !**