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Phenomenological description of electromagnetic and strong interactions in rotating frames at collisions of high energy nuclei

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Main physical result – confining Cornell potential leads to the strong spin-orbit interaction

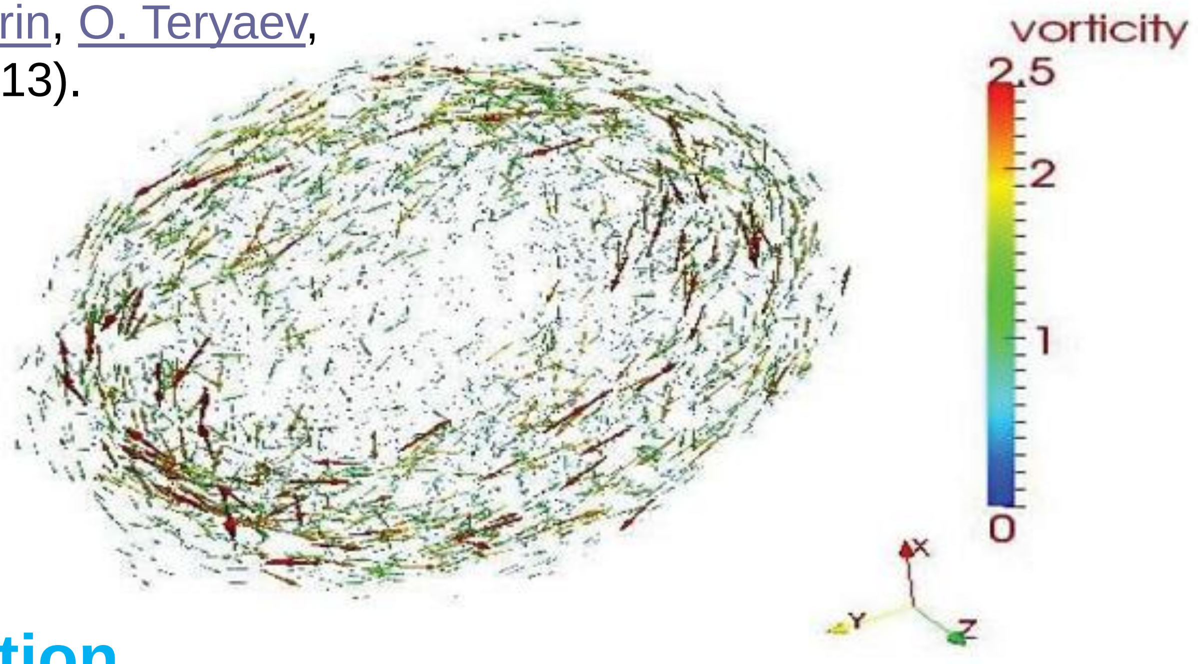
OUTLINE

- **Electromagnetic interactions of a Dirac particle in a rotating frame**
- **Phenomenological description of strong interaction with the four-vector Cornell potential**
- **Phenomenological description of strong interaction with the scalar Cornell potential**
- **Summary**

Rotation in heavy-ion collisions

Heavy-ion collisions – rotation of strongly interacting matter (possible tests at NICA@JINR)

Helicity separation in Heavy-Ion Collisions
M. Baznat, K. Gudima, A. Sorin, O. Teryaev,
Phys. Rev. C **88,** 061901 (2013).



Vorticity distribution
How to describe it in terms of the Dirac equation?

Electromagnetic interactions of a Dirac particle in a rotating frame

We phenomenologically describe the strong interaction of quarks like an electromagnetic one

Dirac equation describing electromagnetic interactions in a Riemannian spacetime

The covariant Dirac equation and the Hermitian Dirac Hamiltonian for a spin-1/2 particle in gravitational and electromagnetic fields

$$\left(i\hbar\gamma^a D_a - mc\right)\psi = 0, \quad D_a = e_a^{\mu}\partial_\mu + \frac{i}{4}\sigma^{bc}\Gamma_{bca},$$

$$\sigma^{bc} = \frac{i}{2}(\gamma^b\gamma^c - \gamma^c\gamma^b), \quad \Gamma_{abc} = -\Gamma_{bac}, \quad a,b,c = 0,1,2,3.$$

tetrad variables

$$\mathcal{H} = \beta mc^2 V + e\Phi + \frac{c}{2}\alpha^a\left(\pi_b \mathcal{F}_a^{\ b} + \mathcal{F}_a^{\ b}\pi_b\right) + \frac{c}{2}\left(\boldsymbol{\pi}\cdot\mathbf{K} + \mathbf{K}\cdot\boldsymbol{\pi}\right) + \frac{\hbar c}{4}\left(\Upsilon\gamma_5 + \boldsymbol{\Xi}\cdot\boldsymbol{\Sigma}\right),$$

metric

$$ds^2 = V^2c^2dt^2 - \delta_{ab}W_i^{\ a}W_j^{\ b}\left(dx^i - cK^i dt\right)\left(dx^j - cK^j dt\right)$$

Here $V = e_0^{\widehat{0}}$, $\mathcal{F}^b{}_a = \sqrt{-g}\, e_{\widehat{a}}^{b} = VW^b{}_{\widehat{a}}$, and

$$\Upsilon = -V\epsilon^{\widehat{a}\widehat{b}\widehat{c}}\Gamma_{\widehat{a}\widehat{b}\widehat{c}} = -V\epsilon^{\widehat{a}\widehat{b}\widehat{c}}\mathcal{C}_{\widehat{a}\widehat{b}\widehat{c}}, \quad \Xi_{\widehat{a}} = \frac{V}{c}\epsilon_{\widehat{a}\widehat{b}\widehat{c}}\Gamma_{\widehat{0}}{}^{\widehat{b}\widehat{c}} = \epsilon_{\widehat{a}\widehat{b}\widehat{c}}\mathcal{Q}^{\widehat{b}\widehat{c}}.$$

Yu. N. Obukhov, A.J. Silenko and O.V. Teryaev, Phys. Rev. D **84**, 024025 (2011).

Dirac Hamiltonian describing electromagnetic interactions in a rotating frame

$$\mathcal{H}=\beta mc^2+e\Phi+c\boldsymbol{\alpha}\cdot\boldsymbol{\pi}-\boldsymbol{\omega}\cdot(\mathbf{r}\times\boldsymbol{\pi})-\frac{\hbar}{2}\boldsymbol{\omega}\cdot\boldsymbol{\Sigma},$$
$$\boldsymbol{\pi}=-i\hbar\nabla-e\mathbf{A}=\mathbf{p}-e\mathbf{A}.$$

We generalize the initial Dirac equation to introduce anomalous magnetic and electric dipole moments

electromagnetic field tensor

$$\left(i\hbar\gamma^a D_a+\frac{\mu'}{2}\sigma^{ab}F_{ab}+\frac{d}{2}\sigma^{ab}G_{ab}-mc\right)\psi=0,\qquad F_{ab}=e_a^\mu e_b^\upsilon F_{\mu\nu}.$$

$$G_{ab}=\frac{1}{2}\eta_{abcd}F^{cd},\quad F_{\mu\nu}=(\mathbf{E},\mathbf{B}),\quad G_{\mu\nu}=(-\mathbf{B},\mathbf{E}).$$

Generalized Dirac Hamiltonian

$$\mathcal{H}=\beta mc^2+e\Phi+c\boldsymbol{\alpha}\cdot\boldsymbol{\pi}-\boldsymbol{\omega}\cdot(\mathbf{r}\times\boldsymbol{\pi})-\frac{\hbar}{2}\boldsymbol{\omega}\cdot\boldsymbol{\Sigma}$$
$$-\beta\left[\boldsymbol{\Sigma}\cdot(\mu'\boldsymbol{\mathcal{B}}+d\boldsymbol{\mathcal{E}})-i\boldsymbol{\alpha}\cdot(\mu'\boldsymbol{\mathcal{E}}-d\boldsymbol{\mathcal{B}})\right].$$

Effective fields in the rotating frame

$$G_{ab}=\frac{1}{2}\eta_{abcd}F^{cd},\quad F_{ab}=\left(\mathcal{E},\mathcal{B}\right),\quad G_{ab}=\left(-\mathcal{B},\mathcal{E}\right).$$

$$\mathcal{E}=\mathbf{E}+\mathbf{G},\quad \mathcal{B}=\mathbf{B},\quad \mathbf{G}=\mathbf{B}\times\left(\boldsymbol{\omega}\times\mathbf{r}\right),$$

$$\mathbf{E}=-\nabla\Phi-\frac{\partial\mathbf{A}}{\partial t},\quad \mathbf{B}=\nabla\times\mathbf{A}.$$

Relativistic Foldy-Wouthuysen transformation in an arbitrarily strong external field

A.J. Silenko, Phys. Rev. A **77**, 012116 (2008).

$$\mathcal{H}_{FW}=\mathcal{H}_{FW}^{(D)}+\mathcal{H}_{FW}^{(add)},\qquad \varepsilon'=\sqrt{m^2c^4+c^2\boldsymbol{\pi}^2},$$

$$\mathcal{H}_{FW}^{(D)}=\beta\varepsilon'+e\Phi-\boldsymbol{\omega}\cdot\left(\mathbf{r}\times\boldsymbol{\pi}\right)-\frac{\hbar}{2}\boldsymbol{\omega}\cdot\boldsymbol{\Sigma}-\frac{e\hbar c^2}{4}\left\{\frac{1}{\varepsilon'},\boldsymbol{\Pi}\cdot\mathbf{B}\right\}$$

$$+\frac{e\hbar c^2}{8}\left\{\frac{1}{\varepsilon'\left(\varepsilon'+mc^2\right)},\left[\boldsymbol{\Sigma}\cdot\left(\boldsymbol{\pi}\times\mathcal{E}-\mathcal{E}\times\boldsymbol{\pi}\right)-\hbar\nabla\cdot\mathcal{E}\right]\right\},$$

$$\mathcal{H}_{FW}^{(add)} = \frac{1}{4}\left\{\frac{1}{\varepsilon'},\left[\boldsymbol{\Sigma}\cdot\left(\boldsymbol{\pi}\times\boldsymbol{\mathcal{P}}-\boldsymbol{\mathcal{P}}\times\boldsymbol{\pi}\right)-\hbar\nabla\cdot\boldsymbol{\mathcal{P}}\right]\right\}-\boldsymbol{\Pi}\cdot\boldsymbol{\mathcal{M}}$$

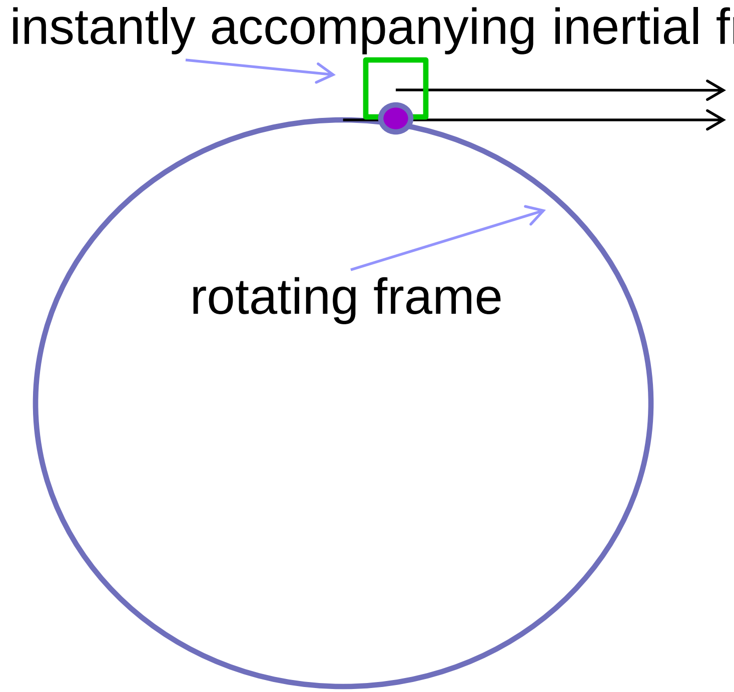
$$+\frac{1}{8}\left\{\frac{1}{\varepsilon'\left(\varepsilon'+mc^2\right)},\left[\left\{\boldsymbol{\Pi}\cdot\boldsymbol{\pi},\left(\boldsymbol{\pi}\cdot\boldsymbol{\mathcal{M}}+\boldsymbol{\mathcal{M}}\cdot\boldsymbol{\pi}\right)\right\}\right.\right.$$

$$\left.\left.+\beta\left(\boldsymbol{\pi}\cdot\boldsymbol{\mathcal{J}}+\boldsymbol{\mathcal{J}}\cdot\boldsymbol{\pi}\right)-\beta\left\{\pi_a,\left(\boldsymbol{\omega}\times\mathbf{r}\right)^b\partial_b\mathcal{P}^a\right\}\right]\right\},$$

$$\boldsymbol{\mathcal{P}}=\mu'\boldsymbol{\mathcal{E}}-d\boldsymbol{\mathcal{B}},\quad \boldsymbol{\mathcal{M}}=\mu'\boldsymbol{\mathcal{B}}+d\boldsymbol{\mathcal{E}},\quad \boldsymbol{\mathcal{J}}=\nabla\times\boldsymbol{\mathcal{M}}-\frac{\partial\boldsymbol{\mathcal{P}}}{\partial t}.$$

When the condition of the Wentzel–Kramers–Brillouin approximation is satisfied, the transition to the classical limit in the relativistic case can be done by replacing the operators in the Foldy-Wouthuysen Hamiltonian and equations of motion by the respective classical quantities

What is a physical meaning of E and B fields?



$$\mathbf{V} = \boldsymbol{\omega} \times \mathbf{r}$$

Let us consider a spinning particle at rest in the both reference frames

$$W_{rot} = -\frac{\hbar}{2}\boldsymbol{\omega}\cdot\boldsymbol{\Sigma} - \mu\boldsymbol{\Pi}\cdot\boldsymbol{\mathcal{B}} - d\boldsymbol{\Pi}\cdot\boldsymbol{\mathcal{E}} \qquad W_{acc} = -\mu\boldsymbol{\Pi}\cdot\mathbf{B}' - d\boldsymbol{\Pi}\cdot\mathbf{E}'$$

A difference between angular velocities of spin precession in the two reference frames is caused by

i) Frame rotation with the angular velocity $\boldsymbol{\omega}$

ii) Thomas precession with the angular velocity $\boldsymbol{\omega}_T = -\frac{\gamma^2}{\gamma+1}\boldsymbol{\beta}\times\dot{\boldsymbol{\beta}}$

However, $\boldsymbol{\beta} \equiv \mathbf{v}/c = 0$ and $\boldsymbol{\omega}_T = 0$ for a particle at rest!

As a result, $\mathcal{B} = \mathbf{B}'$, $\mathcal{E} = \mathbf{E}'$, $\mathbf{E} = \mathbf{E}' - \mathbf{B}' \times (\boldsymbol{\omega} \times \mathbf{r})$

In any local experiment in the Earth's rotating frame, the effective fields acting on the spin coincide with the fields calculated from a distribution of charges and currents

The Earth's rotation does not influence the electromagnetic part of the angular velocity of spin precession

The fields acting on the charged particle differ from the fields acting on its spin

$$\mathbf{F} = e(\mathbf{E} + \mathbf{v} \times \mathbf{B}) + \mathbf{F}_{Cor} + \mathbf{F}_{cf} \qquad (\mathbf{E} \neq \mathcal{E})$$

Coriolis force centrifugal force

$$\mathbf{F}_{cf} = -\varepsilon' \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}), \quad \varepsilon' = \gamma m$$

Phenomenological description of strong interaction with the vector Cornell potential

Interquark Cornell potential

$$V(r) = -\frac{a(r)}{r} + b(r)r$$

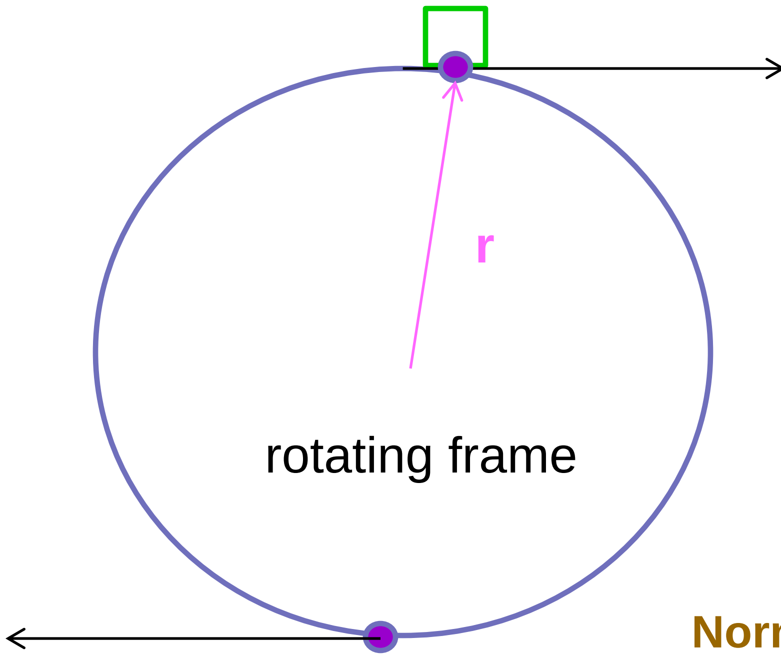
Vector Cornell potential

$$\Phi(r) = -\frac{a(r)}{r} + b(r)r, \quad \mathbf{A}(r) = \Phi(r)\mathbf{V}$$

$$\mathbf{Q} = -\nabla\Phi - \frac{\partial \mathbf{A}}{\partial t}, \quad \mathbf{R} = \nabla \times \mathbf{A}.$$

$$a \approx 100\ \mathrm{MeV} \cdot \mathrm{fm}, \quad b \approx 400\ \mathrm{MeV} \cdot \mathrm{fm}^{-1}$$

Spin-orbit coupling in the lab frame



$$\mathbf{v} = \boldsymbol{\omega} \times \mathbf{r}$$

$$\gamma >> 1$$

$$\omega \approx \frac{c}{r}$$

$$\boldsymbol{\beta} = \frac{\mathbf{v}}{c}$$

$$\mathbf{V} = -\mathbf{v}$$

$$\mathbf{F} = \mathbf{Q} + \boldsymbol{\beta} \times \mathbf{R}, \quad \mathbf{R} = \frac{\mathbf{V}}{c} \times \mathbf{Q}$$

Normal and anomalous quark chromomagnetic moments are of the same order

see N. Kochelev, N. Korchagin, Phys. Lett. B **729,** 117 (2014)

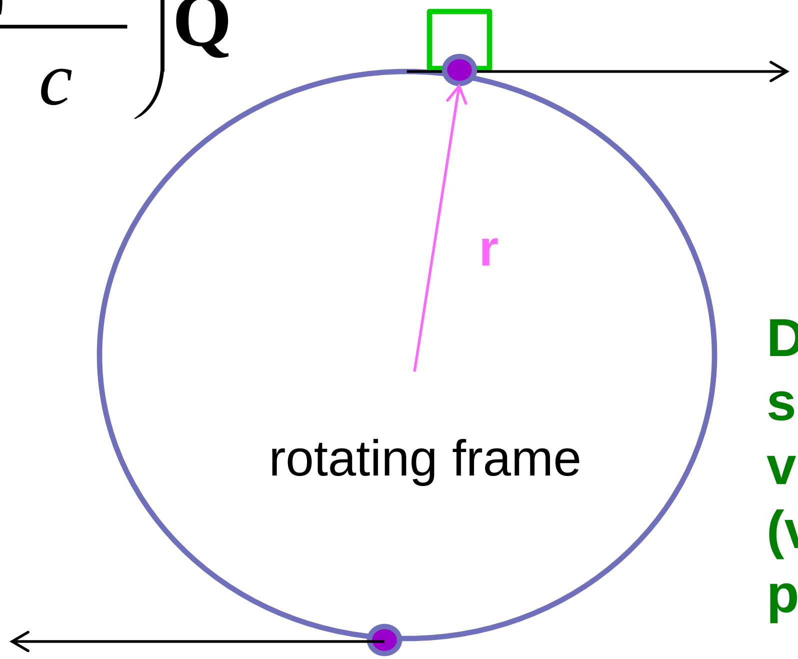
Angular velocity of spin precession

$$\boldsymbol{\Omega} = \frac{G_c}{mc}\left(\boldsymbol{\beta} - \frac{\mathbf{V}}{c}\right) \times \mathbf{Q}.$$

Contributions given by the chromo-electric and chromo-magnetic fields have the same sign

Force $\mathbf{F} = \left(1 - \frac{\boldsymbol{\beta}\cdot\mathbf{V}}{c}\right)\mathbf{Q}$

$\mathbf{v} = \boldsymbol{\omega}\times\mathbf{r}$



$G_c = \frac{g_c - 2}{2}$

Dependence of the spin-orbit interaction via rotation frequency (valid also for NN potentials)

$\mathbf{V} = -\mathbf{v}$

In the stationary case, $\langle\mathbf{F}\rangle = -\langle\mathbf{F}_{cf}\rangle = -\varepsilon'\omega^2\mathbf{r}.$

$$\boldsymbol{\Omega} = G_c\gamma\boldsymbol{\omega}, \quad W_{so} = \mathbf{s}\cdot\boldsymbol{\Omega}.$$

More precise solution can be obtained:

$$\boldsymbol{\Omega} = \frac{2G_c\gamma b}{mc}\cdot\frac{\mathbf{l}}{l}.$$

Since $\omega \sim c/r,$ the estimates agree when $b/(mc) \sim c\cdot 1\ \mathrm{fm}^{-1}.$

An estimate shows the chromomagnetic spin-orbit interaction can be of the order of GeV

The general equation for the angular velocity of spin rotation of quarks (*γ>>1)*:

$$\mathbf{\Omega} = \frac{G_c}{mc}\left(1 - \frac{\boldsymbol{\beta}\cdot\mathbf{V}}{c}\right)^{-1}\left(\boldsymbol{\beta} - \frac{\mathbf{V}}{c}\right)\times\mathbf{F}$$

Phenomenological description of strong interaction with the scalar confining Cornell potential

Scalar Cornell potential

$$V(r) = -\frac{a(r)}{r} + b(r)r$$

$$a \approx 100\ \text{MeV}\cdot\text{fm}, \quad b \approx 400\ \text{MeV}\cdot\text{fm}^{-1}$$

Hermitian Dirac Hamiltonian from the covariant Dirac equation

$$\mathcal{H} = \beta\left(mc^2 + V\right) + \frac{c}{2}\alpha^a\left(\pi_b \mathcal{F}_a^{\ b} + \mathcal{F}_a^{\ b}\pi_b\right) + \frac{c}{2}\left(\mathbf{p}\cdot\mathbf{K} + \mathbf{K}\cdot\mathbf{p}\right) + \frac{\hbar c}{4}\left(\Upsilon\gamma_5 + \boldsymbol{\Xi}\cdot\boldsymbol{\Sigma}\right)$$

Dirac Hamiltonian describing a scalar interaction in a rotating frame

$$\mathcal{H} = \beta\left(mc^2 + V\right) + c\boldsymbol{\alpha}\cdot\mathbf{p} - \boldsymbol{\omega}\cdot\left(\mathbf{r}\times\mathbf{p}\right) - \frac{\hbar}{2}\boldsymbol{\omega}\cdot\boldsymbol{\Sigma}, \qquad \mathbf{p} = -i\hbar\nabla.$$

For the isotropic metric with the Schwinger gauge, the exact Hermitian Dirac Hamiltonian reads

$$\mathcal{H} = \beta\left(mc^2 + V\right) + \frac{c}{2}\left[\boldsymbol{\alpha}\cdot\mathbf{p}\mathcal{F} + \mathcal{F}\boldsymbol{\alpha}\cdot\mathbf{p}\right] + \frac{c}{2}\left(\mathbf{K}\cdot\mathbf{p} + \mathbf{p}\cdot\mathbf{K}\right) + \frac{\hbar c}{4}\left(\nabla\times\mathbf{K}\right)\cdot\boldsymbol{\Sigma}.$$

Yu. N. Obukhov, A.J. Silenko and O.V. Teryaev, Phys. Rev. D **84**, 024025 (2011).

In the considered case, $\mathcal{F} = 1, \quad \mathbf{K} = -\frac{\boldsymbol{\omega}\times\mathbf{r}}{c}.$

Foldy-Wouthuysen Hamiltonian describing a scalar interaction in the rotating frame

$$\mathcal{H}_{FW}=\beta c\varepsilon''-\boldsymbol{\omega}\cdot\mathbf{l}-\frac{\hbar}{2}\boldsymbol{\omega}\cdot\boldsymbol{\Sigma}$$

$$+\beta\frac{\hbar}{4}\left\{\frac{1}{2\varepsilon''^2+\{\varepsilon'',[m+V(r)]\}},\left[\boldsymbol{\Sigma}\cdot\left(\boldsymbol{\mathcal{E}}\times\mathbf{p}-\mathbf{p}\times\boldsymbol{\mathcal{E}}\right)+\hbar\nabla\cdot\boldsymbol{\mathcal{E}}\right]\right\},$$

$$\varepsilon''=\sqrt{[m+V(r)]^2+\mathbf{p}^2},\quad \boldsymbol{\mathcal{E}}=-\nabla V(r)=-V(r)\frac{\mathbf{r}}{r}.$$

Force $\mathbf{F}=\dfrac{[m+V(r)]\boldsymbol{\mathcal{E}}}{\varepsilon''},$ **velocity** $\mathbf{v}=\dfrac{\mathbf{p}}{\varepsilon''},$

Lorentz factor $\gamma=\dfrac{\varepsilon''}{m+V(r)}.$

The general equation for the angular velocity of spin rotation of quarks (*γ>>1)*:

$$\mathbf{\Omega} = -\frac{\gamma}{(\gamma+1)[m+V(r)]c}\boldsymbol{\beta}\times\mathbf{F}$$

This is the universal and practically exact relation

The spin-orbit interaction is also rather strong

Summary

- Relativistic Foldy-Wouthuysen transformation allows to derive Foldy-Wouthuysen Hamiltonians and operator and semiclassical equations of spin motion for a strongly interacting Dirac particle in a rotating frame
- Dirac equations for four-vector and scalar Cornell potential in a rotating frame have been considered
- Spin-orbit interaction of quarks is rather strong
- Angular velocity of spin rotation of quarks has been expressed in terms of the centripetal force


Thank you for your attention