Clothed Particles in Mesodynamics, Electrodynamics and Other Quantum Field Models

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Our motto is reminder:

"... in theories with derivative couplings or spins $j \geq 1$, it is not enough to take Hamiltonian as the integral over space of a scalar interaction density; we also need to add non-scalar terms to the interaction density to compensate non-covariant terms in the propagators ... "
from Chapter IV in [1] Weinberg *The Quantum Theory of Fields* Vol. I (1995).

Outline

- Introductory Remarks
- Underlying Formalism
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- QED Hamiltonian in Coulomb Gauge (CG). Some Parallels
- QED in Clothed Particle Representation (CPR)
- QCD Hamiltonian in CG. Some Similarities
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First, I will show how the notion of clothing in quantum field theory (QFT), put forward by Greenberg and Schweber and developed by M. I. Shirokov, can be employed not only in theory of interacting meson and nucleon fields (see, e.g., our previous works), but in quantum electrodynamics (QED) and, perhaps, in quantum chromodynamics (QCD) too.
As before, using instant form of relativistic dynamics and applying the method of unitary clothing transformations (shortly, UCT method) we have derived a novel analytic expression for the QED Hamiltonian in the clothed particle representation (CPR) in which so-called bad terms are simultaneously removed from Hamiltonian and boosts via one and the same UCT.
Second, we are trying to realize this notion in quantum chromodynamics (QCD) (to be definite for gauge group $SU(3)$) when drawing parallels between QCD and QED.

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Introductory Remarks

- Instant Form of Relativistic Dynamics after Dirac, Particle Number (Fock) Representation
- Mass-Changing Bogoliubov-type Transformations, Bare Particles with Given Mass Values
- Notion of Clothing in Quantum Field Theory (QFT) after Greenberg and Schweber. Its Development by M. I. Shirokov. Removal of Divergences Directly in Field Hamiltonian
- A Novel Family of Relativistic Hermitean and Energy-Independent Interactions for Meson-Nucleon Systems
- A Fresh Look at Calculations of Mass and Vertex Counterterms

- Calculations of S-Matrix in Clothed Particle Representation
- Description of Nucleon-Nucleon Scattering and Deuteron Properties

What follows are fragments of our explorations

Mass-Changing Bogoliubov-type Transformations. Mass Counterterms

Our departure point is Hamiltonian

$$H = H(\overset{\circ}{\alpha}) = H_0(\overset{\circ}{\alpha}) + V_0(\overset{\circ}{\alpha}), \tag{1}$$

where unperturbed Hamiltonian $H_0(\overset{\circ}{\alpha})$ and interaction term $V_0(\overset{\circ}{\alpha})$ depend on creation and destruction operators of “bare” particles with unphysical masses and coupling constants. Here, $\overset{\circ}{\alpha}$ denotes the set of all these operators. For example, in case of a spinor (fermion) field ψ and a neutral pseudoscalar meson field ϕ one has to introduce operators $\overset{\circ}{a}(\mathbf{k})$, $\overset{\circ}{b}(\mathbf{p}, r)$, $\overset{\circ}{d}(\mathbf{p}, r)$ and their adjoint counterparts, respectively, for mesons, nucleons and antinucleons. One has in Schrödinger (S) picture

$$\phi(\mathbf{x}) = (2\pi)^{-3/2} \int d\mathbf{k}\, (2\omega^{\circ}_{\mathbf{k}})^{-1/2} \left[\overset{\circ}{a}(\mathbf{k}) + \overset{\circ}{a}{}^{\dagger}(-\mathbf{k})\right] \exp(i\mathbf{k}\mathbf{x}), \tag{2}$$

$$\psi(\mathbf{x}) = \int d\mathbf{p} \left(m_0/(2\pi)^3 E_{\mathbf{p}}^{\circ}\right)^{1/2} \left[\overset{\circ}{u}(\mathbf{p}r)\, \overset{\circ}{b}(\mathbf{p}r) + \overset{\circ}{v}(-\mathbf{p}r)\, \overset{\circ}{d}{}^{\dagger}(-\mathbf{p}r)\right] \exp(i\mathbf{p}\mathbf{x}), \quad (3)$$

where $\mathbf{k}$, $\mathbf{p}$ and r are particle momenta and fermion polarization index, two Dirac spinors $\overset{\circ}{u}$ and $\overset{\circ}{v}$ satisfy conventional equations $(\hat{p}^{\circ} - m_0)\ \overset{\circ}{u}\ (\mathbf{p}, r) = 0$ and $(\hat{p}^{\circ} + m_0)\ \overset{\circ}{v}\ (\mathbf{p}, r) = 0$ with $\hat{p}^{\circ} = E_{\mathbf{p}}^{\circ}\gamma^0 - \mathbf{p}\boldsymbol{\gamma}$, energies $E_{\mathbf{p}}^{\circ} = \sqrt{\mathbf{p}^2 + m_0^2}$ and $\omega_{\mathbf{k}}^{\circ} = \sqrt{\mathbf{k}^2 + \mu_0^2}$, unknown values m_0 and μ_0 play role of bare (nonrenormalized) masses.
As usually,

$$\begin{aligned} \left[\overset{\circ}{a}(\mathbf{k}), \overset{\circ}{a}{}^{\dagger}(\mathbf{k}')\right] &= \delta(\mathbf{k} - \mathbf{k}'), \\ \left\{\overset{\circ}{b}(\mathbf{p}, r), \overset{\circ}{b}{}^{\dagger}(\mathbf{p}', r')\right\} &= \left\{\overset{\circ}{d}(\mathbf{p}, r), \overset{\circ}{d}{}^{\dagger}(\mathbf{p}', r')\right\} = \delta_{rr'}\delta(\mathbf{p} - \mathbf{p}'). \quad (4) \end{aligned}$$

The corresponding unperturbed Hamiltonian is

$$H_0(\overset{\circ}{\alpha}) = \int d\mathbf{k}\omega_{\mathbf{k}}^{\circ} \overset{\circ}{a}{}^{\dagger}(\mathbf{k}) \overset{\circ}{a}(\mathbf{k}) + \int d\mathbf{p}E_{\mathbf{p}}^{\circ} \left[\overset{\circ}{b}{}^{\dagger}(\mathbf{p},r) \overset{\circ}{b}(\mathbf{p},r) + \overset{\circ}{d}{}^{\dagger}(\mathbf{p},r) \overset{\circ}{d}(\mathbf{p},r) \right]. \tag{5}$$

Now, let us consider a set $\alpha = (a, a^{\dagger}, ...)$ of destruction and creation operators for particles with given masses, e.g., masses of constituent quarks. If m and μ take on some physical values, such a representation refers to “bare particles with physical masses” (see {SheShi01}). By definition, operators α enter

$$\phi(\mathbf{x}) = \int d\mathbf{k} \left(2(2\pi)^3\omega_{\mathbf{k}}\right)^{-1/2} \left[a(\mathbf{k}) + a^{\dagger}(-\mathbf{k})\right] \exp(i\mathbf{kx}), \tag{6}$$

$$\psi(\mathbf{x}) = \int d\mathbf{p} \left(m/(2\pi)^3 E_{\mathbf{p}}\right)^{1/2} \sum_{\mu} \left[\bar{u}(\mathbf{p}\mu)b(\mathbf{p}\mu) + v(-\mathbf{p}\mu)d^{\dagger}(-\mathbf{p}\mu)\right] \exp(i\mathbf{px}). \tag{7}$$

Doing so we find links

$$\frac{\overset{\circ}{a}(\mathbf{k}) + \overset{\circ}{a}{}^{\dagger}(-\mathbf{k})}{\sqrt{\omega_{\mathbf{k}}^{\circ}}} = \frac{a(\mathbf{k}) + a^{\dagger}(-\mathbf{k})}{\sqrt{\omega_{\mathbf{k}}}}, \ \forall\mathbf{k}, \tag{8}$$

and so on.

Moreover, since operators α are assumed to meet the same commutation rules as operators $\overset{\circ}{\alpha}$ do, it allows us to connect them by a similarity (unitary) transformation

$$\overset{\circ}{a}(\mathbf{k}) = Ta(\mathbf{k})\,T^{\dagger},\ \overset{\circ}{b}(\mathbf{p},r) = Tb(\mathbf{p},r)\,T^{\dagger},\ \overset{\circ}{d}(\mathbf{p},r) = Td(\mathbf{p},r)\,T^{\dagger} \tag{9}$$

with $T = T_{mes} \otimes T_{ferm}$, where, e.g.,

$$T_{mes} = \exp\left[-\frac{1}{2}\int d\mathbf{k}\chi_k\left(a^{\dagger}(\mathbf{k})\,a^{\dagger}(-\mathbf{k}) - a(\mathbf{k})\,a(-\mathbf{k})\right)\right], \tag{10}$$

with $\sqrt{\omega_{\mathbf{k}}}\exp\chi_k = \sqrt{\omega_{\mathbf{k}}^{\circ}}$. Then we get

$$\overset{\circ}{a}(\mathbf{k}) = \cosh\chi_k a(\mathbf{k}) + \sinh\chi_k a^{\dagger}(-\mathbf{k}), \tag{11}$$

where

$$\begin{aligned}
\cosh\chi_k &= \frac{1}{2}\left[\sqrt{\frac{\omega_{\mathbf{k}}^{\circ}}{\omega_{\mathbf{k}}}} + \sqrt{\frac{\omega_{\mathbf{k}}}{\omega_{\mathbf{k}}^{\circ}}}\right], \\
\sinh\chi_k &= \frac{1}{2}\left[\sqrt{\frac{\omega_{\mathbf{k}}^{\circ}}{\omega_{\mathbf{k}}}} - \sqrt{\frac{\omega_{\mathbf{k}}}{\omega_{\mathbf{k}}^{\circ}}}\right].
\end{aligned}$$

Explicit expression for operator T_{ferm}, that acts on fermionic sector, is in {KorCanSh
where one can find

$$H_0(\overset{\circ}{\alpha}) = H_F(\alpha) + M_{ren,\, mes}\,(\alpha) + M_{ren,\, ferm}\,(\alpha)\,, \tag{12}$$

where

$$H_F(\alpha) = \int d\mathbf{k}\omega_{\mathbf{k}} a^\dagger\,(\mathbf{k})\,a\,(\mathbf{k}) + \int d\mathbf{p}E_{\mathbf{p}}\left[b^\dagger\,(\mathbf{p},r)\,b\,(\mathbf{p},r) + d^\dagger\,(\mathbf{p},r)\,d\,(\mathbf{p},r)\right], \tag{13}$$

$$M_{ren,\, mes}\,(\alpha) = \frac{\mu_0^2 - \mu^2}{4}\int \frac{d\mathbf{k}}{\omega_{\mathbf{k}}}\left[a^\dagger\,(\mathbf{k})\,a\,(\mathbf{k}) + a\,(\mathbf{k})\,a\,(-\mathbf{k}) + H.c.\right], \tag{14}$$

while fermion mass counterterm has form

$$M_{ferm}\,(\alpha) = m\delta m\left\{b^\dagger M_{11} b + b\dagger M_{12} d^\dagger + dM_{21} b + d^\dagger M_{22} d\right\}, \tag{15}$$

where the matrix M is given by

$$M = \left[\begin{array}{cc} M_{11} & M_{12} \\ M_{21} & M_{22}\end{array}\right] = \frac{\delta\,(\mathbf{p}' - \mathbf{p})}{E_{\mathbf{p}}}\left[\begin{array}{cc} \delta_{r'r} & \bar{u}\,(\mathbf{p}\,',r')\,v\,(-\mathbf{p}\,,r) \\ \bar{v}\,(-\mathbf{p}\,',r')\,u\,(\mathbf{p}\,,r) & \delta_{r'r}\end{array}\right], \tag{16}$$

that is equivalent to

$$M_{ferm} = \delta m\int d\mathbf{x}\bar{\psi}(\mathbf{x})\psi(\mathbf{x}). \tag{17}$$

Underlying formalism

The UCT method is aimed at expressing a field Hamiltonian through clothed-particle creation (annihilation) operators α_c, e.g., $a_c^\dagger(a_c)$, $b_c^\dagger(b_c)$ and $d_c^\dagger(d_c)$ via UCTs $W(\alpha_c) = W(\alpha) = \exp R$, $R = -R^\dagger$ in similarity transformation

$$\alpha = W(\alpha_c)\alpha_c W^\dagger(\alpha_c)$$

that connects set α in BPR with new operators α_c in CPR.
A key point of clothing procedure is to remove bad terms from Hamiltonian

$$H \equiv H(\alpha) = H_F(\alpha) + H_I(\alpha) = W(\alpha_c)H(\alpha_c)W^\dagger(\alpha_c) \equiv K(\alpha_c),$$

By definition, such terms prevent physical vacuum $|\Omega\rangle$ (H lowest energy eigenstate) and one-clothed-particle states $|n\rangle_c = a_c^\dagger(n)|\Omega\rangle$ to be H eigenvectors for all n included. Bad terms occur every time when any normally ordered product

$$a^\dagger(1')a^\dagger(2')...a^\dagger(n'_C)a(n_A)...a(2)a(1)$$

of class [C.A] embodies, at least, one substructure $\in [k.0]$ ($k = 1, 2...$) or/and $[k.1]$ ($k = 2, 3, ...$). In this context all primary Yukawa-type (trilinear) couplings should be eliminated from interaction $V(\alpha)$ that enters

$$H_I(\alpha) = V(\alpha) + \text{mass and vertex counterterms}$$

It results in form

$$H = K_F(\alpha_c) + K_I(\alpha_c) = K, \tag{18}$$

where free part $K_F(\alpha_c) = H_F(\alpha_c)$ while operator $K_I(\alpha_c)$ contains interactions between clothed particles. By construction, latter has property

$$K_I(\alpha_c)|\Omega\rangle = K_I(\alpha_c)|n\rangle_c \equiv 0$$

For boson-fermion (meson-nucleon, photon-electron) system
we have decomposition

$$\begin{aligned} K_I(\alpha_c) = K(ff \to ff) + K(\bar{f}\bar{f} \to \bar{f}\bar{f}) + K(f\bar{f} \to f\bar{f}) \\ + K(bf \to bf) + K(b\bar{f} \to b\bar{f}) + K(f\bar{f} \leftrightarrow bb') \\ + K(ff \leftrightarrow bff) + K(f\bar{f} \leftrightarrow 3b) + K(3f \to 3f) + \cdots , \end{aligned} \tag{19}$$

where separate contributions are responsible for different physical processes so, for instance, operators $K(\gamma e \to \gamma e), K(e\ e \leftrightarrow \gamma e\ e)$ and $K(3N \to 3N)$ can be used in describing Compton scattering on electrons, electron-electron bremsstrahlung and modeling three-nucleon forces, respectively.
In particular, fermion-fermion interaction operator in CPR can be written as

$$K(ff \to ff) = \sum_b K_b(ff \to ff),$$

$$K_b(ff \to ff) = \int \sum_\mu d\vec{p}_1{}' d\vec{p}_2{}' d\vec{p}_1 d\vec{p}_2\, V_b(1', 2'; 1, 2) b_c^\dagger(1') b_c^\dagger(2') b_c(1) b_c(2),$$

where symbol $\sum\limits_\mu$ denotes summation over fermion spin projections, $1 = \{\vec{p}_1, \mu_1\}$, etc.

Interactions of vector fields with other fields

Starting from Lagrangian density $\mathcal{L}(x)$

$$\mathcal{L}(x) = -\frac{1}{4}V^{\mu\nu}(x)V_{\mu\nu}(x) + \frac{1}{2}m_v^2 V^{\mu}(x)V_{\mu}(x) - J^{\mu}(x)V_{\mu}(x)$$

for real massive vector field V_μ with tensor $V^{\mu\nu} = \partial^\mu V^\nu - \partial^\nu V^\mu$, that is coupled via current J^μ with other fields (corresponding "free-particle" terms are omitted). By definition, the current does not involve V_μ. In framework of canonical formalism with such a Lagrangian one gets (see, e.g., [9] S. Weinberg The Quantum Theory of Fields, vol., Cambridge, 1995; {Wein1}) interaction Hamiltonia in S picture

$$V = \int d\vec{x} V(\vec{x}) = \int d\vec{x} \left[J^{\mu}(\vec{x})V_{\mu}(\vec{x}) + \frac{1}{2\, m_v^2} J_0(\vec{x})^2 \right]. \tag{20}$$

Its density in Dirac (D) picture $V_D(x) \equiv V(t,\vec{x}) = V(x)$ does not possess property to be invariant with respect to Poincaré group Π, viz.,

$$U_F(\Lambda, a)V(x)U_F^{-1}(\Lambda, a) = V(\Lambda x + a), \tag{21}$$

$$\forall\ \Lambda \in L_+ \text{ and arbitrary spacetime shifts } a = (a^0, \vec{a}) \tag{22}$$

Correspondence $(\Lambda, a) \to U_F(\Lambda, a)$ between elements $(\Lambda, a) \in \Pi$ and unitary transformations $U_F(\Lambda, a)$ realizes an irreducible representation of Π in Hilbert space of states for free (non–interacting) fields. Here L_+ homogeneous (proper) orthochronous Lorentz group.
Keeping this in mind let us write

$$V(x) = H_{cov}(x) + H_{ncov}(x) \tag{23}$$

with

$$H_{cov}(x) = j^{\mu}(x) v_{\mu}(x),$$

$$H_{ncov}(x) = \frac{1}{2\, m_v^2} j_0(x)^2$$

Following {Wein1} we distinguish via upper (lower) case letters between field operators in Heisenberg and Dirac picture .

We encounter a similar division in theory of interacting $\rho-, \omega-$ meson (φ) and nucleon (ψ) fields, where corresponding densities can be represented as (see Appendix A in {DuShe10})

$$H_{cov}(x) = g_v\bar{\psi}(x)\gamma_\mu\psi(x)\varphi^\mu(x) + \frac{f_v}{4m}\bar{\psi}(x)\sigma_{\mu\nu}\psi(x)\varphi^{\mu\nu}(x)$$

and

$$H_{ncov}(x) \quad = \quad \frac{{g_v}^2}{2m_v^2}\bar{\psi}(x)\gamma_0\psi(x)\bar{\psi}(x)\gamma_0\psi(x) \;+\; \frac{{f_v}^2}{4m^2}\bar{\psi}(x)\sigma_{0i}\psi(x)\bar{\psi}(x)\sigma_{0i}\psi(x).$$

QED Hamiltonian in Coulomb gauge. Parallels

In Coulomb gauge interaction Hamiltonian of spinor QED is given by (cf., for example, Eqs. (8.4.3) and (8.4.23) in {Wein1})

$$V_{qed} = \int d\vec{x} V_{qed}(\vec{x}) = \int d\vec{x} J^k(\vec{x}) A_k(\vec{x}) + V_{Coul}, \tag{24}$$

with electron-positron current $J^\mu(\vec{x}) = e\bar{\psi}(\vec{x})\gamma_\mu\psi(\vec{x})$ and Coulomb part,

$$V_{Coul} = \frac{1}{2}\int d\vec{x}\int d\vec{y}\frac{J^0(\vec{x})J^0(\vec{y})}{4\pi|\vec{x}-\vec{y}|}. \tag{25}$$

Evidently, the corresponding interaction density $V_{qed}(x)$ is not Lorentz scalar and we cannot use the so-called Belinfante ansatz to construct boost generator $\vec{N}$, i.e., put, for example,

$$\mathbf{N}_{qed} = -\int \mathbf{x} V_{qed}(\mathbf{x}) d\mathbf{x}, \tag{26}$$

therefore one has to seek other ways to provide relativistic invariance (RI) in Dirac sense (see, e.g., {SheFro12}).

Further, using Fourier expansions

$$V^{\mu}(\vec{x}) = \int \frac{d\vec{k}}{\sqrt{2(2\pi)^3\omega_{\vec{k}}}} \sum_{s} \left[e^{\mu}(\vec{k},s)a(\vec{k},s) + e^{\mu *}(-\vec{k},s)a^{\dagger}(-\vec{k},s)\right] \exp(i\vec{k}\vec{x}), \tag{27}$$

$$A^{\mu}(\vec{x}) = \int \frac{d\vec{k}}{\sqrt{2(2\pi)^3|\vec{k}|}} \sum_{\sigma} \left[e^{\mu}(\vec{k},\sigma)c(\vec{k},\sigma) + e^{\mu^*}(-\vec{k},\sigma)c^{\dagger}(-\vec{k},\sigma)\right] \exp(i\vec{k}\vec{x}), \tag{28}$$

$$\psi(\vec{x}) = \int d\vec{p}\sqrt{\frac{m}{(2\pi)^3E_{\vec{p}}}} \sum_{\mu} \left[\bar{u}(\vec{p}\mu)b(\vec{p}\mu) + v(-\vec{p}\mu)d^{\dagger}(-\vec{p}\mu)\right] \exp(i\vec{p}\vec{x}), \tag{29}$$

where $E_{\vec{p}} = \sqrt{\vec{p}^2 + m^2}$ fermion energy, one can express Hamiltonians (other generators of Poincaré group, currents, etc.) through set α so following prescriptio given above, we perform first clothing transformation $W^{(1)} = \exp[R^{(1)}]$ ($R^{(1)\dagger} = -R^{(1)}$), which eliminates primary interactions $V^{(1)}$ in first order in coupling constan assuming that these $V^{(1)}$ consist of bad terms only.

It is the case for vector mesons and photons with interactions in D picture

$$V^{(1)}(t) = \int d\vec{x} V^{(1)}(x) \equiv \int d\vec{x} H_{cov}(x) \tag{30}$$

$$V^{(1)}_{qed}(t) = \int d\vec{x} V^{(1)}_{qed}(x) \equiv \int d\vec{x} J^{\mu}(x) A_{\mu}(x) \tag{31}$$

that are trilinear in creation and annihilation operators involved.
Operator $R^{(1)}$ obeys equation for its finding

$$\left[R^{(1)}, H_F\right] + V^{(1)} = 0 \tag{32}$$

which has solution

$$R^{(1)} = -i \lim_{\varepsilon \to 0+} \int\limits_0^{\infty} V^{(1)}_D(t) e^{-\varepsilon t} dt \tag{33}$$

if $m_v < 2m$. Evidently, this inequality is valid with $m_v = m_\gamma$ and $m = m_{e^+} = m_{e^-} = m_e$. One should stress that from this moment all bare-particle operators α are replaced by clothed-particle counterparts.

For this presentation I will confine myself to consideration of $2 \to 2$ processes (in particular, electron-electron scattering). The corresponding interaction operato $V(ee \to ee)$ in CPR is determined by first term of decomposition

$$K_I(\alpha_c) = K(e^-e^- \to e^-e^-) + K(e^+e^+ \to e^+e^+) + K(e^+e^- \to e^+e^-) \\ + K(\gamma e^\pm \to \gamma e^\pm) + K(e^+e^- \leftrightarrow \gamma + \gamma) + K(e^-e^- \leftrightarrow \gamma\, e^-e^-) + \cdots . \quad (34)$$

that yields in e^2 order

$$V(ee \to ee) \equiv K^{(2)}(e^-e^- \to e^-e^-) = \frac{1}{2}\left[R^{(1)}, V^{(1)}\right](e^-e^- \to e^-e^-) \\ + V_{Coul}(e^-e^- \to e^-e^-). \quad (35)$$

and after a simple algebra we get for two clothed electrons

$$\frac{1}{2}\left[R^{(1)}, V^{(1)}\right](e^-e^- \to e^-e^-) = V_{ee} - V_{Coul}(ee), \quad (36)$$

$$V_{ee} \equiv K_\gamma(ee \to ee) = \int \sum_\mu d\vec{p}_1' d\vec{p}_2' d\vec{p}_1 d\vec{p}_2\, V_\gamma(1',2';1,2) b^\dagger(1') b^\dagger(2') b(1) b(2) \quad (37)$$

with c-number matrix

$$V_\gamma(1',2';1,2) = \frac{e^2}{(2\pi)^3}\frac{{m_e}^2}{\sqrt{E_{\vec{p}_1'}E_{\vec{p}_2'}E_{\vec{p}_1}E_{\vec{p}_2}}}\delta\left(\vec{p}_1'+\vec{p}_2'-\vec{p}_1-\vec{p}_2\right)v(1',2';1,2),$$

$$v(1',2';1,2) = \frac{1}{2}\frac{\bar{u}(\vec{p}_1')\gamma^\mu u(\vec{p}_1)\bar{u}(\vec{p}_2')\gamma_\mu u(\vec{p}_2)}{(p_1-p_1')^2-{m_\gamma}^2}.$$

When deriving these formulae we have used completeness condition for photon polarizations and representation,

$$V_{Coul}(e^-e^- \to e^-e^-) \equiv V_{Coul}(ee) = \int\sum_\mu d\vec{p}_1{}'d\vec{p}_2{}'d\vec{p}_1 d\vec{p}_2\, V_{Coul}(1',2';1,2)b^\dagger(1')b^\dagger(2')b(1)b(2), \quad (38)$$

$$V_{Coul}(1',2';1,2) = \frac{e^2}{(2\pi)^3}\frac{{m_e}^2}{\sqrt{E_{\vec{p}_1'}E_{\vec{p}_2'}E_{\vec{p}_1}E_{\vec{p}_2}}}\delta\left(\vec{p}_1'+\vec{p}_2'-\vec{p}_1-\vec{p}_2\right)v_{Coul}(1',2';1,2),$$

$$v_{Coul}(1',2';1,2) = -\frac{1}{2}\frac{\bar{u}(\vec{p}_1')\gamma^0 u(\vec{p}_1)\bar{u}(\vec{p}_2')\gamma^0 u(\vec{p}_2)}{(\vec{p}_1-\vec{p}_1')^2}$$

In order to preserve continuity with vector-meson-nucleon interactions we do not hurry to put $m_\gamma = 0$. Besides, keeping in mind problem of removing infrared divergence sometimes it is convenient to handle infinitesimally small photon mass.
It is time to quote from {Wein1} on p. 355, *viz*., " ... the apparent violation of Lorentz invariance in the instantaneous Coulomb interaction is cancelled by another apparent violation of Lorentz invariance, ... " that arises since photon fields $A_D^\mu(x)$ do not transform as four-vectors, "and therefore have a non-covariant propagator." An important point is that in CPR, unlike {Wein1} , such a cancellation (cf. our results {DuShe10} in mesodynamics) takes place directly in Hamiltonian .

Such a distinct feature of UCT method makes it useful in covariant calculations of S-matrix either by solving two-particle Lippmann-Schwinger equation (LSE) for corresponding T-matrix or using perturbation theory (not obligatorily addressin Dyson-Feynman expansion). In this context, I would like to note an akin approach to problems of relativistic QFT, developed in {[11] Stefanovich Ann. Phys. (2001)}.

Of course, doing so one can find not only S-matrix but eigenstates of operator $K = K_F + V_{ee}$ in Fock subspace $\mathcal{H}_F^{[2]}$ spanned onto clothed-two-particle K_F eigenvectors. In this connection, one has to deal with

$$K_F = \int d\vec{k}\, \omega_{\vec{k}} \sum_{\sigma} c^{\dagger}(\vec{k}\sigma)c(\vec{k}\sigma) + \int d\vec{p}\, E_{\vec{p}} \sum_{\mu} \left[b^{\dagger}(\vec{p},\mu)b(\vec{p},\mu) + d^{\dagger}(\vec{p},\mu)d(\vec{p},\mu) \right] \tag{39}$$

omitting for brevity lower index c at operators in r.h.s. of this expression.

QCD Hamiltonian in CG. Some Similarities

At last, I would like to drawing some parallels between QED and QCD, where we find (see, e.g., survey {Brod98} QCD on the Light Cone by S. Brodsky et al. in Phys. Rep. (1998) 301) QCD Lagrangian density

$$\mathcal{L}_{qcd}(x) = -\frac{1}{4}F_a^{\mu\nu}(x)F^a_{\mu\nu}(x) + \frac{1}{2}[\bar{\tilde{\Psi}}(x)(i\gamma_\mu\tilde{D}^\mu - \tilde{m})\tilde{\Psi}(x) + h.c.] - J^a_\mu(x)A^\mu_a(x) \quad (40)$$

with tensor of color-electro-magnetic fields $F_a^{\mu\nu} = \partial^\mu A_a^\nu - \partial^\nu A_a^\mu + igf^{ars}A_r^\mu A_s^\nu$ (gluon index $a(rs)$ from 1 to $n_c^2 - 1$), color vector potentials A_a^μ, color mass $\tilde{m}_{cc'} = m\delta_{cc'}$ and covariant color derivative $\tilde{D}^\mu_{cc'} = \delta_{cc'}\partial^\mu - ig\tilde{A}^\mu_{cc'}$ $n_c \otimes n_c$ matrices (color indices c and c' run from 1 to n_c), color-Maxwell $\partial_\mu F_a^{\mu\nu} = gJ_a^\nu$ and color-Dirac $(i\gamma_\mu\tilde{D}^\mu - \tilde{m})\tilde{\Psi} = 0$ equations for quark fields $q \equiv \tilde{\Psi}(x)$ in case of $SU(3)$ gauge model with conserved color currents $J_a^\mu = \bar{\tilde{\Psi}}\gamma_\mu T_a\tilde{\Psi} + f^{ars}F_r^{\mu\lambda}A_\lambda^s$, well-known matrices T_a act in color space and for $SU(3)$ are related to Gell-Mann matrices $T_a = \frac{1}{2}\lambda_a$
versus

QED Lagrangian density

$$\mathcal{L}_{qed}(x) = -\frac{1}{4}F^{\mu\nu}(x)F_{\mu\nu}(x) + \frac{1}{2}[\bar{\Psi}(x)(i\gamma_\mu D^\mu - m)\Psi(x) + h.c.] - J^\mu(x)A_\mu(x) \quad (41)$$

with Maxwell equations $\partial_\mu F^{\mu\nu} = gJ^\nu (g = e!)$ and conserved electron-positron current $J^\nu = \bar{\Psi}\gamma^\nu\Psi$ and Dirac equations $(i\gamma_\mu D^\mu - m)\Psi = 0$ with conventional $D^\mu = \partial^\mu - ieA^\mu$.

Going on, one gets color energy-momentum four-vector

$$\mathcal{P}^\nu_{qcd} = \int d\vec{x}(F^{0\lambda}_a F^\nu_{a,\lambda} - g^{0\nu}\mathcal{L}_{qcd} + \frac{1}{2}[i\bar{q}\gamma_0\tilde{D}^\nu q + h.c.]) \quad (42)$$

versus

$$\mathcal{P}^\nu_{qed} = \int d\vec{x}(F^{0\lambda}F^\nu_\lambda - g^{0\nu}\mathcal{L}_{qed} + \frac{1}{2}[i\bar{\psi}\gamma_0 D^\nu\psi + h.c.]) \quad (43)$$

All these quantities are gauge invariant, i.e., remain unchanged with respect to simultaneous transformations $\psi \Rightarrow \psi' = U\psi,\ A_\mu \Rightarrow A'_\mu = UA_\mu U^\dagger - \frac{i}{g}(\partial_\mu U)U^\dagger$ with unitary matrix operator $U = \exp(-ig\epsilon(x))$ (of course, in color space for such a non-abelian gauge theory as QCD).

Correspondingly, Hamiltonians are

$$\mathcal{P}^0_{qcd} = \int d\vec{x}(F^{0\lambda}_a F^{\nu}_{a,\lambda} - \mathcal{L}_{qcd} + \frac{1}{2}[i\bar{q}\gamma_0\tilde{D}^0 q + h.c.]) \quad (44)$$

and

$$\mathcal{P}^0_{qed} = \int d\vec{x}(F^{0\lambda} F^{\nu}_{\lambda} - \mathcal{L}_{qed} + \frac{1}{2}[i\bar{\psi}\gamma_0 D^0\psi + h.c.]) \quad (45)$$

From practical point of view it is convenient to employ CG in which $\partial^k A^a_k = 0$,

$$\mathcal{P}^0_{qcd} = H_F(\alpha) + H_I(\alpha) = H_F(\alpha) + V(\alpha) + \text{mass and vertex counterterms} \quad (46)$$

or introducing transverse color electric fields $E_k \equiv F^{(t)}_{k0}$ one has interaction density in S picture

$$V_{qcd}(\vec{x}) = -j^k_a(\vec{x})A^a_k(\vec{x}) - \frac{1}{2}F^{a(l)}_{k0}F^{k0(l)}_a + \frac{g}{2}\overset{\circ}{F}{}^a_{kn}[A^k, A^n]^a + \frac{g^2}{4}[A_k, A_n]^a[A^k, A^n]^a \quad (47)$$

where $\overset{\circ}{F}{}^{a}_{kn} = \partial_k A^a_n - \partial_n A^a_k$, $j^a_\mu = \bar{q}\gamma_\mu T_a q$, with constraint equation

$$\partial^k F^{a(l)}_{k0} + g[A^k, F^{(l)}_{k0}]^a = -j^a_0 + g[E_k, A^k] \tag{48}$$

Again in Fock representation we have addressed the corresponding set α from

$$A^\mu_a(\vec{x}) = \int \frac{d\vec{k}}{\sqrt{2(2\pi)^3|\vec{k}|}} \sum_\sigma \left[e^\mu(\vec{k},\sigma)c_a(\vec{k},\sigma) + e^{\mu^*}(-\vec{k},\sigma)c^\dagger_a(-\vec{k},\sigma)\right]\exp(i\vec{k}\vec{x}), \tag{49}$$

$$q_{fc}(\vec{x}) = \int d\vec{p}\sqrt{\frac{m}{(2\pi)^3 E_{\vec{p}}}} \sum_\mu \left[\bar{u}(\vec{p}\mu)b_{fc}(\vec{p}\mu) + v(-\vec{p}\mu)d^\dagger_{fc}(-\vec{p}\mu)\right]\exp(i\vec{p}\vec{x}), \tag{50}$$

with the flavor-color label fc, if we want, and canonical commutations

$$[c_a(\vec{k},\sigma), c^\dagger_{a'}(\vec{k}',\sigma')]_+ = \delta(\vec{k}-\vec{k}')\delta_{\sigma\sigma'}\delta_{aa'},$$

$$[b_{fc}(\vec{p}\mu), b^\dagger_{f'c'}(\vec{p}'\mu')]_- = [d_{fc}(\vec{p}\mu), d^\dagger_{f'c'}(\vec{p}'\mu')]_- = \delta(\vec{p}-\vec{p}')\delta_{\mu\mu'}\delta_{ff'}\delta_{cc'}$$

To Conclude

- Using Instant Form of Relativistic Dynamics and relying upon our previous experience we show applications the UCT method for popular field models of interacting mesons and nucleons, photons and electrons, gluons and quarks, etc.
- Mass-Changing Bogoliubov-type Transformations lead to Bare Particles with Given Mass Values
- Within our approach the two-clothed-electron interaction V_{ee} gets covariant form due to cancellation of non-covariant primary Coulomb interaction contribution to QED Hamiltonian. An important point is that in CPR, unlike {Wein1} , such a cancellation (cf. our results {DuShe10} in mesodynamics) takes place directly in Hamiltonian

- Trying to realize the notion of clothing in QCD, we start from well-known QCD Lagrangian density with hermitian and traceless vector potentials, mass and covariant derivative matrices in color space, color-Maxwell equations and color gauge-invariant energy-momentum stress tensor versus their colorless counterparts in QED. Of course, a long way lies ahead.
- A Novel Family of Relativistic Hermitean and Energy-Independent Interactions can be built with hellp of UCT method for each of these systems
- In addition, I'd like to stress: the clothing procedure opens a fresh look at calculations of Mass and Charge Shifts