THE DIRAC THEORY AND CONFINEMENT

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1 Proposal

Before to formulate our proposal we remind some definitions.The coordinate space R^4 is the space of 4-tuples of real numbers (q^1, q^2, q^3, q^4) with usual topology. The mathematical space is differential manifold M whose points are specified by set of one-to-one mappings between the open regions of M and R^4. When this is done the variables q^1, q^2, q^3, q^4 define the system of coordinates of M. It is evident that q^1, q^2, q^3, q^4 should be considered on equal footing. Space-time is a mathematical space whose points must be specified by both space and time coordinates. Our goal is to recognize a regular transition from a mathematical space to space-time and with this derive new information about nature of space and time. To this end we consider the Dirac equation in a mathematical space. A comparison will be produced of the Dirac theory of the electron with spin in a mathematical space and the original Dirac theory in the Minkowski space-time. New representations about nature of space, time, rotation, quark-lepton symmetry and confinement will be derived from this consideration.

2 General consideration

The Dirac equation in a mathematical space reads

$$i\gamma^\mu D_\mu \psi = m\psi, \tag{1}$$

where

$$D_\mu = E^i_\mu \partial_i = E^1_\mu \frac{\partial}{\partial q^1} + E^2_\mu \frac{\partial}{\partial q^2} + E^3_\mu \frac{\partial}{\partial q^3} + E^4_\mu \frac{\partial}{\partial q^4}$$

and E^i_μ are quadruplet of linear independent vector fields which will be considered as components of a tetrad field or simply tetrad. The tetrad has sixteen components.The world indices of vectors are denoted by Latin letters $i, j, k, \dots = 1, 2, 3, 4$, and the Greek letters enumerate the vector fields in question $\mu, \nu \dots = 0, 1, 2, 3$. Since

$$\gamma^\mu D_\mu = \gamma^0 D_0 + \gamma^1 D_1 + \gamma^2 D_2 + \gamma^3 D_3,$$

then to get a regular transition from a mathematical space to space-time we need to introduce the system of coordinates x^1, x^2, x^3, t in which the linear differential operator D_0 takes the form

$$D_0 = E^i_0 \partial_i \to \frac{\partial}{\partial t}$$

To this end let us consider the system of ordinary differential equation

$$\frac{dq^1}{dt} = E^1_0(q^1, q^2, q^3, q^4), \cdots, \frac{dq^4}{dt} = E^4_0(q^1, q^2, q^3, q^4).$$

It is well known, that this system has unique solution

$$q^1 = f_1(q_0^1, q_0^2, q_0^3, q_0^4, t), \cdots, q^4 = f_4(q_0^1, q_0^2, q_0^3, q_0^4, t),$$

which satisfy the condition

$$q_0^1 = f_1(q_0^1, q_0^2, q_0^3, q_0^4, t_0), \cdots, q_0^4 = f_4(q_0^1, q_0^2, q_0^3, q_0^4, t_0).$$

Let initial point $P(q_0^1, q_0^2, q_0^3, q_0^4)$ belongs to the 3d surface S which is parameterised by the coordinates x^1, x^2, x^3

$$q_0^1 = h_1(x^1, x^2, x^3), \cdots, q_0^4 = h_4(x^1, x^2, x^3)$$

The surface S should be chosen such that the variables x^1, x^2, x^3, t define a new system of coordinate in mathematical space and the tetrad take the following form

$$E_0^i = (0,0,0,1), \quad E_1^i = (E_1^1, E_1^2, E_1^3, 0), \quad E_2^i = (E_2^1, E_2^2, E_2^3, 0), \quad E_3^i = (E_3^1, E_3^2, E_3^3, 0).$$

The surface defined in such manner will be called the characteristic surface of space-time, the variables x^1, x^2, x^3 will be called space coordinates and correspondingly t time coordinate. The space-time is causal structure on a mathematical space. In the mathematical space equipped by the causal structure the equation (1) take the Hamiltonian form

$$i\frac{\partial}{\partial t}\psi = H\psi,$$

where operator H does not contain the partial derivative up to t. Now we will concretize this general consideration on the example of a simplest mathematical space that is evidently coordinate space R^4.

3 Symmetry and geometrical aspects of the simplest mathematical space

Points of R^4 have vector

$$\mathbf{q} = (q_1,\ q_2,\ q_3,\ q_4)$$

and quaternion representation

$$q = q_1 i + q_2 j + q_3 k + q_4 1,$$

with usual linear structure. The usual rules are

$$i^2 = j^2 = k^2 = -1, \quad i1 = 1i = i, j1 = 1j = j, k1 = 1k = k,$$

$$ij = -ji = k, jk = -kj = i, ki = -ik = j.$$

Scalar product

$$\mathbf{p} \cdot \mathbf{q} = p_1 q_1 + p_2 q_2 + p_3 q_3 + p_4 q_4$$

$$\mathbf{p} \cdot \mathbf{q} = \frac{1}{2}(p\overline{q} + q\overline{p}) = \frac{1}{2}(\overline{p}q + \overline{q}p), \tag{2}$$

where $\overline{q} = -q_1 i - q_2 j - q_3 k + q_4 1$. Scalar product is invariant with respect to the right and left turn dilatation

$$q \Rightarrow \tilde{q} = s\, q, \quad \Rightarrow \tilde{q} = q\, \bar{t}, \tag{3}$$

since

$$\tilde{\mathbf{p}} \cdot \tilde{\mathbf{q}} = s\overline{s}\, (\mathbf{p} \cdot \mathbf{q}), \quad \tilde{\mathbf{p}} \cdot \tilde{\mathbf{q}} = t\bar{t}\, (\mathbf{p} \cdot \mathbf{q}).$$

We suppose that q and λq, where λ is number are equivalent. For given q equations $q = sq, \quad q = q\bar{t}$ have only trivial solutions $s = \bar{t} = 1$ and this is fundamental property of group symmetry of the space in question.

Now we introduce two natural tetrad associated with mathematical space in question. The standard frame

$$\mathbf{c}_1 = (1,\ 0,\ 0,\ 0) \quad \mathbf{c}_2 = (0,\ 1,\ 0,\ 0)$$
$$\mathbf{c}_3 = (0,\ 0,\ 1,\ 0) \quad \mathbf{c}_4 = (0,\ 0,\ 0,\ 1)$$
$$c_1 = i,\ c_2 = j,\ c_3 = k,\ c_4 = 1$$

gives rise to the pair of right-handled moving frames

$$m_1 = iq,\ m_2 = jq,\ m_3 = kq,\ m_4 = 1q, \quad n_1 = qi,\ n_2 = qj,\ n_3 = qk,\ n_4 = q1.$$

$$\begin{array}{lcllllll}
\mathbf{m}_1 & = & (& q_4, & -q_3, & q_2, & -q_1 &) \\
\mathbf{m}_2 & = & (& q_3, & q_4, & -q_1, & -q_2 &) \\
\mathbf{m}_3 & = & (& -q_2, & q_1, & q_4, & -q_3 &) \\
\mathbf{m}_4 & = & (& q_1, & q_2, & q_3, & q_4 &)
\end{array}$$

$$\begin{array}{lcllllll}
\mathbf{n}_1 & = & (& q_4, & q_3, & -q_2, & -q_1 &) \\
\mathbf{n}_2 & = & (& -q_3, & q_4, & q_1, & -q_2 &) \\
\mathbf{n}_3 & = & (& q_2, & -q_1, & q_4, & -q_3 &) \\
\mathbf{n}_4 & = & (& q_1, & q_2, & q_3, & q_4 &)
\end{array}$$

It is easy to see that

$$\mathbf{m}_a \cdot \mathbf{m}_b = q\bar{q}\, \delta_{ab}, \quad \mathbf{n}_a \cdot \mathbf{n}_b = q\bar{q}\, \delta_{ab}, \quad (a, b = 1, 2, 3, 4).$$

Let us consider the support point $T(q_1,\, q_2,\, q_3,\, q_4)$, and the twelve coherent points

$$A(q_4,\, -q_3,\, q_2,\, -q_1), \quad B(q_3,\, q_4,\, -q_1,\, -q_2), \quad C(-q_2,\, q_1,\, q_4,\, -q_3),$$

$$K(q_4,\, q_3,\, -q_2,\, -q_1), \quad L(-q_3,\, q_4,\, q_1,\, -q_2), \quad M(q_2,\, -q_1,\, q_4,\, -q_3),$$
$$\bar{A}(-q_4,\, q_3,\, -q_2,\, q_1), \quad \bar{B}(-q_3,\, -q_4,\, q_1,\, q_2), \quad \bar{C}(q_2,\, -q_1,\, -q_4,\, q_3),$$
$$\bar{K}(-q_4,\, -q_3,\, q_2,\, q_1), \quad \bar{L}(q_3,\, -q_4,\, -q_1,\, q_2), \quad \bar{M}(-q_2,\, q_1,\, -q_4,\, q_3).$$

It is easy to see that

$$d^2_{AB} = d^2_{AC} = d^2_{BC} = d^2_{TA} = d^2_{TB} = d^2_{TC} = 2q\bar{q}$$

,

$$d^2_{\bar{A}\bar{B}} = d^2_{\bar{A}\bar{C}} = d^2_{\bar{B}\bar{C}} = d^2_{T\bar{A}} = d^2_{T\bar{B}} = d^2_{T\bar{C}} = 2q\bar{q}$$

, and,

$$d^2_{KL} = d^2_{KM} = d^2_{LM} = d^2_{TK} = d^2_{TL} = d^2_{TM} = 2q\bar{q},$$
$$d^2_{\bar{K}\bar{L}} = d^2_{\bar{K}\bar{M}} = d^2_{\bar{L}\bar{M}} = d^2_{T\bar{K}} = d^2_{T\bar{L}} = d^2_{T\bar{M}} = 2q\bar{q},$$

where d_{AB} is distance between the points A and B. We see pair of tetrahedrons and mirror one with common top T: $TABC$ and $TKLM$, $T\bar{A}\bar{B}\bar{C}$ and $T\bar{K}\bar{L}\bar{M}$. These tetrahedron give visual representation of frames in question $\mathbf{m}_\mu$ $\mathbf{n}_\mu$, and $-\mathbf{m}_\mu$ $-\mathbf{n}_\mu$, $\quad (\mu = 1, 2, 3)$.

Let $\mathbf{q} = \mathbf{q}(t)$ is trajectory in R^4. When point T moves along this trajectory the tetrahedrons $TABC$ and $TKLM$ are pulsed and rotated with respect to each other. And the same for the mirror tetrahedrons $T\bar{A}\bar{B}\bar{C}$ $T\bar{K}\bar{L}\bar{M}$.

The matrix of scalar products

$$\mathrm{P}_{\mu\nu} = \mathbf{m}_\mu \cdot \mathbf{n}_\nu, \quad (\mu,\, \nu = 1, 2, 3)$$

describes this movement.

The scalar products of tangent vector $\dot{\mathbf{q}} = d\mathbf{q}/dt$ with vectors of dual frames $\mathbf{m}_a$ $\mathbf{n}_a$, $(a = 1, 2, 3, 4)$

$$\mathbf{m}_1 \cdot \frac{d\mathbf{q}}{dt} = q_4\frac{dq_1}{dt} - q_3\frac{dq_2}{dt} + q_2\frac{dq_3}{dt} - q_1\frac{dq_4}{dt},$$
$$\mathbf{m}_2 \cdot \frac{d\mathbf{q}}{dt} = q_3\frac{dq_1}{dt} + q_4\frac{dq_2}{dt} - q_1\frac{dq_3}{dt} - q_2\frac{dq_4}{dt},$$
$$\mathbf{m}_3 \cdot \frac{d\mathbf{q}}{dt} = -q_2\frac{dq_1}{dt} + q_1\frac{dq_2}{dt} + q_4\frac{dq_3}{dt} - q_3\frac{dq_4}{dt},$$

$$\mathbf{n}_1 \cdot \frac{d\mathbf{q}}{dt} = q_4\frac{dq_1}{dt} + q_3\frac{dq_2}{dt} - q_2\frac{dq_3}{dt} - q_1\frac{dq_4}{dt},$$
$$\mathbf{n}_2 \cdot \frac{d\mathbf{q}}{dt} = -q_3\frac{dq_1}{dt} + q_4\frac{dq_2}{dt} + q_1\frac{dq_3}{dt} - q_2\frac{dq_4}{dt},$$
$$\mathbf{n}_3 \cdot \frac{d\mathbf{q}}{dt} = q_2\frac{dq_1}{dt} - q_1\frac{dq_2}{dt} + q_4\frac{dq_3}{dt} - q_3\frac{dq_4}{dt},$$
$$\mathbf{m}_4 \cdot \frac{d\mathbf{q}}{dt} = \mathbf{n}_4 \cdot \frac{d\mathbf{q}}{dt} = q_1\frac{dq_1}{dt} + q_2\frac{dq_2}{dt} + q_3\frac{dq_3}{dt} + q_4\frac{dq_4}{dt}$$

are invariant with respect to left and right turn dilatations. Invariants

$$\Omega_\mu = \frac{1}{2}\,\mathbf{m}_\mu \cdot \frac{d\mathbf{q}}{dt}, \quad \tilde{\Omega}_\mu = \frac{1}{2}\,\mathbf{n}_\mu \cdot \frac{d\mathbf{q}}{dt}, \quad (\mu = 1, 2, 3)$$

are components of angular velocity of rotation of tetrahedron $TABC$ with respect to tetrahedron $TKLM$. In the theory of rigid body Ω_μ and $\tilde{\Omega}_\mu$.

To quantize this theory we introduce the $4d$ operator ∇

$$\nabla_4 = (\frac{\partial}{\partial q_1}, \frac{\partial}{\partial q_2}, \frac{\partial}{\partial q_3}, \frac{\partial}{\partial q_4})$$

and setting

$$M_\nu = \frac{1}{2}(\mathbf{m}_\nu \cdot \nabla_4), \quad N_\nu = \frac{1}{2}(\mathbf{n}_\nu \cdot \nabla_4), \quad (\nu = 1, 2, 3)$$

do get six antihermitian operators of angular momentum. Factor $\frac{1}{2}$ is essential since natural commutation relations hold valid

$$M_1M_2 - M_2M_1 = M_3, \quad N_1N_2 - N_2N_1 = -N_3$$

and so on. Operator of dilatations

$$D = (\mathbf{m}_4 \cdot \nabla_4) = q_1\frac{\partial}{\partial q_1} + q_2\frac{\partial}{\partial q_2} + q_3\frac{\partial}{\partial q_3} + q_4\frac{\partial}{\partial q_4}$$

have important meaning as well since it commutes with the operators of angular momentum

$$DM_\nu - M_\nu D = 0, \quad DN_\nu - N_\nu D = 0, \quad (\nu = 1,\, 2,\, 3).$$

Now we shall introduce two natural tetrad associated with the mathematical space in question and consider the different form of transition from mathematical space to space-time.

4 Global tetrad

Let

$$\vec{a} = (a^1,\, a^2,\, a^3,\, a^4)$$

be a constant unit vector, then a global tetrad in R^4 is defined as follows

$$\vec{E}_0 = (a^1,\, a^2,\, a^3,\, a^4), \quad \vec{E}_1 = (-a^4,\, -a^3,\, a^2,\, a^1),$$
$$\vec{E}_2 = (a^3,\, -a^4,\, -a^1,\, a^2), \quad \vec{E}_3 = (-a^2,\, a^1,\, -a^4,\, a^3).$$

We put

$$D_0 = \vec{E}_0 \cdot \vec{\nabla}, \quad D_1 = \vec{E}_1 \cdot \vec{\nabla}, \quad D_2 = \vec{E}_2 \cdot \vec{\nabla}, \quad D_3 = \vec{E}_3 \cdot \vec{\nabla},$$

where

$$\vec{\nabla} = (\frac{\partial}{\partial q^1}, \frac{\partial}{\partial q^2}, \frac{\partial}{\partial q^3}, \frac{\partial}{\partial q^4}),$$

then the Dirac equation in the mathematical space R^4 reads

$$i\gamma^\mu D_\mu \psi = \frac{mc}{\hbar}\psi.$$

In accordance with general prescription let us consider the transition to space- time. First of all we need to solve the system of equation

$$\frac{dq^i}{dt} = a^i.$$

The general solution is a straight line that goes through the fixed point $\vec{q_0}$:

$$\vec{q}\,(t) = \vec{a}\,(t - t_0) + \vec{q_0}\,. \tag{4}$$

The 3d surface S in the space of initial data we define as follows

$$\vec{a} \cdot \vec{q}_0 = t_0. \tag{5}$$

The general solution to equation (5) has the form

$$\vec{q}_0 = t_0 \vec{E}_0 + x \vec{E}_1 + y \vec{E}_2 + z \vec{E}_3 .$$

Substituting this representation into formula (4) we get

$$\vec{q} = t \vec{E}_0 + x \vec{E}_1 + y \vec{E}_2 + z \vec{E}_3 .$$

The Dirac equation in the coordinates t, x, y, z has a ordinary form

$$i(\gamma^0 \frac{\partial}{\partial t} + \gamma^1 \frac{\partial}{\partial x} + \gamma^2 \frac{\partial}{\partial y} + \gamma^3 \frac{\partial}{\partial z})\psi = \frac{mc}{\hbar}\psi.$$

One can work in either the coordinates q^1, q^2, q^3, q^4 or in the coordinates t, x, y, z, but in the first case the physical results should not depend on the choice of the constant vector $\vec{a}$.

Now it is important to show how the transition from mathematical space in question to space-time is connected with notion of interval. The interval in the M^4 defined as follows. Let

$$\vec{q}_s = 2 \vec{a} (\vec{a} \cdot \vec{q}) - \vec{q}$$

be the vector symmetrical $\vec{q}$ with respect to the vector $\vec{a}$. Then in the coordinates q^1, q^2, q^3, q^4 the interval can be presented as follows:

$$s^2 = \vec{q} \cdot \vec{q}_s = 2(\vec{a} \cdot \vec{q})^2 - \vec{q} \cdot \vec{q} = -d(\vec{q} \cdot \vec{q}) \cos 2\theta = -q \cos 2\theta,$$

where θ is an angle between $\vec{a}$ and $\vec{q}$. It is easy to see that in the coordinates t, x, y, z,

$$s^2 = t^2 - x^2 - y^2 - z^2.$$

We see that the existence of natural global tetrad in the mathematical space presuppose the existence of Minkowski space-time.

5 Local tetrad

Let

$$\vec{q} = (q^1, q^2, q^3, q^4)$$

be a radius- vector, then a local tetrad in R^4 be the four orthogonal unit vector fields

$$\vec{E}_0 = (\frac{q^1}{\tau}, \frac{q^2}{\tau}, \frac{q^3}{\tau}, \frac{q^4}{\tau}), \quad \vec{E}_1 = (\frac{-q^4}{\tau}, \frac{-q^3}{\tau}, \frac{q^2}{\tau}, \frac{q^1}{\tau}),$$

$$\vec{E}_2 = (\frac{q^3}{\tau}, \frac{-q^4}{\tau}, \frac{-q^1}{\tau}, \frac{q^2}{\tau}), \quad \vec{E}_3 = (\frac{-q^2}{\tau}, \frac{q^1}{\tau}, \frac{-q^4}{\tau}, \frac{q^3}{\tau}),$$

where

$$\tau = \sqrt{(\vec{q} \cdot \vec{q})} = \sqrt{(q^1)^2 + (q^2)^2 + (q^3)^2 + (q^4)^2}.$$

We again put

$$D_0 = \vec{E}_0 \cdot \vec{\nabla}, \quad D_1 = \vec{E}_1 \cdot \vec{\nabla}, \quad D_2 = \vec{E}_2 \cdot \vec{\nabla}, \quad D_3 = \vec{E}_3 \cdot \vec{\nabla},$$

where

$$\vec{\nabla}=(\frac{\partial}{\partial q^1},\frac{\partial}{\partial q^2},\frac{\partial}{\partial q^3},\frac{\partial}{\partial q^4}),$$

and the Dirac equation of rotating matter has the form

$$i\gamma^\mu D_\mu\psi=\frac{mc}{\hbar}\psi. \tag{6}$$

Let us consider the transition from the mathematical space to space-time in this case. The general solution of the system of equations

$$\frac{dq^i}{d\tau}=\frac{u^i}{\sqrt{(u^1)^2+(u^2)^2+(u^3)^2+(u^4)^2}}$$

can be written as follows

$$q^i(\tau)=q_0^i\frac{\tau}{\tau_0}$$

. The 3d surface S in the space of initial data we define as follows

$$\vec{q}_0\cdot\vec{q}_0={\tau_0}^2.$$

This surface can be parameterized by the Euler angles, $\theta,\ \varphi,\gamma$ In the coordinates $\tau,\ \theta,\ \varphi,\gamma,$ $\theta,\ \varphi,\gamma$ we have

$$D_0=\frac{\partial}{\partial\tau},\quad D_1=\frac{1}{\tau}\Big(-\cot\theta\cos\gamma\frac{\partial}{\partial\gamma}-\sin\gamma\frac{\partial}{\partial\theta}+\frac{\cos\gamma}{\sin\theta}\frac{\partial}{\partial\varphi}\Big),$$

$$D_2=\frac{1}{\tau}\Big(-\cot\theta\sin\gamma\frac{\partial}{\partial\gamma}+\cos\gamma\frac{\partial}{\partial\theta}+\frac{\sin\gamma}{\sin\theta}\frac{\partial}{\partial\varphi}\Big),\quad D_3=\frac{1}{\tau}\frac{\partial}{\partial\gamma}.$$

The action for the point particle accosted with rotating can be written in the following form

$$S=-mc\int_p^q\sqrt{1-\tau^2\omega^2}d\tau,$$

where $\omega=dl/d\tau$ and dl is the element of arc on the unit $3d$ sphere. Really, $\vec{du}\cdot\vec{du}=d\tau^2+\tau^2dl^2$, and $\vec{u}\cdot\vec{du}=\tau d\tau$. On this ground we can develop the classical mechanics in new framework.

6 Local tetrad and Maxwell equation

Now we formulate the Maxwell equations in the framework of the new causal structure. Let A_i be the vector potential of the electromagnetic field. Let us define the gauge invariant tensor of electromagnetic field as usual $F_{ij}=\partial_iA_j-\partial_jA_i$. Strength of the electric field is a general covariant and gauge invariant quantity that is defined by the equation $E_i=t^kF_{ik}$, where in our case $t^k=t_k=q^k/\tau$.

A rotor of the vector field $\vec{A}$ is defined as a vector product of $\vec{\nabla}$ and $\vec{A}$

$$rot\ \vec{A}=\vec{\nabla}\times\vec{A},\quad (rot\ \vec{A})^i=e^{ijkl}t_j\partial_kA_l=\frac{1}{2}e^{ijkl}t_j(\partial_kA_l-\partial_lA_k),$$

where e^{ijkl} are contravariant components of the Levi-Chivita tensor normalized as $e_{1234} = \sqrt{g} = 1$. The general covariant and gauge invariant definition of the magnetic field strength is given by the formula $\vec{H}= rot\ \vec{A}$, $H^i = (rot\ \vec{A})^i$. Thus, $H_i = t^k \overset{*}{F}_{ik}$, where $\overset{*}{F}_{ij} = g_{ik}g_{jl}\overset{*}{F}^{kl}$. It is evident that vectors $\vec{E}$ and $\vec{H}$ are orthogonal to $\vec{q}$

$$\vec{q} \cdot \vec{E}= 0, \quad \vec{u} \cdot \vec{H}= 0.$$

Here we write the Maxwell equations in the form that is suitable for its solution:

$$(\vec{D}_0 \cdot \vec{\nabla})\ \vec{H} + \frac{2}{\tau}\ \vec{H}= -rot\ \vec{E}, \tag{7}$$

$$(\vec{D}_0 \cdot \vec{\nabla})\ \vec{E} + \frac{2}{\tau}\ \vec{E}= -rot\ \vec{H} + q\ \vec{J}, \tag{8}$$

$$\vec{\nabla} \cdot \vec{E}= e\bar{\psi}\gamma^0\psi, \quad \vec{\nabla} \cdot \vec{H}= 0, \tag{9}$$

where a current $\vec{J}$ is given by the expression

$$\vec{J}=\vec{D}_1\ \bar{\psi}\gamma^1\psi+ \vec{D}_2\ \bar{\psi}\gamma^2\psi+ \vec{D}_3\ \bar{\psi}\gamma^3\psi.$$

7 Conclusion

Thus, it is shown that on the simplest mathematical space two space-time can be defined. In one case the characteristic surface of space-time represents the 3d plane and congruence of curves is set of parallel straight lines. In another case the characteristic surface is 3d sphere and the congruence of curves is set of rays orthogonal to the 3d sphere. A physical interpretation: the behavior of leptons are defined by the Minkowski space-time and the physics of quarks is tightly connected with new space-time which represents rotating matter. Confinement is new causal structure