Non-Extensive Statistics Effects in Transverse Momentum Spectra of Charged Particles in High-Energy Collisions

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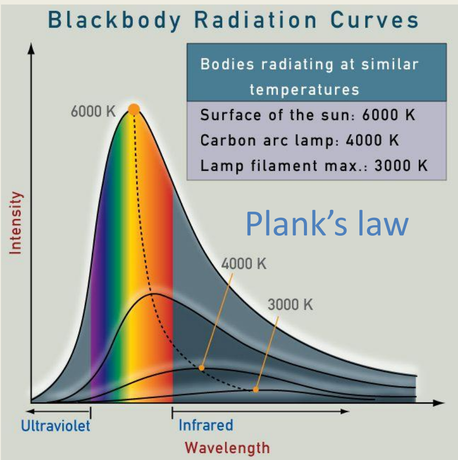
Quantum statistics of particles in high-energy collisions

Equilibrium emission of particles

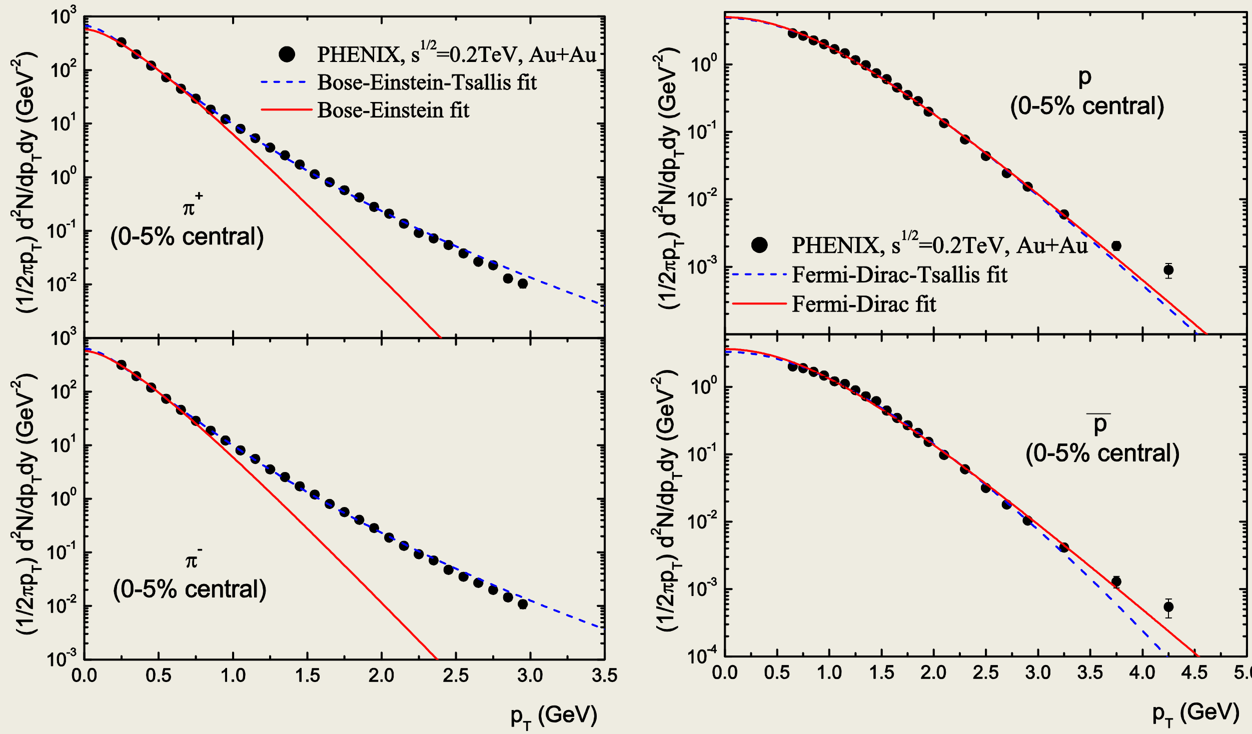


$$\frac{1}{e^{\frac{\varepsilon-\mu}{T}}+\eta} \qquad \eta=-1 \text{ for bosons} \qquad \eta=1 \text{ for fermions}$$

- The transverse momentum distributions for bosons at RHIC violate the Bose-Einstein statistics
- The transverse momentum distributions for fermions at RHIC are surprisingly in a good agreement with Fermi-Dirac statistics

Emission of particles in AA and pp collisions

bosons | fermions



Transverse momentum spectra

The usual Boltzmann-Gibbs statistics

$$\frac{1}{2\pi p_T}\frac{d^2N}{dp_T dy}=\frac{gV}{(2\pi)^3}\frac{m_T\cosh y}{e^{\frac{m_T\cosh y-\mu}{T}}+\eta},$$

The Tsallis-like (Cleymans) statistics

$$\frac{1}{2\pi p_T}\frac{d^2N}{dp_T dy}=\frac{gV}{(2\pi)^3}\frac{m_T\cosh y}{\left(\left[1+(q-1)\frac{m_T\cosh y-\mu}{T}\right]^{\frac{1}{q-1}}+\eta\right)^q}$$

$\eta=1$ for Fermi-Dirac statistics, $\eta=0$ for Maxwell-Boltzmann statistics, $\eta=-1$ for Bose-Einstein statistics

Tsallis statistical mechanics

Tsallis statistics

$$S = -k_B \sum_i \frac{p_i - p_i^q}{1-q}, \qquad \sum_i p_i = 1, \qquad 0 < q < \infty$$

$$\langle A \rangle = \sum_i p_i A_i$$

C. Tsallis, J. Stat. Phys. 52 (1988) 479

Grand Canonical Ensemble (T,V,z,μ)

$$p_i = \left[1 + \frac{1}{z+1} \frac{\Lambda - E_i + \mu N_i}{k_B T}\right]^z, \qquad z = \frac{1}{q-1}$$

$$\sum_i \left[1 + \frac{1}{z+1} \frac{\Lambda - E_i + \mu N_i}{k_B T}\right]^z = 1$$

$$\langle A \rangle = \sum_i A_i \left[1 + \frac{1}{z+1} \frac{\Lambda - E_i + \mu N_i}{k_B T}\right]^z$$

Canonical Ensemble (T,V,z,N)

A.S.P., Phys.Lett. A360 (2006) 26

$$p_i = \left[1 + \frac{1}{z+1} \frac{\Lambda - E_i}{k_B T}\right]^z, \qquad z = \frac{1}{q-1}$$

$$\sum_i \left[1 + \frac{1}{z+1} \frac{\Lambda - E_i}{k_B T}\right]^z = 1$$

$$\langle A \rangle = \sum_i A_i \left[1 + \frac{1}{z+1} \frac{\Lambda - E_i}{k_B T}\right]^z$$

Boltzmann-Gibbs statistics

$$S = -k_B \sum_i p_i \ln p_i, \qquad \sum_i p_i = 1$$

$$\langle A \rangle = \sum_i p_i A_i$$

$$p_i = e^{\frac{\Omega - E_i + \mu N_i}{k_B T}}$$

$$\sum_i e^{\frac{\Omega - E_i + \mu N_i}{k_B T}} = 1, \qquad \left(e^{-\frac{\Omega}{k_B T}} = \sum_i e^{-\frac{E_i - \mu N_i}{k_B T}} \equiv Z_\Omega \right)$$

$$\langle A \rangle = \sum_i A_i e^{\frac{\Omega - E_i + \mu N_i}{k_B T}}$$

$$p_i = e^{\frac{F - E_i}{k_B T}}$$

$$\sum_i e^{\frac{F - E_i}{k_B T}} = 1, \qquad \left(e^{-\frac{F}{k_B T}} = \sum_i e^{-\frac{E_i}{k_B T}} \equiv Z_N \right)$$

$$\langle A \rangle = \sum_i A_i e^{\frac{F - E_i}{k_B T}}$$

Tsallis distributions: Grand canonical ensemble (T,V,z,μ)

$$\frac{d^2N}{dp_T dy} = \frac{V}{h^3} p_T \int_0^{2\pi} d\varphi \varepsilon_{\vec{p}} \langle n_{\vec{p}} \rangle, \qquad \frac{dN}{dy} = \frac{V}{h^3} \int_0^{\infty} p_T dp_T \int_0^{2\pi} d\varphi \varepsilon_{\vec{p}} \langle n_{\vec{p}} \rangle$$

Tsallis statistics

Exact results:

$$\langle n_{\vec{p}} \rangle = \frac{1}{\Gamma(-z)} \int_0^{\infty} dt t^{-z-1} e^{-t+\beta'[\Lambda-\Omega_G(\beta')]} \langle n_{\vec{p}} \rangle_G (\beta'), \qquad z<0$$

$$\frac{1}{\Gamma(-z)} \int_0^{\infty} dt t^{-z-1} e^{-t+\beta'[\Lambda-\Omega_G(\beta')]} = 1$$

$$\langle n_{\vec{p}} \rangle = \Gamma(z+1) \frac{i}{2\pi} \oint_c dt (-t)^{-z-1} e^{-t+\beta'[\Lambda-\Omega_G(\beta')]} \langle n_{\vec{p}} \rangle_G (\beta'), \qquad z>0$$

$$\Gamma(z+1) \frac{i}{2\pi} \oint_c dt (-t)^{-z-1} e^{-t+\beta'[\Lambda-\Omega_G(\beta')]} = 1$$

$$\beta' = -\frac{t\beta}{z+1}, \qquad \beta = \frac{1}{k_B T}$$

Factorization approximation:

$$\langle n_{\vec{p}} \rangle = g \frac{1}{\left[1-\frac{\beta}{(z+1)\lambda}(\varepsilon_{\vec{p}}-\mu)\right]^{-z}+\eta}, \qquad \left(\Lambda = \frac{z+1}{\beta}(\lambda-1)\right)$$

$$z \ln \lambda = -\frac{g}{\eta} \sum_{\vec{p}} \ln\left\{1+\eta\left[1-\frac{\beta}{(z+1)\lambda}(\varepsilon_{\vec{p}}-\mu)\right]^{z}\right\}$$

Boltzmann-Gibbs statistics

Exact results:

$$\Omega_G(\beta) = -\frac{g}{\beta\eta} \sum_{\vec{p}} \ln\left[1+\eta e^{-\beta(\varepsilon_{\vec{p}}-\mu)}\right]$$

$$\langle n_{\vec{p}} \rangle_G (\beta) = g \frac{1}{e^{\beta(\varepsilon_{\vec{p}}-\mu)}+\eta}$$

$\eta = 1$ for Fermi-Dirac statistics

$\eta = 0$ for Maxwell-Boltzmann statistics

$\eta = -1$ for Bose-Einstein statistics

$$\varepsilon_{\vec{p}} = \sqrt{\vec{p}^2+m^2} = \sqrt{p_T^2+m^2} \cosh y$$

$$z = \frac{1}{q-1}$$

Tsallis distributions: Canonical ensemble (T,V,z,N)

Tsallis statistics

Exact results:

$$\langle n_{\vec p}\rangle = \frac{1}{\Gamma(-z)}\int_0^\infty dt\, t^{-z-1} e^{-t+\beta'[\Lambda - F_G(\beta')]} \langle n_{\vec p}\rangle_{G,N}(\beta'), \qquad z<0$$

$$\frac{1}{\Gamma(-z)}\int_0^\infty dt\, t^{-z-1} e^{-t+\beta'[\Lambda - F_G(\beta')]} = 1$$

$$\langle n_{\vec p}\rangle = \Gamma(z+1)\frac{i}{2\pi}\oint_c dt(-t)^{-z-1} e^{-t+\beta'[\Lambda - F_G(\beta')]} \langle n_{\vec p}\rangle_{G,N}(\beta'), \qquad z>0$$

$$\Gamma(z+1)\frac{i}{2\pi}\oint_c dt(-t)^{-z-1} e^{-t+\beta'[\Lambda - F_G(\beta')]} = 1$$

$$\beta' = -\frac{t\beta}{z+1}, \qquad \beta = \frac{1}{k_B T}$$

Factorization approximation:

$$\langle n_{\vec p}\rangle = \frac{1}{Z_N}\sum_{l=1}^{N}(-\eta)^{l+1} g\left[1-\frac{\beta\varepsilon_{\vec p}}{(z+1)\lambda}\right]^{zl} Z_{N-l}, \qquad Z_0 = 1$$

$$z\ln\lambda = -\ln Z_N, \quad Z_N = \frac{1}{N}\sum_{l=1}^{N}\left((-\eta)^{l+1} g\sum_{\vec p}\left[1-\frac{\beta\varepsilon_{\vec p}}{(z+1)\lambda}\right]^{zl}\right) Z_{N-l}$$

$$\langle n_{\vec p}\rangle = N\frac{\left[1-\beta\varepsilon_{\vec p}/(z+1)\lambda\right]^{z}}{\sum_{\vec p}\left[1-\beta\varepsilon_{\vec p}/(z+1)\lambda\right]^{z}} \quad \text{for Maxwell-Boltzmann stat.}$$

$$Z_N = \frac{1}{N!}\left(g\sum_{\vec p}\left[1-\beta\varepsilon_{\vec p}/(z+1)\lambda\right]^{z}\right)^{N}$$

Boltzmann-Gibbs statistics

Exact results:

$$\langle n_{\vec p}\rangle_{G,N}(\beta) = \frac{1}{Z_{G,N}}\sum_{l=1}^{N}(-\eta)^{l+1} g e^{-\beta l\varepsilon_{\vec p}} Z_{G,N-l}, \qquad Z_{G,0} = 1$$

$$e^{-\beta F_G(\beta)} \equiv Z_{G,N} = \frac{1}{N}\sum_{l=1}^{N}\left((-\eta)^{l+1} g\sum_{\vec p} e^{-\beta l\varepsilon_{\vec p}}\right) Z_{G,N-l}$$

$$\langle n_{\vec p}\rangle_{G,N}(\beta) = N\frac{e^{-\beta\varepsilon_{\vec p}}}{\sum_{\vec p} e^{-\beta\varepsilon_{\vec p}}} \quad \text{for Maxwell-Boltzmann statistics}$$

$$Z_{G,N} = \frac{1}{N!}\left(g\sum_{\vec p} e^{-\beta\varepsilon_{\vec p}}\right)^{N}$$

$\eta = 1$ for Fermi-Dirac statistics

$\eta = -1$ for Bose-Einstein statistics

Ultra-relativistic Maxwell-Boltzmann distribution: Canonical ensemble (T,V,z,N)

$$\varepsilon_{\vec{p}} = |\vec{p}|, \qquad m = 0$$

Tsallis statistics

Exact results:

$$\langle n_{\vec{p}} \rangle = \pi^2 \beta^3 \frac{N}{V} \xi_1 \left[1 + \frac{\beta}{z+1} \left(\Lambda - \varepsilon_{\vec{p}} \right) \right]^{z+3(N-1)}$$

$$\xi_1 = Z_{G,N} \frac{(-z-1)^{3(N-1)} \Gamma(-z-3(N-1))}{\Gamma(-z)}, \qquad z < 0$$

$$\xi_1 = Z_{G,N} \frac{(z+1)^{3(N-1)} \Gamma(z+1)}{\Gamma(z+1+3(N-1))}, \qquad z > 0$$

$$1 + \frac{\beta\Lambda}{z+1} = \left[Z_{G,N}^{-1} \frac{\Gamma(-z)}{(-z-1)^{3N} \Gamma(-z-3N)} \right]^{\frac{1}{z+3N}}, \qquad z < 0$$

$$1 + \frac{\beta\Lambda}{z+1} = \left[Z_{G,N}^{-1} \frac{\Gamma(z+1+3N)}{(z+1)^{3N} \Gamma(z+1)} \right]^{\frac{1}{z+3N}}, \qquad z > 0$$

Boltzmann-Gibbs statistics

$$\langle n_{\vec{p}} \rangle_{G,N} = \pi^2 \beta^3 \frac{N}{V} e^{-\beta \varepsilon_{\vec{p}}}$$

$$Z_{G,N} = \frac{1}{N!} \left(\frac{gV}{\pi^2 \beta^3} \right)^N$$

$$\frac{d^2 N}{dp_T dy} = \frac{V}{h^3} 2\pi p_T \varepsilon_{\vec{p}} \langle n_{\vec{p}} \rangle, \qquad \frac{dN}{dy} = \frac{V}{h^3} 2\pi \int_0^\infty p_T dp_T \varepsilon_{\vec{p}} \langle n_{\vec{p}} \rangle$$

Homogeneity properties of Tsallis statistics (extensivity). Statistical ensembles

Equilibrium Thermodynamics!

Variables of state: $(x_1,\ldots,x_n)$

Thermodynamic potential: $f(x)=f(x_1,\ldots,x_n)$

Changes in thermodynamic potential: $\Delta f = df + \frac{1}{2!}d^2 f + \frac{1}{3!}d^3 f + \ldots$

First-order differential: $df = \sum_{i=1}^{n} u_i dx_i$

Observables (First order derivatives) $u_i = \frac{\partial f}{\partial x_i}$

Second-order differential: $d^2 f = \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij} dx_i dx_j$

Observables (Second order derivatives) $a_{ij} = a_{ji} = \frac{\partial^2 f}{\partial x_i \partial x_j} = \frac{\partial u_j}{\partial x_i} = \frac{\partial u_i}{\partial x_j}$

Class of homogeneous functions

$(x_1,\ldots,x_m)$ - extensive variables of state, $(x_{m+1},\ldots,x_n)$ - intensive variables of state

The homogeneous functions of first order (extensive):

$$f_1(\lambda x_1,\ldots,\lambda x_m, x_{m+1},\ldots,x_n) = \lambda f_1(x_1,\ldots,x_m,x_{m+1},\ldots,x_n), \qquad f_1 = (f, u_i, a_{ij})$$

$$\sum_{i=1}^{m} \frac{\partial f_1}{\partial x_i} = f_1$$ - the Euler theorem for extensive functions

The homogeneous function of zero order (intensive):

$$f_0(\lambda x_1,\ldots,\lambda x_m, x_{m+1},\ldots,x_n) = f_0(x_1,\ldots,x_m,x_{m+1},\ldots,x_n), \qquad f_0 = (u_i, a_{ij})$$

$$\sum_{i=1}^{m} \frac{\partial f_0}{\partial x_i} = 0$$ - the Euler theorem for intensive functions

Homogeneity properties of Tsallis statistics (extensivity). Non-relativistic Maxwell-Boltzmann ideal gas: Microcanonical ensemble

Thermodynamic limit $N \to \infty, E \to \infty, V \to \infty, z \to \infty, \varepsilon = \frac{E}{N} = const, v = \frac{V}{N} = const, \tilde{z} = \frac{z}{N} = const$

Tsallis statistics (E, V, z, N)

Non-relativistic Tsallis statistics is extensive!

$$\frac{S(E,V,z,N)}{N} = s(\varepsilon, v, \tilde{z}) = \tilde{z}\left[1 - w^{-1/\tilde{z}}\right]$$

$$T(E,V,z,N) = T(\varepsilon, v, \tilde{z}) = \frac{2}{3}\varepsilon w^{1/\tilde{z}}$$

$$p(E,V,z,N) = p(\varepsilon, v, \tilde{z}) = \frac{2}{3}\frac{\varepsilon}{v}$$

$$\mu(E,V,z,N) = \mu(\varepsilon, v, \tilde{z}) = \frac{2}{3}\varepsilon\left[\frac{5}{2} - \ln w\right]$$

$$X(E,V,z,N) = X(\varepsilon, v, \tilde{z}) = -\frac{2}{3}\varepsilon\left[1 + \frac{1}{\tilde{z}}\ln w - w^{1/\tilde{z}}\right]$$

$TS = E + pV + Xz - \mu N$ - Euler theorem

A.S.P., Phys.Lett. A350 (2006) 331

Boltzmann-Gibbs statistics (E, V, N)

$$W^{1/N}(E,V,N) = w(\varepsilon, v) = v\left(\frac{m\varepsilon e^{5/3}}{3\pi}\right)^{3/2}$$

$$\frac{S(E,V,N)}{N} = s(\varepsilon, v) = \ln w$$

$$T(E,V,N) = T(\varepsilon, v) = \frac{2}{3}\varepsilon$$

$$p(E,V,N) = p(\varepsilon, v) = \frac{2}{3}\frac{\varepsilon}{v}$$

$$\mu(E,V,N) = \mu(\varepsilon, v) = \frac{2}{3}\varepsilon\left[\frac{5}{2} - \ln w\right], \qquad \tilde{z} \to \infty$$

$TS = E + pV - \mu N$ - Euler theorem

Homogeneity properties of Tsallis statistics (extensivity). Non-relativistic Maxwell-Boltzmann ideal gas: Canonical ensemble

Thermodynamic limit $N \to \infty, V \to \infty, z \to \infty, v = \frac{V}{N} = const, \tilde{z} = \frac{z}{N} = const$

Tsallis statistics (T,V,z,N)

Non-relativistic Tsallis statistics is extensive!

$$\frac{F(T,V,z,N)}{N} = f(T,v,\tilde{z})$$

$$= -T\tilde{z}\left[1-\left(\tilde{Z}_G e^{3/2}\right)^{-\frac{1}{\tilde{z}+3/2}}\left(1+\frac{3}{2\tilde{z}}\right)\right]$$

$$\frac{E(T,V,z,N)}{N} = \varepsilon(T,v,\tilde{z}) = \frac{3}{2}T\left(\tilde{Z}_G e^{3/2}\right)^{-\frac{1}{\tilde{z}+3/2}}$$

$$\frac{S(T,V,z,N)}{N} = s(T,v,\tilde{z}) = \tilde{z}\left[1-\left(\tilde{Z}_G e^{3/2}\right)^{-\frac{1}{\tilde{z}+3/2}}\right]$$

$$p(T,V,z,N) = p(T,v,\tilde{z}) = \frac{T}{v}\left(\tilde{Z}_G e^{3/2}\right)^{-\frac{1}{\tilde{z}+3/2}}$$

$$\mu(T,V,z,N) = \mu(T,v,\tilde{z})$$

$$= T\left(\tilde{Z}_G e^{3/2}\right)^{-\frac{1}{\tilde{z}+3/2}}\left[\frac{5}{2}+\tilde{z}\ln\left(\tilde{Z}_G e^{3/2}\right)^{-\frac{1}{\tilde{z}+3/2}}\right]$$

$$X(T,V,z,N) = X(T,v,\tilde{z})$$

$$= T\left[1-\left(\tilde{Z}_G e^{3/2}\right)^{-\frac{1}{\tilde{z}+3/2}}\left(1-\ln\left(\tilde{Z}_G e^{3/2}\right)^{-\frac{1}{\tilde{z}+3/2}}\right)\right]$$

$TS = E + pV + Xz - \mu N$ - Euler theorem

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Boltzmann-Gibbs statistics (T,V,N)

$$Z_G^{1/N}(T,V,N) = \tilde{Z}_G(T,v) = (gve)\left(\frac{mT}{2\pi}\right)^{3/2}$$

$$\frac{F(T,V,N)}{N} = f(T,v) = -T\ln\tilde{Z}_G$$

$$\frac{E(T,V,N)}{N} = \varepsilon(T,v) = \frac{3}{2}T$$

$$\frac{S(T,V,N)}{N} = s(T,v) = \frac{3}{2}+\ln\tilde{Z}_G$$

$$p(T,V,N) = p(T,v) = \frac{T}{v}$$

$$\mu(T,V,N) = \mu(T,v) = T\left[1-\ln\tilde{Z}_G\right] \qquad \tilde{z}\to\infty$$

$TS = E + pV - \mu N$ - Euler theorem

Tsallis-like Statistics

(In collaboration with J. Cleymans, G.I. Lykasov, A.S. Sorin, O.V. Teryaev and D. Worku)

Tsallis-like statistics of grand canonical ensemble

J. Cleymans, D. Worku,
J. Phys. G: Nucl. Part. Phys. 39 (2012) 025006

$$\varepsilon_{\vec{p}} = \sqrt{\vec{p}^2 + m^2} = \sqrt{p_T^2 + m^2}\cosh y$$

Tsallis-like statistics

$$S = -g\sum_{\vec{p}}\left[\langle n_{\vec{p}}\rangle^q \ln_q \langle n_{\vec{p}}\rangle + \frac{1}{\eta}\left(1-\eta\langle n_{\vec{p}}\rangle\right)^q \ln_q\left(1-\eta\langle n_{\vec{p}}\rangle\right)\right]$$

$$S = -g\sum_{\vec{p}}\left[\langle n_{\vec{p}}\rangle^q \ln_q \langle n_{\vec{p}}\rangle - \langle n_{\vec{p}}\rangle\right] \text{ - for Maxwell - Boltzmann stat.}$$

$$g\sum_{\vec{p}}\langle n_{\vec{p}}\rangle^q = N$$

$$g\sum_{\vec{p}}\langle n_{\vec{p}}\rangle^q \varepsilon_{\vec{p}} = E$$

$$\langle n_{\vec{p}}\rangle = \frac{1}{\exp_q\left(\frac{\varepsilon_{\vec{p}}-\mu}{T}\right)+\eta}$$

$$\ln_q(x) \equiv \frac{x^{1-q}-1}{1-q}, \qquad \exp_q(x) \equiv \begin{cases}\left[1+(q-1)x\right]^{\frac{1}{q-1}}, & x>0\\ \left[1+(1-q)x\right]^{\frac{1}{1-q}}, & x\le 0\end{cases}$$

Boltzmann-Gibbs statistics

$$S = -g\sum_{\vec{p}}\left[\langle n_{\vec{p}}\rangle \ln \langle n_{\vec{p}}\rangle + \frac{1}{\eta}\left(1-\eta\langle n_{\vec{p}}\rangle\right) \ln\left(1-\eta\langle n_{\vec{p}}\rangle\right)\right]$$

$$S = -g\sum_{\vec{p}}\left[\langle n_{\vec{p}}\rangle \ln \langle n_{\vec{p}}\rangle - \langle n_{\vec{p}}\rangle\right] \text{- for Maxwell - Boltzmann stat.}$$

$$g\sum_{\vec{p}}\langle n_{\vec{p}}\rangle = N$$

$$g\sum_{\vec{p}}\langle n_{\vec{p}}\rangle \varepsilon_{\vec{p}} = E$$

$$\langle n_{\vec{p}}\rangle = \frac{1}{e^{\frac{\varepsilon_{\vec{p}}-\mu}{T}}+\eta}$$

$\eta = 1$ for Fermi - Dirac statistics

$\eta = 0$ for Maxwell - Boltzmann statistics

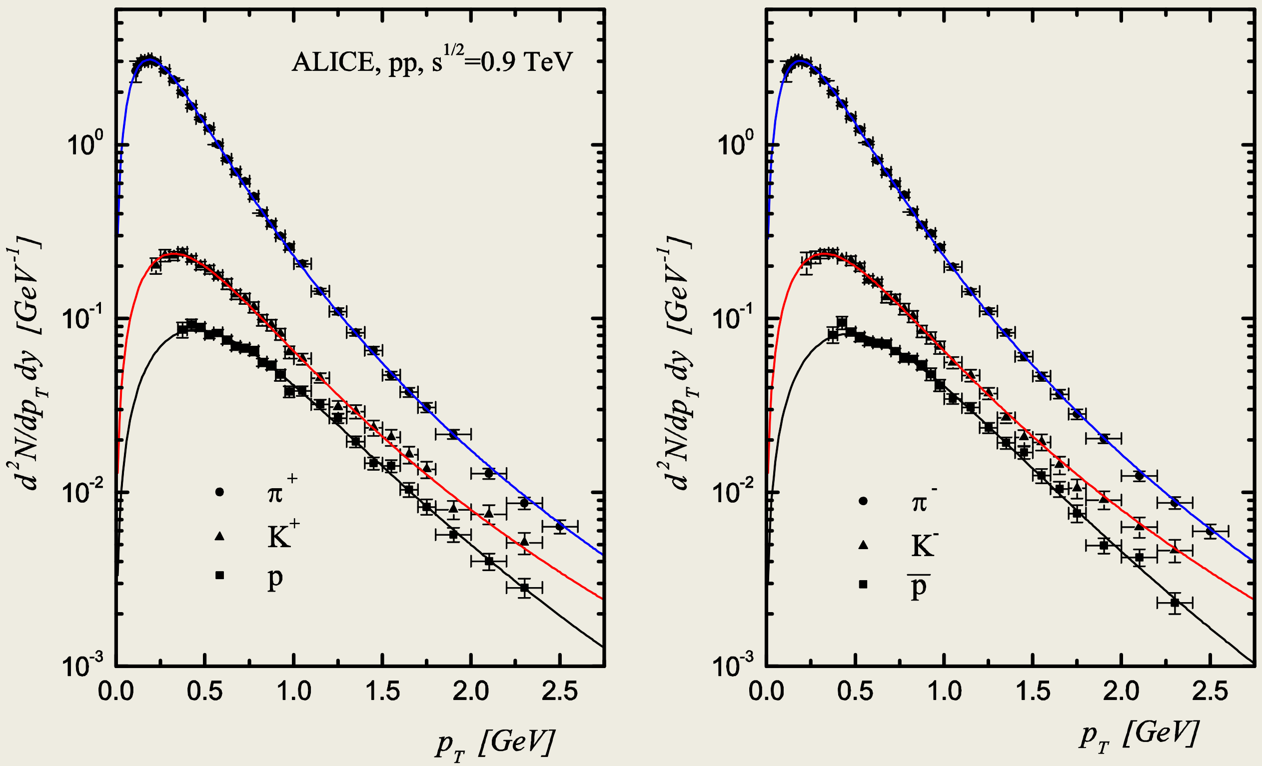
$\eta = -1$ for Bose - Einstein statistics

$$\frac{d^2N}{dp_T dy} = \frac{gV}{(2\pi)^2} p_T \varepsilon_{\vec{p}} \langle n_{\vec{p}}\rangle^q = \frac{gV}{(2\pi)^2}\frac{p_T m_T \cosh y}{\left(\left[1+(q-1)\frac{m_T\cosh y-\mu}{T}\right]^{\frac{1}{q-1}}+\eta\right)^q} \text{ - transverse momentum distribution}$$

$$\frac{dN}{dy} = \frac{gV}{(2\pi)^2}\int_0^\infty p_T dp_T \varepsilon_{\vec{p}} \langle n_{\vec{p}}\rangle^q = \frac{gV}{(2\pi)^2}\int_0^\infty p_T dp_T \frac{m_T \cosh y}{\left(\left[1+(q-1)\frac{m_T\cosh y-\mu}{T}\right]^{\frac{1}{q-1}}+\eta\right)^q} \text{ - rapidity distribution}$$

ALICE Transverse Momentum Spectra

Tsallis-like statistics fit (Maxwell-Boltzmann statistics, η=0)



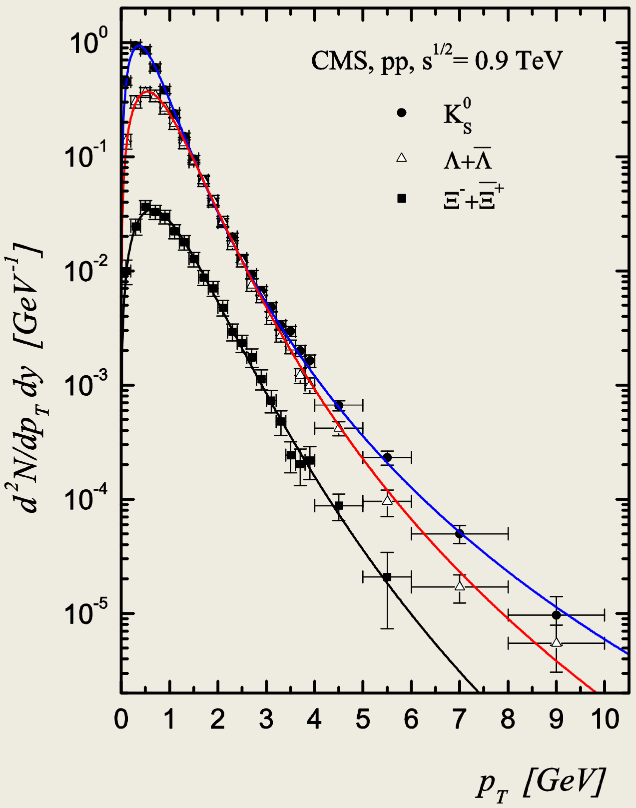
The difference between Tsallis statistics and Tsallis-like one:

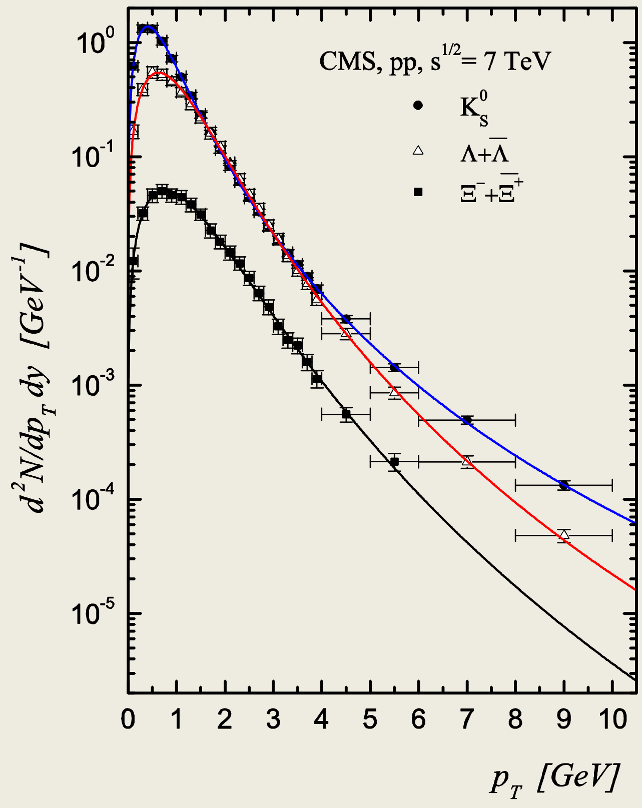
- In the Tsallis statistics the power law form is initially defined for many-body distribution function
- But in the Tsallis-like statistics the power law form is initially defined for one-particle distribution function.

Loose of correlations!

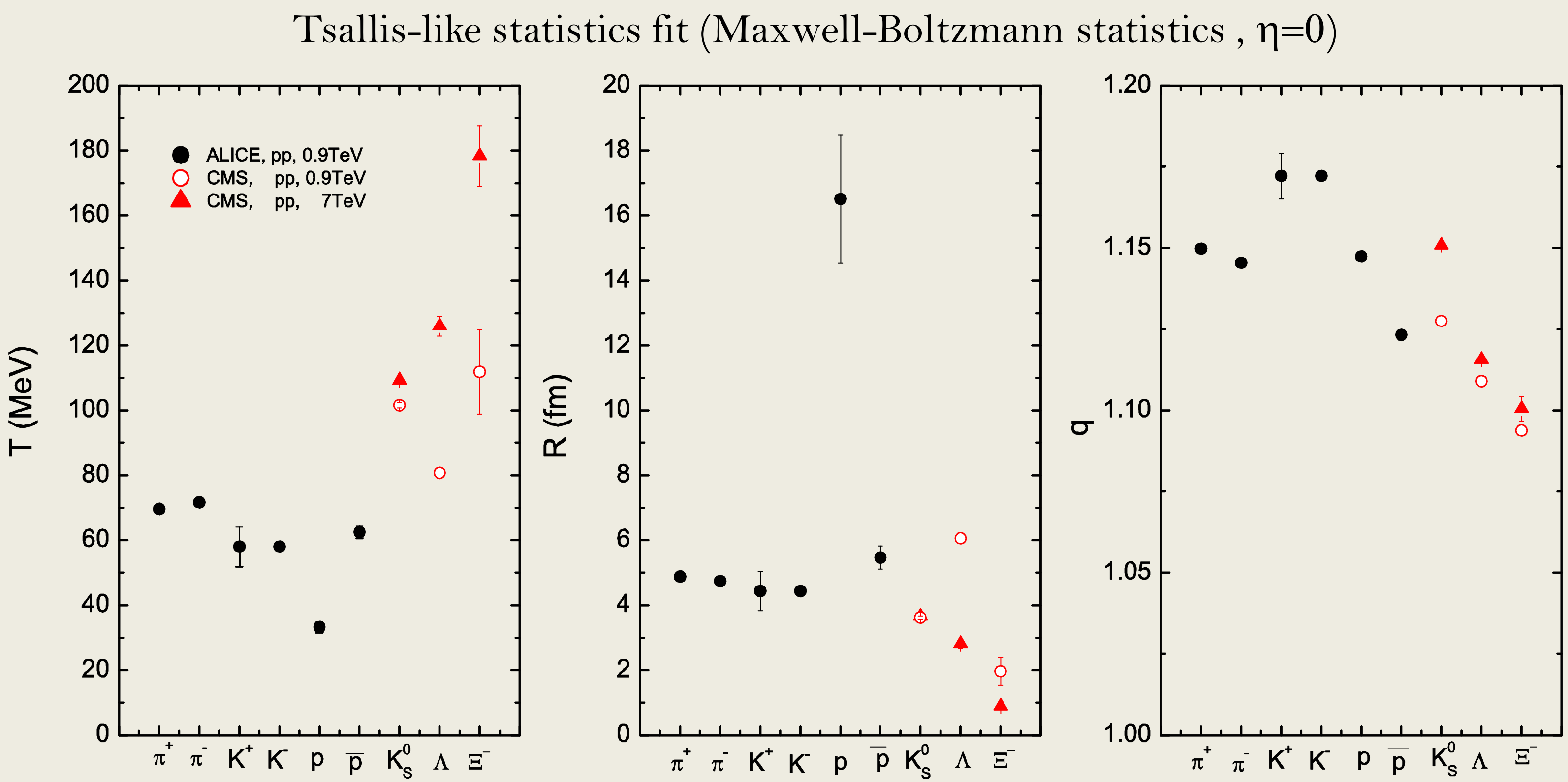
CMS Transverse Momentum Spectra

Tsallis-like statistics fit (Maxwell-Boltzmann statistics , η=0)





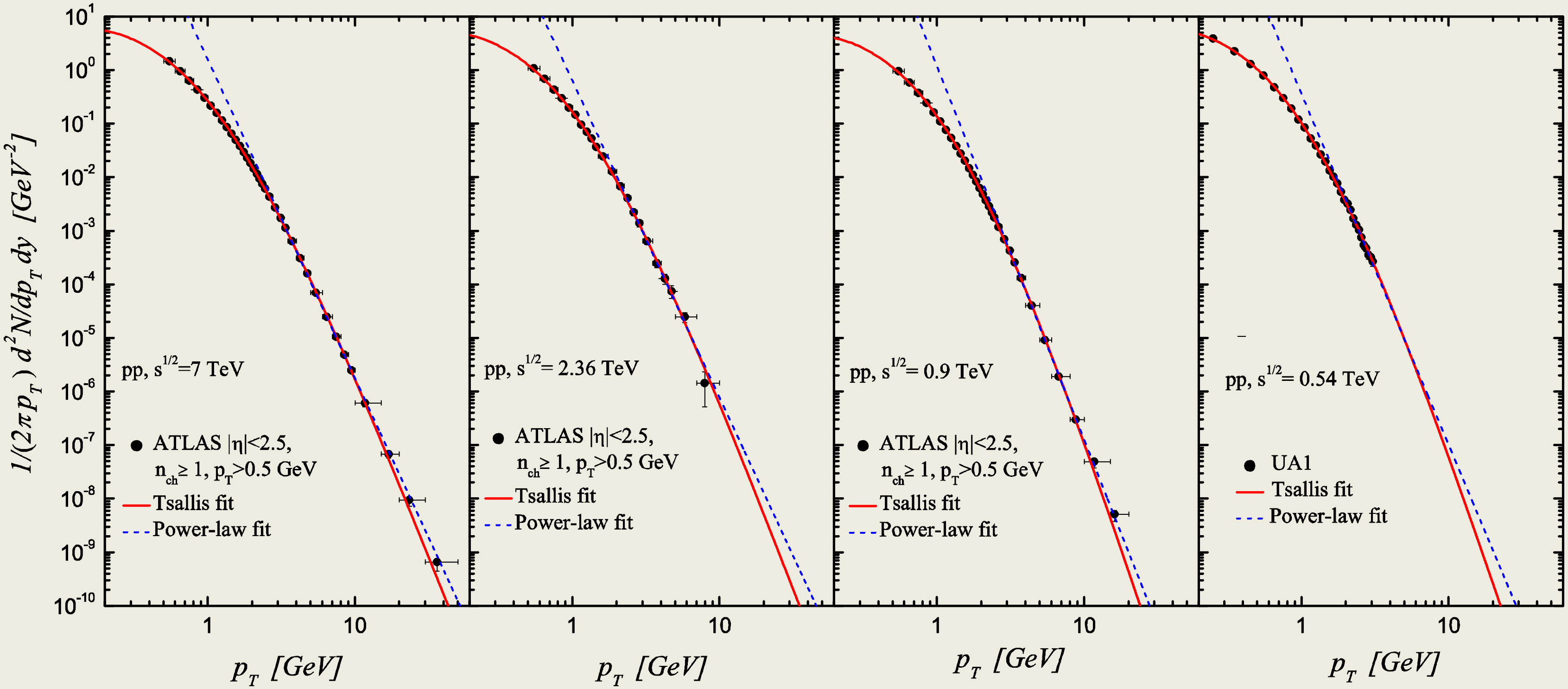
ALICE and CMS parameters



Charged-hadron yield at different energies: ATLAS and UA1

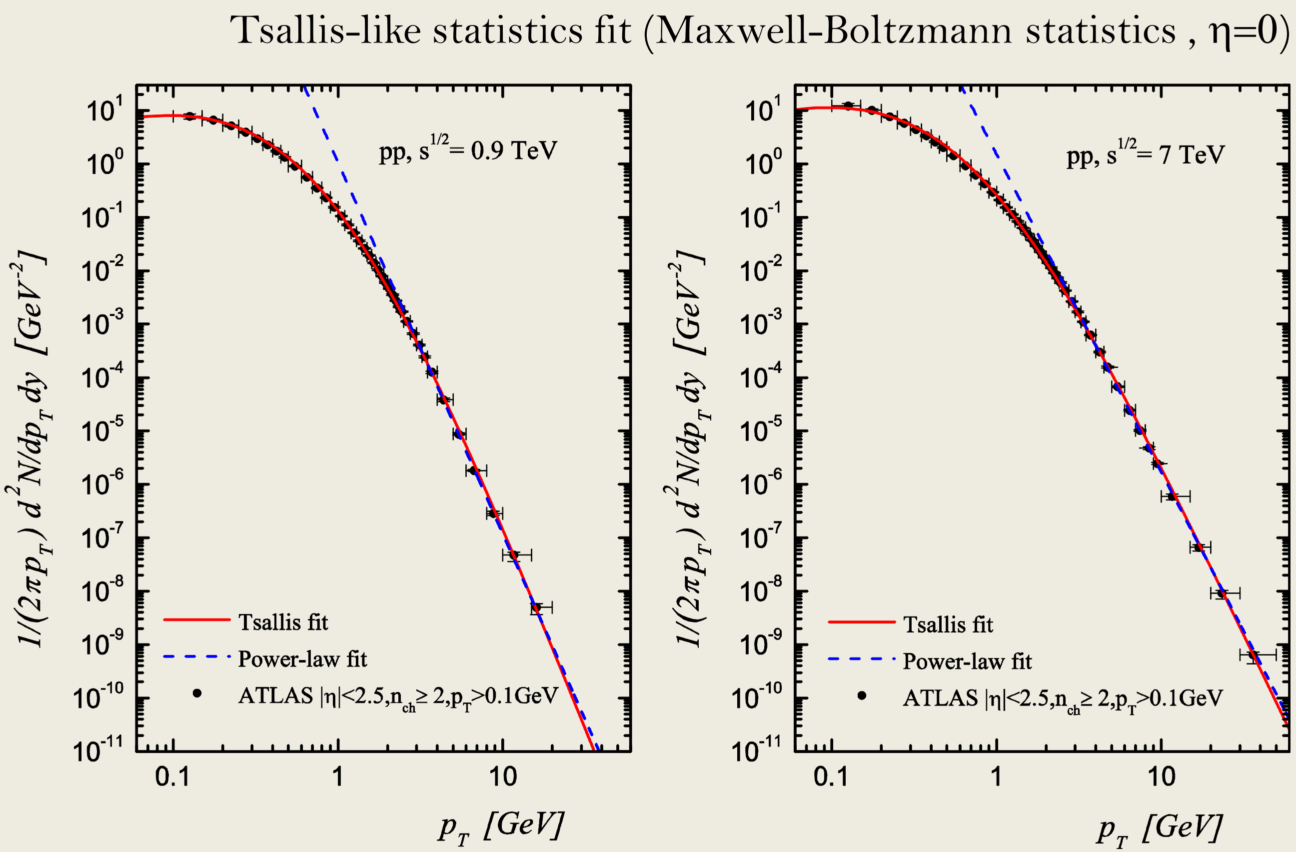
J. Cleymans, G.I. Lykasov, A.P., A.S. Sorin, O.V. Teryaev, D. Worku, Phys. Lett.B 723 (2013) 351

Tsallis-like statistics fit (Maxwell-Boltzmann statistics , η=0)



It has been assumed that all particles are massless charged pions under Maxwell-Boltzmann statistics at y=0

ATLAS charged-hadron yield for $p_T > 0.1 GeV$



It has been assumed that all particles are massless charged pions under Maxwell-Boltzmann statistics at y=0

Charged-hadron yield parameters

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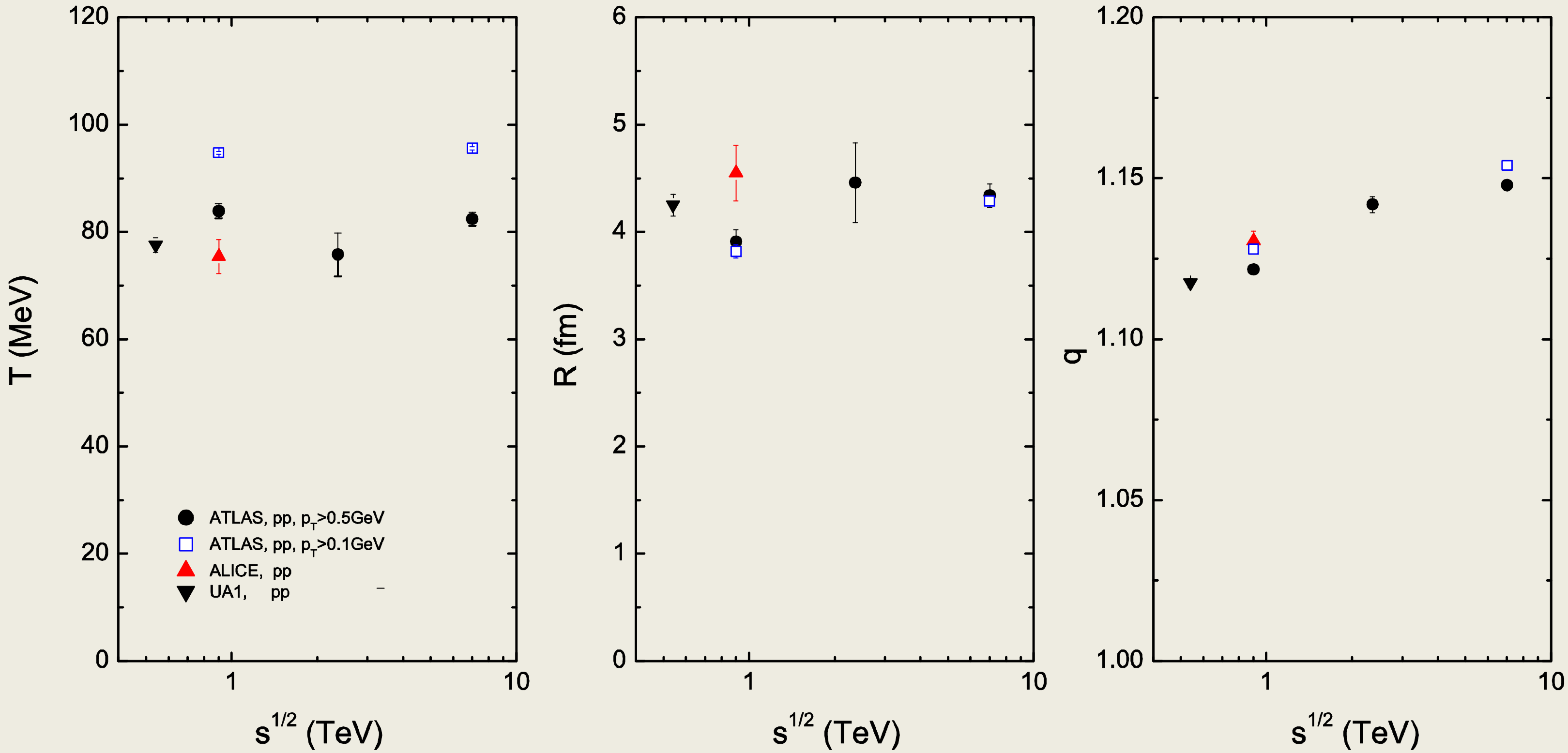
Phys. Lett.B 723 (2013) 351

Tsallis-like statistics fit (Maxwell-Boltzmann statistics , η=0)



In conclusion,

1) The transverse momentum distributions of hadrons in pp collisions at LHC energies are well described by the Tsallis-like distribution functions of Maxwell-Boltzman statistics

2) It was proved that the non-relativistic Tsallis statistics in the canonical and microcanonical ensembles is extensive.

3) The Tsallis-like temperature and volumes for charged hadron yields are consistent with being independent of beam energy within experimental uncertainties.

4) The exact Tsallis statistics have not been applied to describe the experimental transverse momentum spectra yet. It needs very complicated numerical calculations.

Thank you for attention