Proton Form Factors
in
space-like and time-like regions

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XXII International Baldin Seminar on High Energy Physics Problems

Relativistic Nuclear Physics and Quantum Chromodynamics

September 15th-20th, 2014, Dubna, Russia

Agenda

- Nucleon Electromagnetic Form Factors
  - Definition and properties
- The space-like region
  - Proton radius
  - Rosenbluth versus polarization
- The time-like region
  - Unphysical region
  - Threshold
- The asymptotic region
  - An interesting model
- Conclusions

Proton electromagnetic form factors

Nobel Prize in Physics - 1961

Prize motivation: "for his pioneering studies of electron scattering in atomic nuclei and for his thereby achieved discoveries concerning the structure of the nucleons".

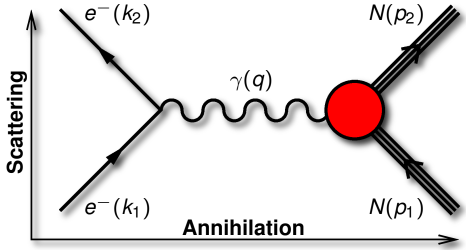
Robert Hofstadter

Stanford University
Stanford, CA, USA

Form factors...

- Characterize the internal structure of a particle ($F_{\text{point-like}} = \text{const.}$).
- Elastic form factors contain information on the hadron ground state.
- In a parity and T-invariant theory, the electromagnetic structure of a particle of spin $S\hbar$ is defined by $2S + 1$ form factors.
- Neutron and proton form factors are different.
- The deuteron has **two** structure functions, but **three** form factors.
- Playground for theory and experiment:
  - at low q^2 probe the size of the nucleons;
  - at high q^2 test QCD scaling.

Dirac and Pauli Form Factors



Scattering: $e^- N \to e^- N$
Space-like kinematic region

$$q^2 = -2\omega_1\omega_2(1-\cos\theta_e) \leq 0$$

Annihilation: $e^+e^- \leftrightarrow N\overline{N}$
Time-like kinematic region

$$q^2 = 4\omega^2 > 0$$

Scattering amplitude in **Born** approximation

$$\mathcal{M} = \frac{1}{q^2}\left[e\,\overline{u}(k_2)\gamma_\mu u(k_1)\right]\underbrace{\left[e\,\overline{U}(p_2)\Gamma^\mu(p_1,p_2)U(p_1)\right]}_{\text{Nucleon EM 4-current: } J_N^\mu}$$

From Lorenz and gauge invariance

$$\Gamma^\mu(p_1,p_2) = \gamma^\mu F_1^N(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2M_N}F_2^N(q^2)$$

- Dirac FF: $F_1^N(q^2)$, $F_1^N(0) = \mathcal{Q}_N$
- Pauli FF: $F_2^N(q^2)$, $F_2^N(0) = \kappa_N$

$\mathcal{Q}_N = N$ electric charge

$\kappa_N = N$ anomalous magnetic moment

Sachs Form Factors

Breit frame

No energy exchanged

$p_1 = (E, -\vec{q}/2)$

$p_2 = (E, \vec{q}/2)$

$q = (0, \vec{q})$

Nucleon elecrtomagnetic four-current

$$J^\mu_N = (J^0_N, \vec{J}_N) \quad \begin{cases} \rho_q = J^0_N = e\left[F^N_1 + \frac{q^2}{4M^2_N} F^N_2\right] \\ \vec{J}_N = e\,\overline{U}(p_2)\vec{\gamma}U(p_1)\left[F^N_1 + F^N_2\right] \end{cases}$$

Sachs Nucleon Form Factors

$$G^N_M(q^2) = F^N_1(q^2) + F^N_2(q^2) \qquad G^N_E(q^2) = F^N_1(q^2) + \frac{q^2}{4M^2_N} F^N_2(q^2)$$

In the Breit frame represent the **Fourier transforms** of **charge** and **magnetic moment spatial distributions** of the nucleon

Normalization at $q^2 = 0$

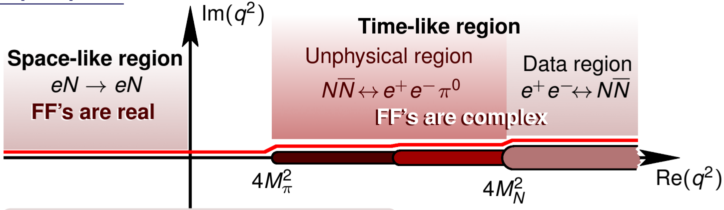
- $G^N_E(0) = \mathcal{Q}_N$
- $G^N_M(0) = \mu_N$

$\mu_N = \mathcal{Q}_N + \kappa_N$

is the nucleon magnetic moment

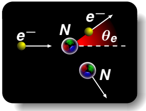
Cross sections and analyticity

q^2-complex plane



Crossing: tot. helicity $= \begin{cases} 1 \Rightarrow G_E \\ 0 \Rightarrow G_M \end{cases}$

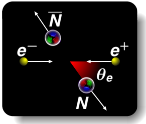
$$G_E(4M_N^2) = G_M(4M_N^2)$$



Elastic scattering

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \omega_2 \cos^2 \frac{\theta_e}{2}}{4\omega_1^3 \sin^4 \frac{\theta_e}{2}} \left[G_E^2 - \tau \left(1 + 2(1-\tau) \tan^2 \frac{\theta_e}{2} \right) G_M^2 \right] \frac{1}{1-\tau}$$

$$\tau = \frac{q^2}{4M_N^2}$$

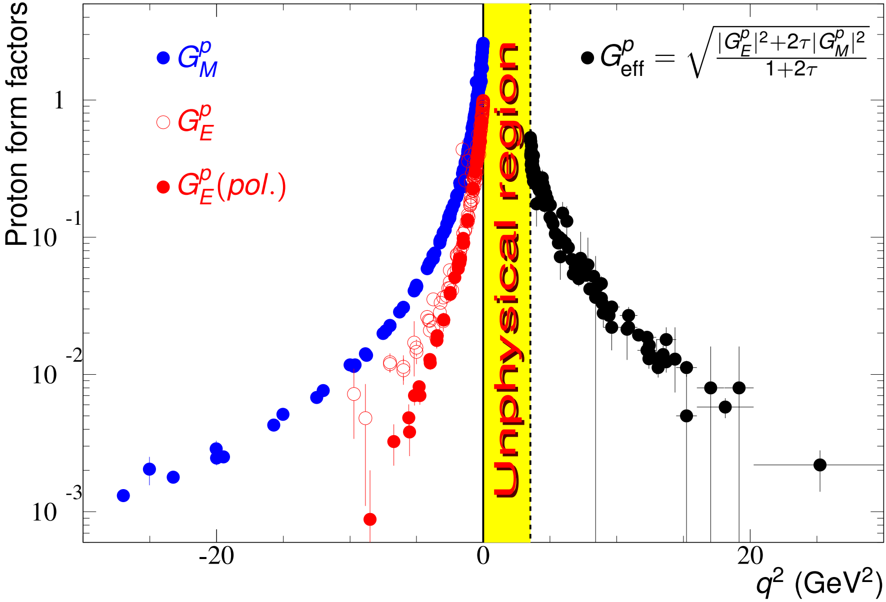


Annihilation

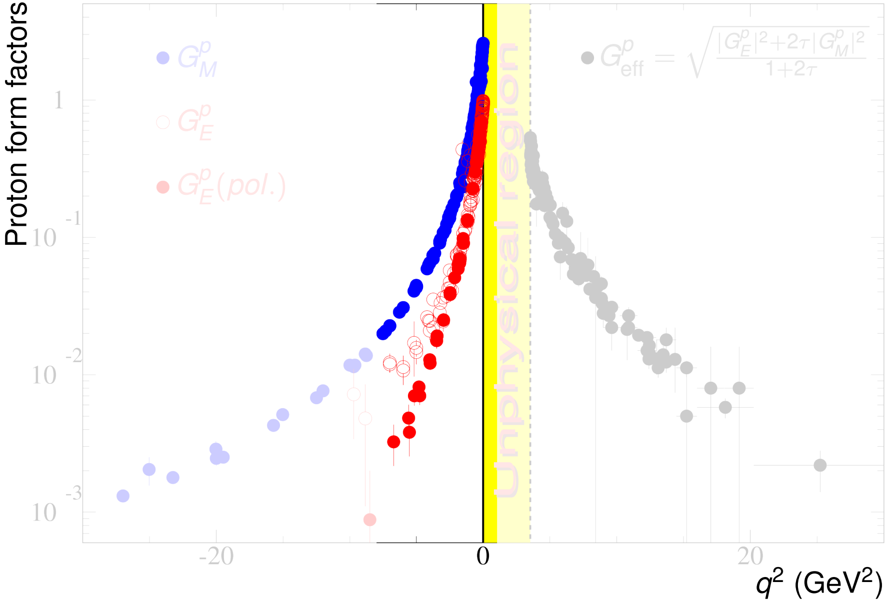
$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \beta C}{4q^2} \left[(1+\cos^2\theta)|G_M|^2 + \frac{1}{\tau} \sin^2\theta |G_E|^2 \right]$$

$$\beta = \sqrt{1 - \frac{1}{\tau}}$$

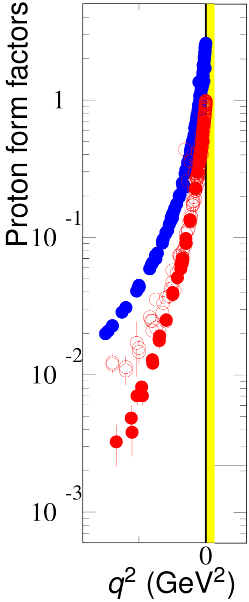
The proton radius



The proton radius



The proton radius



$$G_E^p(q^2) = \int d^3\vec{r}\,\rho(r)\,e^{i\,\vec{q}\cdot\vec{r}} = 1 + \frac{1}{6}q^2\left\langle r_c^2\right\rangle + \mathcal{O}(q^4)$$

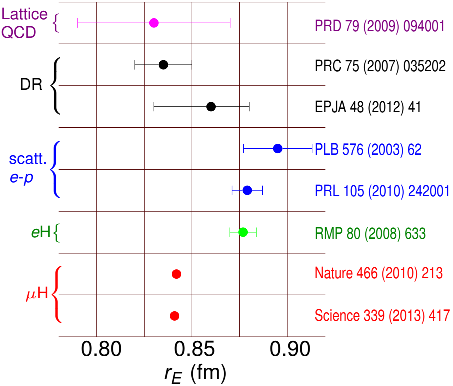
$\rho(r)$: normalized spherical charge density

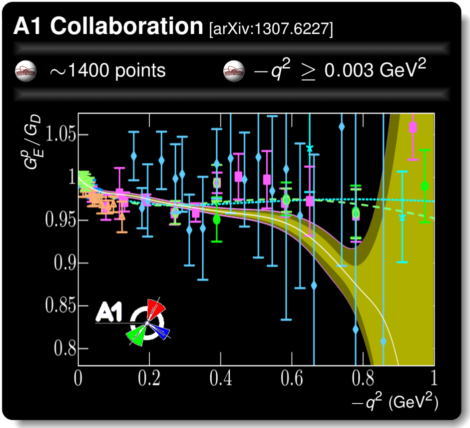
The charge radius

$$r_E = \sqrt{\langle r_c^2\rangle} = \sqrt{4\pi\int_0^\infty r^4\rho(r)\,dr} = \sqrt{\frac{6}{G_E^p(0)}\left.\frac{dG_E^p}{dq^2}\right|_{q^2=0}}$$

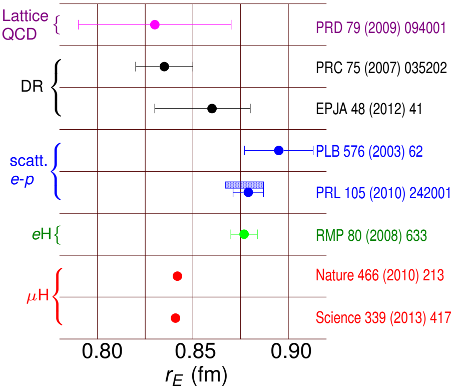
Charge density $\rho(r)$	Form factor $G_E^p(q^2)$	Charge radius r_E	Comments
$\delta^3(r)$	1	0	pointlike
$e^{-\lambda r}$	$\lambda^4/(q^2+\lambda^2)^2$	$2\sqrt{3}/\lambda$	dipole
$e^{-\lambda r}/r$	$\lambda^2/(q^2+\lambda^2)$	$\sqrt{6}/\lambda$	monopole
$e^{-\lambda r^2}/r^2$	$e^{-r^2/(4\lambda^2)}$	$1/\sqrt{2\lambda}$	gaussian

The proton radius



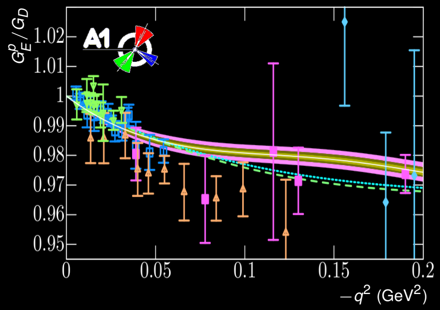


The proton radius



A1 Collaboration [arXiv:1307.6227]

- ∼1400 points
- $-q^2 \geq 0.003$ GeV^2



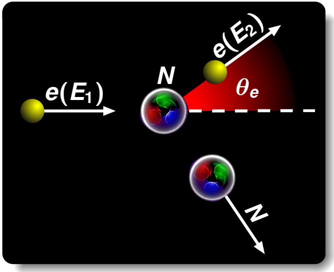
Ongoing discussions…

- Radiative corrections
- Two-photon exchange
- Coulomb corrections

Extrapolating $q^2 \to 0^-$

DR: $$r_E^2 = \frac{12M_\pi^2}{\pi} \int_{4M_\pi^2}^{\infty} \frac{\ln|G_E^p(t)/G_E^p(0)|}{t^2\sqrt{t-4M_\pi^2}}\, dt$$

Rosenbluth separation (one-photon exchange)



Rosenbluth formula

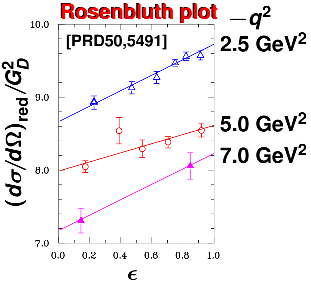
$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\rm Mott} \frac{1}{1-\tau}\left[G_E^2 - \frac{\tau}{\epsilon} G_M^2\right] \qquad \tau = \frac{q^2}{4M_N^2}$$

- Mott pointlike cross section

$$\left(\frac{d\sigma}{d\Omega}\right)_{\rm Mott} = \frac{4\alpha^2}{(-q^2)^2}\frac{E_2^3}{E_1}\cos^2(\theta_e/2)$$

- Photon polarization

$$\epsilon = \left[1 + 2\,(1-\tau)\tan^2(\theta_e/2)\right]^{-1}$$



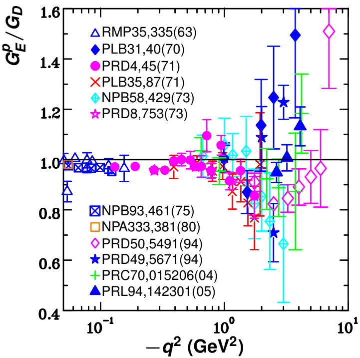
Reduced cross section

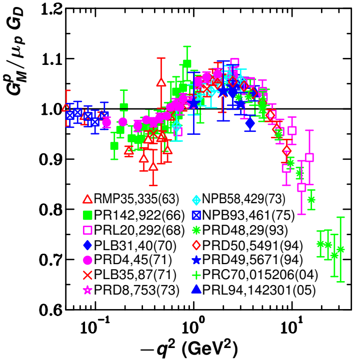
$$\left(\frac{d\sigma}{d\Omega}\right)_{\rm red} = \frac{\epsilon(1-\tau)}{\tau}\frac{(d\sigma/d\Omega)_{\rm exp}}{(d\sigma/d\Omega)_{\rm Mott}} = G_M^2 - \frac{\epsilon}{\tau} G_E^2$$

- $(d\sigma/d\Omega)_{\rm red}\,(\epsilon)$ slope $\longrightarrow G_E$
- $(d\sigma/d\Omega)_{\rm red}\,(\epsilon)$ intercept $\longrightarrow G_M$

G_E^p and G_M^p with Rosenbluth separation

Dipole approximation $G_D(q^2) = \left(1 - q^2/M_D^2\right)^{-2}$ $M_D^2 = 0.71\ \mathrm{GeV}^2$





Classical approach

Form factors, in nonrelativistic approximation, are Fourier transforms of charge and magnetic distributions

The dipole form factor corresponds to an exponential distribution $\rho(r) = \rho_0 e^{-r/r_0}$

$$M_D^2 = 0.71\ \mathrm{GeV}^2 \implies \begin{cases} r_0^2 = (0.24\ \mathrm{fm})^2 \\ \langle r^2 \rangle = (0.81\ \mathrm{fm})^2 \end{cases}$$

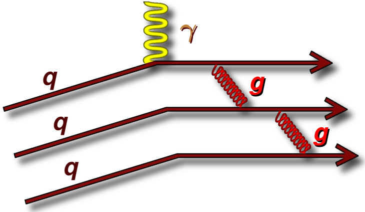
Lett. Nuovo Cim. 7 (1973) 719
Phys. Rev. Lett. 31 (1973) 1153
JETP Lett. 96 (2012) 6-12

Hadron form factor

$$F(q^2) = \frac{C_n}{\left(1 - q^2/M_n^2\right)^{n-1}}$$

- $M_n^2 = n\beta^2$
- β^2 = quark momentum squared
- n = number of constituent quarks

Dimensional scaling



Pion form factor

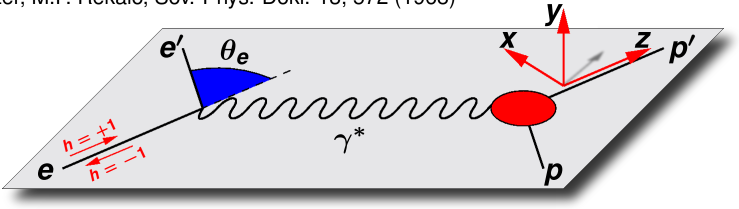
$\Downarrow$

$\beta^2 = (0.471 \pm 0.010)\ \mathrm{GeV}^2$

- $F_\pi(q^2) = \dfrac{C_2}{1 - \dfrac{q^2}{0.471\ \mathrm{GeV}^2}}$ ....pion, $n = 2$
- $F_N(q^2) = \dfrac{C_3}{\left(1 - \dfrac{q^2}{0.71\ \mathrm{GeV}^2}\right)^2}$ ...nucleon, $n = 3$
- $F_d(q^2) = \dfrac{C_6}{\left(1 - \dfrac{q^2}{1.42\ \mathrm{GeV}^2}\right)^5}$ ..deuteron, $n = 6$

Polarization observables

A.I. Akhiezer, M.P. Rekalo, Sov. Phys. Dokl. 13, 572 (1968)



- Elastic scattering of longitudinally polarized ($h = \pm 1$) electrons on nucleon target
- Hadronic tensor: $W_{\mu\nu} = \underbrace{W_{\mu\nu}(0)}_{\text{no pol.}} + \underbrace{W_{\mu\nu}(\vec{P}) + W_{\mu\nu}(\vec{P}')}_{\text{ini. or fin. pol. of } N} + \underbrace{W_{\mu\nu}(\vec{P}, \vec{P}')}_{\text{ini. and fin. pol. of } N}$
- In case of polarized ($h = \pm 1$) electrons on unpolarized nucleon target:

$$P'_x = -\frac{2\sqrt{\tau(\tau-1)}}{G_E^2 - \frac{\tau}{\epsilon} G_M^2} G_E G_M \tan\left(\frac{\theta_e}{2}\right)$$

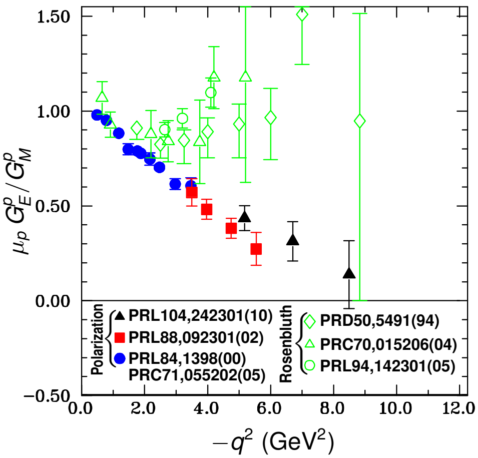
$$P'_z = \frac{(E_e + E'_e)\sqrt{\tau(\tau-1)}}{M\left(G_E^2 - \frac{\tau}{\epsilon} G_M^2\right)} G_M^2 \tan^2\left(\frac{\theta_e}{2}\right)$$

$$\frac{P'_x}{P'_z} = -\frac{2M\cot(\theta_e/2)}{E_e + E'_e} \frac{G_E}{G_M}$$

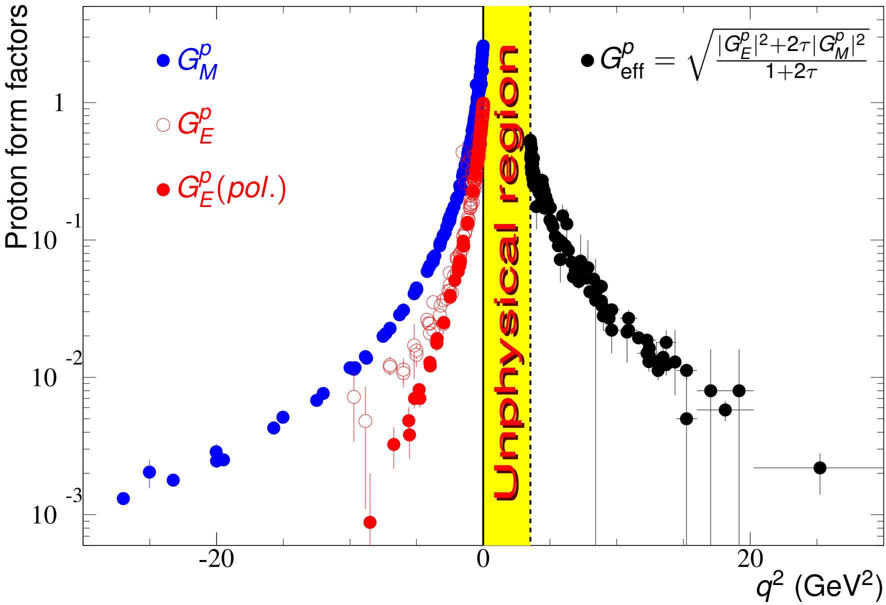
G_E^p / G_M^p in polarization transfer experiments

- "Standard" dipole for the proton magnetic form factors G_M^p
- Linear deviation from the dipole for the electric proton form factor G_E^p
- **QCD scaling still not reached**
- **Zero crossing for G_E^p**
- Polarization data do not agree with old Rosenbluth data (◇)
- New Rosenbluth separation data from JLab **still do not agree** with polarization data

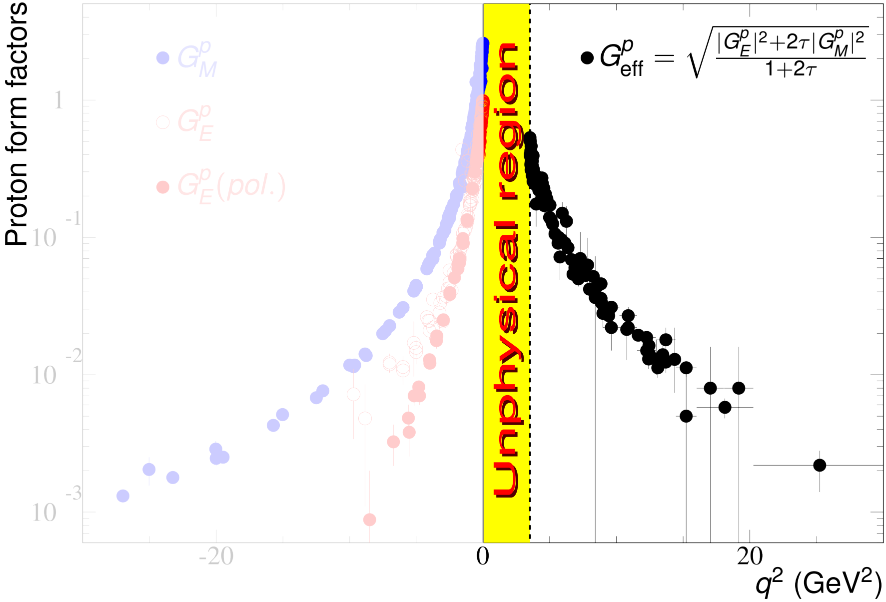
C. Perdrisat *et al*.
JLab-GEp Collaboration



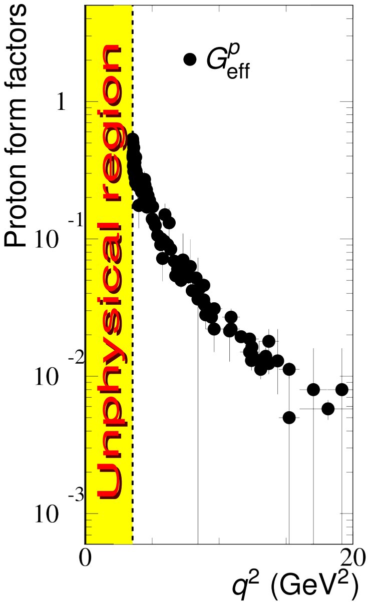
The time-like region



The time-like region



The time-like region



Differential cross section $e^+e^- \to p\bar{p}$

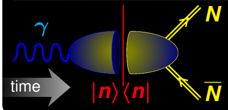
A. Zichichi, S. M. Berman, N. Cabibbo, R. Gatto [NC XXIV (1962) 170]

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \beta C}{4q^2}\left[(1+\cos^2\theta)|G_M^p|^2 + \frac{1}{\tau}\sin^2\theta|G_E^p|^2\right]$$

Optical theorem

$$\mathrm{Im}\langle\overline{N}(p')N(p)|j^\mu|0\rangle \sim \sum_n \langle\overline{N}(p')N(p)|j^\mu|n\rangle\langle n|j^\mu|0\rangle$$

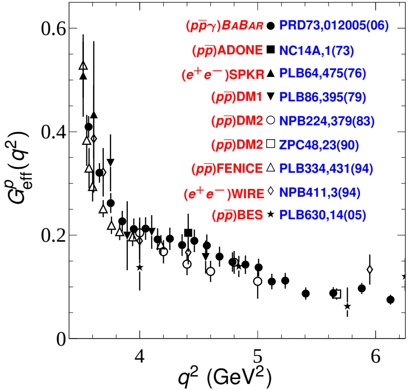
$|n\rangle$ are on-shell intermediate states: 2π, 3π, 4π, ...

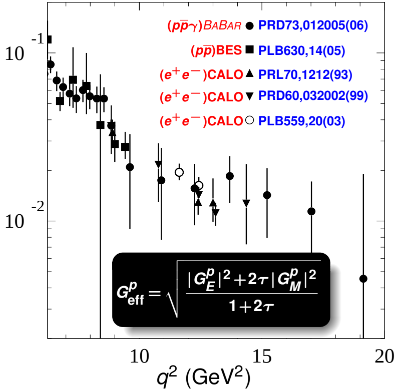


Form factors are complex for $q^2 > 4M_\pi^2$

- The cross section is an even function of $\cos\theta$
- The cross section does not depend on the form factor phases
- At high q^2 the $|G_E^p|^2$ contribution is suppressed
- The unphysical region is not accessible through the annihilations $e^+e^- \leftrightarrow p\bar{p}$

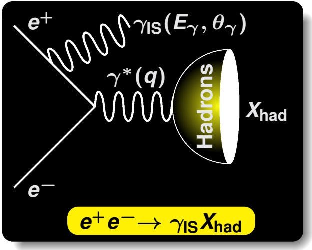
Proton effective form factor





- No individual determination of $|G^p_E|$ and $|G^p_M|$.
- Time-like proton form factors are larger (factor of two) than their space-like values at the same $|q^2|$.
- The threshold behavior is very steep.
- It is not smooth. Structures? Resonances?...

Initial State Radiation



- $$\frac{d^2\sigma}{dE_\gamma \, d\cos\theta_\gamma} = W(E_\gamma, \theta_\gamma)\, \sigma_{e^+e^- \to X_{had}}(s)$$
- $$W(E_\gamma, \theta_\gamma) = \frac{\alpha}{\pi x}\left(\frac{2 - 2x + x^2}{\sin^2\theta_\gamma} - \frac{x^2}{2}\right)$$

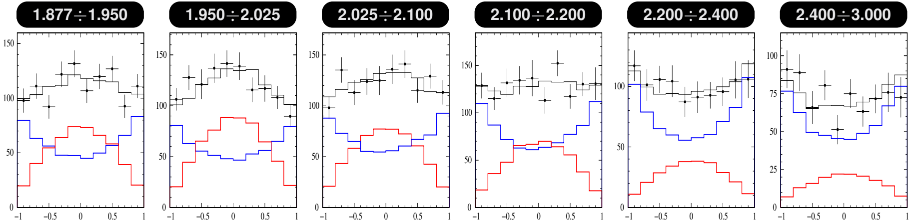
- $s = q^2$, q .... X_{had} momentum
- E_γ, θ_γ . . CM γ_{IS} energy, scatt. ang.
- E_{CM} .... CM e^+e^- energy
- $x = 2E_\gamma / E_{CM}$

- **All energies (q^2) at the same time** $\Rightarrow$ **Better control on systematics (greatly reduced point to point)**
- **Detected ISR at large angles** $\Rightarrow$ **full X_{had} angular coverage**
- **CM boost** $\Rightarrow$ $\begin{cases} \text{efficiency at threshold } \neq 0 \\ \text{energy resolution } \sim 1 \text{ MeV} \end{cases}$

$e^+e^- \to p\bar{p}$ angular distribution (BABAR)

PRD87, 092005

$\cos\theta_p$ distributions form threshold up to 3 GeV [intervals in $E_{CM} \equiv \sqrt{q^2}$ (GeV)]

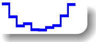


Events/0.2 vs. $\cos\theta_p$

$$\frac{d\sigma}{d\cos\theta_p} = A\left[H_E(\cos\theta_p, q^2)\left|\frac{G_E^p(q^2)}{G_M^p(q^2)}\right|^2 + H_M(\cos\theta_p, q^2)\right]$$

H_E and H_M from MC

G_E^p 

G_M^p 

At low q^2

$\sin^2\theta_p > 1 + \cos^2\theta_p$

$\Rightarrow$

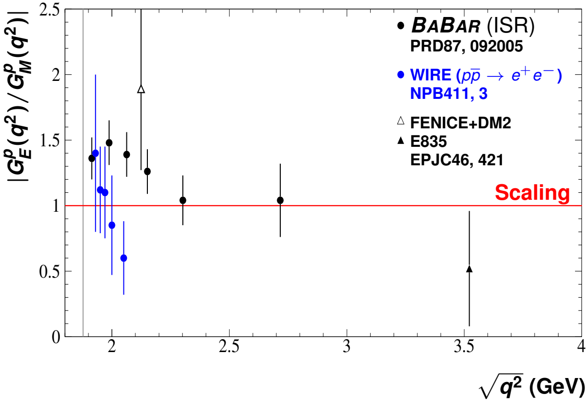
First observation!

$|G_E^p| > |G_M^p|$

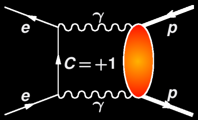
At higher q^2, $|G_E^p| \to |G_M^p|$

Time-like $|G_E^p/G_M^p|$ measurements

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2\beta C}{2q^2}|G_M^p|^2\left[(1+\cos^2\theta)+\frac{4M_p^2}{q^2}\sin^2\theta\left|\frac{G_E^p}{G_M^p}\right|^2\right]$$



$\gamma\gamma$ exchange

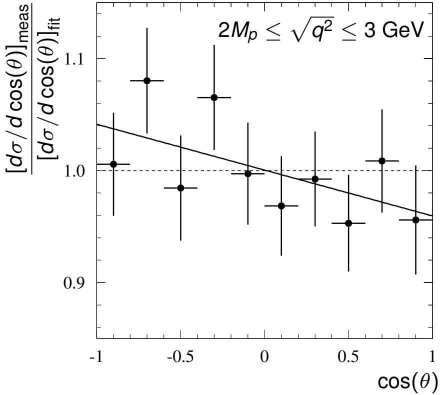


$\gamma\gamma$ exchange interferes with the Born term

Asymmetry in angular distributions

[E. Tomasi-Gustafsson, G.I. Gakh, NPA771,169(06)]

E. Tomasi-Gustafsson, E.A. Kuraev, S. Bakmaev, SP, Phys. Lett. B659 (2008) 197
Phys. Rev. D87 (2013) 092005



Integrated over the $p\bar{p}$-CM energy from threshold up to 3 GeV

The MC-fit assumes **one-photon exchange**

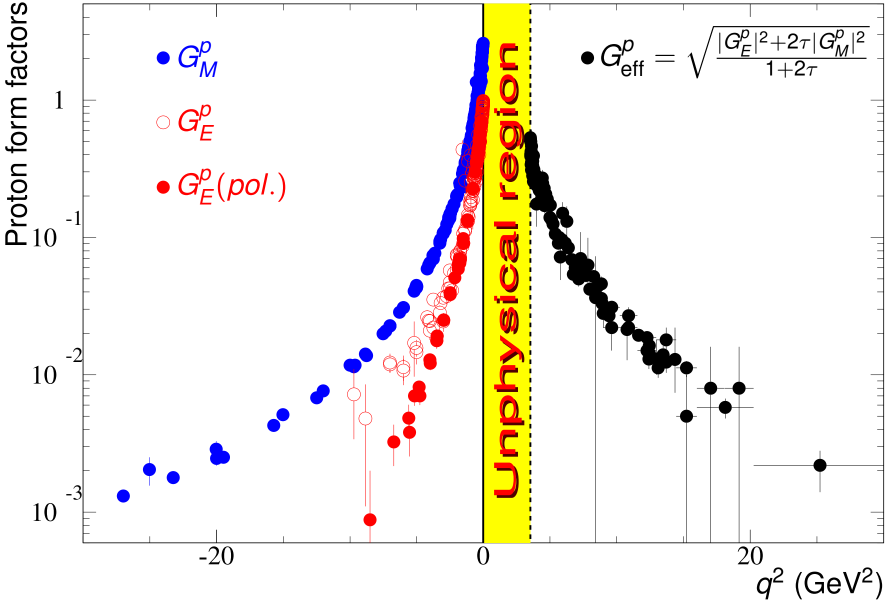
Slope = $-0.041 \pm 0.026 \pm 0.005$

Integral asymmetry

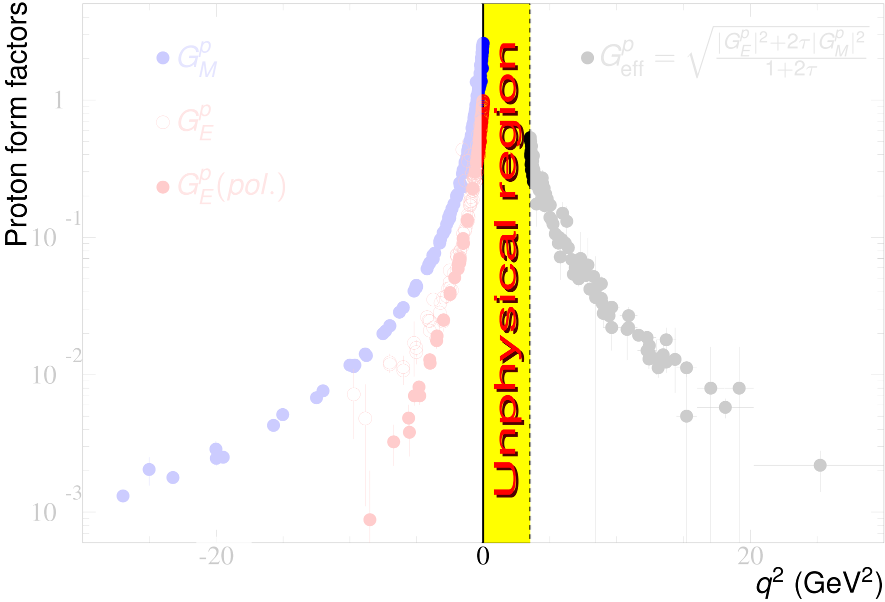
$$\langle \mathcal{A} \rangle_{\cos\theta_p} = \frac{\sigma(\cos\theta_p > 0) - \sigma(\cos\theta_p < 0)}{\sigma(\cos\theta_p > 0) + \sigma(\cos\theta_p < 0)} = -0.025 \pm 0.014 \pm 0.003$$

$\sigma(\cos\theta_p \gtrless 0)$ is the cross section integrated with $\sqrt{q^2} \leq 3$ GeV and $\cos\theta_p \gtrless 0$

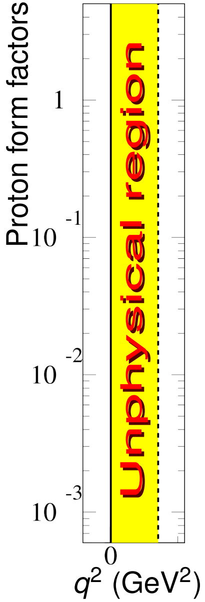
The unphysical region



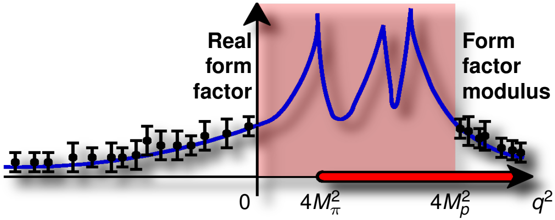
The unphysical region



The unphysical region



Unphysical region goes from $q^2 = 0$ up to the physical threshold $q^2 = 4M_p^2$



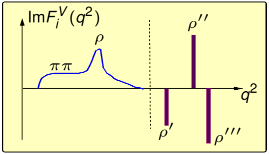
In that region, form factors

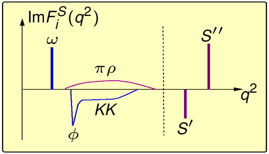
- are still well defined but not (directly) experimentally accessible
- are complex and, following VMD-based models, receive their main contribution from intermediate resonances

Model dependent **disclosing** [Höler, Mergell, Meissner, Hammer]

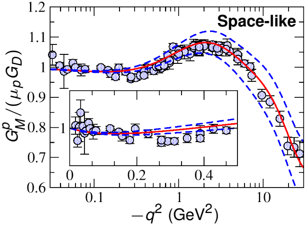
Optical theorem .... $\mathrm{Im}\langle \overline{N}(p')N(p)|j^\mu|0\rangle \sim \sum_n \langle \overline{N}(p')N(p)|j^\mu|n\rangle\langle n|j^\mu|0\rangle$

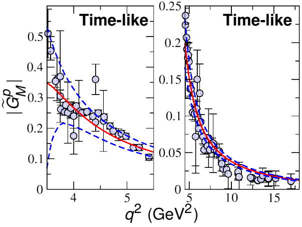
Dispersion relations for the imaginary part .... $F(q^2_{\mathrm{SL}}) = \frac{1}{\pi}\int_{4M_\pi^2}^{\infty} \frac{\mathrm{Im}F(q^2_{\mathrm{TL}})}{q^2_{\mathrm{TL}} - q^2_{\mathrm{SL}}} dq^2_{\mathrm{TL}}$





- 2π and $2K$ continua are known
- The ρ resonance with finite width
- Dirac delta poles for higher mass states





- Super convergence relations
  $\int_{4M_\pi^2}^{\infty} \mathrm{Im}\, F_{1,2}(q^2)dq^2 = 0$
  $\int_{4M_\pi^2}^{\infty} q^2 \mathrm{Im}\, F_2(q^2)dq^2 = 0$
- Asymptotic behaviors from perturbative QCD

Model independent disclosing [EPJC11 709]

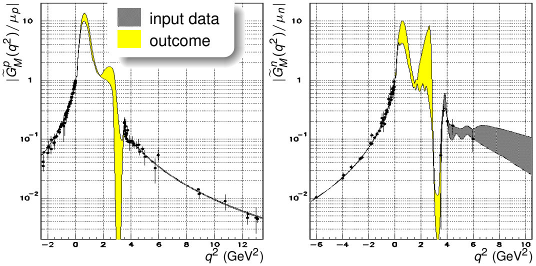
Dispersion relation subtracted at $t = 0$

$$\ln G(t) = \frac{t\sqrt{4M_\pi^2 - t}}{\pi}\int_{4M_\pi^2}^{\infty}\frac{\ln|G(s)|ds}{s\sqrt{s-4M_\pi^2}(s-t)}$$

- **Less dependent on the asymptotic behavior of the FF**
- **$\ln G(0) = 0 \Longrightarrow$ no further terms have to be considered**

Splitting the integral $\int_{4M_\pi^2}^{\infty}$ into $\int_{4M_\pi^2}^{4M_p^2} + \int_{4M_p^2}^{\infty}$ we obtain the integral equation

$$\overbrace{\ln G(t) - I_{\rm phy}^{\infty}(t)}^{\text{Data and Theory}} = \frac{t\sqrt{4M_\pi^2 - t}}{\pi}\int_{4M_\pi^2}^{4M_p^2}\frac{\overbrace{\ln|G(s)|}^{\text{Unknown}}\,ds}{s\sqrt{s-4M_\pi^2}(s-t)}$$



Solutions are regularized by requiring their total curvature in the unphysical region to be limited

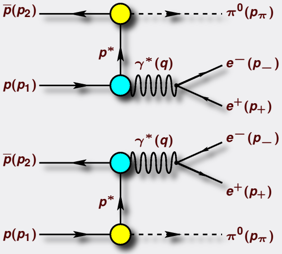
Steep behavior at thresholds

Solutions show resonant structures similar to the expected vector mesons

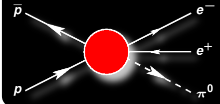
Accessing the unphysical region

[C. Adamuscin, E.A. Kuraev, E. Tomasi-Gustafsson, F. Maas]

The initial state π-production
$\boldsymbol{p\bar{p} \to \pi^0 e^+ e^-}$

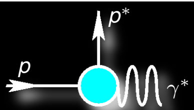


The process $p\bar{p} \to \pi^0 e^+ e^-$



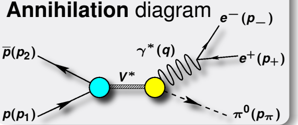
Described in general by **six** amplitudes which depend on **three** kinematical variables

Hadronic current **[PRC75 045205]**



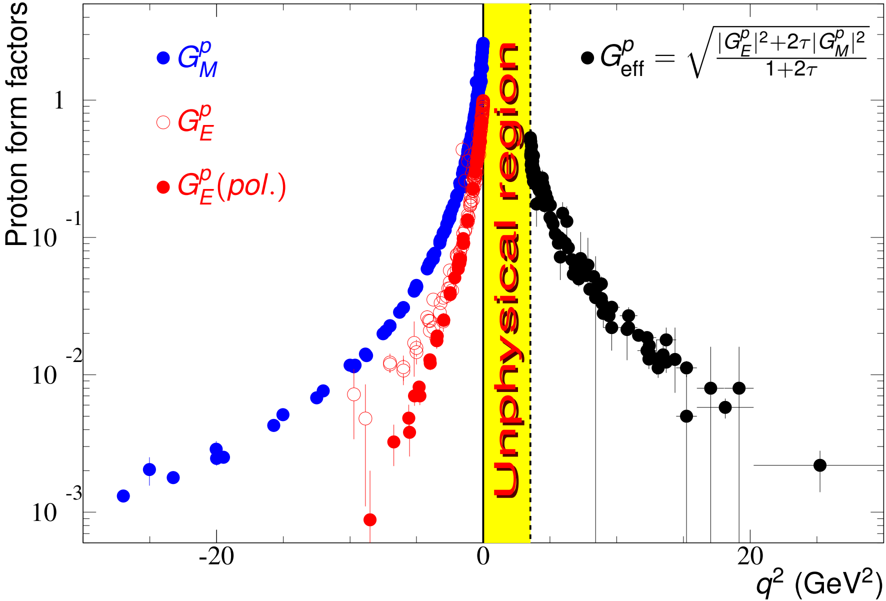
- $J_\mu = \phi_\pi(p_\pi)\,\bar{v}(p_2)\,O_\mu\,u(p_1)$
- $O_\mu = O_\mu[\Gamma_\mu(q)]$
- $\langle N(p')|\Gamma_\mu(q)|N(p)\rangle = \bar{u}(p')\left[F_1(q^2)\gamma_\mu + \frac{i\,\sigma_{\mu\nu}q^\nu}{4M_p^2}F_2(q^2)\right]u(p)$

Background
Annihilation diagram

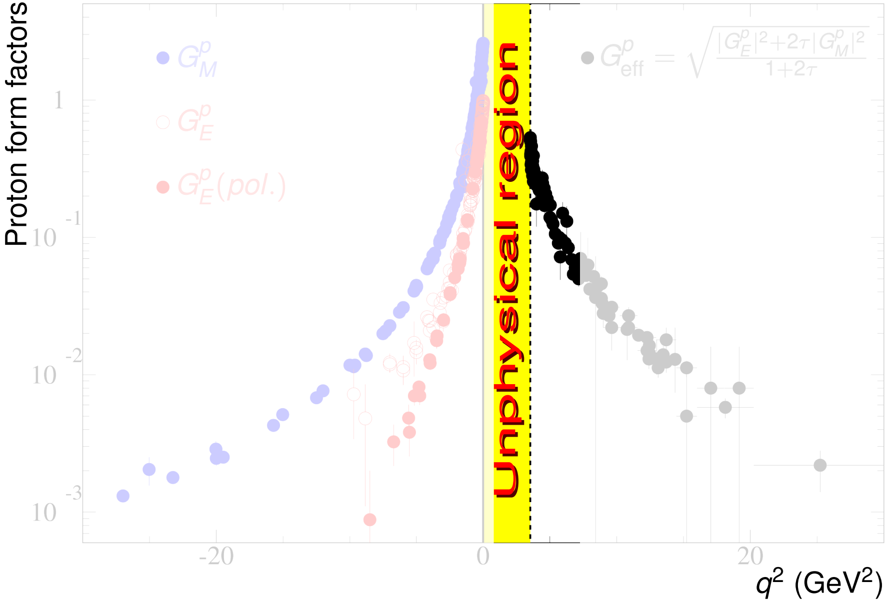


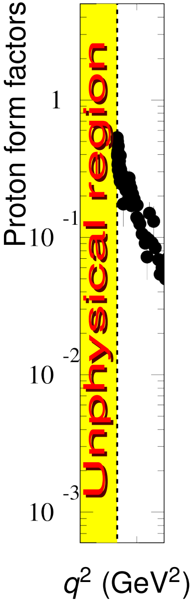
Polarization observables help in disentangle reaction mechanisms
[E. A. Kuraev *et al*., J. Exp. Theor. Phys. 115 (2012) 93
G.I. Gakh, E. Tomasi-Gustafsson, A. Dbeyssi, A.G. Gakh
PhysRevC86 (2012) 025204]

The threshold region₁



The threshold region$_1$

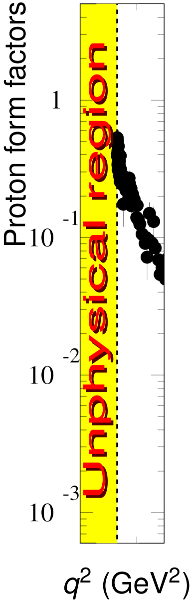




Annihilation cross section

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \beta C}{4q^2}\left[(1+\cos^2\theta)|G_M|^2 + \frac{1}{\tau}\sin^2\theta|G_E|^2\right]$$

The threshold region$_1$



Annihilation cross section

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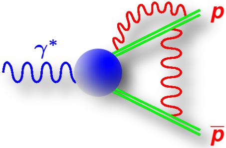


$p\bar{p}$ Coulomb interaction as FSI

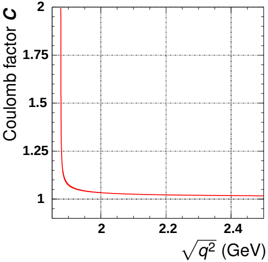
[Sommerfeld, Sakharov, Schwinger, Fadin, Khoze]

Schrödinger wave function

$$C = |\Psi_{\text{Coul}}(0)|^2$$

- **S-wave:** $C = \dfrac{\frac{\pi\alpha}{\beta}}{1 - \exp\left(-\frac{\pi\alpha}{\beta}\right)} \xrightarrow[\beta \to 0]{} \dfrac{\pi\alpha}{\beta}$
- **D-wave:** $C = 1$

No Coulomb factor for boson pairs (P-wave)

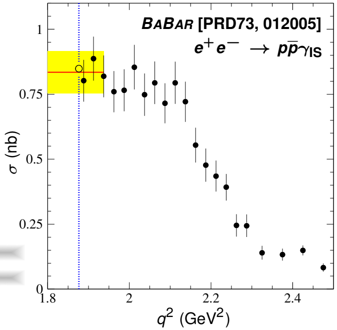


Partial wave form factors

$$G_S^p = \frac{1}{3}\left(2G_M^p\sqrt{\frac{q^2}{4M_p^2}} + G_E^p\right) \ , \ G_D^p = \frac{1}{3}\left(G_M^p\sqrt{\frac{q^2}{4M_p^2}} - G_E^p\right)$$

Cross section

$$\sigma(q^2) = 2\pi\alpha^2\beta\,\frac{4M_p^2}{(q^2)^2}\left[C\,|G_S^p(q^2)|^2 + 2|G_D^p(q^2)|^2\right]$$

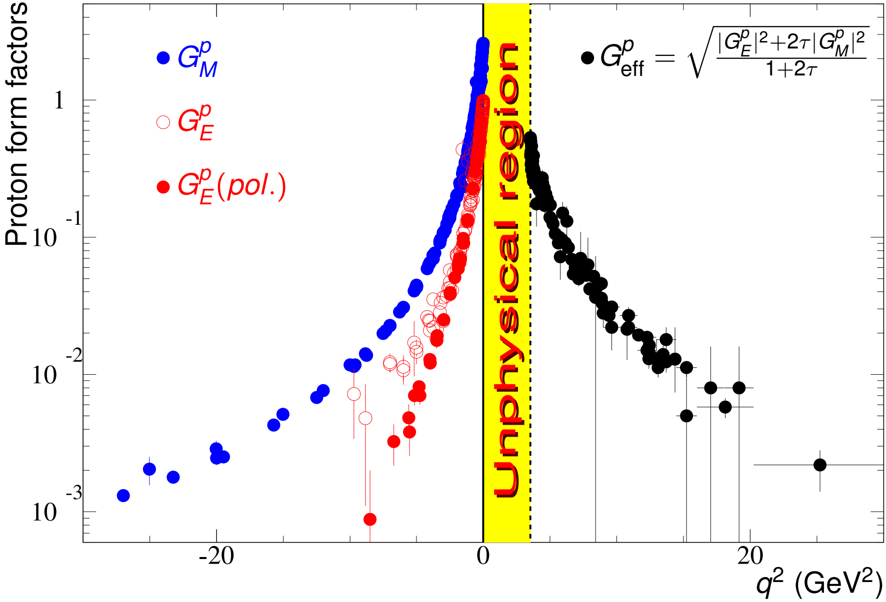


Step at threshold

$$\sigma(4M_p^2) = \frac{\pi^2\alpha^3}{2M_p^2}\frac{\cancel{\beta}}{\cancel{\beta}}|G_S^p(4M_p^2)|^2 = 0.85\,|G_S^p(4M^2)|^2 \text{ nb}$$

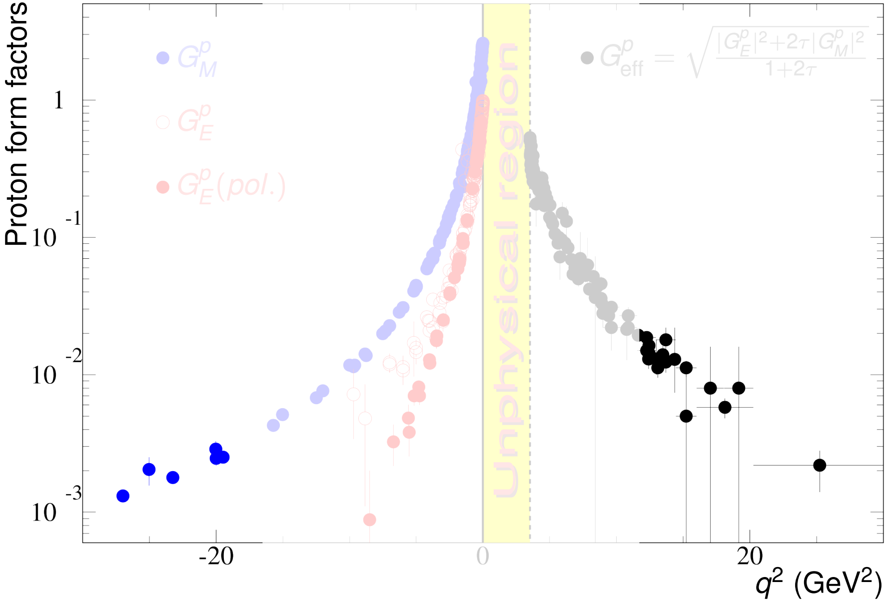
$$|G_S^p(4M_p^2)| = 1$$

$$|G_S^p(4M_p^2)| = 0.99 \pm 0.05$$



Proton form factors
G_M^p
G_E^p
$G_E^p(pol.)$
Unphysical region
$G_{\rm eff}^p = \sqrt{\frac{|G_E^p|^2+2\tau|G_M^p|^2}{1+2\tau}}$
1
10^{-1}
10^{-2}
10^{-3}
-20
0
20
q^2 (GeV^2)

The asymptotic regions$_1$

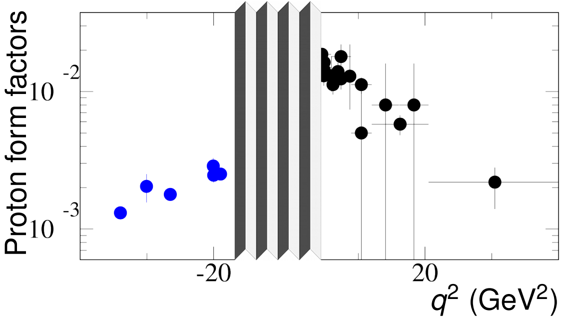


Time-like asymptotic behavior

Phragmèn Lindelöf theorem:

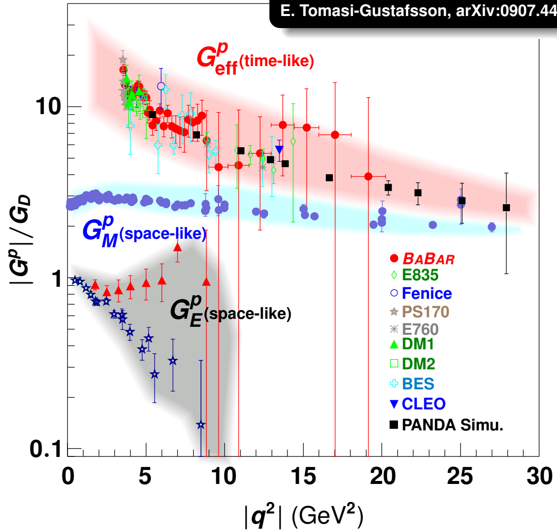
If a function $f(z) \to a$ as $z \to \infty$ along a straight line, and $f(z) \to b$ as $z \to \infty$ along another straight line, and $f(z)$ is regular and bounded in the angle between, then $a = b$ and $f(z) \to a$ uniformly in this angle.

- $$\underbrace{\lim_{q^2 \to -\infty} G_{E,M}(q^2)}_{\text{space-like}} = \underbrace{\lim_{q^2 \to +\infty} G_{E,M}(q^2)}_{\text{time-like}}$$
- $$G_{E,M} \underset{q^2 \to +\infty}{\sim} (q^2)^{-2}$$ **real**



The asymptotic regions$_2$

E. Tomasi-Gustafsson and M. P. Rekalo, PLB504,291
E. Tomasi-Gustafsson, arXiv:0907.4442



pQCD

$$G^p_{eff}(q^2) \underset{q^2\to\infty}{\sim} G^p_M(q^2)$$

Phragmèn Lindelöf

$$\lim_{q^2\to\infty} \frac{G^p_{eff}(q^2)}{G^p_M(-q^2)} = 1$$

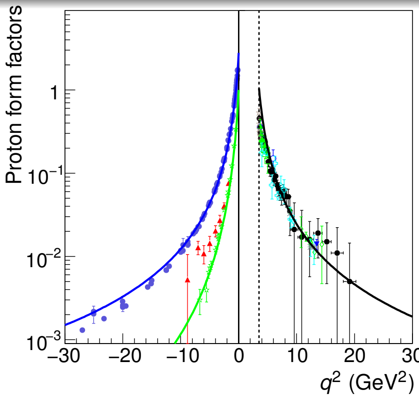
An interesting model

E. A. Kuraev, E. Tomasi-Gustafsson, A. Dbeyssi PLB712 240

Assumption
Pauli principle pulls away from the internal region of strong chromo-electromagnetic field quarks of same flavor because the color quantum number does not play any role (stochastic averaging).

Outer spatial region $|p\rangle = \epsilon^{ijk}|u_i u_j d_k\rangle$ charge = 1

Central region $|p\rangle \neq \epsilon^{ijk}|u_i u_j d_k\rangle$ charge = 0



space-like

Counting rule on the vector part of interaction

$$G_M^p(q^2) = \mu_p\, G_D(q^2)$$

A screening effect from the central region provides an additional suppression for the electric form factor

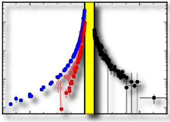
$$G_E^p(q^2) = \frac{G_D(q^2)}{1 - q^2/q_1^2}$$

time-like

$$G_M^p(q^2) = \frac{\theta(q^2 - 4M_p^2)}{\left[1 + (q^2 - 4M_p^2)^2/q_2^2\right]^2}$$

$$G_E^p(q^2) = \frac{G_M^p(q^2)\,\theta(q^2 - 4M_p^2)}{1 + (q^2 - 4M_p^2)^2/q_2^2}$$

Conclusions



Global models for proton and neutron, electric and magnetic form factors must be encouraged. They can help in understanding…

- the threshold behavior
- the proton radius
- the presence of zeros
- the asymptotic behavior
- the unphysical region
- …

To measure…

- zero of G_E^p in space-like region
- moduli of G_E and G_M in time-like region
- complex structure of form factors (polarization)
- unphysical time-like form factors ($p\bar{p} \to \pi^0 e^+ e^-$)
- …

Experiments: now and future

Space-like region

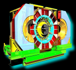
A1 Mainz
- G_E^n at $-q^2 = 1.5$ GeV^2 (Pol. ^{3}He)
- G_E^p and G_M^p for $-q^2 \leq 2.0$ GeV^2

Jefferson Lab
- [Hall A] G_E^p/G_M^p up to 14 GeV^2
- [Hall A] G_M^n (ratio) up to 18 GeV^2
- [Hall A] G_E^n/G_M^n up to 10.2 GeV^2
- [Hall B] G_M^n (deuterium) up to 14 GeV^2
- [Hall C] G_E^n up to 7 GeV^2

Time-like region

SND, CMD at VEPP-2000 e^+e^- collider
$|G_{\rm eff}^p|$, $|G_{\rm eff}^n|$ (scan)
$q^2 \leq (4\text{ GeV})^2$

BESIII at BEPCII e^+e^- collider



$|G_E^p|$, $|G_M^p|$, $|G_{\rm eff}^n|$ (scan and ISR)
$q^2 \leq (3.5\text{ GeV})^2$

PANDA at FAIR $p\bar{p}$ collider
$|G_E^p|$, $|G_M^p|$, G_E^p/G_M^p phase ($\bar{p}$ polarization)
$(2.4\text{ GeV})^2 \leq q^2 \leq (3.7\text{ GeV})^2$

Belle II ? at SuperKEKB e^+e^- collider
$|G_E^p|$, $|G_M^p|$, (ISR)
$q^2 \leq (4.5\text{ GeV})^2$

Additional slides

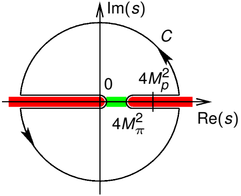
Suppressing unphysical contributions
[Geshkenbeĭn, Ioffe, Shifman]

Dispersion relation for

$$\phi(s) = f(s)\frac{\ln G(s)}{s\sqrt{4M_\pi^2 - s}}$$

$f(s)$ suppression function

$$\int_0^{4M_p^2} f^2(s)ds \ll 1$$



Assuming $G(q^2) \neq 0$
Cauchy theorem gives

$$\oint_C \phi(s)ds = 0$$

$$-\int_{-\infty}^{0}\frac{\mathrm{Im}[f(t)]\ln G(t)}{t\sqrt{4M_\pi^2 - t}}dt = \int_{4M_\pi^2}^{\infty}\frac{f(s)\ln|G(s)|}{s\sqrt{s - 4M_\pi^2}}ds$$

Handling the unphysical region$_3$

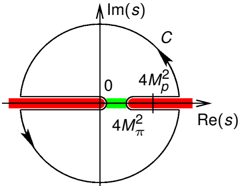
Suppressing unphysical contributions
[Geshkenbeĭn, Ioffe, Shifman]

Dispersion relation for

$$\phi(s) = f(s) \frac{\ln G(s)}{s\sqrt{4M_\pi^2 - s}}$$

$f(s)$ suppression function

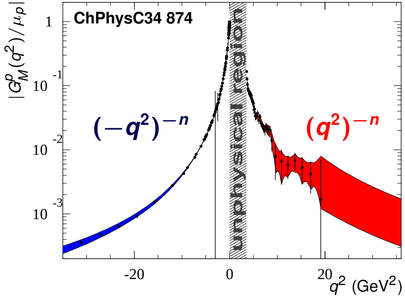
$$\int_0^{4M_p^2} f^2(s)ds \ll 1$$



Assuming $G(q^2) \neq 0$
Cauchy theorem gives

$$\oint_C \phi(s)ds = 0$$

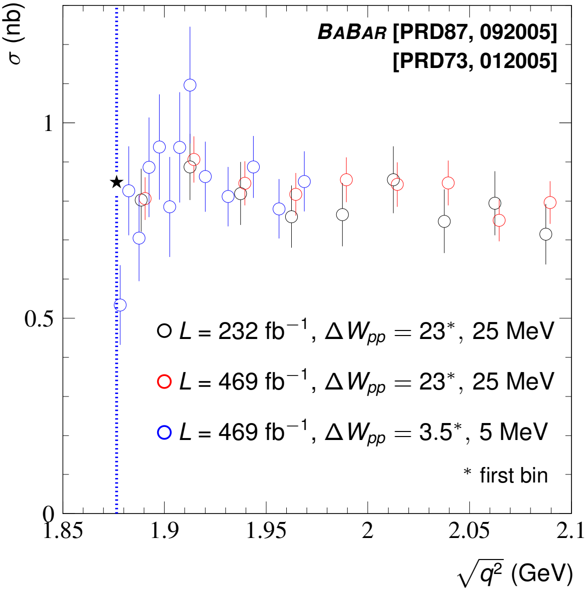
$$\underbrace{-\int_{-\infty}^{0} \frac{\mathrm{Im}[f(t)] \ln G(t)}{t\sqrt{4M_\pi^2 - t}} dt}_{\text{space-like data} + (-t)^n} \simeq \underbrace{\int_{4M_p^2}^{\infty} \frac{f(s) \ln |G(s)|}{s\sqrt{s - 4M_\pi^2}} ds}_{\text{time-like data} + s^n}$$



Sum rule for the asymptotic power law

$$G_M^p(q^2) \underset{|q^2|\to\infty}{\propto} |q^2|^{-(2.27\pm0.36)}$$

The threshold region₃



The threshold region$_3$