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DOMAIN WALL NETWORK AS QCD VACUUM: CONFINEMENT, CHIRAL SYMMETRY, HADRONIZATION

Confinement, chiral symmetry, hadronization under ”normal” conditions

Deconfinement, chiral symmetry restoration under ”extreme” conditions

Dubna, September 15-20, 2014

An overall task pursued by most of the approaches to QCD vacuum structure is an identification of the properties of nonperturbative gauge field configurations able to provide a coherent resolution of the confinement, the chiral symmetry breaking, the $U_\mathrm{A}(1)$ symmetry realization and the strong CP problems, both in terms of color-charged fields and colorless hadrons.

The other side of this task is identification of the conditions for deconfinement and chiral symmetry restoration, if any.

Classical YM action | **Effective quantum action**

"Pseudo-particles":topological localized field configurations $\Leftrightarrow$ **Global minima of the effective quantum action: homogeneous background gauge fields**

Condensation of instantons, monopoles, vortices, double-layer domain walls $\Leftrightarrow$ **Defects in the initally homogeneous background**

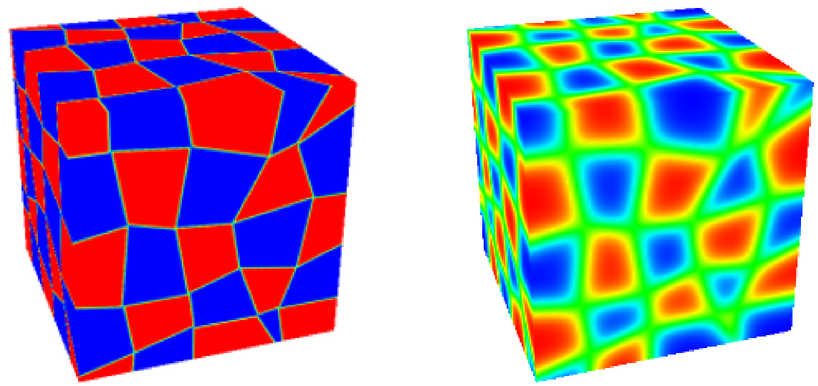
Regge spectrum of hadrons, QCD String, Linear potential, Dual Meissner effect, Area law

Condensates, Spectrum of quantum excitations, Analytical properties of quark and gluon propagators, Regge spectrum of hadrons

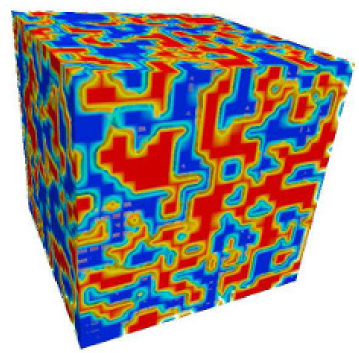
At first sight, both ways seem to lead to a similar outcome - a class of nonperturbative gluon field configurations with a self-consistent balance of order and disorder, characterized by gluon and quark condensates,
keywords: Abelian, (anti)selfdual

Pure Yang-Mills vacuum (no quarks present):

$$\langle : g^2F^2 : \rangle \neq 0, \quad \chi = \int d^4x \langle Q(x)Q(0)\rangle \neq 0, \quad \langle Q(x)\rangle = 0$$



Topological charge density $Q(x) = \frac{g^2}{32\pi^2} F^a_{\mu\nu}(x)\tilde{F}^a_{\mu\nu}(x)$



Topological charge density $Q(x) = \frac{g^2}{32\pi^2} F^a_{\mu\nu}(x)\tilde{F}^a_{\mu\nu}(x)$

Visualizations of Quantum Chromodynamics (QCD) Buried treasure in the sand of the QCD vacuum
P.J. Moran,Derek B. Leinweber, CSSM, University of Adelaide, Australia arXiv:0805.4246v1 [hep-lat] 2008

- **Confinement of both static and dynamical quarks** $\longrightarrow W(C) = \langle \mathrm{Tr}\ \mathrm{P}\ e^{i\int_C dz_\mu \hat{A}_\mu} \rangle$

  $S(x,y) = \langle \psi(y)\bar{\psi}(x) \rangle$

- **Dynamical Breaking of chiral $SU_L(N_f) \times SU_R(N_f)$ symmetry** $\longrightarrow \langle \bar{\psi}(x)\psi(x) \rangle$

- $U_A(1)$ **Problem** $\longrightarrow \eta'$ (χ, Axial Anomaly)

- **Strong CP Problem** $\longrightarrow Z(\theta)$

- **Colorless Hadron Formation:** $\longrightarrow$ Effective action for colorless collective modes:

  hadron masses, formfactors, scattering

  Light mesons and baryons, **Regge spectrum** of excited states of light hadrons,
  heavy-light hadrons, **heavy quarkonia**

What would be a formalism for coherent simultaneous description of all these nonperturbative features of QCD?

QCD vacuum as a medium characterized by certain condensates,

quarks and gluons - elementary coloured excitations (confined),

mesons and baryons - collective colourless excitations (masses, form factors, etc)

Deconfinement $\longrightarrow$ Rearrangement of the vacuum gluon configurations ???

Quarks and gluons has to be activated as quasiparticle elementary excitations (deconfined!!!),

- Effective action of $SU(3)$ YM theory, global minima of the effective action.
- Gluon condensates, Weyl reflections, CP: kink-like gauge field configurations
- Domain wall network as QCD vacuum
- Confinement and the spectrum of charged field fluctuations
- Impact of the strong electromagnetic fields on the QCD vacuum structure and chromomagnetic trap formation, color charged quasiparticles.
- Domain model of confinement.
- Testing the model on the standard set of problems of pure gluodynamics: σ, χ, $\langle F^2 \rangle$.
- Chirality of quark modes. Realisation of chiral symmetry and quark condensate: $U_A(1)$ and $SU_{\rm L}(N) \times SU_{\rm R}(N)$.
- $U_A(1)$ and the strong CP problem. Anomalous Ward Identities.
- Nonlocal effective meson Lagrangian: hadronization scheme and meson masses.

General definition

$$\exp\left(W[J]\right) = N\ \mathrm{Ren} \int \mathcal{D}\phi \exp\left(-S[\phi] + \int dx J\phi\right)$$

$$\Gamma[\varphi] = \int dx J\varphi - W[J], \quad \varphi = \frac{\delta W[J]}{\delta J}$$

$$\Gamma^{(n)}(x_1 \dots x_n) \equiv \left. \frac{\delta^{(n)}\Gamma[\varphi]}{\delta\phi_1(x_1)\cdots\delta\phi_n(x_n)}\right|_{\phi=0}$$

$$\Gamma[\varphi] = \sum_n \int dx_1 \dots \int dx_n \phi_1(x_1)\cdots\phi_n(x_n)\Gamma^{(n)}(x_1 \dots x_n)$$

▶ $\Gamma[\varphi]$ - generating functional of the vertex functions.

▶ $\Gamma[\varphi]$ describes interactions between "classical" fields φ taking into account all quantum effects.

▶ $U_{\text{eff}}(\phi_0) = \Gamma[\varphi_0]/V$ describes the energy density of the quantum system (effective potential) in the presence $\varphi_0 =$"const". Different global minima correspond to different phases of the quantum field system - dynamical symmetry rearrangement, etc.

Tools to compute and study???

P. Minkowski, Nucl. Phys. B**177**, 203 (1981).
H. Leutwyler, Phys. Lett. B**96**, 154 (1980); Nucl. Phys. B**179**, 129 (1981).
H. Pagels, and E. Tomboulis // Nucl. Phys. B143. 1978.
H. D. Trottier and R. M. Woloshyn// Phys. Rev. Lett. 70. 1993.
L. D. Faddeev,"Mass in Quantum Yang-Mills Theory", arXiv:0911.1013v1[math-ph]

S.N. Nedelko, V.E. Voronin, arXiv:1403.0415v1 [hep-ph] (2014)
B. Galilo, S.N. Nedelko, Phys. Rev D 84 (2011)
B. Galilo, S.N. Nedelko, PEPAN Lett. 8 (2011) [arXiv:1006.0248v2],
J. M. Pawlowski, D. F. Litim, S. Nedelko and L. von Smekal, Phys. Rev. Lett. **93** (2004)
A. Eichhorn, H. Gies, J. M. Pawlowski, Phys. Rev. D83 (2011) [arXiv:1010.2153 [hep-ph]]

G.V. Efimov, Ja.V. Burdanov, B.V. Galilo, A.C. Kalloniatis, S.N. Nedelko , L. von Smekal, S.A. Solunin,
Phys. Rev. D 73 (2006), 71 (2005), 70 (2004), 69 (2004), 66 (2002), 51 (1995), 54 (1996).

follows

$$e^{iW(A_{\mathrm{in}},A_{\mathrm{out}})} = \int_{A\to \substack{A_{\mathrm{in}},t\to-\infty \\ A_{\mathrm{out}},t\to+\infty}} e^{iS(A)} dA, \tag{2}$$

where $S(A)$ is the classical action (1). Symbol dA denotes the integration measure and we shall make it more explicit momentarily.

The only functional integral one can deal with is a gaussian one. To reduce (2) to this form and, in particular, to identify corresponding quadratic form we make shift of integration variable

$$A = B + ga,$$

where the external variable B should take into account the asymptotic boundary conditions and new integration variable a has zero incoming and outgoing components.

We can consider both A and B as connections, then a will have only homogeneous gauge transformation

$$a(x) \to h^{-1}(x)ah(x).$$

However, for fixed B the transformation law for a is nonhomogeneous

$$a \to a^h = \frac{1}{g}(A^h - B). \tag{3}$$

Thus the functional $S(B+a) - S(B)$ is constant along such "gauge orbits". Integration over a is to take this into account. We shall denote $W(A_{\mathrm{in}}, A_{\mathrm{out}})$ as $W(B)$, having in mind that B is defined by A_{in}, A_{out} via some differential equation. Here is the answer detailing the formula (2)

$$\begin{aligned} e^{iW(B)} = e^{iS(B)} \int \exp i\{S(B+a) - S(B) + \int \frac{1}{2}\,\mathrm{tr}(\nabla_\mu a_\mu)^2 dx\} \\ \times \det\big((\nabla_\mu + ga_\mu)\nabla_\mu\big) \prod_x da(x). \end{aligned} \tag{4}$$

L. D. Faddeev,"Mass in Quantum Yang-Mills Theory", arXiv:0911.1013v1[math-ph]

In Euclidean functional integral for YM theory **one has to allow the gluon condensate to be nonzero**:

$$Z = N \int_{\mathcal{F}_B} DA \exp\{-S[A]\}$$

$$\mathcal{F}_B = \{A : \lim_{V\to\infty} \frac{1}{V} \int_V d^4x g^2 F^a_{\mu\nu}(x) F^a_{\mu\nu}(x) = B^2\}, \quad B^2 \neq 0.$$

Separation of the long range modes B^a_μ **and local fluctuations** Q^a_μ **in the background** B^a_μ, background gauge fixing condition ($D(B)Q = 0$): $A^a_\mu = B^a_\mu + Q^a_\mu$

$$1 = \int_{\mathcal{B}} DB\Phi[A,B] \int_{\mathcal{Q}} DQ \int_{\Omega} D\omega \delta[A^\omega - Q^\omega - B^\omega]\delta[D(B^\omega)Q^\omega]$$

Q^a_μ – local (perturbative) fluctuations of gluon field **with zero gluon condensate:** $Q \in \mathcal{Q}$;
B^a_μ are **long range field configurations with nonzero condensate:** $B \in \mathcal{B}$**.**

$$Z = N' \int_{\mathcal{B}} DB \int_{\mathcal{Q}} DQ \det[D(B)D(B+Q)]\delta[D(B)Q] \exp\{-S[B+Q]\}$$

Self-consistency: the character of long range fields has yet to be identified by the dynamics of fluctuations:

$$Z = N' \int_{\mathcal{B}} DB \exp\{-S_{\text{eff}}[B]\}.$$

Global minima of $S_{\text{eff}}[B]$ – field configurations that are dominant in the thermodynamic limit $V \to \infty$.

$$Z = N' \int_{\mathcal{B}} DB \int_{\mathcal{Q}} DQ \det[D(B)D(B+Q)]\delta[D(B)Q] \exp\{-S[B+Q]+S[B]\}.$$

***H. Leutwyler*: Covariantly constant Abelian (anti-)self-dual fields**

$$B^a_\mu = -\frac{1}{2} n^a B_{\mu\nu} x_\nu, \quad \tilde{B}_{\mu\nu} = \pm B_{\mu\nu}$$

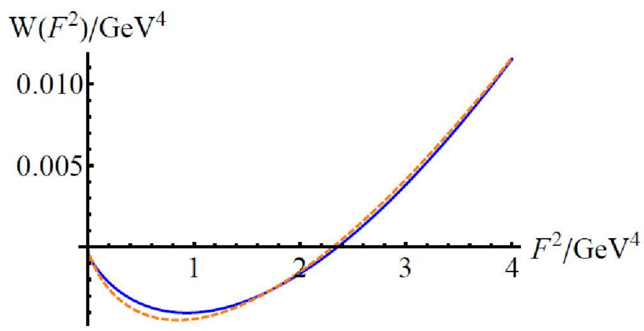
are the only stable against local fluctuations Q constant fields.

Explicit **one-loop effective action:**

$$S^{1-\mathrm{loop}}_{\mathrm{eff}} = B^2 \left[\frac{11}{24\pi^2} \ln \frac{\lambda B}{\Lambda^2} + \varepsilon_0 \right].$$

H. Leutwyler (1980,1981); *P. Minkowski* (1981); *H. Pagels, and E. Tomboulis* (1978); *H. D. Trottier and R. M. Woloshyn* (1993).

Effective potential for covariantly constant Abelian (anti-)self-dual field within the Functional RG:



A. Eichhorn, H. Gies, J. M. Pawlowski, Phys. Rev. D83 (2011) [arXiv:1010.2153 [hep-ph]]

Ginsburg-Landau Effective Lagrangian

Consider the following Ginsburg-Landau effective Lagrangian for the soft gauge fields satisfying the requirements of invariance under the gauge group $SU(3)$ and space-time transformations,

$$L_{\rm eff} = -\frac{\Lambda^2}{4}\left(D^{ab}_\nu F^b_{\rho\mu} D^{ac}_\nu F^c_{\rho\mu} + D^{ab}_\mu F^b_{\mu\nu} D^{ac}_\rho F^c_{\rho\nu}\right) - U_{\rm eff}$$

$$U_{\rm eff} = \frac{1}{12}{\rm Tr}\left(C_1 \hat{F}^2 + \frac{4}{3}C_2\hat{F}^4 - \frac{16}{9}C_3\hat{F}^6\right),$$

where

$$D^{ab}_\mu = \delta^{ab}\partial_\mu - i\hat{A}^{ab}_\mu = \partial_\mu - iA^c_\mu (T^c)^{ab}, \quad F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - if^{abc}A^b_\mu A^c_\nu,$$

$$\hat{F}_{\mu\nu} = F^a_{\mu\nu}T^a, \quad T^a_{bc} = -if^{abc} \quad {\rm Tr}\left(\hat{F}^2\right) = \hat{F}^{ab}_{\mu\nu}\hat{F}^{ba}_{\nu\mu} = -3F^a_{\mu\nu}F^a_{\mu\nu} \le 0,$$

$$gA_\mu \to A_\mu.$$

$$\lim_{V\to\infty} V^{-1}\int_V d^4x \langle F^2\rangle \neq 0 \longrightarrow C_1 > 0,\ C_2 > 0,\ C_3 > 0.$$

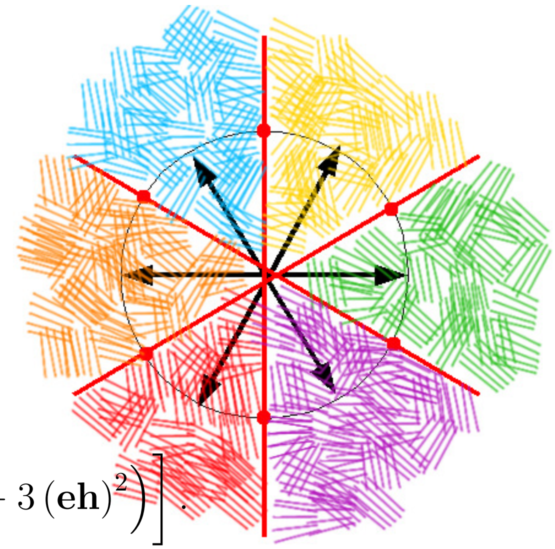
$$F^a_{\mu\nu}F^a_{\mu\nu} = 4b^2_{\rm vac}\Lambda^4 > 0,\ \ b^2_{\rm vac} = \left(\sqrt{C_2^2 + 3C_1C_3} - C_2\right)/3C_3.$$

Consider A_μ fields with the **Abelian field strength**

$$\hat{F}_{\mu\nu} = \hat{n} B_{\mu\nu},$$

where matrix $\hat{n}$ can be put into Cartan subalgebra

$$\hat{n} = T^3 \cos\xi + T^8 \sin\xi,\ 0 \le \xi < 2\pi.$$



It is convenient to introduce the following notation:

$$\hat{b}_{\mu\nu} = \hat{n} B_{\mu\nu}/\Lambda^2 = \hat{n} b_{\mu\nu},\ b_{\mu\nu} b_{\mu\nu} = 4b_{\rm vac}^2,$$

$$e_i = b_{4i},\ h_i = \frac{1}{2}\varepsilon_{ijk} b_{jk},\ \mathbf{e}^2 + \mathbf{h}^2 = 2b_{\rm vac}^2.$$

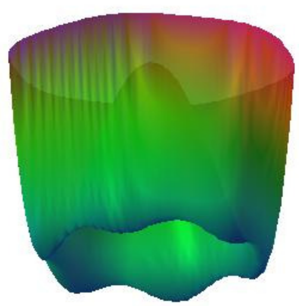
$$(\mathbf{eh}) = |\mathbf{e}|\,|\mathbf{h}| \cos\omega,\ \ (\mathbf{eh})^2 = \mathbf{h}^2 \left(2b^2 - \mathbf{h}^2\right) \cos^2\omega.$$

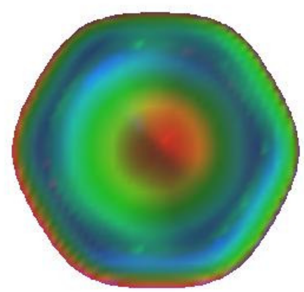
Hence the effective potential takes the form

$$U_{\rm eff} = \Lambda^4 \left[-C_1 b_{\rm vac}^2 + C_2 \left(2b_{\rm vac}^4 - (\mathbf{eh})^2\right) + \frac{1}{9} C_3 b^2 \left(10 + \cos 6\xi\right) \left(4b_{\rm vac}^4 - 3\,(\mathbf{eh})^2\right) \right].$$

There are twelve discrete global degenerated minima at the following values of the variables h, ω and ξ

$$\mathbf{h}^2 = b_{\rm vac}^2 > 0,\ \ \omega = \pi k\ \ (k = 0, 1),\ \ \xi = \frac{\pi}{6}(2n+1)\ (n = 0, \ldots, 5).$$





Domain wall defects in homogeneous background

Discrete minima mean that there exist kink-like field configurations interpolating between these minima. For instance, for the angle ω

$$L_{\rm eff} = -\frac{1}{2}\Lambda^2 b_{\rm vac}^2 \partial_\mu\omega\partial_\mu\omega - b_{\rm vac}^4\Lambda^4\left(C_2 + 3C_3 b_{\rm vac}^2\right)\sin^2\omega,$$

with the sine-Gordon equation of motion

$$\partial^2\omega = m_\omega^2 \sin 2\omega, \quad m_\omega^2 = b_{\rm vac}^2\Lambda^2\left(C_2 + 3C_3 b_{\rm vac}^2\right),$$

with kink solution

$$\omega = 2\arctan\left(\exp\left(\sqrt{2}m_\omega x_1\right)\right),$$

which can be treated as domain wall.

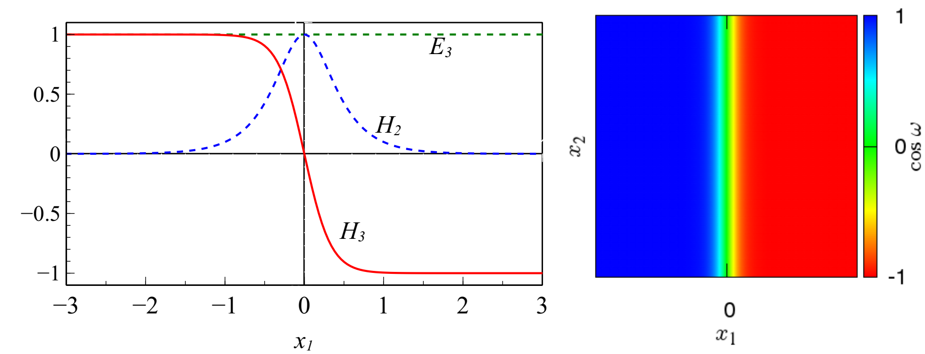


Figure 1: Kink profile in terms of the components of the chromomagnetic and chromoelectric field strength (left), and a two-dimensional slice for the topological charge density in the presence of a single kink measured in units of $g^2 F^b_{\alpha\beta}F^b_{\alpha\beta}$ (right). Chromomagnetic and chromoelectric fields are orthogonal to each other inside the wall (green color).

Domain wall network

Denote the general kink configuration:

$$\zeta(\mu_i, \eta^i_\nu x_\nu - q^i) = \frac{2}{\pi} \arctan \exp(\mu_i(\eta^i_\nu x_\nu - q^i))$$

μ_i – inverse width of the kink, η^i_ν – a normal vector to the plane of the wall, $q^i = \eta^i_\nu x^i_\nu$ with x^i_ν - coordinates of the wall.

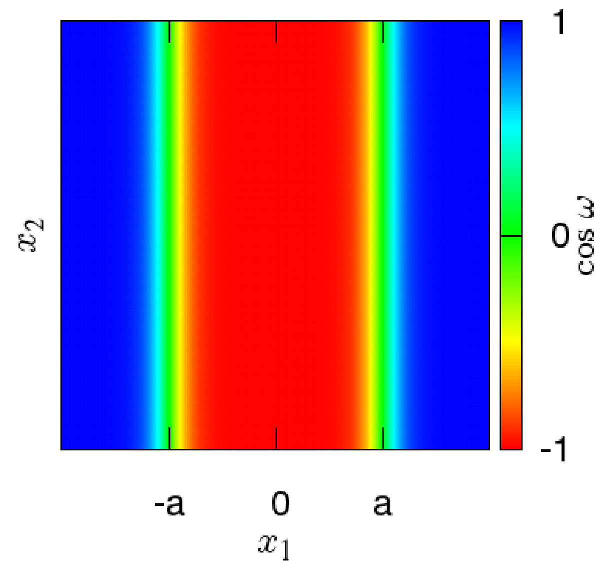


Figure 2: Two-dimentional slice of a multiplicative superposition of two kinks with normal vectors anti-parallel to each other $\omega(x_1) = \pi\zeta(\mu_1, x_1 - a_1)\zeta(\mu_2, -x_1 - a_2)$.

Additive superposition of infinitely many such pairs

$$\omega(x_1) = \pi \sum_{j=1}^{\infty} \zeta(\mu_j, x_1 - a_j)\zeta(\mu_{j+1}, -x_1 - a_{j+1})$$

gives a layered topological charge structure in R^4, Fig.3.

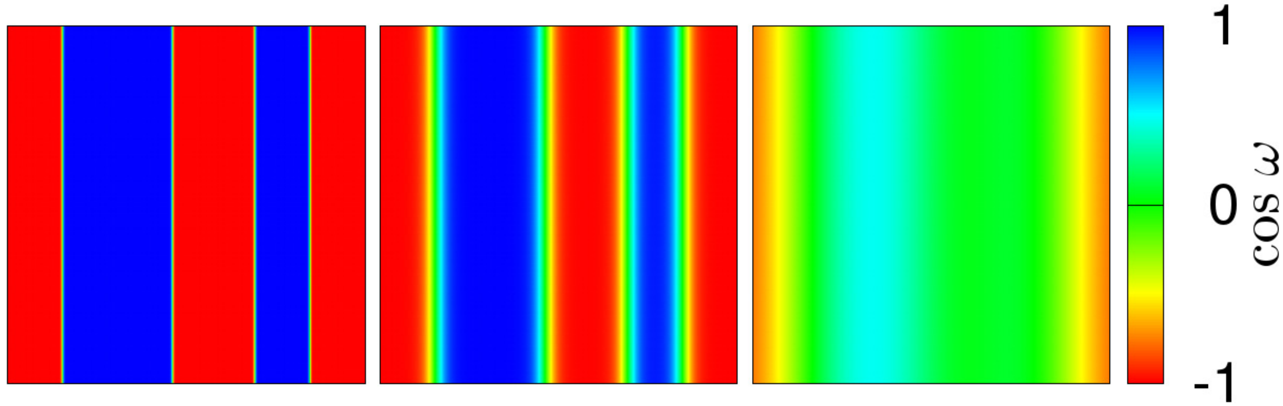


Figure 3: Two-dimentional slice of layered topological charge distribution in R^4 . The action density is equal to the same nonzero constant value for all three configurations. The LHS plot represents configuration with infinitely thin planar Bloch domain wall defects, which is Abelian homogeneous (anti-)self-dual field almost everywhere in R^4, characterized by the nonzero absolute value of topological charge density almost everywhere proportional to the value of the action density. The most RHS plot shows the opposite case of very thik kink network. Green color corresponds to the gauge field with infinitesimally small topological charge density. The most LHS configuration is confining (only colorless hadrons can be excited) while the most RHS one supports the color charged quasiparticles as elementary excitations.

One may go further and consider a product

$$\omega(x) = \pi \prod_{i=1}^{k} \zeta(\mu_i, \eta^i_\nu x_\nu - q^i). \tag{1}$$

For an appropriate choice of normal vectors η^i this superposition represents a lump of anti-selfdual field in the background of the selfdual one, in two, three and four dimensions for $k = 4, 6, 8$ respectively.

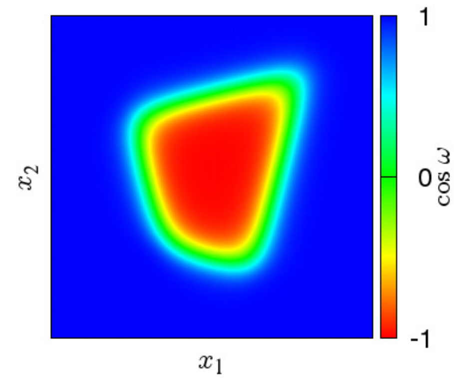


Figure 4: A two-dimensional slice of the four-dimentional lump of anti-selfdual field in the background of the self-dual configuration. The domain wall surrounding the lamp in the four-dimensional space is given by the multiplicative superposition of eight kinks as it is defined by Eq.(1).

$$\mathrm{div}\mathbf{H} \neq 0$$

The general kink network is then given by additive superposition of lumps (1)

$$\omega = \pi \sum_{j=1}^{\infty} \prod_{i=1}^{k} \zeta(\mu_{ij}, \eta_{\nu}^{ij} x_{\nu} - q^{ij}).$$

Correponding topological charge density is shown in Fig.5. The LHS plot in Figs.5 and 3 represents configuration with infinitely thin domain walls, that isAbelian homogeneous (anti-)self-dual field almost everywhere in R^4, characterized by the nonzero absolute value of topological charge density which is constant and proportional to the value of the action density almost everywhere.

The most RHS plots Figs.3 and 5 show the opposite case of the network composed of very thik kinks. Green color corresponds to the gauge field with infinitesimally small topological charge density. Study of the spectrum of clorless and color charged fluctuations indicates that the most LHS configuration is expected to be confining (only colorless hadrons can be excited as particles) while the most RHS one (crossed orthogonal field) supports the color charged quasi-particles as the dominant elementary excitations.

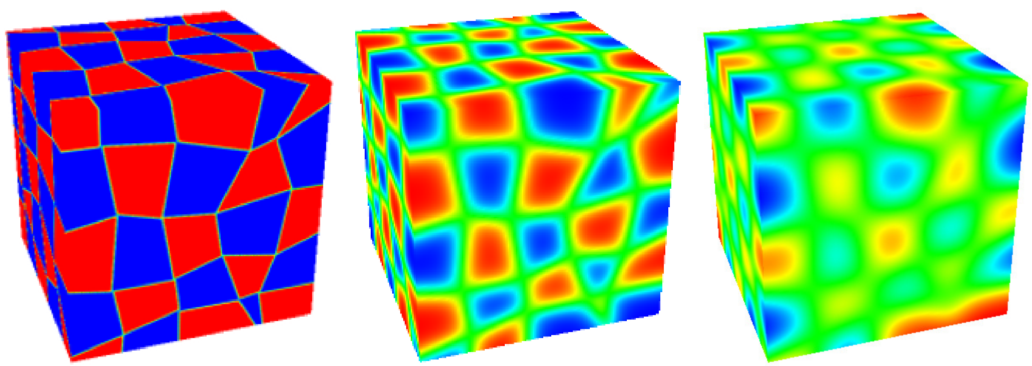


Figure 5: Three-dimensional slices of the kink network - additive superposition of numerous four-dimensional lamps.

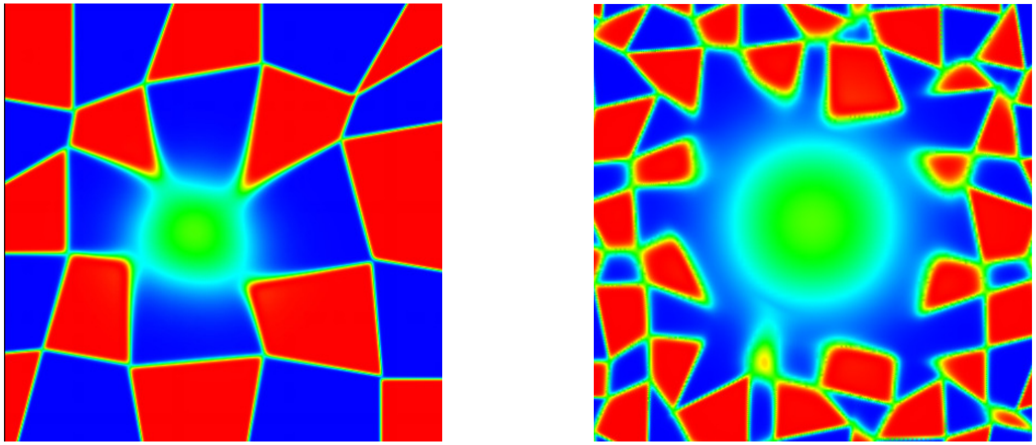


Figure 6: Examples of two-dimensional slice of the cylindrical thick domain wall junctions. The correspondence of colors is the same as in Fig.3. Blue and red regions represent self-dual and anti-self-dual lumps. Confinement is lost in the green region where $g^2\tilde{F}_{\mu\nu}(x)F_{\mu\nu}(x) = 0$. The scalar condensate density $g^2F_{\mu\nu}(x)F_{\mu\nu}(x)$ is nonzero and homogeneous everywhere.

Separation of the Abelian part – gauge field parameterization

The Abelian $\hat{B}_\mu(x)$ part of the gauge fields

$$\hat{A}_\mu(x) = \hat{B}_\mu(x) + \hat{X}_\mu(x), \quad [\hat{B}_\mu(x), \hat{B}_\nu(x)] = 0$$

Cho-Faddev-Niemi-Shabanov-Kondo: the Abelian part $\hat{V}_\mu(x)$ of the gauge field $\hat{A}_\mu(x)$ is separated manifestly,

$$\hat{A}_\mu(x) = \hat{V}_\mu(x) + \hat{X}_\mu(x),$$

$$\hat{V}_\mu(x) = \hat{B}_\mu(x) + \hat{C}_\mu(x),$$

$$\hat{B}_\mu(x) = [n^a A^a_\mu(x)]\hat{n}(x) = B_\mu(x)\hat{n}(x),$$

$$\hat{C}_\mu(x) = g^{-1}\partial_\mu \hat{n}(x) \times \hat{n}(x),$$

$$\hat{X}_\mu(x) = g^{-1}\hat{n}(x) \times \left(\partial_\mu \hat{n}(x) + g\hat{A}_\mu(x) \times \hat{n}(x)\right),$$

$$\hat{A}_\mu(x) = A^a_\mu(x)t^a, \quad \hat{n}(x) = n_a(x)t^a, \quad n^a n^a = 1$$

$$\partial_\mu \hat{n} \times \hat{n} = -if^{abc}t^a n^b \partial_\mu n^c, \; [t^a, t^b] = if^{abc}t^c.$$

The field $\hat{V}_\mu$ is the Abelian field:

$$[\hat{V}_\mu(x), \hat{V}_\nu(x)] = 0$$

$$\hat{F}_{\mu\nu}(x) = \hat{n}(x)\left[\partial_\mu B_\nu - \partial_\nu B_\mu + ig^{-2}f^{abc}n^a\partial_\mu n^b(x)\partial_\nu n^c(x)\right]$$

L. D. Faddeev, A. J. Niemi // Nucl. Phys. B. 776. 2007
Kri-Ichi Kondo, Toru Shinohara, Takeharu Murakami // arXiv:0803.0176v2 [hep-th] 2008
Y.M. Cho, Phys. Rev. D 21, 1080(1980); Y.M. Cho, Phys. Rev. D 23, 2415(1981).
S.V. Shabanov, Phys. Lett. B 458, 322(1999); Phys. Lett. B 463, 263(1999); Teor. Mat. Fiz., Vol. 78, 411 (1989)

The eigenmodes of the scalar charged field in (anti-)selfdual background

$$-\left(\partial_\mu - iB_\mu(x)\right)^2 \phi = \lambda\phi.$$

The homogeneous (anti-)self-dual fields

$$B_\mu(x) = B_{\mu\nu}x_\nu,\;\; \tilde{B}_{\mu\nu} = \pm B_{\mu\nu},\; B_{\mu\alpha}B_{\nu\alpha} = B^2\delta_{\mu\nu},\; B = \Lambda^2 b_{\rm vac},$$

The eigenvalue equation can be rewritten as follows

$$\left[\beta_\pm^+\beta_\pm + \gamma_+^+\gamma_+ + 1\right]\phi = \frac{\lambda}{4B}\phi$$

where creation and annihilation operators $\beta_\pm$, $\beta_\pm^+$, $\gamma_\pm$, $\gamma_\pm^+$ are expressed in terms of the operators α^+, α,

$$\beta_\pm = \frac{1}{2}\left(\alpha_1 \mp i\alpha_2\right),\; \gamma_\pm = \frac{1}{2}\left(\alpha_3 \mp i\alpha_4\right),\; \alpha_\mu = \frac{1}{\sqrt{B}}(Bx_\mu + \partial_\mu),$$

$$\beta_\pm^+ = \frac{1}{2}\left(\alpha_1^+ \pm i\alpha_2^+\right),\; \gamma_\pm^+ = \frac{1}{2}\left(\alpha_3^+ \pm i\alpha_4^+\right),\; \alpha_\mu^+ = \frac{1}{\sqrt{B}}(Bx_\mu - \partial_\mu).$$

The eigenvalues and the square integrable eigenfunctions are

$$\lambda_r = 4B\,(r+1)\,,\;\; r = k + n \;(\text{for self}-\text{dual field}),\;\; r = l + n \;(\text{for anti}-\text{self}-\text{dual field}) \qquad (2)$$

$$\phi_{nmkl}(x) = \frac{1}{\sqrt{n!m!k!l!\pi^2}}\left(\beta_+^+\right)^k\left(\beta_-^+\right)^l\left(\gamma_+^+\right)^n\left(\gamma_-^+\right)^m\phi_{0000}(x),\; \phi_{0000}(x) = e^{-\frac{1}{2}Bx^2}, \qquad (3)$$

Discrete spectrum. Absence of periodic solutions is treated as confinement of the charged field.

$$D^2(x)G(x,y) = -\delta(x-y) \quad G(x,y) = e^{ixBy}H(x-y) \quad \tilde{H}(p^2) = \frac{1-e^{-p^2/B}}{p^2}$$

Charged field fluctuations in the background of a domain wall
A single planar domain wall of the Bloch type.

Confined fluctuations in the bulk: $x_1 \neq 0$

Consider the eigenvalue problem

$$-\tilde{D}^2\Phi = \lambda\Phi.$$

for the functions square integrable in R^4 and satisfying the integral current continuity condition .
For all $x_1 \neq 0$ the operator $\tilde{D}^2$ takes the form

$$\tilde{D}^2 = (\partial_1 \pm i\hat{n}Bx_2)^2 + \partial_2^2 + \partial_3^2 + (\partial_4 + i\hat{n}Bx_3)^2$$

"+" corresponds to the anti-selfdual configuration ($x_1 > 0$) and "-" is for the self-dual one ($x_1 < 0$).

$$\Phi_{kl} = \int dp_1 dp_4 f(p_1) g(p_4) \exp\left\{\pm ip_1x_1 + ip_4x_4 - \frac{1}{2}|\hat{n}|B(x_2 + p_1/|\hat{n}|B)^2 - \frac{1}{2}|\hat{n}|B(x_3 + p_4/|\hat{n}|B)^2\right\}$$
$$\times H_k\left(\sqrt{|\hat{n}|B}\left[x_2 + \frac{p_1}{|\hat{n}|B}\right]\right) H_l\left(\sqrt{|\hat{n}|B}\left[x_3 + \frac{p_4}{|\hat{n}|B}\right]\right),$$

where H_m are the Hermite polynomials. The eigenvalues are

$$\lambda_{kl} = 2|\hat{n}|B(k + l + 1), \ \ k, l = 0, 1, \ldots .$$

The eigenfunctions are of the bound state type with the purely discrete spectrum. Field fluctuations of this type can be seen as confined. The eigenvalues coincide with those for the purely homogeneous (anti-)selfdual Abelian field. In this sense domain wall defect does not destroy confinement of dynamical color charged fields. The eigenfunctions are restricted by the correlated evenness condition, while in the case of the the homogeneous field the properties of the amplitude $f(p_1)$ and the polynomial H_k are mutually independent.

Color charged quasi-particles on the wall: $x_1 = 0$

On the wall the chromomagnetic and chromoelectric fields are orthogonal to each other (see Fig.??). In conformity with integral current continuitythe absence of the charged current off the infinitely thin domain wall requires

$$\partial_1 \Phi|_{x_1=0} = 0,$$

and **the eigenvalue problem on the wall takes the form**

$$\left[-\partial_2^2 - \partial_3^2 + \hat{n}^2 B^2 x_3^2 + (i\partial_4 - \hat{n} B x_3)^2\right] \Phi = \lambda \Phi$$

with the solution

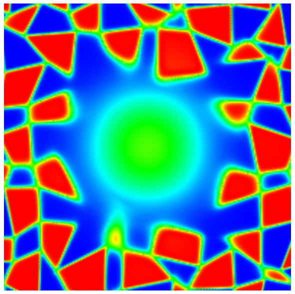
$$\Phi_{kp_2p_4}(x_2, x_3, x_4) = e^{ip_2x_2 + ip_4x_4} e^{-\frac{|\hat{n}|B}{\sqrt{2}}\left(x_3 - \frac{p_4}{2|\hat{n}|B}\right)^2} H_k \left[\sqrt{\sqrt{2}|\hat{n}|B}\left(x_3 - \frac{p_4}{2|\hat{n}|B}\right)\right],$$

$$\lambda_k(p_2^2, p_4^2) = \sqrt{2}|\hat{n}|B(2k+1) + \frac{p_4^2}{2} + \frac{p_2^2}{2}, \ k = 0, 1, 2, \ldots$$

The spectrum of the eigenmodes on the wall is continuous, it depends on the momentum p_2 longitudinal to the chromomagnetic field and Euclidean energy p_4, the corresponding eigenfunctions are oscillating in x_2 and x_4.
In the transverse to chromomagnetic field direction x_3 the eigenfunctins are bounded and the eigenvalues display the Landau level structure.
This can be treated as the lack of confinement - the color charged quasi-particles can be excited on the wall.
The continuation $p_4 = -p_0$ leads to the dispersion relation for the quasi-particles with the masses μ_n

$$p_0^2 = p_2^2 + \mu_k^2, \quad \mu_k^2 = 2\sqrt{2}(2k+1)|\hat{n}|B, \ k = 0, 1, 2, \ldots.$$

Cylindrical trap



Scalar field eigenmodes

Consider the eigenvalue problem for the massless scalar field Φ^a

$$-\left(\partial_\mu - i\breve{B}_\mu\right)^2 \Phi(x) = \lambda^2\Phi(x) \tag{4}$$

in the cylindrical region

$$x \in \mathcal{T} = \left\{x_1^2 + x_2^2 < R^2,\ (x_3, x_4) \in \mathrm{R}^2\right\}$$

with the homogeneous Dirichlet condition at the boundary

$$\Phi(x) = 0,\ x \in \partial\mathcal{T}, \quad \partial\mathcal{T} = \left\{x_1^2 + x_2^2 = R^2,\ (x_3, x_4) \in \mathrm{R}^2\right\}.$$

Here $\breve{B}_\mu$ stays for adjoiunt representaion of the homogeneous chromomagnetic field $H_i^a = \delta_{i3} n^a H$ with the vector potential taken in the symmetric gauge

$$\breve{B}_\mu = -\frac{1}{2}\breve{n} B_{\mu\nu} x_\nu, , \quad \breve{B}_4 = \breve{B}_3 = 0,\ B_{12} = -B_{21} = H, \quad \breve{n} = T_3\cos(\xi) + T_8\sin(\xi).$$

Solution of the problem (4) is straightforward. It is convenient to introduce dimensionless variables using the strength of the chromomagnetic field as a basic scale. Below all quantities are assumed to be measured in terms of this scale, for instance

$$\sqrt{H}x_\mu \equiv x_\mu, \quad \frac{\lambda}{\sqrt{H}} \equiv \lambda.$$

After diagonalization with respect to color indices and transformation to the cylindrical coordinates Eq. (4) takes the form

$$-\left[\partial_4^2 + \partial_3^2 + \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \vartheta^2} - i\kappa^a v \frac{\partial}{\partial \vartheta} - \frac{1}{4}v^2 r^2\right]\Phi^a = \lambda^2 \Phi^a, \tag{5}$$

The variables in Eq. (5) are separated by substitution

$$\Phi^a = \phi^a(r) e^{il\vartheta} \exp\left(ip_3 x_3 + ip_4 x_4\right).$$

Periodicity of the solution in angle $\vartheta \in [0, 2\pi]$ requires integer values of parameter l. The radial part $\phi(r)$ should satisfy equation

$$-\left[\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} - \frac{1}{r^2}\left(\frac{1}{2}\breve{v} r^2 - l\right)^2\right]\phi = \mu^2 \phi, \tag{6}$$

where μ is related to the original eigenvalue λ,

$$\lambda^2 = p_4^2 + p_3^2 + \mu^2.$$

By means of the substitution

$$\phi = r^l e^{-\frac{1}{4}\breve{v} r^2}\chi$$

one arrives at the Kummer equation ($z = \breve{v} r^2/2$)

$$\left[z\frac{d^2}{dz^2} + (l + 1 - z)\frac{d}{dz} - \frac{\breve{v} - \mu^2}{2\breve{v}}\right]\chi = 0. \tag{7}$$

General solution of equation (6) takes the form

$$\phi_l(r) = e^{-\frac{1}{4}\breve{v}r^2}\left[C_1 r^l M\left(\frac{\breve{v}-\mu^2}{2\breve{v}}, 1+l, \frac{1}{2}\breve{v}r^2\right) + C_2 r^{-l} M\left(\frac{\breve{v}-\mu^2}{2\breve{v}} - l, 1-l, \frac{1}{2}\breve{v}r^2\right)\right].$$

The first term is regular at $r = 0$ provided $l \geqslant 0$ while the second one is well-defined for $l \leqslant 0$. Therefore, the solution regular inside the cylinder is

$$\phi_{al} = e^{-\frac{1}{4}\breve{v}_a r^2} r^l M\left(\frac{\breve{v}_a-\mu^2}{2\breve{v}_a}, 1+l, \frac{1}{2}\breve{v}_a r^2\right), \quad l \geqslant 0, \tag{8}$$

$$\phi_{al} = e^{-\frac{1}{4}\breve{v}_a r^2} r^{-l} M\left(\frac{\breve{v}_a-\mu^2}{2\breve{v}_a} - l, 1-l, \frac{1}{2}\breve{v}_a r^2\right), \quad l < 0, \tag{9}$$

where the color index a has been explicitly indicated. The color matrix elements $\breve{v}_a$ can be negative. In this case one has to apply Kummer transformation

$$M(a,b,z) = e^z M(b-a, b, -z).$$

Dirichlet boundary condition (5) defines the infinite discrete set of eigenvalues as the solutions μ^2_{alk} $(k = 0, 1\ldots\infty)$ of the equations

$$M\left(\frac{\hat{v}_a-\mu^2}{2\hat{v}_a}, 1+l, \frac{1}{2}\hat{v}_a R^2\right) = 0, \quad l \geqslant 0, \quad M\left(\frac{\hat{v}_a-\mu^2}{2\hat{v}_a} - l, 1-l, \frac{1}{2}\hat{v}_a R^2\right) = 0, \quad l < 0. \tag{10}$$

If μ^2_{alk} satisfies equation (10), than $\tilde{\mu}^2_{alk} = \mu^2_{alk} - 2\hat{v}_a l$ is a solution of (10).

Finally the complete orthogonal set of eigenfunctions for the problem (4) and (5) reads

$$\Phi_{alk}(p_3, p_4 | r, \vartheta, x_3, x_4) = e^{ip_3x_3 + ip_4x_4} e^{il\vartheta} \phi_{alk}(r),$$
$$\lambda^2_{alk} = p_4^2 + p_3^2 + \mu^2_{akl}, \quad k = 0, 1, \ldots, \infty, \quad l = -\infty \ldots \infty,$$

where functions ϕ_{alk} are defined by (9) with $\mu^2 = \mu^2_{akl}$ solving the boundary condition (10).

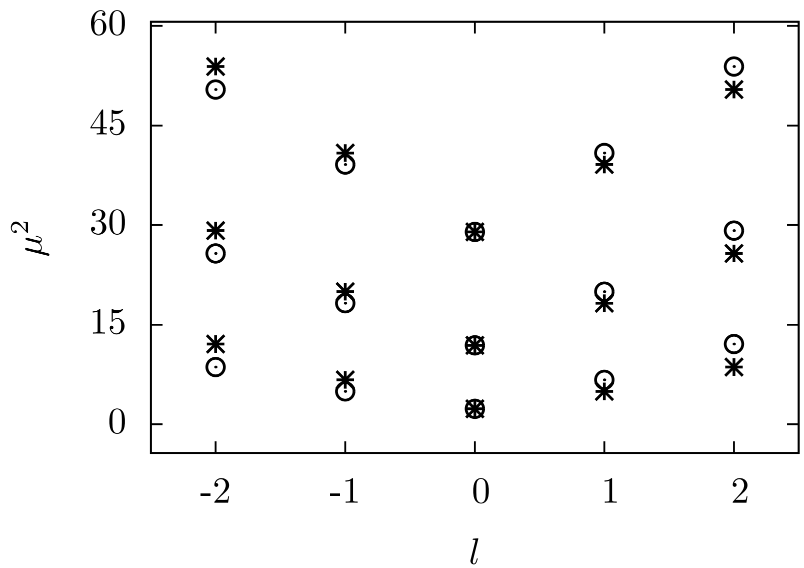


Figure 7: Eigenvalues μ^2_{alk} for the scalar field problem, $l = -2, -1, 0, 1, 2$ and $k = 0, 1, 2$, for $\sqrt{H}R = 1.6$.

Unlike Landau levels in the infinite space the eigenvalues μ^2_{akl} are not equidistant in k and non-degenerate in l as it is illustrated in Fig.7. The dependence of several low-lying eigenvalues μ^2_{akl} on the dimensionless size parameter $\sqrt{H}R$ is shown in Fig.8.

Vector field eigenmodes

The problem for adjoint representation vector field takes the form

$$\left[-\breve{D}^2\delta_{\mu\nu} + 2i\breve{n}B_{\mu\nu}\right]Q_\nu = \lambda Q_\mu, \tag{11}$$

and the boundary conditions are

$$\breve{n}Q_\mu(x) = 0,\ x \in \partial\mathcal{T},\quad \partial\mathcal{T} = \left\{x_1^2 + x_2^2 = R^2,\ (x_3, x_4) \in \mathrm{R}^2\right\}. \tag{12}$$

In terms of the eigenvectors $\breve{Q}^a_\mu$ of matrices $B_{\mu\nu}$ and $\breve{n}$ Eqs. (11) and (12) take the form

$$\left[-\breve{D}^2 + 2s_\mu\breve{v}H\right]^a\breve{Q}^a_\mu = \lambda_{a\mu}\breve{Q}^a_\mu,\qquad \breve{v}\breve{Q}_\mu(x) = 0,\ x \in \partial\mathcal{T}. \tag{13}$$

Omitting obvious well-known details we just note that equation (13) describes sixteen charged with respect to $\breve{n}$ spin-color polarizations of the gluon fluctuations with $(s_1 = 1, s_2 = -1, s_3 = s_4 = 0)$ and $\breve{v}^a \neq 0$ as well as sixteen "color neutral" with respect to $\breve{n}$ modes

$$-\partial^2\breve{Q}^{(0)}_\mu = p^2\breve{Q}^{(0)}_\mu.$$

Neutral mode $\breve{Q}^{(0)}_\mu$ is a zero mode of $\breve{n}$, and it is insensitive to the boundary condition (12). We shall briefly discuss the possible role of the neutral modes in the last section.

Equations for the color charged modes have the same form as the scalar field equation in the previous subsection. The only essential difference is that the eigenvalues $\lambda_{alk\nu}$ for nonzero v^a have an addition $\pm 2vH$ to the eigenvalues μ^2_{akl} of the scalar case:

$$\lambda^2_{alk\nu} = p_4^2 + p_3^2 + \mu^2_{alk} + 2s_\nu\kappa_a v, \quad k = 0, 1, \ldots, \infty, \ l \in Z, \ s_1 = 1, \ s_2 = -1, \ s_3 = s_4 = 0, \ \kappa_a = \pm 1,$$

where μ^2_{akl} are the same as in the scalar case. If we were considering the square integrable solutions in R^4 then the lowest mode $\lambda^2_{a00\nu}$ with $s_\nu\kappa_a = -1$ would be tachyonic. In the finite trap the lowest eigenvalue is

$$\lambda^2_{a00\nu} = p_4^2 + p_3^2 + \mu^2_{a00} - 2v, \ s_\nu\kappa_a = -1.$$

The dependence of μ^2_{a00} on dimensionless size parameter $\sqrt{H}R$ is strongly nonlinear. If the dimensionless size $\sqrt{H}R$ of the trap is sufficiently small

$$\sqrt{H}R < \sqrt{H}R_c \approx 1.91,$$

then there are no unstable tachyonic modes in the spectrum of color charged vector fields.

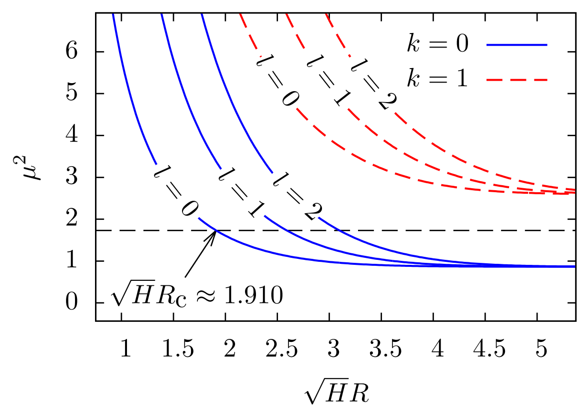


Figure 8: The lowest eigenvalues corresponding to positive color orientation $\kappa^a = 1$ as functions of $\sqrt{H}R$.

To estimate the critical size one may use the mean phenomenological value of the gluon condensate

$$\langle g^2 F^a_{\mu\nu} F^{a\mu\nu} \rangle = 2H^2 \approx 0.5\mathrm{GeV}^4 \quad \longrightarrow \quad R_c \approx 0.51 \text{ fm } (2R_c \approx 1 \text{ fm}).$$

The tachyonic mode is absent if the diameter of the cylindrical trap is less or equal to 1 fm.

Quark field eigen modes

In this subsection we address the eigenvalue problem for Dirac operator in the cylindrical region in the presence of chromomagnetic background field ()

$$\not{D}\psi(x) = \lambda\psi(x), \ \ D_\mu = \partial_\mu + \frac{i}{2}\hat{n}B_{\mu\nu}x_\nu, \tag{14}$$

$$\hat{n} = t_3\cos\xi + t_8\sin\xi = \frac{1}{2}\mathrm{diag}\left(\cos\xi + \frac{\sin\xi}{\sqrt{3}}, -\cos\xi + \frac{\sin\xi}{\sqrt{3}}, -\frac{2\sin\xi}{\sqrt{3}}\right). \tag{15}$$

Euclidean Dirac matrices are taken in the anti-hermitian representation.
The angle ξ is assumed to take one of the vacuum values ξ_k, the following forms of the matrix $\hat{n}$ can occur

$$\hat{n} = \left\{\pm\frac{1}{\sqrt{3}}\mathrm{diag}\left(1, -\frac{1}{2}, -\frac{1}{2}\right), \pm\frac{1}{\sqrt{3}}\mathrm{diag}\left(\frac{1}{2}, \frac{1}{2}, -1\right), \pm\frac{1}{\sqrt{3}}\mathrm{diag}\left(-\frac{1}{2}, 1, -\frac{1}{2}\right)\right\}.$$

The boundary conditions are

$$i\,\not{\eta}(x)e^{i\theta\gamma_5}\hat{n}\psi(x) = \hat{n}\psi(x), \quad \bar{\psi}(x)e^{i\theta\gamma_5}\hat{n}i\not{\eta}(x) = -\bar{\psi}(x)\hat{n}, \ x \in \partial\mathcal{T}, \tag{16}$$

where η_μ is a unit vector normal to the cylinder surface $\partial\mathcal{T}$, see Eq. (12). These are simply the bag boundary conditions. This choice appears to be rather natural. Indeed, inside the thick domain wall junction one expects an existence of the color charged quasiparticles (quarks) being the carriers of the color current, but outside the junction gluon configurations are confining (see Fig. 6)) and the current has to vanish at the boundary. Unlike the adjoint representation of color matrix (**??**) the matrix $\hat{n}$ in fundamental representation (15) has no zero eigenvalues for any value of the angle ξ corresponding to the boundaries of Weyl chambers. Boundary condition (16) restricts all three color components of the quark field.

Substitution

$$\psi = (\not{D} + \lambda)\,\varphi \tag{17}$$

leads to the equation

$$-\left(D^2 + \hat{u}H\Sigma_3\right)\varphi = \lambda^2\varphi, \tag{18}$$

where it has been used that in the pure chromomagnetic field

$$\frac{1}{2}\sigma_{\mu\nu}\hat{B}_{\mu\nu} = \Sigma_3 H\hat{u},\ \Sigma_i = \frac{1}{2}\varepsilon_{ijk}\sigma_{jk}.$$

Equation (18) is essentially the same as (4). Its solution in cylindrical coordinates (2π-periodic in ϑ and regular at $r = 0$) is given by four independent components φ_l^α ($\alpha = 1, \ldots, 4$, $l \in Z$):

$$\varphi_l^\alpha = e^{-ip_3x_3 - ip_4x_4} e^{il\vartheta}\phi_l^\alpha(r)$$

$$\phi_l^\alpha = e^{-\frac{1}{4}\hat{u}r^2} r^l M\left(\frac{1+s_\alpha}{2} - \frac{\mu^2}{2\hat{u}}, 1+l, \frac{\hat{u}r^2}{2}\right). \quad l \geqslant 0$$

$$\phi_l^\alpha = e^{-\frac{1}{4}\hat{u}r^2} r^{-l} M\left(\frac{1+s_\alpha}{2} - \frac{\mu^2}{2\hat{u}} - l, 1-l, \frac{\hat{u}r^2}{2}\right). \quad l < 0$$

Here

$$s_\alpha = (-1)^\alpha,\ \alpha = 1, \ldots, 4$$

denotes the sign of the quark spin projection on the direction of chromomagnetic field, and therefore

$$\phi_l^3 = \phi_l^1 = \Phi_l^{\uparrow\uparrow}(r),\ \phi_l^4 = \phi_l^2 = \Phi_l^{\uparrow\downarrow}(r).$$

The variable μ is related to the Dirac eigenvalues as

$$\mu^2 = \lambda^2 - p_3^2 - p_4^2.$$

Finally the Dirac operator eigenfunction ψ take the form for $l \geqslant 0$

$$\psi_l^{(1)} = e^{-ip_3x_3 - ip_4x_4} \begin{pmatrix} \lambda \Phi_l^{\uparrow\uparrow}(r) e^{il\vartheta} \\ 0 \\ (p_4 + ip_3)\Phi_l^{\uparrow\uparrow}(r) e^{il\vartheta} \\ \frac{\mu^2}{2(l+1)} \Phi_{l+1}^{\uparrow\downarrow}(r) e^{i(l+1)\vartheta} \end{pmatrix}, \quad \psi_l^{(2)} = e^{-ip_3x_3 - ip_4x_4} \begin{pmatrix} 0 \\ \lambda \Phi_{l+1}^{\uparrow\downarrow}(r) e^{i(l+1)\vartheta} \\ -2(l+1)\Phi_l^{\uparrow\uparrow}(r) e^{il\vartheta} \\ (p_4 - ip_3)\Phi_{l+1}^{\uparrow\downarrow}(r) e^{i(l+1)\vartheta} \end{pmatrix},$$

$$\psi_l^{(3)} = e^{-ip_3x_3 - ip_4x_4} \begin{pmatrix} (p_4 - ip_3)\Phi_l^{\uparrow\uparrow}(r) e^{il\vartheta} \\ -\frac{\mu^2}{2(1+l)} \Phi_{l+1}^{\uparrow\downarrow}(r) e^{i(l+1)\vartheta} \\ \lambda \Phi_l^{\uparrow\uparrow}(r) e^{il\vartheta} \\ 0 \end{pmatrix}, \quad \psi_l^{(4)} = e^{-ip_3x_3 - ip_4x_4} \begin{pmatrix} 2(l+1)\Phi_l^{\uparrow\uparrow}(r) e^{il\vartheta} \\ (p_4 + ip_3)\Phi_{l+1}^{\uparrow\downarrow}(r) e^{i(l+1)\vartheta} \\ 0 \\ \lambda \Phi_{l+1}^{\uparrow\downarrow}(r) e^{i(l+1)\vartheta} \end{pmatrix},$$

and for $l < 0$

$$\psi_l^{(1)} = e^{-ip_3x_3 - ip_4x_4} \begin{pmatrix} \lambda \Phi_l^{\uparrow\uparrow}(r) e^{il\vartheta} \\ 0 \\ (p_4 + ip_3)\Phi_l^{\uparrow\uparrow}(r) e^{il\vartheta} \\ 2l \Phi_{l+1}^{\uparrow\downarrow}(r) e^{i(l+1)\vartheta} \end{pmatrix}, \quad \psi_l^{(2)} = e^{-ip_3x_3 - ip_4x_4} \begin{pmatrix} 0 \\ \lambda \Phi_{l+1}^{\uparrow\downarrow}(r) e^{i(l+1)\vartheta} \\ -\frac{\mu^2}{2l}\Phi_l^{\uparrow\uparrow}(r) e^{il\vartheta} \\ (p_4 - ip_3)\Phi_{l+1}^{\uparrow\downarrow}(r) e^{i(l+1)\vartheta} \end{pmatrix},$$

$$\psi_l^{(3)} = e^{-ip_3x_3 - ip_4x_4} \begin{pmatrix} (p_4 - ip_3)\Phi_l^{\uparrow\uparrow}(r) e^{il\vartheta} \\ -2l \Phi_{l+1}^{\uparrow\downarrow}(r) e^{i(l+1)\vartheta} \\ \lambda \Phi_l^{\uparrow\uparrow}(r) e^{il\vartheta} \\ 0 \end{pmatrix}, \quad \psi_l^{(4)} = e^{-ip_3x_3 - ip_4x_4} \begin{pmatrix} \frac{\mu^2}{2l}\Phi_l^{\uparrow\uparrow}(r) e^{il\vartheta} \\ (p_4 + ip_3)\Phi_{l+1}^{\uparrow\downarrow}(r) e^{i(l+1)\vartheta} \\ 0 \\ \lambda \Phi_{l+1}^{\uparrow\downarrow}(r) e^{i(l+1)\vartheta} \end{pmatrix}.$$

All four spinors are eigenfunctions of the total momentum projection operator onto x_3,

$$J_3 \psi_l^{(m)} = \left(l + \frac{1}{2}\right) \psi_l^{(m)}, \quad J_i = L_i + S_i, \quad L_i = -i\varepsilon_{ijk} x_j \partial_k, \quad S_i = \frac{1}{2}\Sigma_i.$$

Only two of these solutions at given l are linearly independent. We select

$$\psi_l = A\psi_l^{(1)} + B\psi_l^{(4)}.$$

Boundary condition (16) with $\theta = \pi/2$ leads to the equation defining the values of the parameter μ as well as the ratio of A and B. For $l \geqslant 0$ one gets

$$A\left(\frac{\mu^2}{2(1+l)}\Phi_{l+1}^{\uparrow\downarrow}(R) + \lambda\Phi_l^{\uparrow\uparrow}(R)\right) + B\left(\lambda\Phi_{l+1}^{\uparrow\downarrow}(R) + 2(l+1)\Phi_l^{\uparrow\uparrow}(R)\right) = 0, \quad A\Phi_l^{\uparrow\uparrow}(R) + B\Phi_{l+1}^{\uparrow\downarrow}(R) = 0.$$

This system has a nontrivial solution for A and B if the determinant of the matrix composed of the coefficients in front of them is equal to zero

$$\left[\Phi_l^{\uparrow\uparrow}(R)\right]^2 = \left[\frac{\mu}{2(1+l)}\Phi_{l+1}^{\uparrow\downarrow}(R)\right]^2. \tag{19}$$

This equation defines the spectrum of μ^2. States with definite spin orientation with respect to the chromomagnetic field are mixed in the boundary condition, and the spin projection onto the direction of the field is not a good quantum number unlike the projection of the total momentum j_3 as it is taken into account in Fig.9. As is illustrated in Fig.9 there is a discrete set of solutions $\mu_{ilk} > 0$ which depend also on the color orientation $\hat{u}_i$ ($j_3 = (2l+1)/2$ with $l \in Z$, $k \in N$, $j = 1,2,3$). As a rule one can omit the color index j assuming that μ_{lk} is a diagonal color matrix for any l, k. The values μ_{lk}^2 has to be used to find the relation between A and B

$$\frac{B_{lk}}{A_{lk}} = -\left.\frac{\Phi_l^{\uparrow\uparrow}(R)}{\Phi_{l+1}^{\uparrow\downarrow}(R)}\right|_{\mu^2=\mu_{lk}^2} = (-1)^{k+1}\frac{\mu_{lk}}{2(l+1)}, \tag{20}$$

$$\lambda_{lk} = \pm\sqrt{\mu_{lk}^2 + p_3^2 + p_4^2} = \pm|\lambda_{lk}|.$$

Here μ_{lk} is taken to be positive, and λ_{lk} takes both positive and negative values. Equation (19) has been used in combination with observation (by inspection) that the sign of the ratio B_{lk} and A_{lk} depends on $k \in N$ as it is indicated in (20) irrespectively to l and color orientation.

Analogous consideration for the case $l < 0$ leads to the equation for μ

$$\left[\Phi^{\uparrow\downarrow}_{l+1}(R)\right]^2 = \left[\frac{\mu}{2l}\Phi^{\uparrow\uparrow}_{l}(R)\right]^2, \quad \frac{A_{lk}}{B_{lk}} = -\left.\frac{\Phi^{\uparrow\uparrow}_{l}(R)}{\Phi^{\uparrow\downarrow}_{l+1}(R)}\right|_{\mu^2=\mu^2_{lk}} = (-1)^k\frac{\mu_{lk}}{2l},$$

$$\lambda_{lk} = \pm\sqrt{\mu^2_{lk} + p_3^2 + p_4^2} = \pm|\lambda_{lk}|.$$

The orthogonal normalized set of solutions has the form

$$\psi^{(\pm)}_{lk} = \frac{A_{lk}}{(2\pi)^{\frac{3}{2}}\sqrt{2|\lambda_{lk}|}}\begin{pmatrix} \frac{\pm|\lambda_{lk}|+(-1)^{k+1}\mu_{lk}}{\sqrt{p_4+ip_3}}\Phi^{\uparrow\uparrow}_{l}(r)e^{il\vartheta} \\ (-1)^{k+1}\frac{\mu_{lk}\sqrt{p_4+ip_3}}{2(l+1)}\Phi^{\uparrow\downarrow}_{l+1}(r)e^{i(l+1)\vartheta} \\ \sqrt{p_4+ip_3}\Phi^{\uparrow\uparrow}_{l}(r)e^{il\vartheta} \\ \frac{\mu_{lk}(\mu_{lk}\pm(-1)^{k+1}|\lambda_{lk}|)}{2(l+1)\sqrt{p_4+ip_3}}\Phi^{\uparrow\downarrow}_{l+1}(r)e^{i(l+1)\vartheta}\end{pmatrix} e^{-ip_3x_3-ip_4x_4}, \quad l \geqslant 0$$

$$\psi^{(\pm)}_{lk} = \frac{B_{lk}}{(2\pi)^{\frac{3}{2}}\sqrt{2|\lambda_{lk}|}}\begin{pmatrix} \frac{\mu_{lk}(\mu_{lk}\pm(-1)^{k}|\lambda_{lk}|)}{2l\sqrt{p_4+ip_3}}\Phi^{\uparrow\uparrow}_{l}(r)e^{il\vartheta} \\ \sqrt{p_4+ip_3}\Phi^{\uparrow\downarrow}_{l+1}(r)e^{i(l+1)\vartheta} \\ (-1)^{k}\frac{\mu_{lk}\sqrt{p_4+ip_3}}{2l}\Phi^{\uparrow\uparrow}_{l}(r)e^{il\vartheta} \\ \frac{\pm|\lambda_{lk}|+(-1)^{k}\mu_{lk}}{\sqrt{p_4+ip_3}}\Phi^{\uparrow\downarrow}_{l+1}(r)e^{i(l+1)\vartheta}\end{pmatrix} e^{-ip_3x_3-ip_4x_4}, \quad l < 0.$$

The spinors $\psi^{(+)}_{lk}$ and $\psi^{(-)}_{lk}$ correspond to the positive and negative eigenvalues λ_{lk} in (20) respectively, they are eigenfunctions of J_3 with $j_3 = l + 1/2$. Normalization constants are

$$A^{-2}_{jlk}(R) = \int\limits_0^R drr\left[\left(\frac{\mu_{jlk}}{2(l+1)}\Phi^{\uparrow\downarrow}_{l+1}(r)\right)^2 + \left(\Phi^{\uparrow\uparrow}_{l+1}(r)\right)^2\right], \quad B^{-2}_{jlk}(R) = \int\limits_0^R drr\left[\left(\frac{\mu_{jlk}}{2l}\Phi^{\uparrow\uparrow}_{l+1}(r)\right)^2 + \left(\Phi^{\uparrow\downarrow}_{l+1}(r)\right)^2\right].$$

The same procedure applied to the equation

$$\bar\psi(x)\overleftarrow{\not{D}} = \lambda\bar\psi(x)$$

leads to the solutions

$$\bar{\psi}_{lk}^{(\pm)} = \frac{A_{lk}}{(2\pi)^{\frac{3}{2}}\sqrt{2|\lambda_{lk}|}} \begin{pmatrix} \pm\sqrt{p_4+ip_3}\Phi_l^{\uparrow\uparrow}(r)e^{-il\vartheta} \\ \frac{\mu_{lk}(\mp\mu_{lk}+(-1)^{k+1}|\lambda_{lk}|)}{2(1+l)\sqrt{p_4+ip_3}}\Phi_{l+1}^{\uparrow\downarrow}(r)e^{-i(l+1)\vartheta} \\ \frac{\pm(-1)^k\mu_{lk}+|\lambda_{lk}|}{\sqrt{p_4+ip_3}}\Phi_l^{\uparrow\uparrow}(r)e^{-il\vartheta} \\ \mp(-1)^k\frac{\mu_{lk}\sqrt{p_4+ip_3}}{2(1+l)}\Phi_{l+1}^{\uparrow\downarrow}(r)e^{-i(l+1)\vartheta} \end{pmatrix}^{\mathrm{T}} e^{ip_3x_3+ip_4x_4}, \qquad l \geqslant 0$$

$$\bar{\psi}_{lk}^{(\pm)} = \frac{B_{lk}}{(2\pi)^{\frac{3}{2}}\sqrt{2|\lambda_{lk}|}} \begin{pmatrix} \pm(-1)^k\frac{\mu_{lk}\sqrt{p_4+ip_3}}{2l}\Phi_l^{\uparrow\uparrow}(r)e^{-il\vartheta} \\ \frac{|\lambda_{lk}|\mp(-1)^k\mu_{lk}}{\sqrt{p_4+ip_3}}\Phi_{l+1}^{\uparrow\downarrow}(r)e^{-i(l+1)\vartheta} \\ \frac{\mu_{lk}(\mp\mu_{lk}+(-1)^k|\lambda_{lk}|)}{2l\sqrt{p_4+ip_3}}\Phi_l^{\uparrow\uparrow}(r)e^{-il\vartheta} \\ \pm\sqrt{p_4+ip_3}\Phi_{l+1}^{\uparrow\downarrow}(r)e^{-i(l+1)\vartheta} \end{pmatrix}^{\mathrm{T}} e^{ip_3x_3+ip_4x_4}, \qquad l < 0.$$

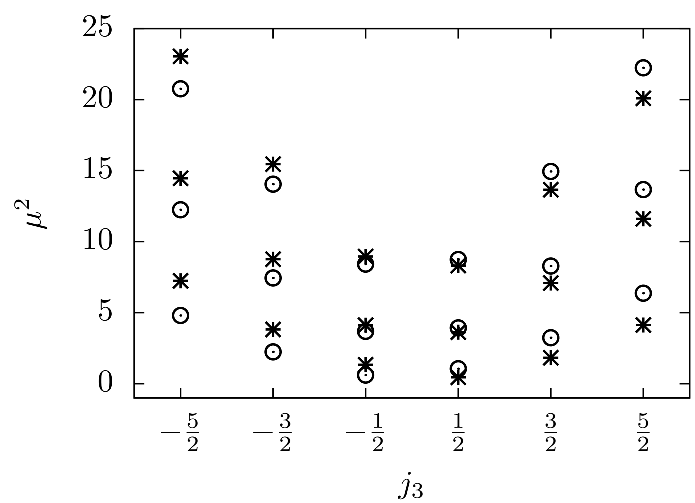


Figure 9: The lowest values of μ solving Eq. (19) for $\sqrt{H}R = 1.6$. Here $j_3 = l + 1/2$ is the projection of the total momentum on the direction of the chromomagnetic field. Eigenvalues are denoted by asterisks in the case of positive u_j and by circles in the case of negative u_j.

Quasiparticles

Adjoint representation: color charged bosons

In Minkowski space-time the problem (4) and (5) turns to the wave equation

$$-\left(\partial_\mu - i\breve{B}_\mu\right)^2 \phi(x) = 0$$

for color charged adjoint spin zero field inside a cylindrical wave guide. Here ϕ^a is a complex scalar field, and the corresponding solution of (21) satisfying boundary condition (5) takes the form

$$\phi^a(x) = \sum_{lk} \int_{-\infty}^{+\infty} \frac{dp_3}{2\pi} \frac{1}{\sqrt{2\omega_{alk}}} \left[a^+_{akl}(p_3) e^{ix_0\omega_{akl} - ip_3x_3} + b_{akl}(p_3) e^{-ix_0\omega_{akl} + ip_3x_3}\right] e^{il\vartheta} \phi_{alk}(r), \quad (21)$$

$$\phi^{a\dagger}(x) = \sum_{lk} \int_{-\infty}^{+\infty} \frac{dp_3}{2\pi} \frac{1}{\sqrt{2\omega_{alk}}} \left[b^+_{akl}(p_3) e^{-ix_0\omega_{akl} + ip_3x_3} + a_{akl}(p_3) e^{ix_0\omega_{akl} - ip_3x_3}\right] e^{-il\vartheta} \phi_{alk}(r),$$

$$p_0^2 = p_3^2 + \mu^2_{akl}, \quad p_0 = \pm\omega_{akl}(p_3), \ \omega_{akl} = \sqrt{p_3^2 + \mu^2_{akl}}, \qquad k = 0, 1, \ldots, \infty, \quad l \in Z, \quad (22)$$

$$\int_0^\infty drr \int_0^{2\pi} d\vartheta e^{i(l-l')\vartheta} \phi_{alk}(r) \phi_{al'k'}(r) = \delta_{ll'}\delta_{kk'}.$$

with $\phi_{alk}(r)$ defined in (9). Equation (22) can be treated as the dispersion relation between energy p_0 and momentum p_3 for the quasiparticles with masses μ_{akl}. These quasiparticles are extended in x_1 and x_2 directions and are classified by the quantum numbers l, k. The orthogonality, normalization and completeness of the set of functions $e^{il\vartheta}\phi_{alk}(r)$ guarantees the standard canonical commutation relations for the field ϕ^a and its canonically conjugated momentum if $a^\dagger_{akl}(p_3)$, $a_{akl}(p_3)$, $b^\dagger_{akl}(p_3)$ and $b_{akl}(p_3)$ are assumed to satisfy the standard commutation relations for creation and annihilation operators. The Fock space of states for the quasiparticles with masses μ_{akl} can be constructed by means of the standard QFT methods.

The **vector adjoint field** can be elaborated in the similar to the scalar case way. A modification relates just to the inclusion of polarization vectors. As it has already been mentioned the most important feature is the absence of tachyonic mode of the vector color charged field if $R < R_{\mathrm{c}}$. Disappearance of the tachyonic mode for subcritical size of the trap is one of the most important observations of this paper.

Fundamental representation: color charged fermions

Neither the background field nor the boundary condition involve the time coordinate. The solution of the Dirac equation

$$i\not{D}\psi(x) = 0,$$

satisfying bag boundary condition can be obtained from Euclidean solutions unnormalized solutions by the analytical continuation $p_4 \to ip_0$, $x_4 \to ix_0$ and the requirement $\lambda_{lk} = 0$, which leads to the energy-momentum relation for the solutions with definite j_3, k and color j

$$p_0^2 = p_3^2 + \mu_{jlk}^2, \quad p_0 = \pm\omega_{jlk}(p_3), \quad \omega_{jlk} = \sqrt{p_3^2 + \mu_{jlk}^2}.$$

Finally the solution of the Dirac equation takes the form

$$\psi^j(x) = \sum_{lk} \int_{-\infty}^{+\infty} \frac{dp_3}{2\pi} \frac{1}{\sqrt{2\omega_{jlk}}} \left[a_{jlk}^\dagger(p_3)\chi_{jlk}(p_3|r,\vartheta)e^{ix_0\omega_{jlk} - ix_3p_3} + b_{jlk}(p_3)\upsilon_{jlk}(p_3|r,\vartheta)e^{-ix_0\omega_{jlk}+ix_3p_3} \right],$$

$$\bar\psi^j(x) = \sum_{lk} \int_{-\infty}^{+\infty} \frac{dp_3}{2\pi} \frac{1}{\sqrt{2\omega_{jlk}}} \left[b_{jlk}^\dagger(p_3)\bar\chi_{jlk}(p_3|r,\vartheta)e^{-ix_0\omega_{jlk} + ix_3p_3} + a_{jlk}(p_3)\bar\upsilon_{jlk}(p_3|r,\vartheta)e^{ix_0\omega_{jlk}-ix_3p_3} \right].$$

Here the pair of spinors for positive χ_{lk} and negative υ_{lk} energy solutions are

$$\chi_{lk} = A_{lk} \begin{pmatrix} (-1)^{k+1}\frac{\mu_{lk}}{\sqrt{\omega_{lk}+p_3}}\Phi_l^{\uparrow\uparrow}(r)e^{il\vartheta} \\ i(-1)^{k+1}\frac{\mu_{lk}\sqrt{\omega_{lk}+p_3}}{2(l+1)}\Phi_{l+1}^{\uparrow\downarrow}(r)e^{i(l+1)\vartheta} \\ i\sqrt{\omega_{lk}+p_3}\Phi_l^{\uparrow\uparrow}(r)e^{il\vartheta} \\ \frac{\mu_{lk}^2}{2(l+1)\sqrt{\omega_{lk}+p_3}}\Phi_{l+1}^{\uparrow\downarrow}(r)e^{i(l+1)\vartheta} \end{pmatrix}, \quad \upsilon_{lk} = A_{lk} \begin{pmatrix} (-1)^{k+1}\frac{\mu_{lk}}{\sqrt{\omega_{lk}+p_3}}\Phi_l^{\uparrow\uparrow}(r)e^{il\vartheta} \\ i(-1)^{k}\frac{\mu_{lk}\sqrt{\omega_{lk}+p_3}}{2(l+1)}\Phi_{l+1}^{\uparrow\downarrow}(r)e^{i(l+1)\vartheta} \\ -i\sqrt{\omega_{lk}+p_3}\Phi_l^{\uparrow\uparrow}(r)e^{il\vartheta} \\ \frac{\mu_{lk}^2}{2(l+1)\sqrt{\omega_{lk}+p_3}}\Phi_{l+1}^{\uparrow\downarrow}(r)e^{i(l+1)\vartheta} \end{pmatrix}, \quad l \geqslant 0$$

$$\chi_{lk} = B_{lk} \begin{pmatrix} \frac{\mu_{lk}^2}{2l\sqrt{\omega_{lk}+p_3}} \Phi_l^{\uparrow\uparrow}(r) e^{il\vartheta} \\ i\sqrt{\omega_{lk}+p_3}\Phi_{l+1}^{\uparrow\downarrow}(r) e^{i(l+1)\vartheta} \\ i(-1)^k \frac{\mu_{lk}\sqrt{\omega_{lk}+p_3}}{2l} \Phi_l^{\uparrow\uparrow}(r) e^{il\vartheta} \\ (-1)^k \frac{\mu_{lk}}{\sqrt{\omega_{lk}+p_3}} \Phi_{l+1}^{\uparrow\downarrow}(r) e^{i(l+1)\vartheta} \end{pmatrix}, \quad \upsilon_{lk} = B_{lk} \begin{pmatrix} \frac{\mu_{lk}^2}{2l\sqrt{\omega_{lk}+p_3}} \Phi_l^{\uparrow\uparrow}(r) e^{il\vartheta} \\ -i\sqrt{\omega_{lk}+p_3}\Phi_{l+1}^{\uparrow\downarrow}(r) e^{i(l+1)\vartheta} \\ i(-1)^{k+1} \frac{\mu_{lk}\sqrt{\omega_{lk}+p_3}}{2l} \Phi_l^{\uparrow\uparrow}(r) e^{il\vartheta} \\ (-1)^k \frac{\mu_{lk}}{\sqrt{\omega_{lk}+p_3}} \Phi_{l+1}^{\uparrow\downarrow}(r) e^{i(l+1)\vartheta} \end{pmatrix}, \quad l < 0$$

The spinors are normalized as

$$\int_0^{2\pi} d\vartheta \int_0^R dr r \chi_{jlk}^\dagger(p_3|r,\vartheta)\chi_{jlk}(p_3|r,\vartheta) = \int_0^{2\pi} d\vartheta \int_0^R dr r \upsilon_{jlk}^\dagger(p_3|r,\vartheta)\upsilon_{jlk}(p_3|r,\vartheta) = 2\omega_{jlk}.$$

The Dirac conjugated spinors are

$$\bar\psi^j(x) = \psi^{j\dagger}(x)\gamma_0$$

The Fock space can be constructed by means of the creation and annihilation operators $\left\{a_{jlk}^\dagger(p_3), a_{jlk}(p_3), b_{jlk}^\dagger(p_3), b_{jlk}(p_3)\right\}$ satisfying the standard anticommutation relations. The one-particle state is characterized by a color orientation j, momentum p_3, projection $j_3 = (l+1/2)$ of the total angular momentum and the energy $\omega_{jlk} = \sqrt{p_3^2 + \mu_{jlk}^2}$. Since the boundary condition mixes the states with spin parallel and anti-parallel to the chromomagnetic field the spin projection is not a good quantum number unlike the half-integer valued projection of the total angular momentum j_3.

Inside chromomagnetic trap!!!

$$\langle F^2\rangle \neq 0, \ \ \langle F\tilde F\rangle = 0, \ \ \langle \bar\psi\psi\rangle = 0.$$

Formulation of the Domain Model

Euclidean partition function is defined as

$$\mathcal{Z}(\theta) = \lim_{V,N\to\infty} \mathcal{N} \int\limits_{\Omega_{\alpha,\vec\beta}} d\Omega_{\alpha,\vec\beta} \prod_{i=1}^{N} \int\limits_{\mathcal{B}} dB_i \int\limits_{\mathcal{F}^i_\psi} \mathcal{D}\psi^{(i)}\mathcal{D}\bar\psi^{(i)} \int\limits_{\mathcal{F}^i_Q} \mathcal{D}\mu[Q^i] e^{-S^{\rm QCD}_{V_i}\left[Q^{(i)}+B^{(i)},\psi^{(i)},\bar\psi^{(i)}\right]-i\theta Q_{V_i}[Q^{(i)}+B^{(i)}]}$$

$$\mathcal{D}\mu = \delta[D(B^{(i)})Q^{(i)}]\Delta_{\rm FP}[B^{(i)},Q^{(i)}]$$

The thermodynamic limit: $v^{-1} = N/V$ = const, as $V, N \to \infty$. **Functional spaces** $\mathcal{F}^i_Q$ and $\mathcal{F}^i_\psi$ are specified by BCs at $(x-z_i)^2 = R^2$

$$\breve n_i Q^{(i)}(x) = 0, \quad i\,\not\eta_i(x) e^{i(\alpha+\beta^a\lambda^a/2)\gamma_5}\psi^{(i)}(x) = \psi^{(i)}(x), \quad \bar\psi^{(i)} e^{i(\alpha+\beta^a\lambda^a/2)\gamma_5} i\,\not\eta_i(x) = -\bar\psi^{(i)}(x),$$

$$\eta_i^\mu = \frac{(x-z_i)^\mu}{|x-z_i|}, \quad \breve n_i = n_i^a T^a, T^a\text{—adjoint representation.}$$

$\int\limits_\Sigma d\sigma_i \mathcal{Z}_i(\sigma) \Longrightarrow$ **Ensemble of** "domain-" or "cluster-like" structured **background fields** with the field strength tensor

$$F^a_{\mu\nu}(x) = \sum_{j=1}^N n^{(j)a} B^{(j)}_{\mu\nu}\theta(1-(x-z_j)^2/R^2), \quad B^{(j)}_{\mu\nu}B^{(j)}_{\mu\rho} = B^2\delta_{\nu\rho}, \quad B^2 = \text{const}$$

$$\tilde B^{(j)}_{\mu\nu} = \pm B^{(j)}_{\mu\nu}, \quad \hat n^{(j)} = t^3\cos\xi_j + t^8\sin\xi_j, \quad \xi_j \in \{\frac{\pi}{6}(2k+1),\ k=0,\dots,5\}$$

Free parameters: the field **strength** B and the **radius** R. Domains are **hyperspherical, centered at random points** z_j.

$$\int\limits_{\mathcal{B}} dB_i \cdots = \frac{1}{24\pi^2}\int\limits_V \frac{d^4 z_i}{V}\int\limits_0^{2\pi} d\varphi_i \int\limits_0^{\pi} d\theta_i \sin\theta_i \int\limits_0^{2\pi} d\xi_i \sum_{l=0,1,2}^{3,4,5}\delta(\xi_i - \frac{(2l+1)\pi}{6})\int\limits_0^{\pi} d\omega_i \sum_{k=0,1}\delta(\omega_i - \pi k)\dots$$

Testing The Model In Pure Yang-Mills System.

• Mean **topological charge** is zero. At finite volume $V = vN$ the distribution of the topological charge $-Nq \le Q \le Nq$ is symmetric about $Q = 0$.

$$Q(x) = \frac{g^2}{32\pi^2}\tilde{F}(x)F(x), \quad \mathcal{P}_N(Q) = \frac{N!}{2^N\,(N/2 - Q/2q)!\,(N/2 + Q/2q)!},$$

$$Q = \int\limits_V d^4x Q(x) = q(N_+ - N_-), \quad q = \frac{B^2R^4}{16}, \quad N_+ + N_- = N$$

• **Gluon condensate**: $\langle : g^2 F^a_{\mu\nu}(x)F^a_{\mu\nu}(x) :\rangle = 4B^2$
• **Topological susceptibility for pure YM** $\chi = \int d^4x \langle Q(x)Q(0)\rangle = B^4R^4/128\pi^2$
• **The area law**: Wilson loop for a circular contour with radius $L >> R$ for $N_c = 3$

$$W(L) = \lim_{V,N\to\infty} \prod_{j=1}^{N} \int d\sigma_j \frac{1}{N_c} \mathrm{Tr} e^{i\int_{S_L} d\sigma_{\mu\nu}(x)\hat{B}_{\mu\nu}(x)} = e^{-\sigma\pi L^2 + O(L)},$$

$$\sigma = Bf(\pi BR^2), \quad f(z) = \frac{2}{3z}\left(3 - \frac{\sqrt{3}}{2z}\int\limits_0^{2z/\sqrt{3}} \frac{dx}{x}\sin x - \frac{2\sqrt{3}}{z}\int\limits_0^{z/\sqrt{3}} \frac{dx}{x}\sin x\right)$$

Fitting $(B, R) : \sqrt{B} = 947\mathrm{MeV}, R = (760\mathrm{MeV})^{-1} = 0.26\mathrm{fm}$

$$\sigma = (420\mathrm{MeV})^2, \quad \chi = (197\mathrm{MeV})^4, \quad \frac{\alpha_s}{\pi}\langle F^2\rangle = .081\mathrm{GeV}^4 \; ; \quad q = 0.15, \; v^{-1} = 42\mathrm{fm}^{-4}$$

Here q is a **fraction of top. charge per domain**, and v^{-1} is the **density of domains** .

Including Quark Fields

- **Eigen modes**

$$\psi(x) = \sum_n b_n \psi_n(x), \quad \bar{\psi}(x) = \sum_n \bar{b}_n \bar{\psi}_n(x)$$

$$\not{D}\psi_n(x) = \lambda_n \psi_n(x),$$
$$i \not{\eta}(x) e^{i\alpha\gamma_5}\psi(x) = \psi(x), \ x^2 = R^2$$
$$\bar{\psi}(x) e^{i\alpha\gamma_5} i \not{\eta}(x) = -\bar{\psi}(x), \ x^2 = R^2.$$

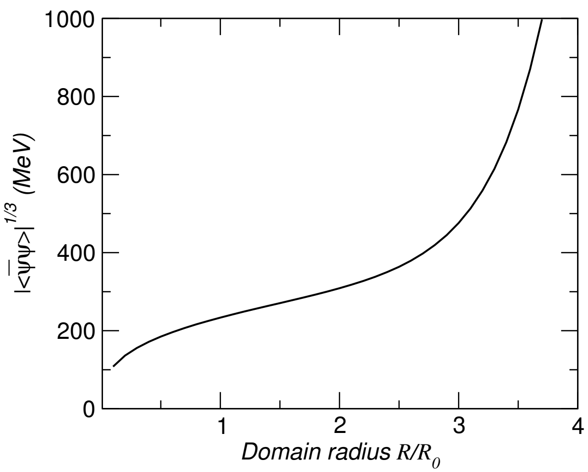
- **Quark determinant and realisation of** $U_{\rm A}(1)$ and $SU_{\rm L}(N_{\rm f}) \times SU_{\rm L}(N_{\rm f})$

Anomaly reduces $U_A(1)$ to a discrete subgroup. Unlike $U_A(1)$ flavour chiral symmetry is broken **spontaneously.**

$$\langle \bar{\psi}(x)\psi(x)\rangle = -\frac{1}{\pi^2 R^3} {\rm Tr} \sum_{k=1}^{\infty} \frac{k}{k+1}\left[M(1,k+2,z) - \frac{z}{k+2} M(1,k+3,-z) - 1\right]$$

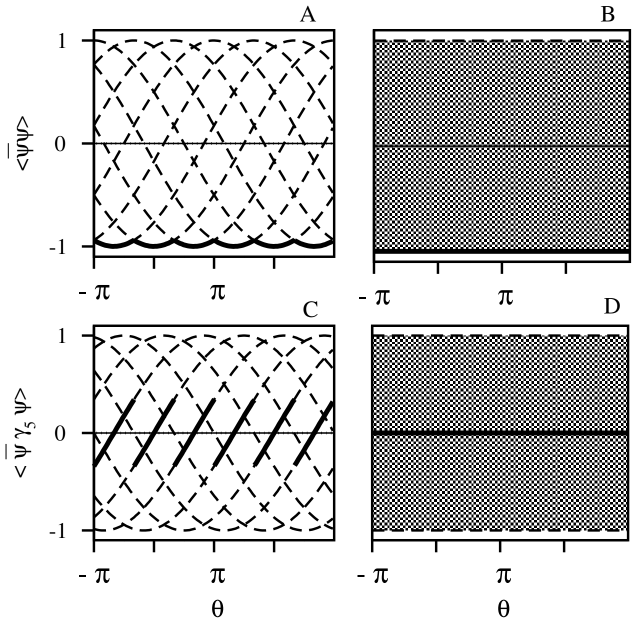
$z = \hat{n} B R^2/2$, Tr – color trace (matrix $\hat{n}$ - diagonal).

With B, R determined from pure gluodynamics $\langle\bar{\psi}\psi\rangle = -(237.8{\rm MeV})^3$



Absolute value of quark condensate as a function of domain radius in units of $R_0 = (760{\rm MeV})^{-1}$.

Poincaré Recurrence Theorem and The Strong CP-problem

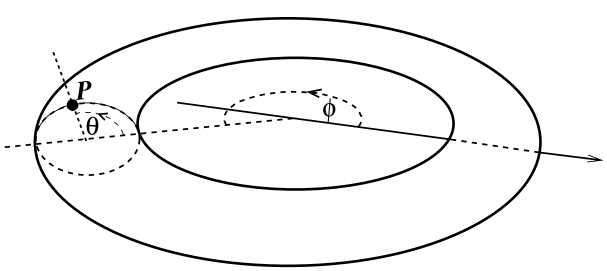


The scalar (A and B) and pseudoscalar (C and D) quark condensates as functions of θ for $N_f = 3$ in units of $\aleph$. The plots A and C are for rational $q = 0.15$, while B and D correspond to any irrational q, for instance to $q = \frac{3}{2.02\pi^2} = 0.15047\dots$ which is numerically only slightly different from 0.15. The dashed lines in A and C correspond to discrete minima of the free energy density which are degenerate for $m \equiv 0$. The solid bold lines denote the minimum which is chosen by an infinitesimally small mass term for a given θ. Points on the solid line in A, where two dashed lines cross each other, correspond to critical values of θ, at which CP is broken spontaneously. This is signalled by the discontinuity in the pseudoscalar condensate in C. For irrational q, as illustrated in B and D, the dashed lines densely cover the strip between 1 and -1, and the set of critical values of θ is dense in $\mathbb{R}$.

The set of critical values of the θ is dense in the interval $[-\pi, \pi]$:

$$Z = \lim_{V\to\infty} Z_V(\theta) = \lim_{V\to\infty} Z_V(0), \quad \lim_{V\to\infty} \partial_\theta^n Z_V(\theta) \neq \partial_\theta^n \lim_{V\to\infty} Z_V(\theta), \quad \lim_{V\to\infty} \langle \mathbb{E} \rangle_V^\theta \equiv \lim_{V\to\infty} \langle \mathbb{E} \rangle_V^{\theta=0}, \quad \lim_{V\to\infty} \langle \mathbb{O} \rangle_{V,\theta} \equiv 0,$$

for **any CP-even and CP-odd operators** $\mathbb{E}$ and $\mathbb{O}$ respectively, which **resolves the problem of CP-violation.**



A simple example: the Poincaré theorem for uniform motion on a torus,

$$\dot\theta = b_\theta, \quad \dot\phi = b_\phi,$$

where θ and ϕ are the latitude and longitude of a point on the torus. If $b = b_\theta/b_\phi$ – **rational** number, trajectory is **closed** and can be characterised by an integer winding number. In the case of **irrational** b the trajectory is **dense on the torus**.

Collective modes - hadronization

Nonlocal Wick-Cutkosky Model

$$\mathcal{L} = -\Phi^\dagger S^{-1}(-\partial^2)\Phi - \frac{1}{2}\phi D^{-1}(-\partial^2)\phi - g\Phi^\dagger\Phi\phi.$$

The Wick-Cutkosky model:

$$D^{-1}(p^2) = p^2, \ S^{-1}(p^2) = p^2 + m^2$$

ϕ– real massless scalar field , Φ – massive charged scalar field.
A prototype theory for studying the relativistic bound state problem in **QED**

The nonlocal version of Wick-Cutkosky model:

$D(p^2)$, $S(p^2)$ – entire analytical functions

$$S(p^2) \sim D(p^2) \sim \frac{1 - e^{-p^2/\Lambda^2}}{p^2}$$

Euclidean momentum space propagators

$$S(p^2) = D(p^2) = \frac{1}{\Lambda^2} e^{-p^2/\Lambda^2}$$

Spectrum of relativistic collective excitations (”bound states”) is analytically soluble in the one-boson exchange approximation.
The nonlocal model – soluble prototype of a confining theory, with **confined fundamental fields** ϕ and Φ and a **Regge spectrum of relativistic bound states** representing the physical particle spectrum.

$$M_{nl}^2 = M_0^2 + (2n + l) \ln(2 + \sqrt{3})^2 \Lambda^2$$

is linear both in radial number n and angular momentum l.

The Effective Action for Composite Fields

The starting point is the Euclidean functional integral

$$\mathcal{Z}[I] = \mathcal{N}\int \mathcal{D}\phi\mathcal{D}\Phi\mathcal{D}\Phi^\dagger \exp\left\{\int d^4x \left[\mathcal{L}(x) - \Phi^\dagger(x)\Phi(x)I(x)\right]\right\},$$

the external current $I(x)$

$$I(x) = e^2\int d^4y G(x-y)\bar{l}(y)l(y)$$

interaction with the scalar lepton current $\bar{l}(y)l(y)$ by virtue of scalar particle exchange reflected in the propagator $G(z)$.

$$L_2[\Phi] = \frac{g^2}{2}\int d^4x_1 d^4x_2 \Phi^\dagger(x_1)\Phi(x_1)D(x_1-x_2)\Phi^\dagger(x_2)\Phi(x_2).$$

$$L_2 = \frac{g^2}{2}\sum_{\mathcal{Q}}\int d^4x J_{\mathcal{Q}}(x)J_{\mathcal{Q}}(x)$$

$$\begin{aligned} J_{\mathcal{Q}}(x) &= \Phi^\dagger(x)V_{\mathcal{Q}}(\overleftrightarrow{\partial})\Phi(x) \\ V_{\mathcal{Q}}(\overleftrightarrow{\partial}) &= \int d^4y\sqrt{D(y)}U_{\mathcal{Q}}(y)e^{\frac{y}{2}\overleftrightarrow{\partial}} \end{aligned}$$

$\mathcal{Q} = \{n, l, \mu\}$, $\alpha = g^2/(4\pi\Lambda)^2$

$$Z = \prod_{\mathcal{Q}}\int \mathcal{D}\Psi_{\mathcal{Q}} \exp\left\{-\frac{\Lambda^2}{2}\int d^4p \Psi_{\mathcal{Q}}(-p)(\delta_{\mathcal{Q}\mathcal{Q}'} - \alpha\Sigma_{\mathcal{Q}\mathcal{Q}'}(p))\Psi_{\mathcal{Q}'}(p) + W_I[g\Psi]\right\}$$

$$W_I[g\Psi] = -\mathrm{Tr}[\ln(1 - g\Psi_{\mathcal{Q}}V_{\mathcal{Q}}S - IS) + \frac{g^2}{2}\Psi_{\mathcal{Q}}V_{\mathcal{Q}}SV_{\mathcal{Q}'}S\Psi_{\mathcal{Q}'}]$$

$$\alpha\tilde{\Sigma}_{\mathcal{Q}\mathcal{Q}'}(x-y) = \frac{g^2}{2\Lambda^2}V_{\mathcal{Q}}(\overset{\leftrightarrow}{\partial}_x)S(x-y)V_{\mathcal{Q}'}(\overset{\leftrightarrow}{\partial}_y)S(y-x)$$

$U_{\mathcal{Q}}$ – complete orthonormal set: $\Sigma_{\mathcal{Q}\mathcal{Q}'}(p) = E_{\mathcal{Q}}(-p^2)\delta_{\mathcal{Q}\mathcal{Q}'}$

Composite field masses are then real solutions to

$$1 = \alpha E_{\mathcal{Q}}(M^2_{\mathcal{Q}}). \tag{23}$$

Diagonalisation of the self-energy is equivalent to the solution of the Bethe-Salpeter equation in the one-boson exchange approximation. In the momentum representation propagators and vertices involved in the effective action have the form:

$$\begin{aligned} S(p) &= \Lambda^{-2}e^{-p^2/\Lambda^2}, \\ V^{nl}_{\mu_1\ldots\mu_l}(K) &= e^{i(q-p)x}\left[e^{ipx}V^{nl}_{\mu_1\ldots\mu_l}\left(\overset{\leftrightarrow}{\partial}_x\right)e^{-iqx}\right], \\ &= (-1)^{n+l}\Lambda C_{nl}T^l_{\mu_1,\ldots,\mu_l}(K)L^{l+1}_n(aK^2)e^{-bK^2} \end{aligned}$$

$K = (p+q)/2\Lambda$, $a = 2\sqrt{3}$, $b = 2/(1+\sqrt{3}) = 4/(2+a)$,

$$C_{nl} = \frac{2^{n+2+2l}\sqrt{3}^{l/2+1}}{(1+\sqrt{3})^{2n+l+2}}\sqrt{\frac{n!(l+1)}{(n+l+1)!}}.$$

The angular part of the vertex $T^l_{\mu_1\ldots\mu_l}$

$$\begin{aligned} &\int\limits_{\Omega}\frac{d\omega}{2\pi^2}T^l_{\mu_1\ldots\mu_l}(n_y)T^k_{\nu_1\ldots\nu_k}(n_y) = \frac{1}{2^l(l+1)}\delta^{lk}\delta_{\mu_1\nu_1}\ldots\delta_{\mu_l\nu_l} \\ &T^l_{\mu_1\ldots\mu\ldots\nu\ldots\mu_l}(n_y) = T^l_{\mu_1\ldots\nu\ldots\mu\ldots\mu_l}(n_y),\ T^l_{\mu\mu\ldots\mu_l}(n_y) = 0, \\ &T^l_{\mu_1\ldots\mu_l}(n_y)T^l_{\mu_1\ldots\mu_l}(n_x) = \frac{1}{2^l}C^{(1)}_l(n_yn_x),\ \ n^2_x = n^2_y = 1 \end{aligned} \tag{24}$$

$C_l^{(1)}$– Gegenbauer polynomials. The radial part – Laguerre polynomials L_n^{l+1}

$$\int_0^\infty du u^{l+1} e^{-u} L_n^{l+1}(u) L_{n'}^{l+1}(u) = \delta_{nn'}$$

The fields are rescaled

$$\Psi_{\mathcal{Q}} = \Psi_{\mathcal{Q}} g^{-1} \Lambda h_{\mathcal{Q}},$$
$$h_{\mathcal{Q}}^{-2} = \frac{\Lambda^2}{(4\pi)^2} \frac{dE_{\mathcal{Q}}(-p^2)}{dp^2}\Big|_{p^2=-M_{\mathcal{Q}}^2}$$

The final form of the functional integral for composite fields is

$$Z = \prod_{\mathcal{Q}} \int \mathcal{D}\Psi_{\mathcal{Q}} \exp\left\{-\frac{\Lambda^4 h_{\mathcal{Q}}^2}{2g^2} \int d^4p \Psi_{\mathcal{Q}}(-p)(1 - \alpha E_{\mathcal{Q}}(-p^2))\Psi_{\mathcal{Q}}(p) + W_I[h_{\mathcal{Q}}\Psi]\right\}.$$

The original coupling constant g enters only the quadratic part of the effective action, while the remaining terms contain the effective coupling constant $h_{\mathcal{Q}}$.

$$E_{nl}(-p^2) = \frac{e^{-\frac{p^2}{2\Lambda^2}}}{(2+\sqrt{3})^{2n+l+2}},$$

and the square of the bound state masses read

$$M_{nl}^2 = 2\Lambda^2 \left[\ln\frac{(2+\sqrt{3})^2}{\alpha} + (2n+l)\ln(2+\sqrt{3})\right]$$

which manifests a linear Regge spectrum.

Hadronization: Effective meson action, meson spectrum

$$Z = \mathcal{N} \lim_{V\to\infty} \int D\Phi_{\mathcal{Q}} \exp\left\{ -\frac{B}{2}\frac{h^2_{\mathcal{Q}}}{g^2 C_{\mathcal{Q}}} \int dx \Phi^2_{\mathcal{Q}}(x) - \sum_k \frac{1}{k} W_k[\Phi] \right\},$$

$$C_{Jnl} = C_J \frac{l+1}{2^l n!(l+n)!}, \quad C_{S/P} = \frac{1}{9}, \quad C_{S/P} = \frac{1}{18}$$

$$1 = \frac{g^2 C_{\mathcal{Q}}}{B} \tilde{\Gamma}^{(2)}_{\mathcal{Q}\mathcal{Q}}(-M^2_{\mathcal{Q}}|B), \quad h^{-2}_{\mathcal{Q}} = \frac{d}{dp^2}\tilde{\Gamma}^{(2)}_{\mathcal{Q}\mathcal{Q}}(p^2)|_{p^2=-M^2_{\mathcal{Q}}}.$$

$$\begin{aligned}
W_k[\Phi] &= \sum_{\mathcal{Q}_1\dots\mathcal{Q}_k} h_{\mathcal{Q}_1}\dots h_{\mathcal{Q}_k} \int dx_1 \dots \int dx_k \Phi_{\mathcal{Q}_1}(x_1)\dots\Phi_{\mathcal{Q}_k}(x_k)\Gamma^{(k)}_{\mathcal{Q}_1\dots\mathcal{Q}_k}(x_1,\dots,x_k|B),\\
\Gamma^{(2)}_{\mathcal{Q}_1\mathcal{Q}_2} &= \overline{G^{(2)}_{\mathcal{Q}_1\mathcal{Q}_2}(x_1,x_2)} - \Xi_2(x_1-x_2)\overline{G^{(1)}_{\mathcal{Q}_1}G^{(1)}_{\mathcal{Q}_2}},\\
\Gamma^{(3)}_{\mathcal{Q}_1\mathcal{Q}_2\mathcal{Q}_3} &= \overline{G^{(3)}_{\mathcal{Q}_1\mathcal{Q}_2\mathcal{Q}_3}(x_1,x_2,x_3)} - \frac{3}{2}\Xi_2(x_1-x_3)\overline{G^{(2)}_{\mathcal{Q}_1\mathcal{Q}_2}(x_1,x_2)G^{(1)}_{\mathcal{Q}_3}(x_3)}\\
&+ \frac{1}{2}\Xi_3(x_1,x_2,x_3)\overline{G^{(1)}_{\mathcal{Q}_1}(x_1)G^{(1)}_{\mathcal{Q}_2}(x_2)G^{(1)}_{\mathcal{Q}_3}(x_3)},\\
\Gamma^{(4)}_{\mathcal{Q}_1\mathcal{Q}_2\mathcal{Q}_3\mathcal{Q}_4} &= \overline{G^{(4)}_{\mathcal{Q}_1\mathcal{Q}_2\mathcal{Q}_3\mathcal{Q}_4}(x_1,x_2,x_3,x_4)} - \frac{4}{3}\Xi_2(x_1-x_2)\overline{G^{(1)}_{\mathcal{Q}_1}(x_1)G^{(3)}_{\mathcal{Q}_2\mathcal{Q}_3\mathcal{Q}_4}(x_2,x_3,x_4)}\\
&- \frac{1}{2}\Xi_2(x_1-x_3)\overline{G^{(2)}_{\mathcal{Q}_1\mathcal{Q}_2}(x_1,x_2)G^{(2)}_{\mathcal{Q}_3\mathcal{Q}_4}(x_3,x_4)}\\
&+ \Xi_3(x_1,x_2,x_3)\overline{G^{(1)}_{\mathcal{Q}_1}(x_1)G^{(1)}_{\mathcal{Q}_2}(x_2)G^{(2)}_{\mathcal{Q}_3\mathcal{Q}_4}(x_3,x_4)}\\
&- \frac{1}{6}\Xi_4(x_1,x_2,x_3,x_4)\overline{G^{(1)}_{\mathcal{Q}_1}(x_1)G^{(1)}_{\mathcal{Q}_2}(x_2)G^{(1)}_{\mathcal{Q}_3}(x_3)G^{(1)}_{\mathcal{Q}_4}(x_4)}.
\end{aligned}$$

The **vertices** $\Gamma^{(k)}$ are **expressed** *via* **quark loops** $G^{(n)}_{\mathcal{Q}}$ with n quark-meson vertices

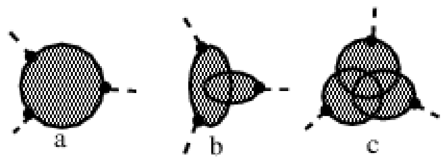
$$\overline{G^{(k)}_{\mathcal{Q}_1\ldots\mathcal{Q}_k}(x_1,\ldots,x_k)} = \int_\Sigma d\sigma_j \mathrm{Tr} V_{\mathcal{Q}_1}(x_1|B^{(j)})S(x_1,x_2|B^{(j)})\ldots V_{\mathcal{Q}_k}(x_k|B^{(j)})S(x_k,x_1|B^{(j)})$$

$$\overline{G^{(l)}_{\mathcal{Q}_1\ldots\mathcal{Q}_l}(x_1,\ldots,x_l)G^{(k)}_{\mathcal{Q}_{l+1}\ldots\mathcal{Q}_k}(x_{l+1},\ldots,x_k)} = \int_\Sigma d\sigma_j$$

$$\times\mathrm{Tr}\left\{V_{\mathcal{Q}_1}(x_1|B^{(j)})S(x_1,x_2|B^{(j)})\ldots V_{\mathcal{Q}_k}(x_l|B^{(j)})S(x_l,x_1|B^{(j)})\right\}$$

$$\times\mathrm{Tr}\left\{V_{\mathcal{Q}_{l+1}}(x_{l+1}|B^{(j)})S(x_{l+1},x_{l+2}|B^{(j)})\ldots V_{\mathcal{Q}_k}(x_k|B^{(j)})S(x_k,x_{l+1}|B^{(j)})\right\},$$

bar denotes **integration over** all configurations of the **background field with measure** $d\sigma_j$.



All the elements of the effective action are fixed: nonlocal meson-quark vertices $V_{\mathcal{Q}_1}(x|B)$ and quark propagators $S(x,y|B)$ and background field correlators $\Xi_n(x)$ are given in explicit analytical form.

The quark propagator

$$S(x,y) = \exp\left(-\frac{i}{2}x_\mu \hat{B}_{\mu\nu}y_\nu\right)H(x-y),$$

$$\tilde{H}(p) = \frac{1}{2v\Lambda^2}\int_0^1 ds e^{-p^2/2v\Lambda^2}\left(\frac{1-s}{1+s}\right)^{m^2/4v\Lambda^2}$$

$$\times\left[p_\alpha\gamma_\alpha \pm is\gamma_5\gamma_\alpha f_{\alpha\beta}p_\beta + m\left(P_\pm + P_\mp\frac{1+s^2}{1-s^2} - \frac{i}{2}\gamma_\alpha f_{\alpha\beta}\gamma_\beta\frac{s}{1-s^2}\right)\right].$$

in the presence of **the (anti-)self-dual homogeneous** field

$$\hat{B}_\mu(x) = -\frac{1}{2}\hat{n}B_{\mu\nu}x_\nu,\ \hat{B}_{\mu\nu}\hat{B}_{\mu\rho} = 4v^2\Lambda^4\delta_{\nu\rho},$$
$$f_{\alpha\beta} = \frac{\hat{n}}{v\Lambda^2}B_{\mu\nu},\ v = \mathrm{diag}(1/6, 1/6, 1/3),$$

$$\Lambda^2 = \frac{\sqrt{3}}{2}B.$$

Quark-meson vertices

$$V_{\mathcal{Q}} \propto \Gamma\lambda T^{(l)}(\overset{\leftrightarrow}{\nabla}/i\Lambda)F_{nl}(\overset{\leftrightarrow}{\nabla}^2/\Lambda^2)$$

$$F_{nl}(s) = \int_0^1 dt t^{l+n}e^{st},\ s = \overset{\leftrightarrow}{\nabla}^2/\Lambda^2,$$
$$\overset{\leftrightarrow}{\nabla}_{ff'} = \xi_f\overset{\leftarrow}{\nabla} - \xi_{f'}\overset{\rightarrow}{\nabla},\ \xi_f = m_f/(m_f + m_{f'}),$$
$$\overset{\leftarrow}{\nabla}_\mu = \overset{\leftarrow}{\partial}_\mu + iB_\mu,\ \overset{\rightarrow}{\nabla}_\mu = \overset{\rightarrow}{\partial}_\mu - iB_\mu.$$

$$\Xi_2(x-y) = \frac{N}{V}\int_V dz\theta(x-z)\theta(y-z) = \frac{2}{3\pi}\phi\left(\frac{(x-y)^2}{4R^2}\right),$$
$$\phi(\rho^2) = \left[\frac{3\pi}{2} - 3\arcsin(\rho) - 3\rho\sqrt{1-\rho^2} - 2\rho(1-\rho^2)\sqrt{1-\rho^2}\right].$$

m_u (MeV)	m_d (MeV)	m_s (MeV)	m_c (MeV)	m_b (MeV)	Λ (MeV)	g
198.3	198.3	413	1650	4840	319.5	9.96

Meson	ℓ	j	M	$M^{\rm exp}$
π	0	0	140	140
b_1	1	1	1252	1235
K	0	0	496	496
$K_1(1270)$	1	1	1263	1270
ρ	0	1	770	770
	1	0	1238	
a_1	1	1	1311	1260
a_2	1	2	1364	1320
K^*	0	1	890	890
	1	0	1274	
$K_1(1400)$	1	1	1342	1400
K_2^*	1	2	1388	1430

Meson	π	ρ	K	K^*	ω	ϕ
M	140	770	496	890	770	1034
$M^{\rm exp}$	140	770	496	890	786	1020
f_P	126	-	145	-	-	-
$f_P^{\rm exp}$	132	-	157	-	-	-
h	6.51	4.16	7.25	4.48	4.16	4.94
M^*	630	864	743	970	864	1087

Meson	D	D^*	D_s	D_s^*	B	B^*	B_s	B_s^*
M	1766	1991	1910	2142	4965	5143	5092	5292
$M^{\rm exp}$	1869	2010	1969	2110	5278	5324	5375	5422
f_P	149	-	177	-	123	-	150	-

Meson	η_c	J/ψ	χ_{c_0}	χ_{c_1}	χ_{c_2}	ψ'	ψ''
n	0	0	0	0	0	1	2
ℓ	0	0	1	1	1	0	0
j	0	1	0	1	2	1	1
M (MeV)	3000	3161	3452	3529	3531	3817	4120
$M^{\rm exp}$ (MeV)	2980	3096	3415	3510	3556	3770	4040

Meson	Υ	χ_{b_0}	χ_{b_1}	χ_{b_2}	Υ'	χ'_{b_0}	χ'_{b_1}	χ'_{b_2}	Υ''
n	0	0	0	0	1	1	1	1	2
ℓ	0	1	1	1	0	1	1	1	0
j	1	0	1	2	1	0	1	2	1
M (MeV)	9490	9767	9780	9780	10052	10212	10215	10215	10292
$M^{\rm exp}$ (MeV)	9460	9860	9892	9913	10230	10235	10255	10269	10355

$$M_\eta = 640 \text{ MeV}, \ M_{\eta'} = 950 \text{ MeV}, \ \ h_\eta = 4.72, \ h_{\eta'} = 2.55, \ \ \sqrt{B}R = 1.56.$$

Features of **the spectrum of light vector and pseudoscalar mesons** are driven by the chiral symmetries and are correctly reproduced by the model quantitatively.

$$B^a_\mu = n^a B_{\mu\nu} x_\nu, \quad \tilde{B}_{\mu\nu} = \pm B_{\mu\nu}, \quad B_{\mu\alpha} B_{\alpha\nu} = \delta_{\mu\nu} B^2, \quad B^2 = \mathrm{const}$$

$$D^2(x) G(x,y) = -\delta(x-y) \quad G(x,y) = e^{ixBy} H(x-y) \quad \tilde{H}(p^2) = \frac{1 - e^{-p^2/B}}{p^2}$$

$$\tilde{H}_f(p \mid B) \to O\left(\exp\left\{\frac{p^2}{\Lambda^2}\right\}\right), \; F_{n\ell}\left(p^2\right) \to O\left(\exp\left\{\frac{p^2}{\Lambda^2}\right\}\right),$$

Regge behaviour of the spectrum is due to nonlocality of the vertices and propagators.

- $M^2_{aJ\ell n} = \frac{8}{3} \ln\left(\frac{5}{2}\right) \cdot \Lambda^2 \cdot n + O(\ln n)$, for $n \gg \ell$, $\qquad M^2_{aJ\ell n} = \frac{4}{3} \ln 5 \cdot \Lambda^2 \cdot \ell + O(\ln \ell)$, for $\ell \gg n$.

Heavy-light mesons and heavy quarkonia

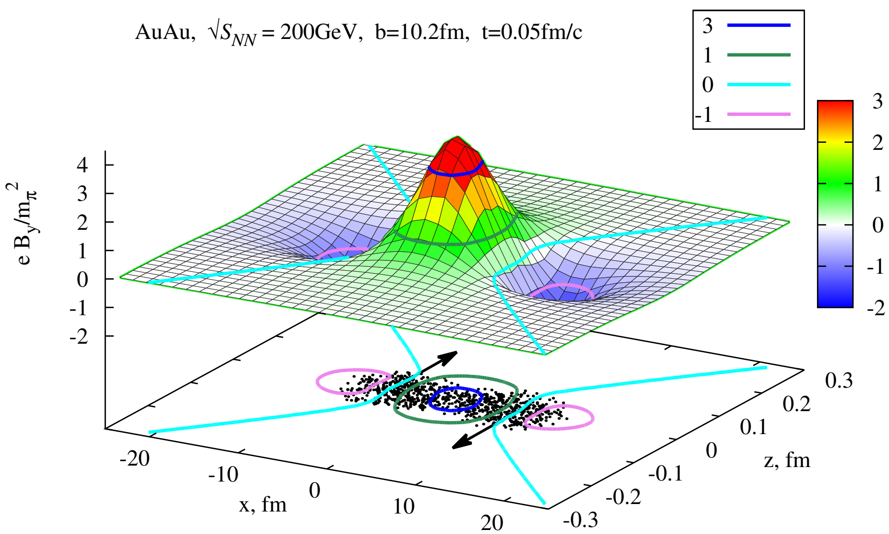
- $m_Q \gg \Lambda, m_Q \gg m_q, \quad M_{Q\bar{q}} = m_Q + \Delta^{(J)}_{Q\bar{q}} + O(1/m_Q)$
- $m_Q \gg \Lambda, \quad M_{Q\bar{Q}} = 2m_Q - \Delta_{Q\bar{Q}}, \quad \Delta^{(P)}_{Q\bar{Q}} = 2\Delta^{(V)}_{Q\bar{Q}}$

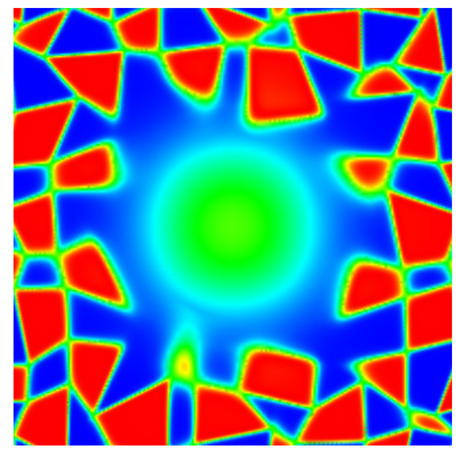
Impact of electromagnetic fields on “QCD vacuum“.

- **Relativistic heavy ion collisions - extremely strong electromagnetic fields**

V. Voronyuk, V. D. Toneev, W. Cassing, E. L. Bratkovskaya,

V. P. Konchakovski and S. A. Voloshin, Phys. Rev C 84 (2011)





One-loop quark contribution to the effective potential in the presence of arbitrary homogenous Abelian fields

$$U_{\rm eff}(G) = -\frac{1}{V}\ln\frac{\det(i\not{D}-m)}{\det(i\not{\partial}-m)} = \frac{1}{V}\int\limits_V d^4x \mathrm{Tr}\int\limits_m^\infty dm'\left[S(x,x|m') - S_0(x,x|m')\right]|$$

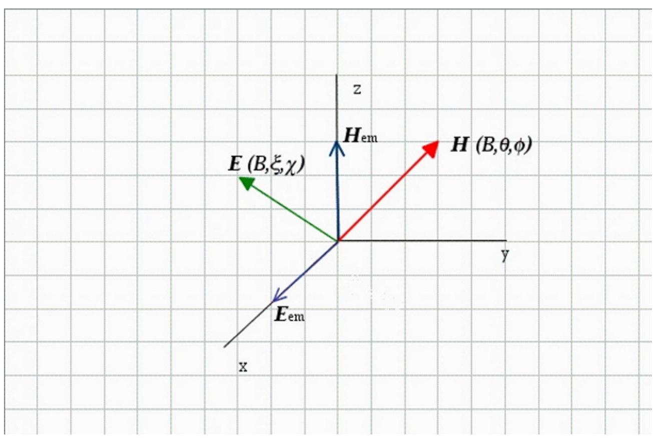
$$U_{\rm eff}^{\rm ren}(G) = \frac{B^2}{8\pi^2}\int\limits_0^\infty \frac{ds}{s^3}\mathrm{Tr}_n\left[s\varkappa_+\coth(s\varkappa_+)s\varkappa_-\coth(s\varkappa_-) - \mathbf{1} - \frac{s^2}{3}(\varkappa_+^2+\varkappa_-^2)\right]e^{-\frac{m^2}{B}s},$$

$$\varkappa_\pm = \frac{1}{2B}\sqrt{\mathcal{Q}\sigma_\pm} = \frac{1}{2B}\left(\sqrt{2(\mathcal{R}+\mathcal{Q})}\pm\sqrt{2(\mathcal{R}-\mathcal{Q})}\right),$$

$$\mathcal{R} = (H^2-E^2)/2 + \hat{n}^2B^2 + \hat{n}B(H\cos(\theta) + iE\cos(\chi)\sin(\xi))$$

$$\mathcal{Q} = \hat{n}BH\cos(\xi) + i\hat{n}BE\sin(\theta)\cos(\phi) + \hat{n}^2B^2(\sin(\theta)\sin(\xi)cos(\phi-\chi) + cos(\theta)cos(\xi))$$

Y. M. Cho and D. G. Pak, Phys.Rev. Lett., 6 (2001) 1047

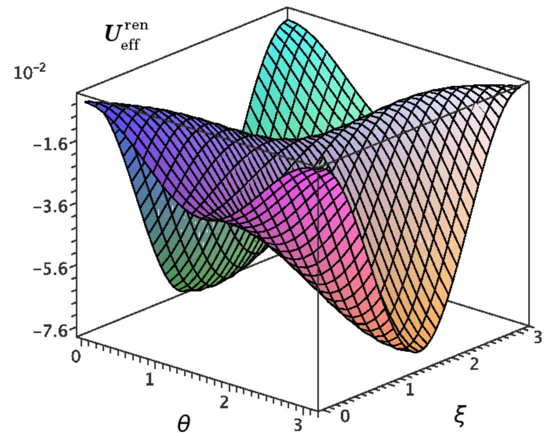


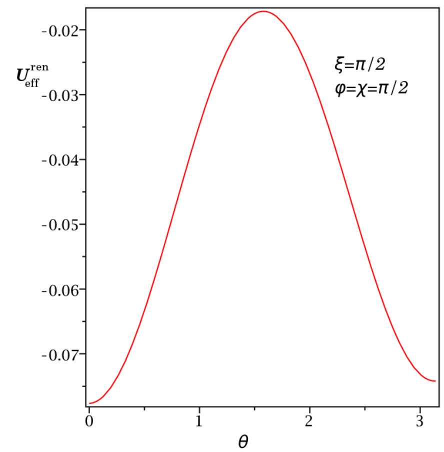
$$H_i = H\delta_{i3},\quad E_j = E\delta_{j1},\quad H^c = \{B,\theta,\phi\},\quad E^c = \{B,\xi,\chi\}$$

$H \neq 0$, $E \neq 0$ and arbitrary gluon field

$$\Im(U_{\text{eff}}) = 0 \Longrightarrow \cos(\chi)\sin(\xi) = 0, \ \sin(\theta)\cos(\phi) = 0$$

Effective potential (in units of $B^2/8\pi^2$) for the electric $E = .5B$ and the magnetic $H = .9B$ fields as functions of angles θ and ξ ($\phi = \chi = \pi/2$)





Minimum is at $\theta = \pi$ and $\xi = \pi/2$:

orthogonal to each other chromomagnetic and chromoelectric fields: $\mathcal{Q} = 0$.

Strong electro-magnetic field plays catalyzing role for deconfinement and anisotropies!

B.V. Galilo and S.N. Nedelko, Phys. Rev. D84 (2011) 094017.

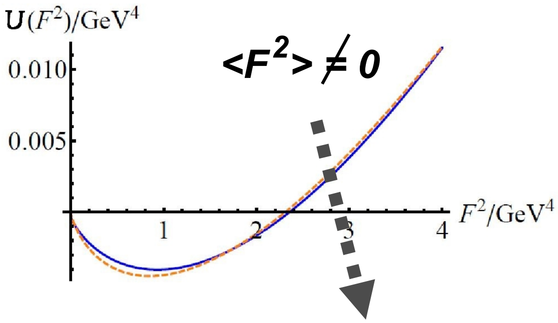
M. D'Elia, M. Mariti and F. Negro, Phys. Rev. Lett. **110**, 082002 (2013)

G. S. Bali, F. Bruckmann, G. Endrodi, F. Gruber and A. Schaefer, JHEP **1304**, 130 (2013)

A. Eichorn, et al, Phys.Rev.D83, 2011

Weyl group, CP and the kink-like field configurations in the effective SU(3) gauge theory

B. Galilo, S. Nedelko, *Phys.Part.Nucl.Lett.*, 8, 2011; *Phys. Rev. D* 84,2011.

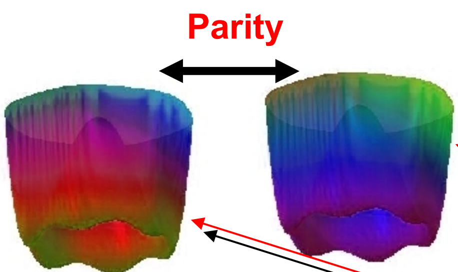


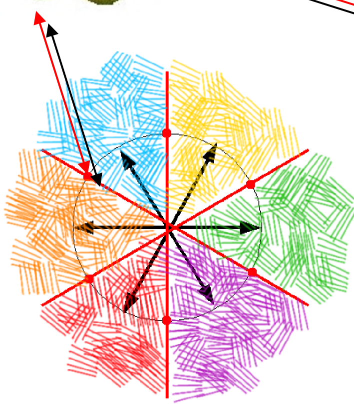
Dynamical breaking of CP and colour gauge symmetry:

SU(3)* → *U(1) x U(1) ↔ **Weyl reflections ?**

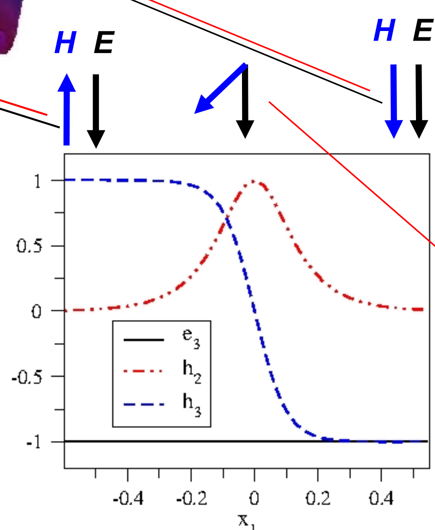
Topological defects(domain walls and lower dim. deffects at their intersections) bring disorder into ensemble of vacuum gluon configurations — on average SU(3) and CP <u>are not broken!</u>

Ensemble of domain structured vacuum fields: dynamical quark confinement, chiral symmetry breaking. In bulk of domain color is confined, color charged quasi-particles are localized at the boundaries.



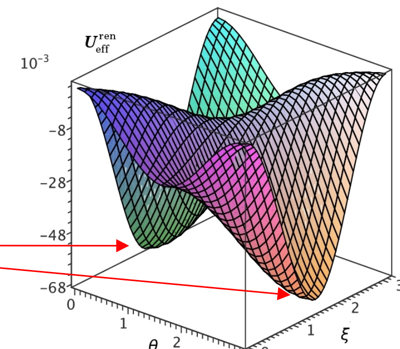


Weyl reflections in the root space of *color su*(3) — kink between boundaries of Weyl chambers



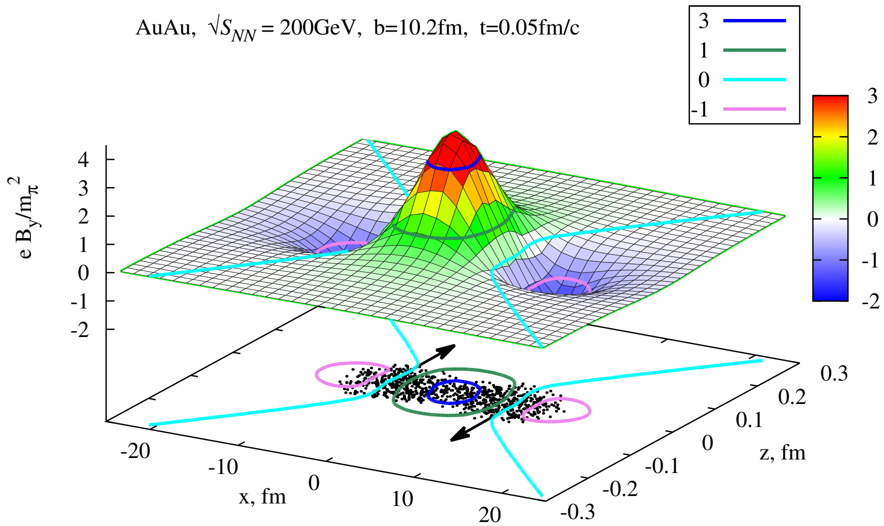
<u>Parity transformation</u> - kink interpolates Between self- and antiself-dual Abelian gluon configurations

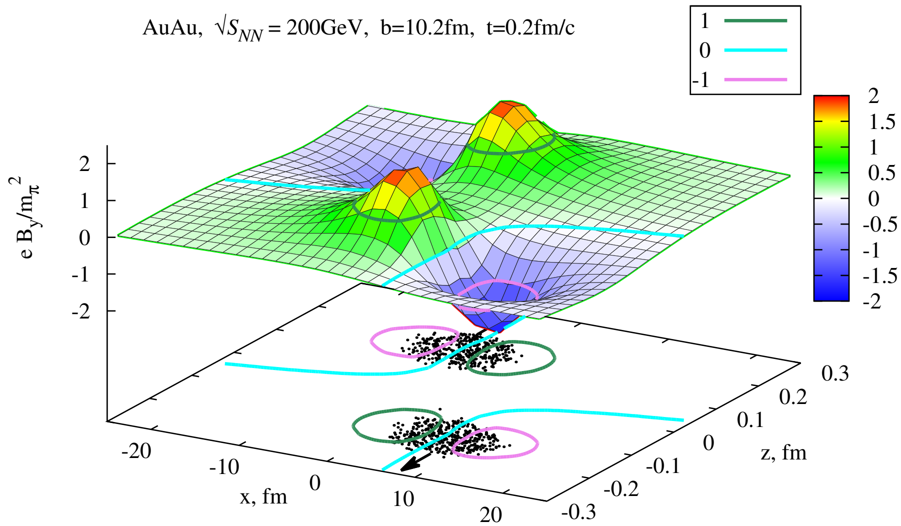
Strong crossed electromagnetic field creates relatively stable domain wall defect and thus triggers deconfinement of color charged particles in the space-time region of the relativistic heavy ion collision



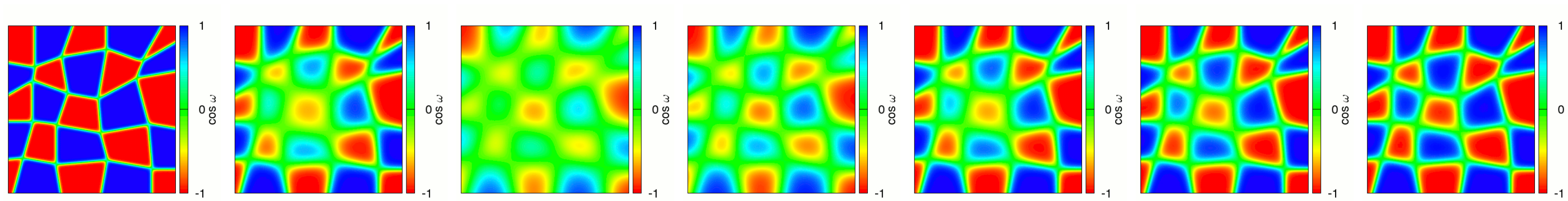
Quark contribution to QCD effective potential for Abelian gluon field in the presence of the strong crossed electromagnetic field

V. Voronyuk, V. D. Toneev, W. Cassing, E. L. Bratkovskaya,
V. P. Konchakovski and S. A. Voloshin, Phys. Rev C 84 (2011)





Magnetic field $eB \gtrsim m_\pi^2$ in the region $5fm \times 5fm \times .2fm \times .2fm/c$



A "bag" filled by hundreds of color charged quasi-particles, azimuthal asymmetry

K. A. Bugaev, V. K. Petrov and G. M. Zinovjev, Phys. Atom. Nucl. **76** (2013) 341.

Summary

Starting with

$$\lim_{V\to\infty}\frac{1}{V}\int_V d^4x g^2 F^a_{\mu\nu}(x)F^a_{\mu\nu}(x)\neq 0.$$

one arrives at the importance of the lumpy structured gluon configurations (almost everywhere homogeneous abelian (anti-)selfdual field) and correctly implemented:

- **Domain wall network as QCD vacuum:** almost everywhere homogeneous Abelian (anti-)selfdual gluon fields.
- **Domain wall network as QCD vacuum:** deconfinement can occur only in the restricted region of the space - chromomagnetic trap formation as a thick domain wall junction.
- **Confinement of both static and dynamical quarks** $\longrightarrow W(C)=\langle \mathrm{Tr}\ \mathrm{P}\ e^{i\int_C dz_\mu \hat{A}_\mu}\rangle,$
  $$S(x,y)=\langle\psi(y)\mathrm{P}e^{i\int_y^x dz_\mu\hat{A}_\mu}\bar\psi(x)\rangle$$
- **Dynamical Breaking of $SU_L(N_f)\times SU_R(N_f)$** $\longrightarrow \langle\bar\psi(x)\psi(x)\rangle$
- $U_A(1)$ **Problem** $\longrightarrow \eta'$, χ, Axial Anomaly
- **Strong CP Problem** $\longrightarrow \lim_{V\to\infty}\partial^n_\theta Z_V(\theta)\neq\partial^n_\theta\lim_{V\to\infty}Z_V(\theta)$
- **Colorless Hadron Formation:** $\longrightarrow$ Effective action for colorless collective modes: spectrum, formfactors (**Light** mesons and baryons, **Regge spectrum** of excited states of light hadrons, **heavy-light** hadrons, **heavy quarkonia**)
- **QCD vacuum is characterized as heterophase mixed state with corresponding phase transition mechanism.**

  V. I. Yukalov and E. P. Yukalova, PoS ISHEPP 2012, 046 (2012) [arXiv:1301.6910 [hep-ph]]; Phys. Rep. 208 (1991) 395;
- **Impact of a strong electromagnetic field as a trigger of deconfinement is indicated.**