HEAVY-LIGHT QUARKS INTERACTIONS IN QCD VACUUM

Yousuf Musakhanov

National University of Uzbekistan

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HEAVY-LIGHT QUARKS INTERAC-TIONS IN QCD VACUUM

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QCD vacuum
QCD instanton vacuum

Quarks in the instanton vacuum

Light quarks in the instanton background

Light quark determinant

Light quarks partition function

Heavy quarks in the instanton vacuum

Heavy quark propagator in the instanton vacuum with light quarks

Heavy–light quarks interactions at any number of light quarks N_f

Heavy–light quarks interactions at $N_f = 1$

Outline

1. QCD vacuum
   - QCD instanton vacuum

2. Quarks in the instanton vacuum
   - Light quarks in the instanton background
   - Light quark determinant
   - Light quarks partition function
   - Heavy quarks in the instanton vacuum
   - Heavy quark propagator in the instanton vacuum with light quarks
   - Heavy–light quarks interactions at any number of light quarks N_f
   - Heavy–light quarks interactions at $N_f = 1$
   - Heavy quark–antiquark system

3. Discussion

$S\chi$SB in QCD instanton vacuum

- Correct description of the spontaneous breaking of the chiral symmetry ($S\chi$SB), which is responsible for properties of most light hadrons.

- $S\chi$SB is due to the delocalization of single-instanton quark zero modes in the instanton medium.

- Only two parameters:
  - average instanton size $\rho \sim 0.3\,\mathrm{fm}$,
  - average inter-instanton distance $R \sim 1\,\mathrm{fm}$,
  – suggested phenomenologically (Shuryak1981),
  – derived variationally from $\Lambda_{\overline{\mathrm{MS}}}$ (Diakonov,Petrov1983)
  – confirmed by lattice measurements (Negele et al1998, DeGrand et al2001, Faccioli et al2003, Bowman etal2004).

- The model provided a consistent description of the light quark physics (Diakonov et al, Goeke et al, Musakhanov et al).

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QCD instantons

Instantons –self-dual classical solutions of the equations of motion in Euclidean space (Belavin *et.al.*, 1975):

$$A_\mu^{I,a}(x) = \frac{2\rho^2 \bar{\eta}_{\mu a}^\nu (x-z)_\nu}{(x-z)^2[\rho^2 + (x-z)^2]}, \ G_{\mu\nu}^a = \tilde{G}_{\mu\nu}^a.$$

- The topological charge $Q = \frac{1}{32\pi^2} \int d^4x \, G_{\mu\nu}^a \tilde{G}_{\mu\nu}^a = 1$.
- For the antiinstanton the t'Hooft symbol $\bar{\eta} \to \eta$, $G_{\mu\nu}^a = -\tilde{G}_{\mu\nu}^a, \ Q = -1$.
- Chern-Simons number collective coordinate

$$N_{CS} = \frac{1}{16\pi^2} \int d^3x \, \epsilon^{ijk} \left(A_i^a \partial_j A_k^a + \frac{1}{3} \epsilon^{abc} A_i^a A_j^b A_k^c \right),$$

- Large gauge transformations
$N_W = \frac{1}{24\pi^2} \int d^3x \epsilon_{ijk} \left\langle \left(U^\dagger \partial_i U\right) \left(U^\dagger \partial_j U\right) \left(U^\dagger \partial_k U\right) \right\rangle.$
$N_{CS} \Rightarrow N_{CS} + N_W.$
- Number of collective coordinates:

$$4 \, (centre) \ + \ 1 \, (size) \ + \ (4N_c - 5) \, (orientations) \ = \ 4N_c$$

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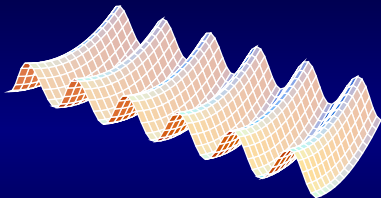Musakhanov

Dependence on N_{CS}

Figure: Vacuum gluon energy vs Chern-Simons number collective coordinate N_{CS}. The amplitude of quantum tunneling $\sim \exp(-S_I)$ between the states with $|\Delta N_{CS}| = 1$. Here the action $S_I = \frac{8\pi^2}{g^2}$.

Instanton vacuum model

- Sum ansatz $A = \sum_I A^I + \sum_{\bar{I}} A^{\bar{I}}$ for dilute gas approximation.
- Inter-instantons interactions have to stabilize instanton size ρ and inter-instanton distance R.

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Parameters of instanton vacuum

- Averaged instanton size $\bar{\rho}$;
- Averaged inter-instanton distance $\bar{R}$.
- Results:
  - Lattice estimate: $\bar{R} \approx 0.89 \, fm$, $\bar{\rho} \approx 0.36 \, fm$,
  - Phenomenological estimate: $\bar{R} \approx 1 \, fm$, $\bar{\rho} \approx 0.33 \, fm$,
  - Our estimate (with account of $1/N_c$ corrections):
    $\bar{R} \approx 0.76 \, fm$, $\bar{\rho} \approx 0.32 \, fm$, correspond
    $F_{\pi, m=0} = 88 MeV$, $\langle \bar{q} q \rangle_{m=0} = -(255 MeV)^3$

  Thus within $10 - 15\%$ uncertainty different approaches
  give similar estimates
- Packing parameter $\pi^2 (\frac{\bar{\rho}}{\bar{R}})^4 \sim 0.1 - 0.3$
  $\Rightarrow$ Independent averaging over instanton positions and
  orientations.

QCD vacuum on the lattice

Figure: Action and topological charge densities in different configurations on the lattice.

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Light quarks in the single instanton background

Very strong modification of the light quark propagator

$$S(x, y) \approx \frac{|\Phi_0(x, \zeta)\rangle \langle\Phi_0(y, \zeta)|}{im} + \frac{1}{i\hat{\partial}},$$

due to the zero mode

$$(i\hat{\partial} + g\hat{A})\Phi_0(x, \zeta) = 0.$$

Here collective coordinates ζ: a instanton position z and color orientation U.

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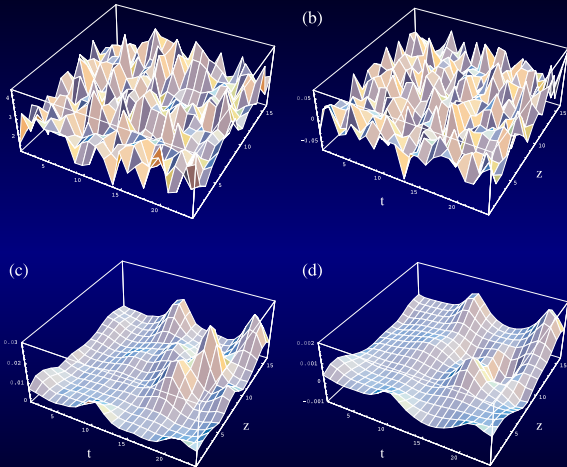quarks
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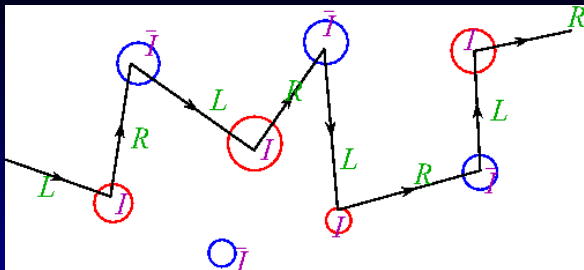
Spontaneous Breaking of the Chiral Symmetry

Figure: Light quark in the instanton vacuum. $N_f = 1$.

Sum-up of multi-scattering series $\Rightarrow$ full light quark propagator:

$$S - S_0 = -S_0 \sum_{i,j} \hat{p} |\Phi_{0i}\rangle \left\langle \Phi_{0i} \left| \left(\frac{1}{B(m)} \right) \right| \Phi_{0j} \right\rangle \langle \Phi_{0j} | \hat{p} S_0,$$

$$B(m) = \hat{p} S_0 \hat{p}$$

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Low-frequency part of the light quark determinant with the quark sources

We was able to find $\mathrm{Det}_{low}(\hat{P} + im)e^{(-\xi^+ S\xi)}$:

$$= \int \prod_f D\psi_f D\psi_f^\dagger \exp \int \left(\psi_f^\dagger(\hat{p} + im_f)\psi_f + \psi_f^\dagger \xi_f + \xi_f^+ \psi_f \right)$$

$$\times \prod_f \left\{ \prod_+^{N_+} V_{+,f}[\psi^\dagger, \psi] \prod_-^{N_-} V_{-,f}[\psi^\dagger, \psi] \right\} ,$$

where $V_{\pm,f}[\psi^\dagger, \psi] =$

$$= i \int dx \left(\psi_f^\dagger(x)\, \hat{p}\Phi_{\pm,0}(x; \zeta_\pm) \right) \int dy \left(\Phi_{\pm,0}^\dagger(y; \zeta_\pm)(\hat{p}\, \psi_f(y)) \right).$$

Fermionic fields $\psi^\dagger, \psi$ has a meaning of constituent quarks.

Partition function

Averaging over instantons collective coordinates $\Rightarrow$ partition function $Z[\xi_f, \xi_f^+] =$

$$= \int \prod_f D\psi_f D\psi_f^\dagger \exp \int \left(\psi_f^\dagger (\hat{p} + im_f)\psi_f + \psi_f^\dagger \xi_f + \xi_f^+ \psi_f \right)$$

$$\times \int D\zeta \prod_+^{N_+} \prod_f V_{+,f}[\psi^\dagger, \psi] \prod_-^{N_-} \prod_f V_{-,f}[\psi^\dagger, \psi] ,$$

Small packing parameter provided here independent averaging:

$$\overline{\prod_f V_{\pm,f}[\psi^\dagger, \psi]} = \int d\zeta_\pm \prod_f V_{\pm,f}[\psi^\dagger, \psi]$$

$\Rightarrow$ non-local t'Hooft-like vertex with $2N_f$-legs. Nonlocality range $\sim \rho$.

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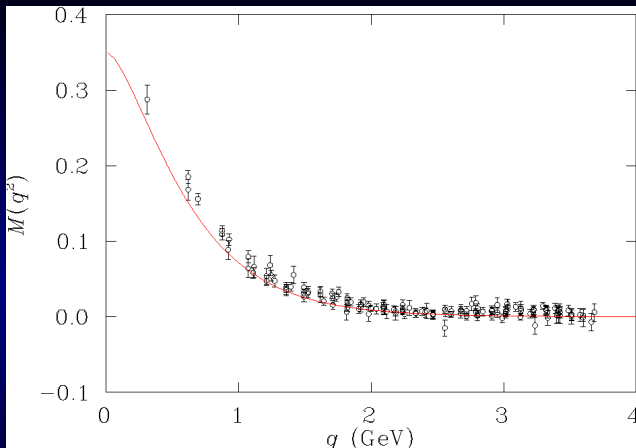quarks
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Dynamical quark mass $M(q)$

Figure: Light quark dynamical mass M(q) (red line) in comparison with lattice results (Bowman et al 2004). M(q)-dependence is entirely defined by zero-mode!!! $M(0) \approx 365\,MeV$ gives a strength of the light quark interactions with QCD vacuum instantons.

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Heavy quarks in the instanton vacuum

$L_\Psi = \Psi^+(\hat{P} + im_H)\Psi \Rightarrow Q^+\gamma_4 P_4 Q + Q^+ Q_1 Q,$
$Q_1 = \frac{\vec{P}^2}{2m_H} - \frac{\vec{\sigma}\vec{B}}{2m_H}, \quad P = p - gA, \quad \vec{B} = rot\vec{A}.$
(Infinitely) heavy quark propagator (Wilson line) in instanton vacuum defined as

$$w = \int D\zeta \frac{1}{\theta^{-1} - \sum_i a_i}$$

where $a_i(t) = iA_{i,\mu}(x(t))\frac{d}{dt}x_\mu(t), w_\pm = \frac{1}{\theta^{-1} - a_\pm}, < t|\theta|t' > = \theta(t - t').$
Pobylica Eq. for the w^{-1} has a solution

$$w^{-1} = \theta^{-1} - \frac{N}{2}\sum_\pm \theta^{-1}(w_\pm - \theta)\theta^{-1} + O(N^2/V^2),$$

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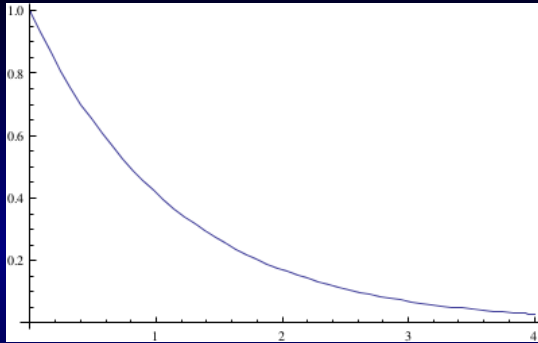# Heavy quark interaction with instanton vacuum

Figure: Form-factor $i_0[q\rho]/i_0[0]$, $i_0[0] = 0.55197$. The split of heavy quark mass in instanton vacuum is $\Delta M = 16\pi i_0[0](\rho^4/R^4)\rho^{-1}/N_c$. At $\rho = 0.32 fm$, $R = 0.76 fm$ (ChPT) $\Delta M = 148 MeV$ gives a strength of the heavy quark interactions with QCD vacuum instantons at the range $\sim \rho$.

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Heavy quark propagator at light quark number $N_f = 1$

Extension of DPP89 solution (planar graphs) is $w^{-1}[\psi, \psi^\dagger] =$

$$= \theta^{-1} - \frac{N}{2} \sum_\pm \frac{1}{V_\pm[\psi^\dagger, \psi]} \Delta_{H,\pm}[\psi^\dagger, \psi] + O(N^2/V^2),$$

$$\Delta_{H,\pm}[\psi^\dagger, \psi] = \int d\zeta_\pm \ V_\pm[\psi^\dagger, \psi]\theta^{-1}(w_\pm - \theta)\theta^{-1}.$$

Then heavy quark propagator at light quark number $N_f = 1$

$$S_H = \frac{1}{\theta^{-1} - \lambda \sum_\pm \Delta_{H,\pm}[\frac{\delta}{\delta\xi}, \frac{\delta}{\delta\xi^+}]} \exp\left[-\xi^+ \left(\hat{p} + iM(p)\right)^{-1} \xi\right]|_{\xi=\xi^+}$$

DPP89 solution is reproduced at the approximation:

$$S_H^{-1} \approx \theta^{-1} - \lambda \sum_\pm \Delta_{H,\pm}[\frac{\delta}{\delta\xi}, \frac{\delta}{\delta\xi^+}] \exp\left[-\xi^+ \left(\hat{p} + iM(p)\right)^{-1} \xi\right]|_{\xi=}$$

At any N_f and in saddle-point approximation no an essential difference with $N_f = 1$.

Heavy–light quarks interactions at any number of light quarks N_f

is given by the expression $-\lambda \sum_{\pm} Q^\dagger \Delta_{H,\pm}[\psi^\dagger, \psi] Q$

$$= -i\lambda \sum_{\pm} \int d^4 z_\pm dU_\pm \prod_{f=1}^{N_f} \frac{d^4 k_f}{(2\pi)^4} \frac{d^4 q_f}{(2\pi)^4} \exp(i(q_f - k_f)z_\pm)$$

$$\psi^+_{f,a_f\alpha_f}(k_f)(\gamma_{\mu_f}\gamma_{\nu_f} \frac{1 \pm \gamma_5}{2})_{\alpha_f\beta_f} (U^{a_f}_{\pm,i_f}(\tau^{\mp}_{\mu_f}\tau^{\pm}_{\nu_f})^{i_f}_{j_f} U^{\dagger j_f}_{\pm,b_f} \psi^{b_f}_{f,\beta_f}(q_f)$$

$$\frac{(2\pi\rho)^2 F(k_f)F(q_f)}{8} Q^+_{a_3} U^{a_3}_{\pm,i_3} (\theta^{-1}(w_\pm - \theta)\theta^{-1})^{i_3}_{j_3} U^{\dagger j_3}_{\pm,b_3} Q^{b_3}$$

At any N_f the interaction term have 2 heavy and $2N_f$ light quark legs. The actual structure is defined by the color orientation integration and it have to have $SU_L(N_f) \times SU_R(N_f)$ symmetry.

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Heavy–light quarks interactions at $N_f = 1$

is

$$-\lambda \sum_{\pm} Q^{\dagger} \Delta_{H,\pm}[\psi^{\dagger}, \psi] Q$$

$$= i \int \frac{d^4 k_1}{(2\pi)^4} \frac{d^4 k_2}{(2\pi)^4} \frac{d^3 q}{(2\pi)^3} (2\pi)^4 \delta^3(\vec{k}_2 + \vec{k}_1 - \vec{q}) \delta(k_{2,4} - k_{1,4})$$

$$(M(k_1) M(k_2))^{1/2} 8\pi \rho^3 i_0(q\rho) \left[\frac{2N_c - 1}{N_c^2 - 1} \psi^+(k_1) \psi(k_2) Q^+ Q \right.$$

$$\left. + \frac{N_c - 2}{N_c^2 - 1} (\psi^+(k_1) Q Q^+ \psi(k_2) + \psi^+(k_1) \gamma_5 Q Q^+ \gamma_5 \psi(k_2)) \right]$$

Bosonizaion of the first term provide the light-quark exchange QQ potential V_{lq}, while second and third terms – Qq mesons degenerated on parity.

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Heavy quark–antiquark system correlator, $N_f = 1$

The correlator $C(L_1, L_2) =$

$$\frac{1}{Z} \int D\psi D\psi^\dagger \prod_\pm^{N_\pm} \bar{V}_\pm[\psi^\dagger, \psi] \exp \int \left(\psi^\dagger (p \hat{+} im)\psi \right) W[\psi, \psi^\dagger],$$

$$< T|W[\psi, \psi^\dagger]|0 > = \left(\prod_\pm^{N_\pm} \bar{V}_\pm[\psi^\dagger, \psi] \right)^{-1} \int D\zeta \prod_\pm^{N_\pm} V_\pm[\psi^\dagger, \psi]$$

$$\times < T|\frac{1}{\theta^{-1} - \sum_i a_i^{(1)}}|0 > < 0|\frac{1}{\theta^{-1} - \sum_i a_i^{(2)}}|T > .$$

is a Wilson loop along the rectangular contour $L \times r$. The sides $L_1 = (0, T), L_2 = (T, 0)$ are parallel to x_4 axes and separated by the distance r. The $a^{(1)}, a^{(2)}$ are the projections of the instantons onto the lines L_1, L_2.

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Heavy quark–antiquark system correlator, $N_f = 1$

The extension of DPP89 solution is $W^{-1}[\psi, \psi^\dagger] =$

$$= w_1^{-1}[\psi, \psi^\dagger] \times w_2^{-1, T}[\psi, \psi^\dagger] - \frac{N}{2} \sum_\pm \bar{V}_\pm^{-1}[\psi^\dagger, \psi] \int d\zeta_\pm$$

$$\times V_\pm[\psi^\dagger, \psi] \theta^{-1} \left(w_\pm^{(1)} - \theta \right) \theta^{-1} (\times) \left(\theta^{-1} \left(w_\pm^{(2)} - \theta \right) \theta^{-1} \right)^T$$

where, superscript T means the transposition, $(\times)$ – tensor product and

$$w^{(1,2)-1}[\psi, \psi^\dagger] = \theta^{-1} - \frac{N}{2} \sum_\pm \frac{1}{\bar{V}_\pm[\psi^\dagger, \psi]} \Delta_{H,\pm}^{(1,2)}[\psi^\dagger, \psi] + O(\frac{N^2}{V^2}).$$

The integration of the first term in $W^{-1}[\psi, \psi^\dagger]$ over $\psi, \psi^\dagger$ leads to heavy quark–antiquark potential V_{lq}, generated by light quarks.

Heavy quark–antiquark potential V_{lq}, generated by light quarks, $N_f = 1$
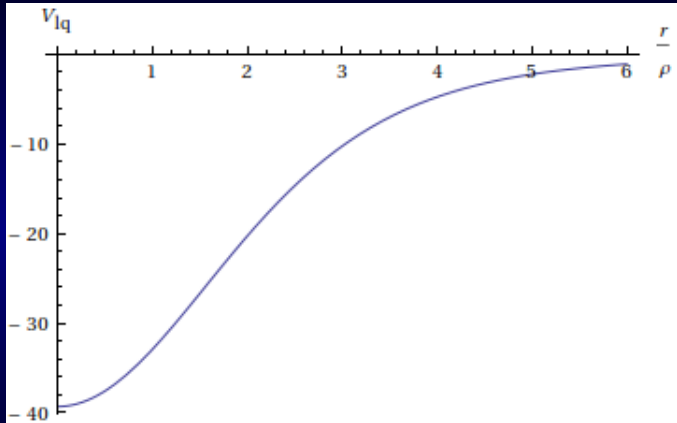

Figure: Heavy quark–antiquark potential $V_{lq}(r/\rho)$ (in MeV), generated by light quarks, its range is controlled by dynamical light quark mass $M \sim 0.36$ GeV.

Discussion

QCD instanton vacuum naturally lead to the consistent treatment of light quark physics. It is applied to heavy quarks, too. Within this framework we find:

- Light quarks strongly interact with QCD vacuum instantons due to zero-modes. These interactions are responsible for the dynamical quark mass $M \sim 360\,MeV$ together with $S\chi SB$ and the most important properties of light hadrons and nuclei.

- Heavy quarks interact with these instantons moderately and it leads to heavy quark mass shift $\Delta M \sim 148\,MeV$.

- QCD vacuum instantons generate also heavy-light quarks interactions, responsible for the traces of the chiral symmetry of the light quarks.

- There is a consistent way to estimate the couplings in the phenomenological chiral lagrangian for heavy and light mesons, accounting $S\chi SB$ and heavy quark symmetries.