Self-similarity in the description of inclusive spectra of hadrons produced in pp- and AA- collisions

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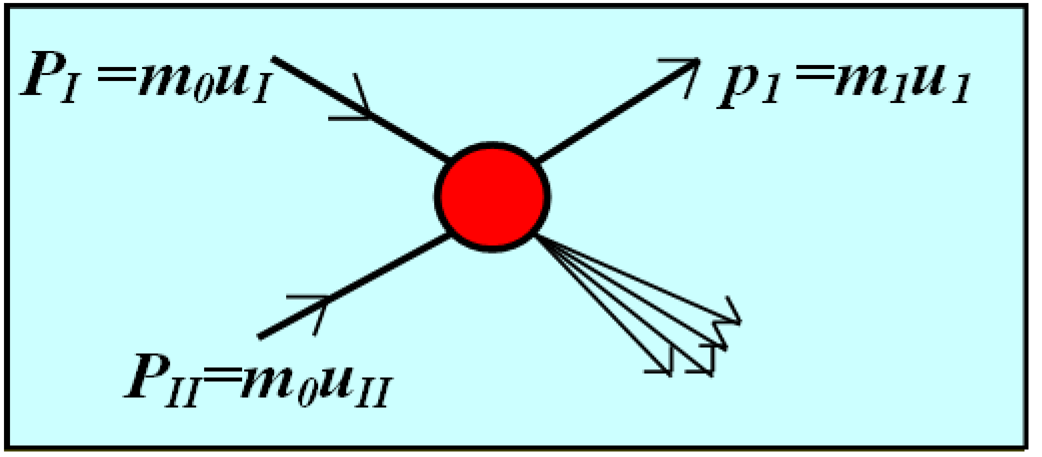
- **Baldin approach in description of nuclear interactions in four velocity space and self-similarity parameter**

$I + II \rightarrow 1 + \ldots$



$$(N_I P_I + N_{II} P_{II} - p_1)^2 = (N_I m_0 + N_{II} m_0 + M)^2$$

N_I and N_{II} are nucleon effective numbers participating in interaction (cumulative numbers) for nuclei I and II.

M is the mass of the particle providing conservation of the baryon number, strangeness and other quantum numbers.

For antinuclei and **K^-** mesons **$M = m_1$**, for **nuclear fragments $M = -m_1$**. For **K^+** mesons **$M = m_\Lambda - m_0$**. For the particles produced without accompanying antiparticles (**π mesons**) **$M = 0$**.

Using the principle of symmetries, in particular, symmetries of the solutions, allowed A.M.Baldin to introduce the self-similarity parameter of Π for the nuclear interaction:

$$\mathbf{\Pi = \min[\tfrac{1}{2}\sqrt{(u_I N_I + u_{II} N_{II})^2}],}$$

where N_I and N_{II} are nucleon effective numbers participating in interaction (cumulative numbers) for nuclei I and II, and u_I and u_{II} are 4-velocities of these nuclei.

A.M.Baldin, A.A.Baldin. Phys. Particles and Nuclei, v.29, No.3 (1998) p.232.

A.M.Baldin, A.I.Malakhov, and A.N.Sissakian. Physics of Particles and Nuclei, Vol.32. Suppl. 1, 2001, pp.S4-S30.

The invariant cross-sections of the output inclusive particles of different types at nuclear interactions can be represented as follows:

$$E(d^3\sigma/dp^3) \sim \varphi(\Pi).$$

One of possible ways is given in the work: **A.M.Baldin, A.A.Baldin. Phys. Particles and Nuclei, v.29, No.3 (1998) p.232.**

In this case the invariant cross-sections of the output inclusive particles of different types at nuclear interactions with atomic numbers A_I and A_{II}, are described by universal dependence in a wide energy range and different atomic numbers of colliding nuclei:

$$Ed^3\sigma/dp^3 = C_1 A_I^{\alpha(N_I)} A_{II}^{\alpha(N_{II})} \exp(-\Pi/C_2),$$

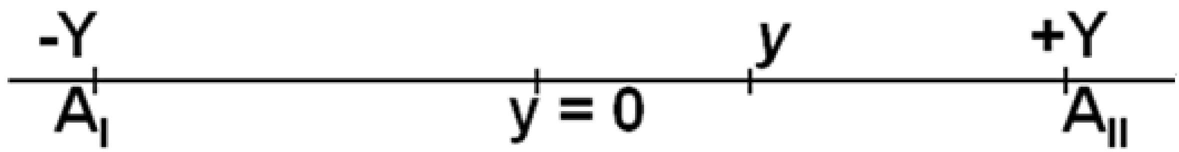
where $\alpha(N_I) = 1/3 + N_I/3$, $\alpha(N_{II}) = 1/3 + N_{II}/3$,
$C_1 = 1.9 \cdot 10^4$ mb GeV^{-2}c^{3}st^{-1} и $C_2 = 0.125 \pm 0.002$

- **Analytical solution for self-similarity parameter in central rapidity region**

$$\Pi^{min} \Rightarrow d\Pi/dN_I = 0; \; d\Pi/dN_{II} = 0$$

In the central rapidity region ($y = 0$)

$$(u_1 \cdot u_I) = (u_1 \cdot u_{II})$$



$$N_I = N_{II} = N = [1 + \sqrt{1 + (\Phi_0/\Phi^2)}]\cdot\Phi,$$

where

$$\Phi = (1/m_0)\cdot[m_T\cdot chY + M]/(2\cdot sh^2Y)$$

$$\Phi_0 = (M^2 - m_1^2)/(4\cdot m_0^2\cdot sh^2Y)$$

and

$$\Pi^{min} = N\cdot chY$$

Baldin A.M., Malakhov A.I. JINR Rapid Communications, No.1(87)-98, 1998, pp.5-12.

- **Calculation of yield ratio of antibaryons to baryons**

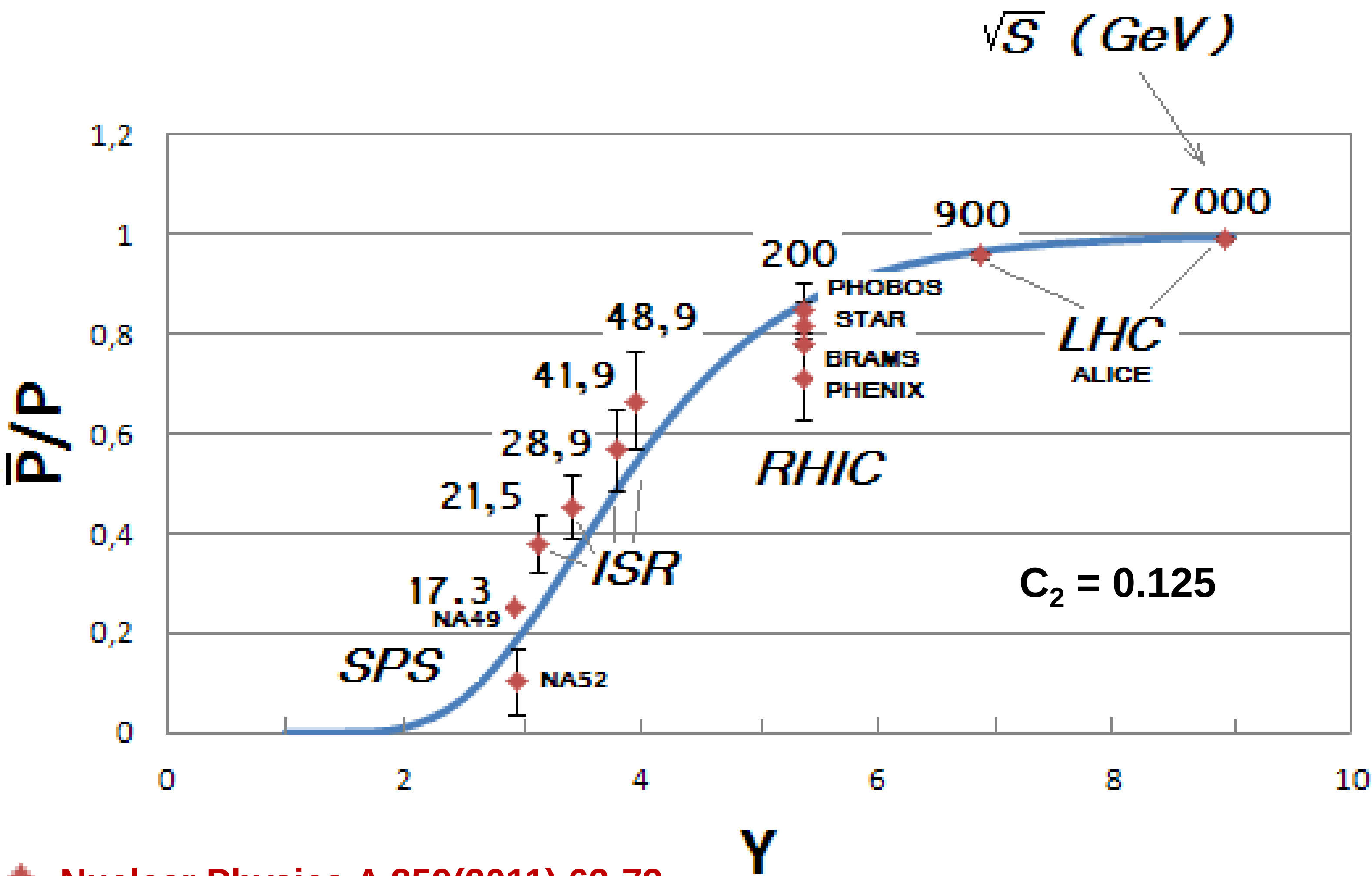
For nuclei and nuclear fragments $M = -m_1$ ⟶

$$\Pi_1 = \left[\frac{m_{1T}}{m_0} chY - \frac{m_1}{m_0}\right] \frac{chY}{sh^2 Y}$$

For antinuclei and antinulear fragments $M = m_1$ ⟶

$$\Pi_2 = \left[\frac{m_{1T}}{m_0} chY + \frac{m_1}{m_0}\right] \frac{chY}{sh^2 Y}$$

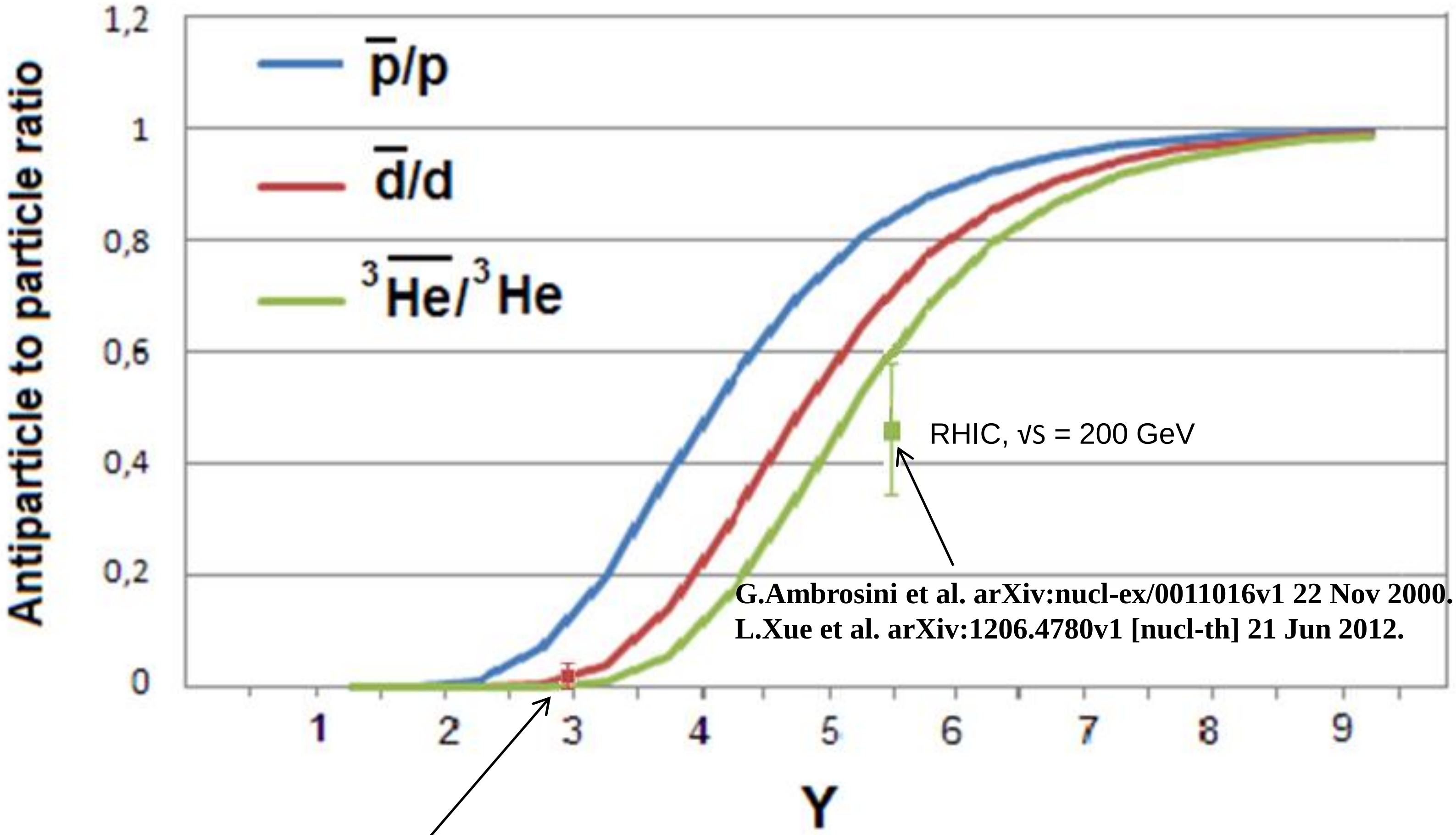
$$\text{Ratio}\left(\frac{\text{antinuclei}}{\text{nuclei}}\right) = \frac{\int_0^\infty m_{1T} \cdot C_1 \cdot A_I^{\alpha(N_I)} \cdot A_{II}^{\alpha(N_{II})} \cdot \exp\left(-\frac{\Pi_2}{C_2}\right) \cdot dm_{1T}}{\int_0^\infty m_{1T} \cdot C_1 \cdot A_I^{\alpha(N_I)} \cdot A_{II}^{\alpha(N_{II})} \cdot \exp\left(-\frac{\Pi_1}{C_2}\right) \cdot dm_{1T}} = \exp\left(-\frac{2}{C_2} \cdot \frac{m_1}{m_0} \cdot \frac{chY}{sh^2 Y}\right)$$



Nuclear Physics A 859(2011) 63-72.

http://hepdata.cedar.ac.uk/view/p7907.

Nuclear Physics A610 (1996) 306c-316c.



G.Ambrosini et al. arXiv:nucl-ex/0011016v1 22 Nov 2000.
L.Xue et al. arXiv:1206.4780v1 [nucl-th] 21 Jun 2012.

Physical Review C 85 (2012) 044913 (SPS, NA49). SPS, √S = 7,3 GeV

- **Further development of the Baldin approach in the description of the inclusive cross sections**

$$E(d^3\sigma/dp^3)_q = \rho_q(y=0,\ p_t) = \varphi_q(y=0,p_t)\cdot\sum_{n=1}^{\infty}[n\cdot\sigma_n(s)] =$$

$$= \varphi_q(y=0,p_t)\cdot g(s/s_0)^{\Delta}$$

K.A.Ter-Martirosyan. Sov.J.Nucl.Phys., 44, 817 (1986).

Inclusive hadron production in central region and the AGK (Abramovsky, Gribov, Kanchelly) cancellation

$$E(d^3\sigma/dp^3)_g = \rho_g(y=0,\ p_t) = \varphi_g(y=0,p_t)\cdot\sum_{n=2}^{\infty}[(n-1)\sigma_n(s)] =$$

$$= \varphi_g(y=0,p_t)\cdot(g(s/s_0)^{\Delta} - \sigma_{nd})$$

V.A. Bednyakov, A.A. Grinyuk, G.I. Lykasov, M. Poghosyan, Int.J.Mod.Phys., A27, (2012) 1250042; A.A. Grinyuk, G.I. Lykasov, A.V. Lipatov, N.P. Zotov, Phys.Rev.D87 (2013) 074017.

$$E(d^3\sigma/dp^3) = [\varphi_q(y,p_t) + \varphi_g(y,p_t)\cdot(1- \sigma_{nd}/g(s/s_0)^{\Delta})]\cdot g(s/s_0)^{\Delta}$$

σ_n – cross-section of hadron production by the exchange of n-pomerons.
$\varphi = \varphi(\Pi)$.
g – constant (~20 mbarn).
S_0 ~ 1 GeV^2.
$\Delta = [\alpha_p(0)-1]$ ~ 0,08

$$E(d^3\sigma/dp^3)_q = \rho_q(y=0,\ p_t) = \varphi_q(y=0,p_t)\cdot\sum_{n=1}^{\infty}[n\cdot\sigma_n(s)] =$$

$$= \varphi_q(y=0,p_t)\cdot g(s/s_0)^{\Delta}$$

K.A.Ter-Martirosyan. Sov.J.Nucl.Phys., 44, 817 (1986).

Inclusive ha… …ral region

$$E(d^3\sigma/dp^3)_g = \rho_g(y=0,\ p\ldots$$

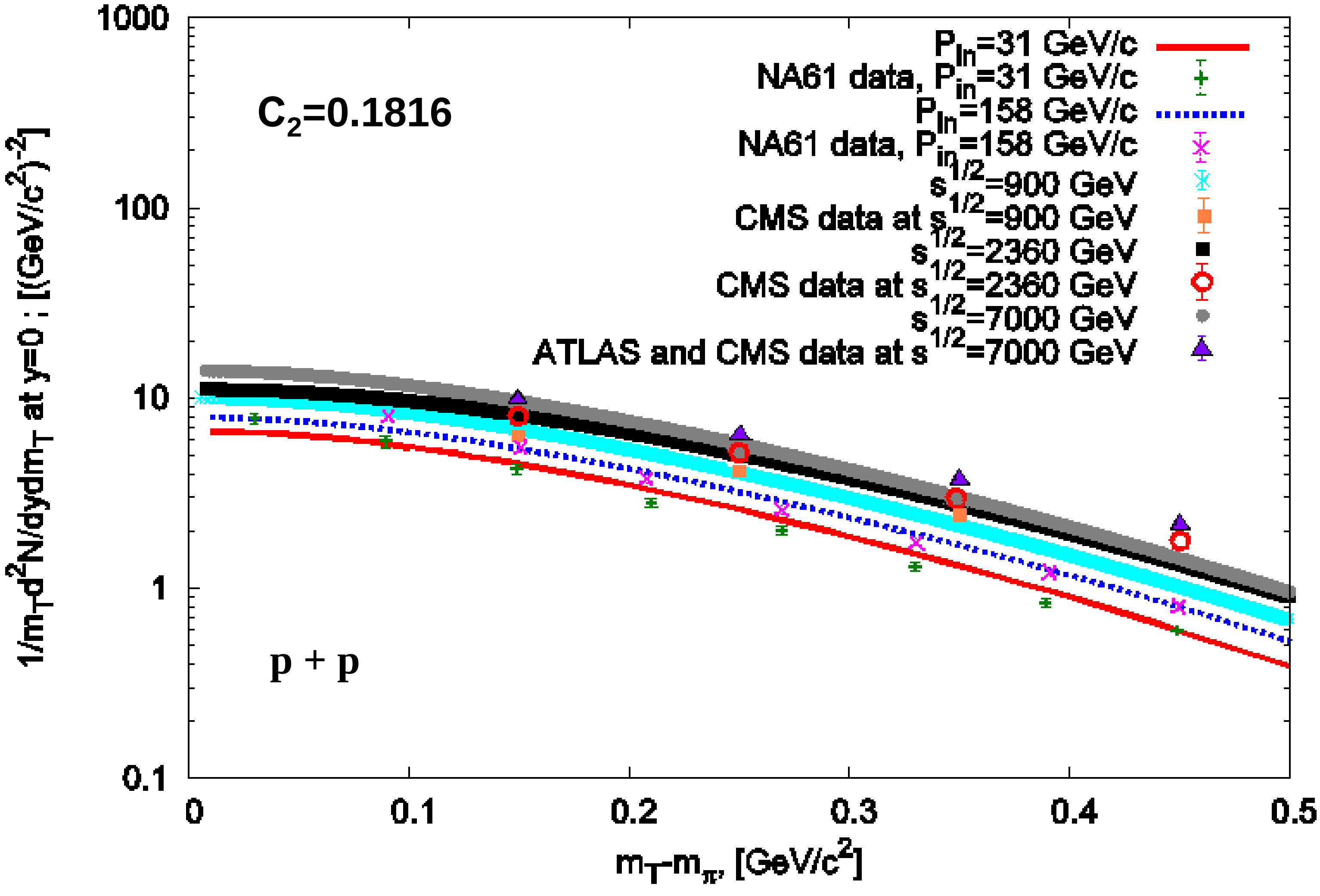
$$= \varphi_g(y=0,p_t)\cdot(\ldots$$

Main idea $\varphi = \varphi(\Pi)$.

…nyuk, G.I. …osyan, …ys., A27, (2012) 1250042; …nyuk, G.I. Lykasov, A.V. Lipatov, N.P. Zotov, Phys.Rev.D87 (2013) 074017.

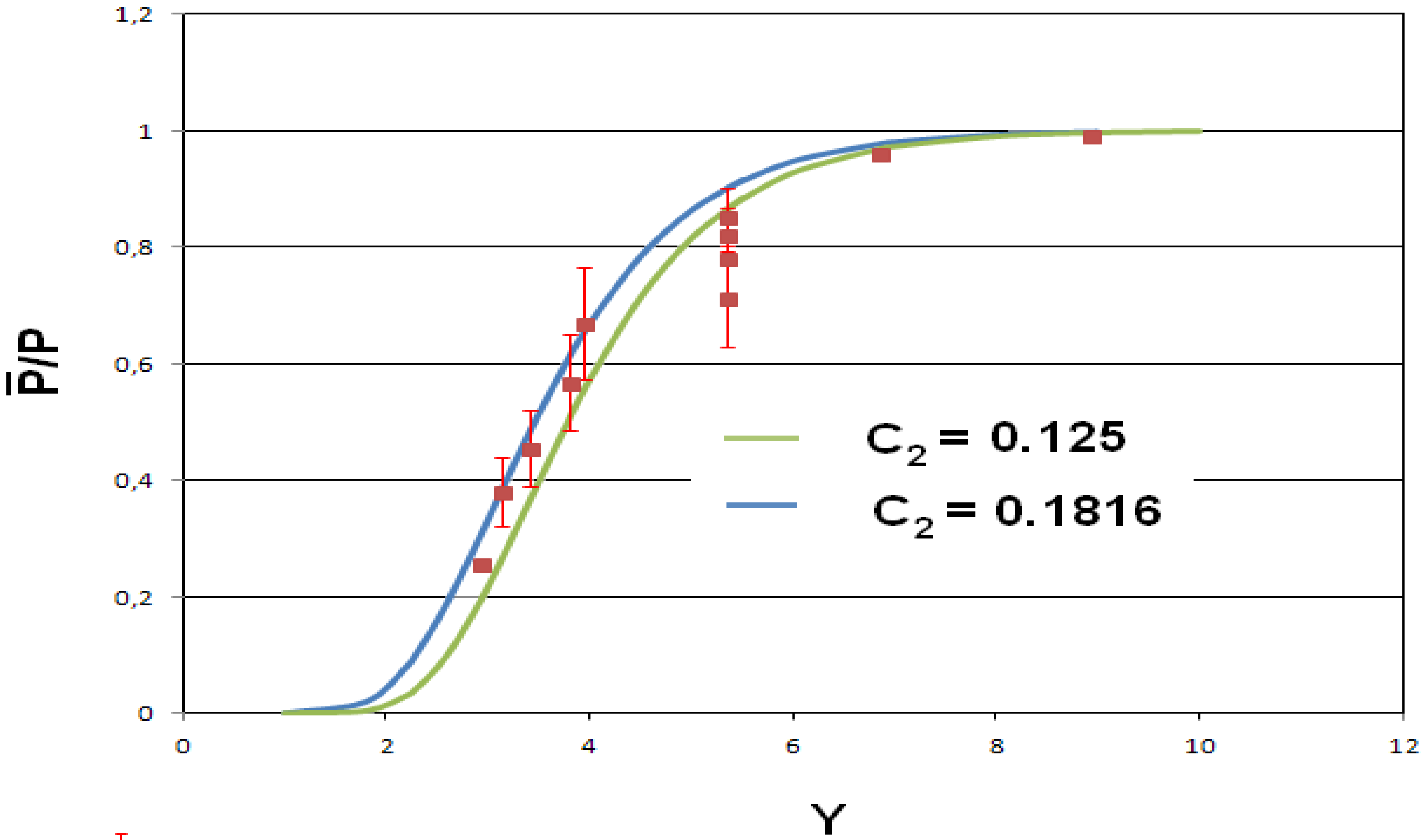
$$E(d\ldots\ \varphi_g(y,p_t)\cdot(1-\sigma_{nd}/g(s/s_0)^{\Delta})]\cdot g(s/s_0)^{\Delta}$$

σ_n – cros… …ron production by the exchange of n-pomerons.
$\varphi = \varphi(\Pi)$.
g – consta… (~20 mbarn).
$S_0 \sim 1$ GeV^2.
$\Delta = [\alpha_p(0)-1] \sim 0{,}08$



Normalized on the NA61 data at the energy of 31 GeV and multiplied by the factor of $(S/S_0)^{\Delta}$,

$\Delta = [\alpha_p(0)-1] \sim 0{,}08$.



- Nuclear Physics A 859(2011) 63-72.

http://hepdata.cedar.ac.uk/view/p7907.

Nuclear Physics A610 (1996) 306c-316c.

$\Pi(\pi) = (1+\mathrm{sqrt}[1- (m_\pi^2/m_T^2)(Sh^2Y/Ch^2Y)](m_T Ch^2Y/2m_0 Sh^2Y) \approx$ при $m_T > m_\pi$

$\approx m_T Ch^2Y/m_0 Sh^2Y = m_T/m_0(1-1/Ch^2Y) = m_T/m_0\,[1-4m_0^2 s/(s-m_0^2)^2] \approx$ при $s > m_0$

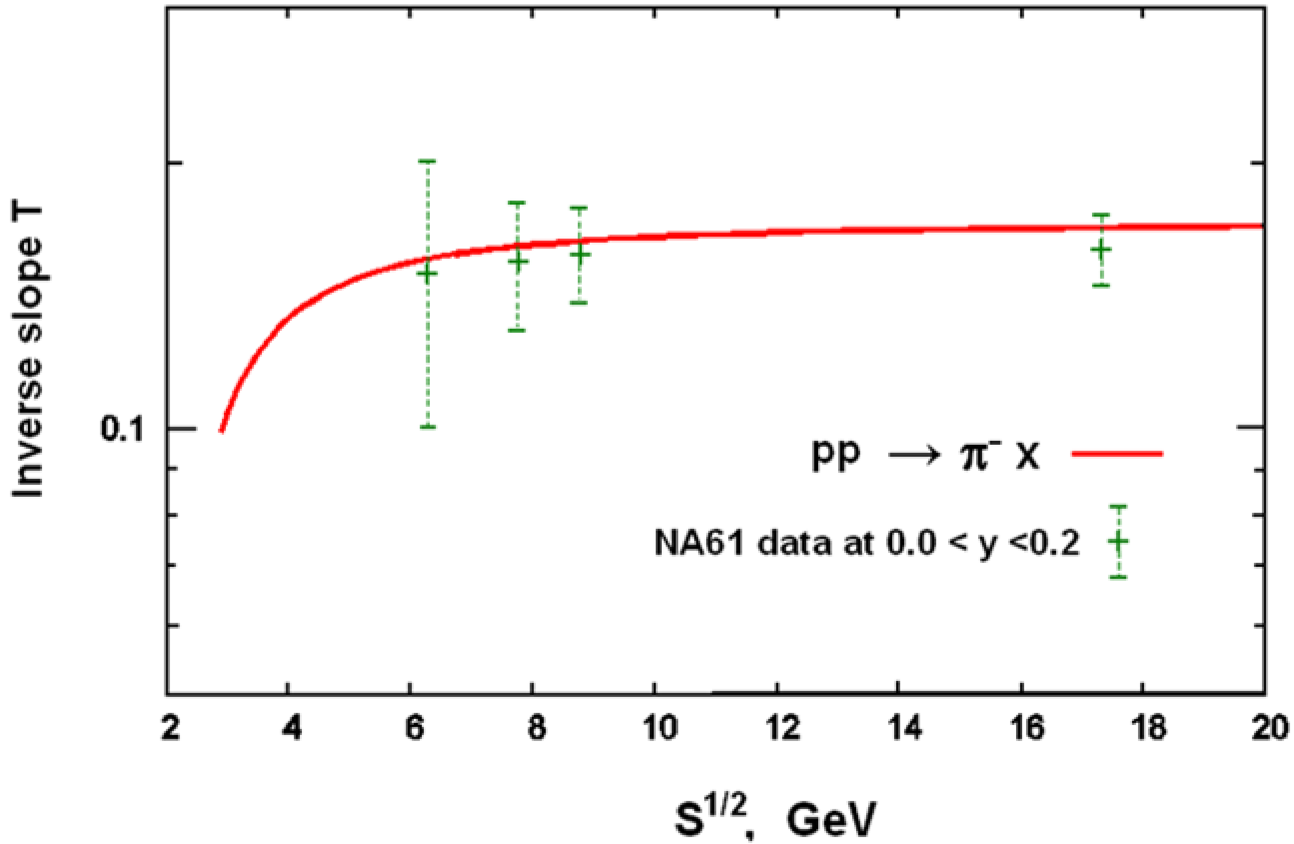
$\approx m_T/m_0(1- 4m_0^2/s).$

$$\exp[-\Pi(\pi)/C_2] = \exp[- m_T/C_2 m_0(1- 4m_0^2/s)]$$

$$T = C_2 m_0(1- 4m_0^2/s)$$

$m_T > m_\pi \quad \rightarrow \quad E(d^3\sigma/dp^3) \approx C\exp(-m_T/T)$

$T = C_2\,(1 - 4m_0^2/s),\ C_2 = 0.1816$



A.A.Abgrall et al. Measurements of negatively charged pion spectra in inelastic p+p interactions at p_{lab}= 20,31, 40, 80 and 158 GeV/c. Eur.Phys.J. C74 (2014) 2794.

Conclusions

- The use of the self-similarity allows us to describe rather well the ratio of the proton to anti-proton yields in A-A collisions as a function of the energy in a wide range from 10-20 GeV to a few TeV.
- To study the similar ratio of light nuclei to anti-nuclei, we need more detailed experimental information.
- The inclusive spectra in the space of four-velocities depend on relativistic invariant variables and the initial energy.
- Modification of our approach taking into account the energy dependence of inclusive spectra of hadrons allows us to describe the differential cross section as a function of m_T.
- We have got a satisfactory description of the slopes of the hadron spectra as functions of the energy in a wide range from AGS to LHC.

Thank you for the attention!

$$\Pi(K^-) = (m_T \cdot ChY + m_k) \cdot ChY / (m_0 \cdot Sh^2Y) \approx \text{ при } m_T > m_k$$

$$\approx m_T \cdot Ch^2Y / (m_0 \cdot Sh^2Y) = m_T / [m_0 \cdot (1 - 1/Ch^2Y)] \approx$$

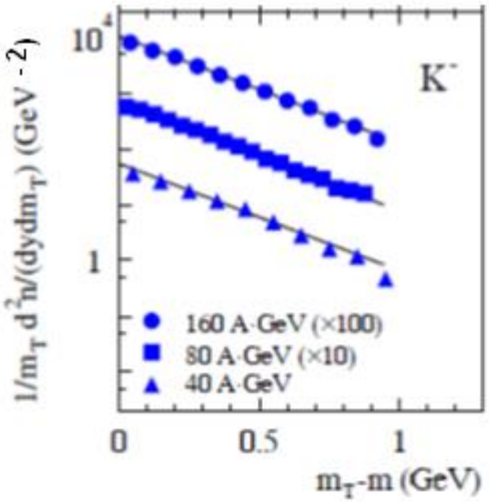
при $s > m_0$

$$\approx m_T / [m_0 \cdot (1 - 4m_0^2/s)].$$

$$\exp[-\Pi(K^-)/C_2] = \exp\{- m_T / [C_2 \cdot m_0 \cdot (1 - 4m_0^2/s)]\}$$

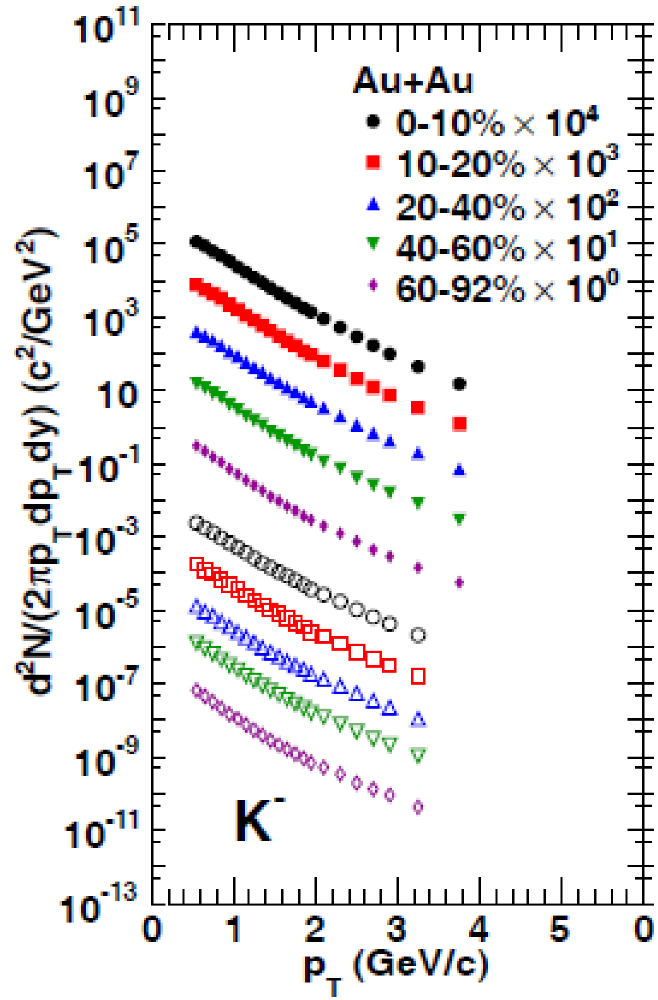
$$T = C_2 \cdot m_0 \cdot (1 - 4m_0^2/s)$$

S.V.Afanasiev et al. Phys. Rev. C66, 054902 (2002)



E_{lab} (AGeV)	$S^{1/2}$ (AGeV)	T
40	8.8	0.205
80	12.25	0.210
159	17.3	0.230

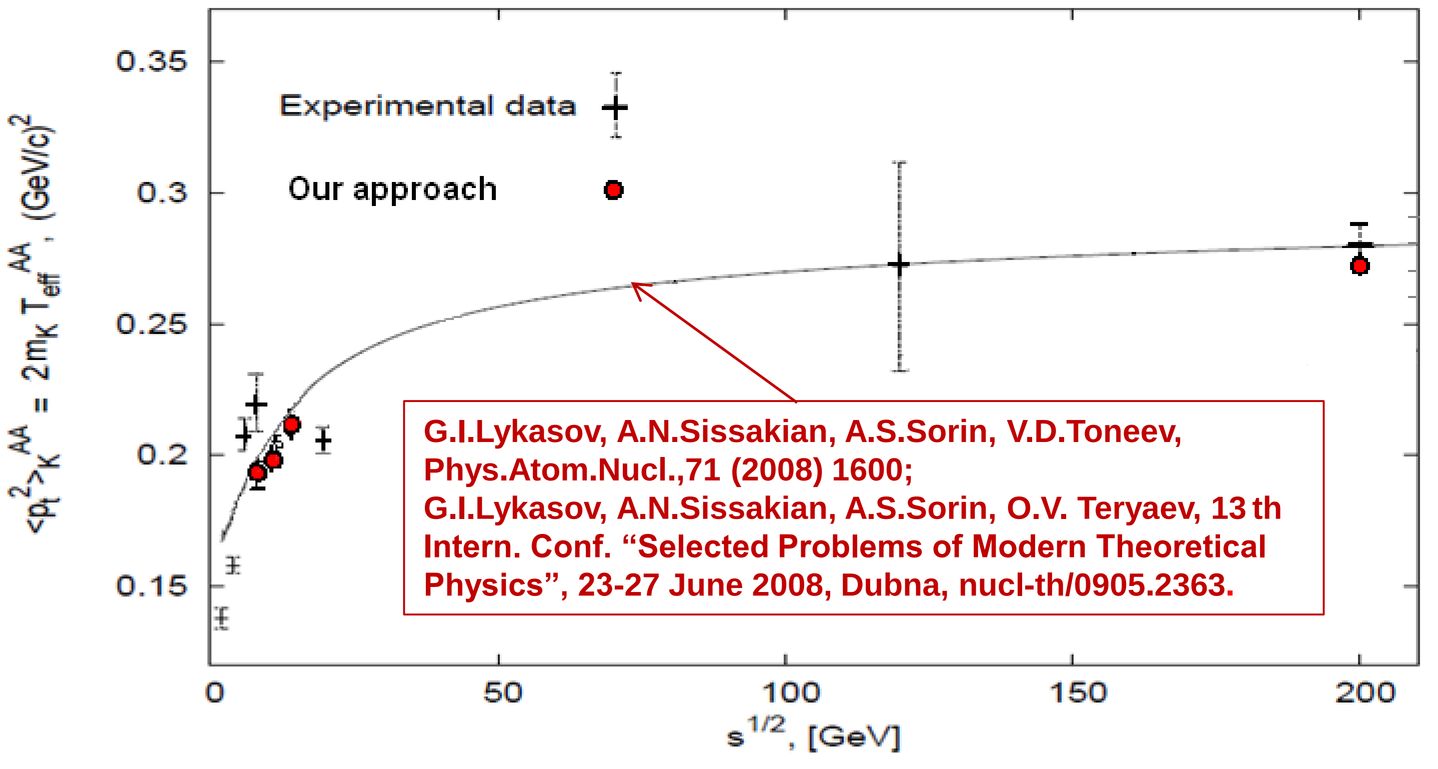
A.Adare et al. Phys.Rev. C88 (2013) 2, 024906



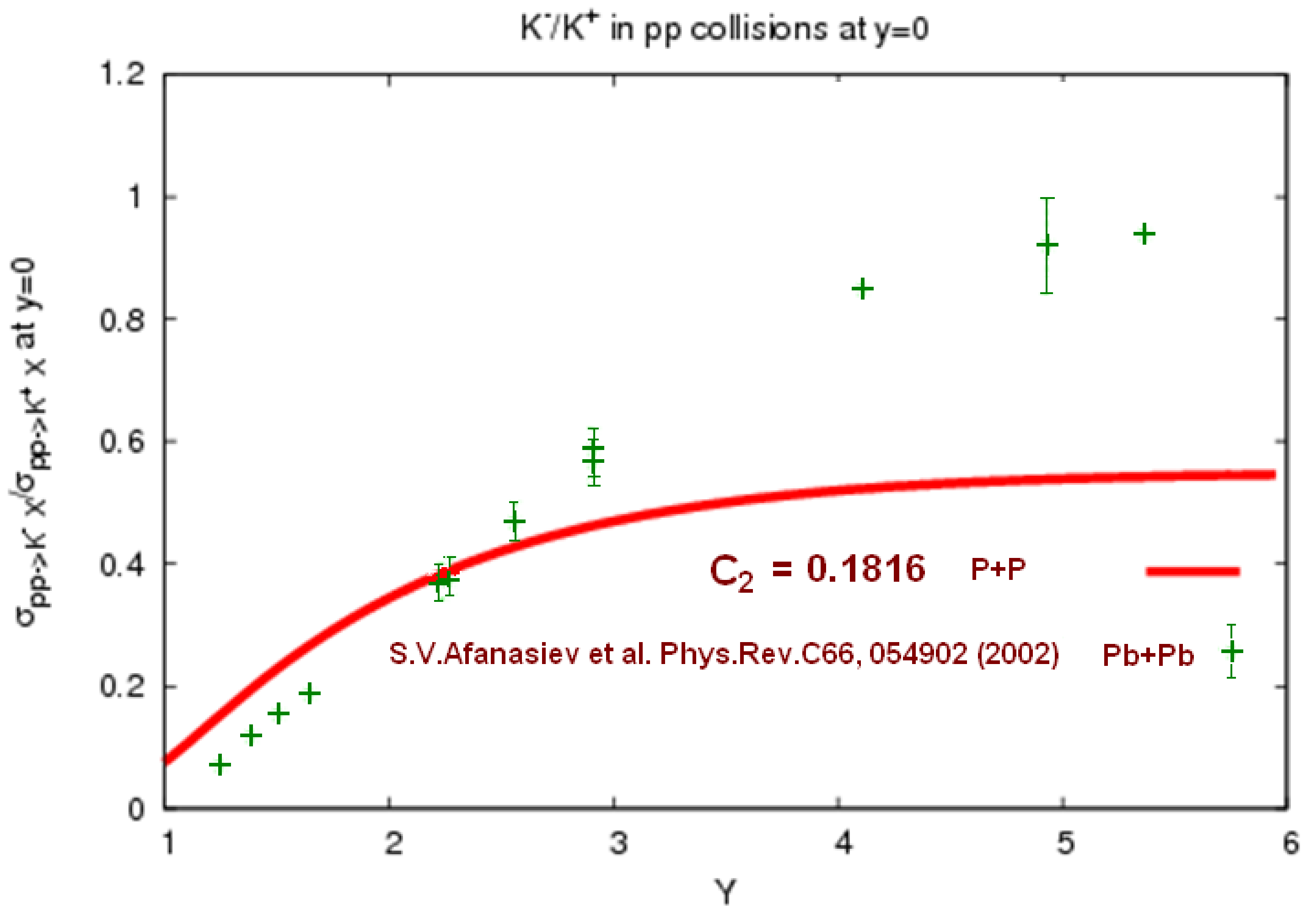
$S^{1/2}$ (AGeV)	T
200	0,290

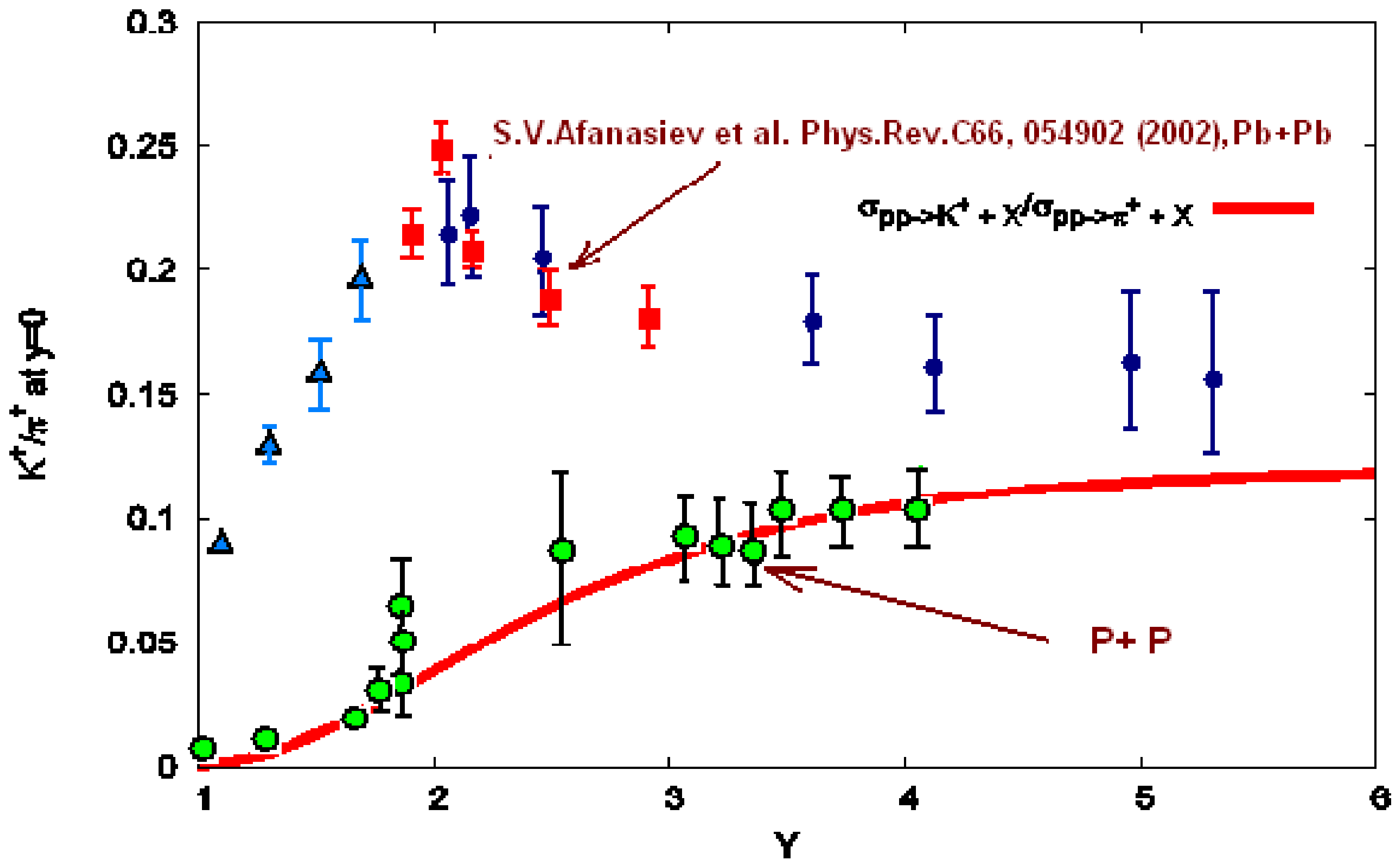
The mean transverse momentum squared of the K^- meson produced in the central A-A collisions.

$$\langle P_T^2\rangle = 2m_k T = 2m_k m_0 C_2(1-4m_0^2/s)$$

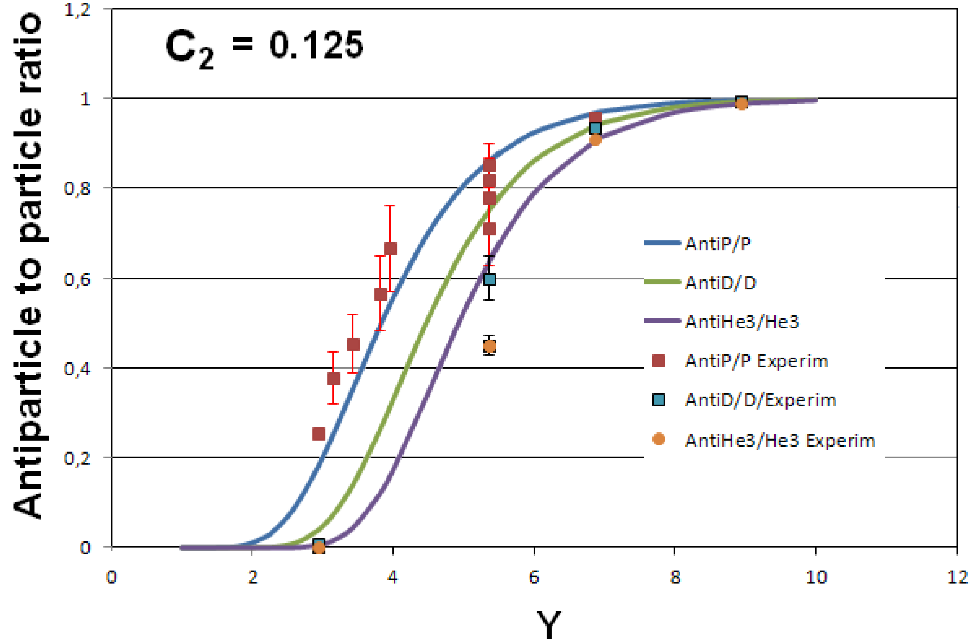


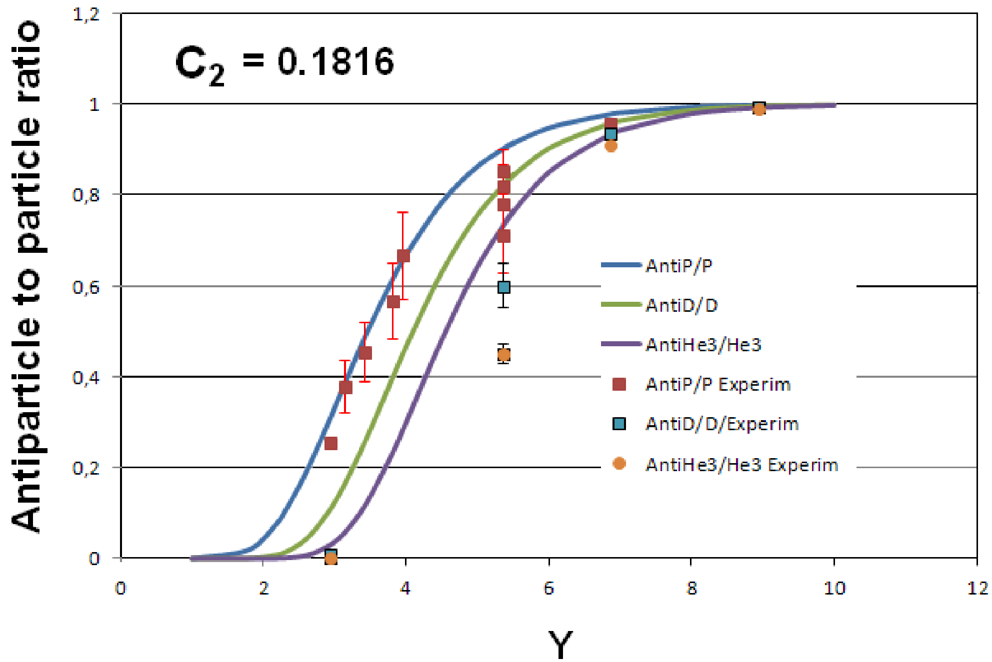
G.I.Lykasov, A.N.Sissakian, A.S.Sorin, V.D.Toneev, Phys.Atom.Nucl.,71 (2008) 1600;
G.I.Lykasov, A.N.Sissakian, A.S.Sorin, O.V. Teryaev, 13 th Intern. Conf. “Selected Problems of Modern Theoretical Physics”, 23-27 June 2008, Dubna, nucl-th/0905.2363.


K⁻/K⁺ in pp collisions at y=0
$\sigma_{pp\to K^- X}/\sigma_{pp\to K^+ X}$ at y=0
Y
1.2
1
0.8
0.6
0.4
0.2
0
1
2
3
4
5
6
C_2 = 0.1816
P+P
S.V.Afanasiev et al. Phys.Rev.C66, 054902 (2002)
Pb+Pb



C_2=0.1816 and our results are devided by a factor of 1.5


C2 = 0.125
Antiparticle to particle ratio
1,2
1
0,8
0,6
0,4
0,2
0
0
2
4
6
8
10
12
Y
AntiP/P
AntiD/D
AntiHe3/He3
AntiP/P Experim
AntiD/D/Experim
AntiHe3/He3 Experim


C2 = 0.1816
Antiparticle to particle ratio
1,2
1
0,8
0,6
0,4
0,2
0
0
2
4
6
8
10
12
Y
AntiP/P
AntiD/D
AntiHe3/He3
AntiP/P Experim
AntiD/D/Experim
AntiHe3/He3 Experim

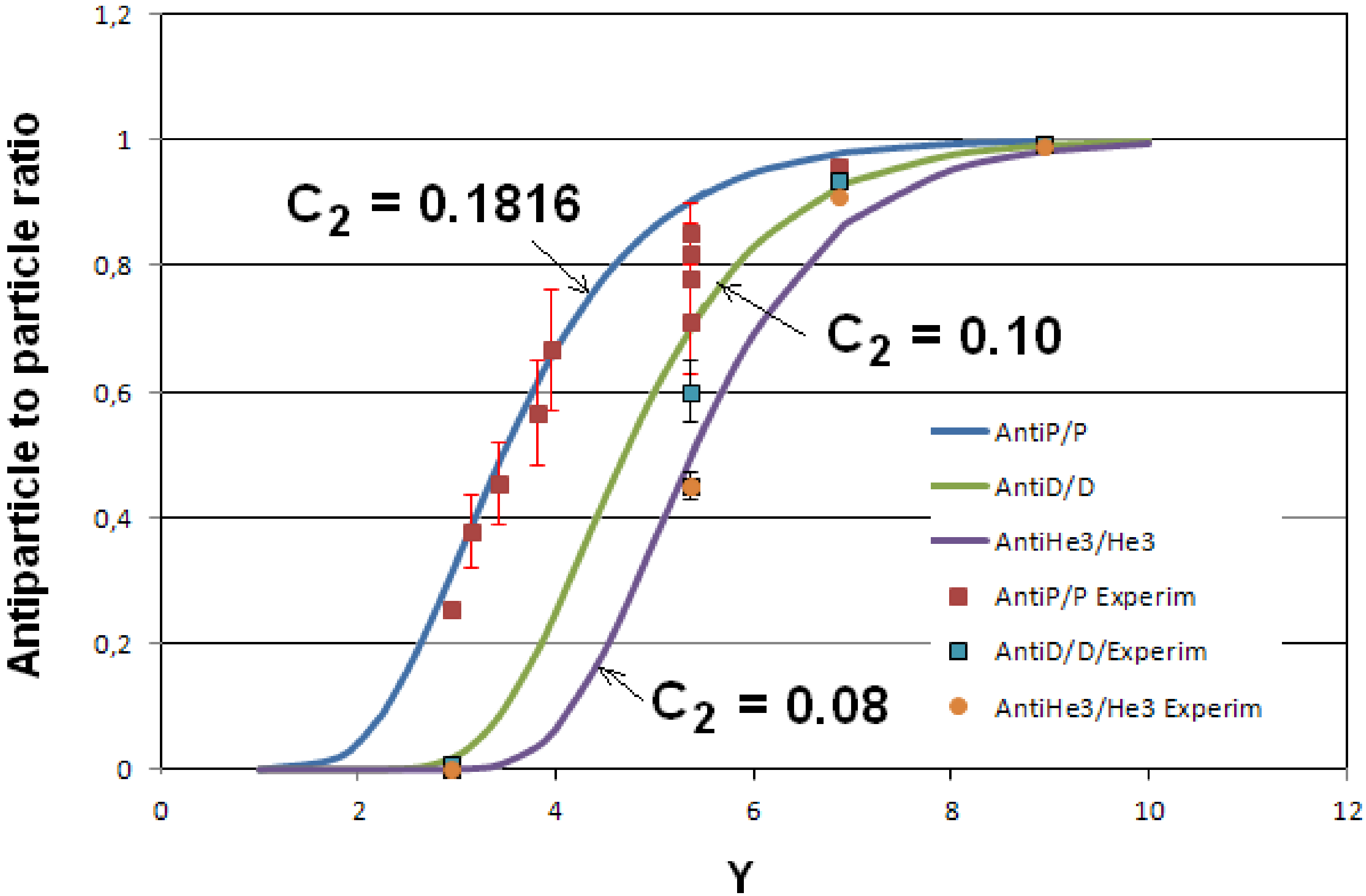
$$\Pi_2 = \left[\frac{m_{1T}}{m_0}\,chY + \frac{m_1}{m_0}\right]\frac{chY}{sh^2 Y} = a\cdot m_{1T} + b\,; \qquad \Pi_1 = \left[\frac{m_{1T}}{m_0}\,chY - \frac{m_1}{m_0}\right]\frac{chY}{sh^2 Y} = a\cdot m_{1T} - b$$

$$a = \frac{ch^2 Y}{sh^2 Y}\cdot\frac{1}{m_0} \qquad b = \frac{m_1}{m_0}\cdot\frac{chY}{sh^2 Y}$$

$$\text{Ratio}\left(\frac{\text{antinuclei}}{\text{nuclei}}\right) = \frac{\int_0^\infty m_{1T}\cdot C_1\cdot A_I^{\alpha(N_I)}\cdot A_{II}^{\alpha(N_{II})}\cdot \exp\left(-\frac{\Pi_2}{C_2}\right)\cdot dm_{1T}}{\int_0^\infty m_{1T}\cdot C_1\cdot A_I^{\alpha(N_I)}\cdot A_{II}^{\alpha(N_{II})}\cdot \exp\left(-\frac{\Pi_1}{C_2}\right)\cdot dm_{1T}} = \frac{\int_0^\infty m_{1T}\cdot \exp\left(-\frac{\Pi_2}{C_2}\right)\cdot dm_{1T}}{\int_0^\infty m_{1T}\cdot \exp\left(-\frac{\Pi_1}{C_2}\right)\cdot dm_{1T}} =$$

$$= \frac{\int_0^\infty m_{1T}\cdot \exp\left[-\frac{(a\cdot m_{1T}+b)}{C_2}\right]\cdot dm_{1T}}{\int_0^\infty m_{1T}\cdot \exp\left[-\frac{(a\cdot m_{1T}-b)}{C_2}\right]\cdot dm_{1T}} = \frac{\exp\left[-\frac{b}{C_2}\right]\cdot \int_0^\infty m_{1T}\cdot \exp\left[-\frac{a\cdot m_{1T}}{C_2}\right]\cdot dm_{1T}}{\exp\left[\frac{b}{C_2}\right]\cdot \int_0^\infty m_{1T}\cdot \exp\left[-\frac{a\cdot m_{1T}}{C_2}\right]\cdot dm_{1T}} = \frac{\exp\left[-\frac{b}{C_2}\right]}{\exp\left[\frac{b}{C_2}\right]} = \exp\left[-\frac{2b}{C_2}\right] =$$

$$= \exp\left(-\frac{2}{C_2}\cdot\frac{m_1}{m_0}\cdot\frac{chY}{sh^2 Y}\right)$$



1,2
1
0,8
0,6
0,4
0,2
0
0
2
4
6
8
10
12
Y
Antiparticle to particle ratio
C2 = 0.1816
C2 = 0.10
C2 = 0.08
AntiP/P
AntiD/D
AntiHe3/He3
AntiP/P Experim
AntiD/D/Experim
AntiHe3/He3 Experim