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on High Energy Physics Problems
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Hyperfragments from Light p-shell Hypernuclei

II. Recent Progress & next steps

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Outline

1. Introduction
   - Hypernuclei
   - Domain
2. Motivation
3. Approach
   - Shell model
   - Extensions
   - Cluster structure
4. Suggestions

Hypernuclei

A very interesting phenomenon in nuclear physics is
the existence of nuclei containing hyperon(Y)
(Λ-hyperon).
When hyperon replaces one of the nucleons(N=*n* or *p*)
in the nucleus,
the original nuclear structure can change
to a bound system formed by
the hyperon and remaining nucleons.

Hypernuclei in Dubna

Podgoretski's JETP 17 ('63) ingenious idea
to use the Strangeness Exchange Reaction $(K^-_{\mathrm{in-flight}}, \pi^-)$

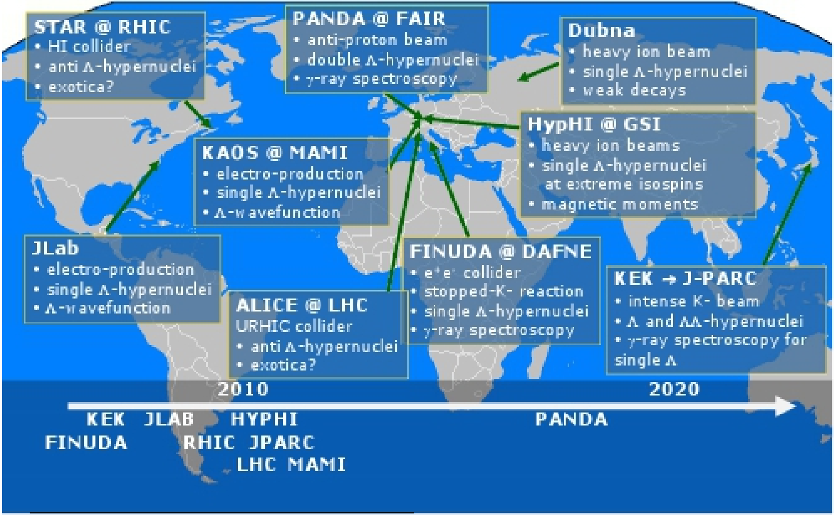
Now hypernuclei are
part of intermediate energy nuclear physics
(interactions of μ, π, $K \ldots$ with light nuclei)

Khorozov & Lukstins's NP A 547 ('92) unique
experiment: production of relativistic hypernuclei

Lukstins PAN **71** (2008) 2137
Studying of Hypernuclei in Nuclotron Beams
identification ${}^{6}_{\Lambda}$H; ${}^{10}_{\Lambda}$B, ${}^{10}_{\Lambda}$Be

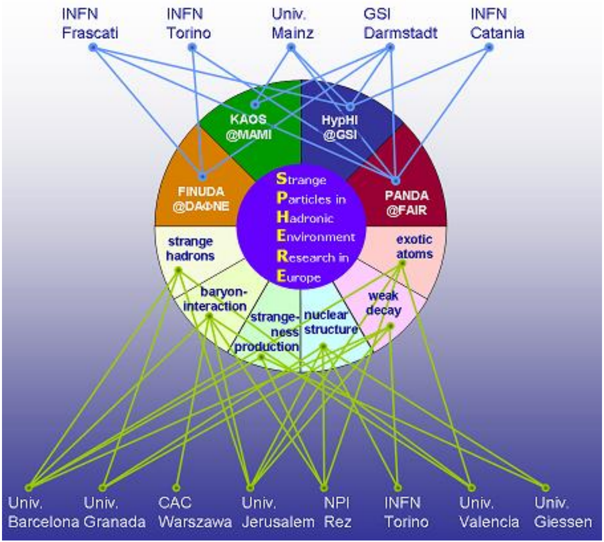
NICA Program Kekelidze's Monday talk

International Hypernuclear Network [1]



[1] Pochodzalla, Zakopane 2010

Sphere Network



SPHERE Network

Hadron Physics$_2$ Study of Strongly Interacting Matter

SPHERE Network :

Strange **P**articles in **H**adronic **E**nvironment **R**esearch in **E**urope

coordinates studies of hypernuclei at

FINUDA@DAΦNE (Frascati)

KAOS@MAMI (Mainz)

HypHI@GSI (Darmstadt)

$\bar{\mathrm{P}}$ANDA@FAIR

extended by including

J-PARC@Tokai and CEBAF@J-Lab

close cooperation with Network LEANNIS :

Low **E**nergy **A**ntikaon **N**ucleon and **N**uclei **I**nteraction **S**tudies

Motivation

Experiment : Mainz Microtron **MAMI-C** :

- pionic decay of ${}^{4}_{\Lambda}\mathrm{H}$ in **electroproduction** process
- the possibility to identify **hyperfragments** in the reaction $(e, e' K^+)$:

$$e + {}^{9}\mathrm{Be} \to {}^{9}_{\Lambda}\mathrm{Li} + K^+$$

$$(e + p \to \Lambda + K^+)$$

where Λ may be bound/unbound;
unbound $\to$ pick up few nucleons and forms hyperfragment

Cluster structure of ${}^{9}\mathrm{Be} \stackrel{?}{\Rightarrow}$ **hyperfragment** ${}^{4}_{\Lambda}\mathrm{H}$
was **really** recognized

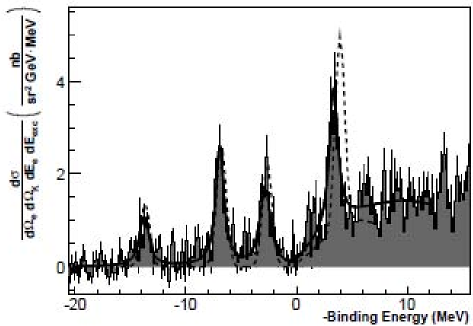
Is it possible to estimate yield of other hyperfragments?

"Particle-hole" excitation : Shell model

Example : closed shell nuclei

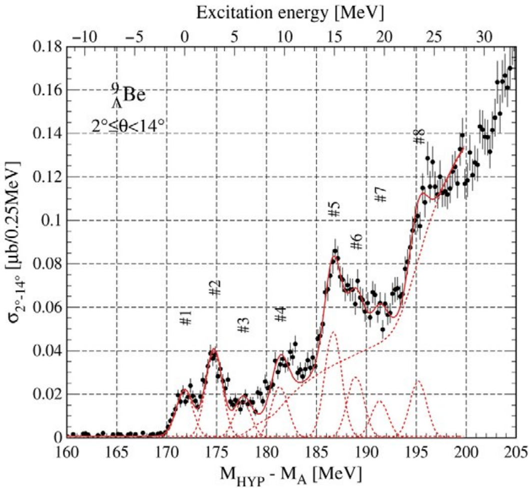
$^{16}_{\Lambda}$N : F. Cusanno *et al.*, PRL **103**, 202501 (2009)

$p_{\frac{1}{2}}^{-1}$ s_Λ $p_{\frac{3}{2}}^{-1}$ s_Λ $p_{\frac{1}{2}}^{-1}$ p_Λ $p_{\frac{3}{2}}^{-1}$ p_Λ

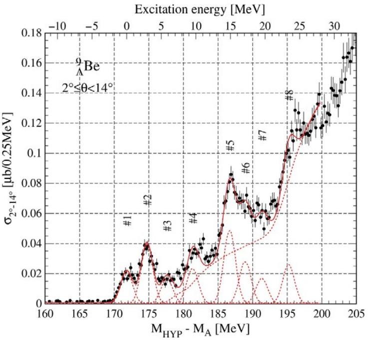


Example : Spectrum of $^{9}_{\Lambda}$Be

Spectrum of $^{9}_{\Lambda}$Be measured using the (π^+, K^+) reaction.
From O. Hashimoto and H. Tamura



Deciphering spectrum of $^{9}_{\Lambda}$Be



. p^{-1} s_Λ p_Λ

. p^{-1} s_Λ p_Λ

. s^{-1} s_Λ s_Λ

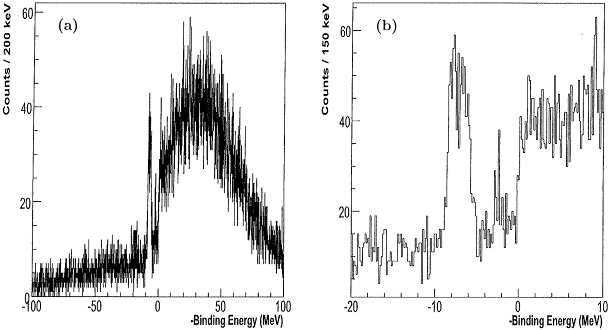
^{8}Be :

J^π ; T	E_{ex}	shell model		"hole"
0^+; 0	0.0	s^4p^4 [44]	S	p^{-1}
2^+; 0	3.0	s^4p^4 [44]	D	p^{-1}
. . .				
2^+; 0+1	16.7	s^4p^4 [431]	D	p^{-1}
. . .				
2^- ; 0	18.9	s^3 p^5[44]	P	s^{-1}
. . .				
$(1,2)^-$; 1	24.0	s^3p^5 [431]	P	s^{-1}

No		l_n^{-1} l_Λ
1,2		p^{-1} s_Λ
3,4		p^{-1} p_Λ
5	}	p^{-1} s_Λ
6		s^{-1} s_Λ
7		p^{-1} p_Λ
8		s^{-1} s_Λ

"Our" hypernucleus: $^{9}\mathrm{Be}(e, e' K^{+})^{9}_{\Lambda}\mathrm{Li}$

arXiv:1405.5839



- the whole energy range — the region of interest

E_{thr}	1.2	3.3	4.5	5.3	9.7	23	30
	$^{6}_{\Lambda}\mathrm{He}$	$^{4}_{\Lambda}\mathrm{H}$	$^{7}_{\Lambda}\mathrm{He}$	$^{8}_{\Lambda}\mathrm{He}$	$^{3}_{\Lambda}\mathrm{H}$	$^{4}_{\Lambda}\mathrm{He}$	$^{6}_{\Lambda}\mathrm{H}$

Shell Model - 1

Seminal papers:

Gal, Soper, Dalitz, AP **63** (1971), **72** (1972) **113** (1978)
$\Psi({}^{A+1}_{\Lambda}Z) = \Psi({}^{A}Z) \cdot \psi^{\Lambda}$ "weak coupling"

for *p*-shell hypernuclei:
$\underbrace{|s^4\ p^{A-4} : J_A^{\pi} T_A}_{\textit{Cohen Kurath,}} \quad \otimes \quad s_{\Lambda} : J >$

Discrete part of spectra ($0\hbar\omega$ excitations) :
Millener, Lecture Notes in Physics, vol. 724, 2007, Springer

Shell Model 1

REMINDER 1:

L S coupling

$$|p^k : ETJ > = \sum \alpha^{ETJ}_{fLS} \, |p^k : [f]T(S\ L)J >$$

$[f]$: permutation group

α - intermediate coupling:

. Cohen & Kurath; Barker; Boyarkina; ...

$$|p^k : [f]TSL > = |p^k : [f](\lambda\mu)L > \cdot |(\underbrace{\frac{1}{2}\frac{1}{2}}_{\tau})^k : [\bar{f}]TS >$$

Shell Model 1

REMINDER 2:

Fractional parentage decomposition - one particle

$$|p^k : [f](\lambda\mu)TSL >=$$

$$\sum \sqrt{\frac{nf'}{nf}}\, g_\ell\, g_{ST} \cdot |p^{k-1} : [f'](\lambda\mu)'T'S'L' > \cdot |p >$$

$$g_\ell \equiv$$

$$< p^{k-1}(\lambda\mu)'L'; p|p^k(\lambda\mu)L >= \left(\begin{array}{cc|c} (\lambda\mu)' & (10) & (\lambda\mu) \\ L' & 1 & L \end{array} \right)$$

$$g_{ST} \equiv$$

$$< \tau^{k-1}[\bar{f}]'T'S'; \tau|\tau^k[\bar{f}]TS >= \left(\begin{array}{cc|c} [\bar{f}'] & [\bar{1}] & [\bar{f}] \\ T'S' & \frac{1}{2}\,\frac{1}{2} & TS \end{array} \right)$$

SU(3), SU(4) Clebsch Gordan Coefficients

Shell Model - 2

$$\text{EXCITED nuclear states :}$$

$$\text{no core shell model}$$

$$|0s^{\,ks}\,[f_s];\quad 1p^{\,kp}\,[f_p];\quad \ell^{\,k\ell}\,[f_\ell] : \quad [f]\,TS \quad (\lambda\mu)L \quad : J >$$

$$SU(4) \quad SU(3)$$

$$\textit{Wigner} \quad \textit{Elliott}\ ('58)$$

$$\text{constrain :}\quad ks + kp + k\ell = A$$

$$\text{notation :}\quad \ell = 2d, 2s; \quad 3f, 3p; \ldots$$

Shell Model - 2

TRANSLATION INVARIANT SM

First step - rewriting

$$|p^{\,kp}[f_p]: \quad [f_p]LST >$$

$$|s^4[4]; \quad p^{\,kp}[f_p]: \quad [f_A]=[4f_p], LST > \quad \cdot \quad \underset{\mathrm{CM}}{\Psi_0(R_A)}$$

Young tableaux [f]

Wave functions are classified with Young tableaux [f]

Incorporating clusters in shell model:

Cluster structure – labeled by Young tableaux:

d: s^2 [2] $({}^3_\Lambda\mathrm{H}: \; |s^2\, s_\Lambda)$

t: s^3 [3] $({}^4_\Lambda\mathrm{H}: \; |s^3\, s_\Lambda)$

α: s^4 [4]

Excited states of p-shell nuclei

$0\hbar\omega_N \ (k)$:

$$|0s^4[4]\ 1p^k[f_p] \qquad : [f_A]\ (\lambda\mu) > \quad = \quad \Phi_k^{(A)}[f_A](\lambda\mu)\cdot\Psi_0(R_A)$$

$$[f_A] = [4f_p]$$

$1\hbar\omega_N \ (k+1)$:

$$|0s^4[4]\ 1p^{k-1}[f_p] \quad 2d\ [1] \qquad : [f_A]\ (\lambda\mu) > \qquad \Phi_k^{(A)}[f_A](\lambda\mu)\cdot\Psi_1(R_A)$$

$$|0s^3[3]\ 1p^{k+1}[f'_p] \qquad : [f_A]\ (\lambda\mu) > \qquad \Phi_{k+1}^{(A)}[f_A](\lambda\mu)\cdot\Psi_0(R_A)$$

$$[f_A] = [4f_p],\ [3f'_p]$$

$2\hbar\omega_N \ (k+2)$:

$$|0s^4[4]\ 1p^{k-1}[f_p] \quad 3f\ [1] \qquad : [f_A]\ (\lambda\mu) > \qquad \Phi_k^{(A)}[f_A](\lambda\mu)\cdot\Psi_2(R_A)$$

$$|0s^4[4]\ 1p^{k-2}[f'_p] \quad 2d^2[f_d] \qquad : [f_A]\ (\lambda\mu) > \qquad \Phi_k^{(A)}[f_A](\lambda\mu)\cdot\Psi_2(R_A)$$

$$|0s^3[3]\ 1p^{k}\ [f''_p] \quad 2d\ [1] \qquad : [f_A]\ (\lambda\mu) > \qquad \Phi_{k+1}^{(A)}[f_A](\lambda\mu)\cdot\Psi_1(R_A)$$

$$|0s^2[2]\ 1p^{k+2}[f'''_p] \qquad : [f_A]\ (\lambda\mu) > \qquad \Phi_{k+2}^{(A)}[f_A](\lambda\mu)\cdot\Psi_0(R_A)$$

$$[f_A] = [4f_p],\ [3f'_p],\ [2f''_p]$$

^{4}He

Due to NON-CENTRAL forces,
the w.f. for the ^{4}He $J^\pi = 0^+$ g.s.
is a mixture of $^1S_0$, $^3P_0$ and $^5D_0$ states

Recent calculations:

ref.	interaction	$^{11}S_0$	$^{13}P_0$	$^{15}D_0$
[1]	AV8':	85.7 %	0.4 %	13.9 %
[2]	AV18 + UIX:	83.2 %	0.8 %	16.0 %

[1] K. Kamada *et al.* PR C 64, 044001 ('01)
Benchmark test calculation of a 4N bound state
[2] A. Nogga *et al.*: PR C 65, 054003 ('02)
. α *particle based on modern forces*

in SHELL MODEL:

$$^{15}D_0 : |0s^2\, 1p^2 : [22] >$$

Excited states of p-shell HYPERNUCLEI:

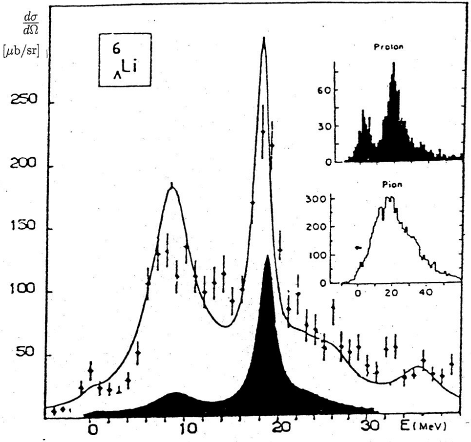
$\hbar\omega$	: $\hbar\omega_k + \hbar\omega_\Lambda$		
0	$0+0$	$\Phi_k^{(A)} \cdot \varphi_0^\Lambda(R_A - r_\Lambda)$	$(\varphi_0^\Lambda \equiv 0s_{\frac{1}{2}}^\Lambda)$
1	$0+1$	$\Phi_k^{(A)} \cdot \varphi_1^\Lambda$	$(\varphi_1^\Lambda \equiv 1p_{\frac{3}{2}}^\Lambda,\ 1p_{\frac{1}{2}}^\Lambda)$
	$1+0$	$\Phi_{k+1}^{(A)} \cdot \varphi_0^\Lambda$	
2	$0+2$	$\Phi_k^{(A)} \cdot \varphi_2^\Lambda$	$(\varphi_2^\Lambda \equiv 2d_{\frac{5}{2}}^\Lambda,\ 2d_{\frac{3}{2}}^\Lambda,\ 2s_{\frac{1}{2}}^\Lambda)$
	$1+1$	$\Phi_{k+1}^{(A)} \cdot \varphi_1^\Lambda$	
	$2+0$	$\Phi_{k+2}^{(A)} \cdot \varphi_0^\Lambda$	

Example : structure $^{7}_{\Lambda}$He

N	^{6}He	$\Phi^{(6)}_{N}$	$[f_6]$			$^{7}_{\Lambda}$He		
2	$s^4\ p^2$	$\Phi^{(6)}_{2}$	[42], ...			s_Λ	p_Λ	d_Λ
3	$s^4\ p\ d$	$\Phi^{(6)}_{3}$	[42], ...				s_Λ	p_Λ
3	$s^3\ p^3$	$\Phi^{(6)}_{3}$	[42], ...	[33], ...			s_Λ	p_Λ
4	$s^4\ p\ f$	$\Phi^{(6)}_{4}$	[42], ...					s_Λ
4	$s^4\ d^2$	$\Phi^{(6)}_{4}$	[42], ...					s_Λ
4	$s^3\ p^2\ d$	$\Phi^{(6)}_{4}$	[42], ...	[33], ...				s_Λ
4	$s^2\ p^4$	$\Phi^{(6)}_{4}$	[42], ...	[33], ...	[222]			s_Λ

Spectrum of ${}^{6}_{\Lambda}$Li

deciphering:	$p^{-1}s_\Lambda$	$p^{-1}p_\Lambda$	$s^{-1}s_\Lambda$
.	[41]	[41]	[32]



${}^{6}_{\Lambda}$Li - discussion

In Phys. Lett. B **92** 256 (1980)
we have discussed hypernucleus ${}^{6}_{\Lambda}$Li in terms of an

extended supermultiplet scheme

which combines the 1*s* and 1*p* orbitals, and classifies the nuclear and Λ- hypernuclear states by Young tableaux $[f]$.

The lower state has symmetry [41] for its nuclear core, so its break-up to ${}^{5}_{\Lambda}\mathrm{He}+p$ or to ${}^{5}\mathrm{Li}+\Lambda$ is ALLOWED both energetically and by supermultiplet symmetry.

The upper state has symmetry [32] for its nuclear core so that its decay to ${}^{5}\mathrm{Li}+\Lambda$ or ${}^{5}_{\Lambda}\mathrm{He}+p$, is FORBIDDEN by the selection rules for the supermultiplet symmetry.

ALLOWED : ${}^{4}_{\Lambda}\mathrm{H}+2p$, ${}^{4}_{\Lambda}\mathrm{He}+d$, ${}^{3}_{\Lambda}\mathrm{H}+t$

Cluster wave function in Shell Model

A = a + b

$$\Phi_N^{(A)}[f](\lambda\mu)\ LST =$$

$$\sum \sqrt{\frac{n_{f'} \cdot n_{f''}}{n_f}}\ GL^{(\lambda\mu)}_{(\lambda\mu)',(\lambda\mu)''}\ GT^{[f]}_{[f'],[f'']}$$

$$\phi^{(a)}_{n'}[f'](\lambda\mu)' \ \cdot\ \phi^{(b)}_{n''}[f''](\lambda\mu)'' \ \cdot\ \varphi_\nu(r_{a,b})$$

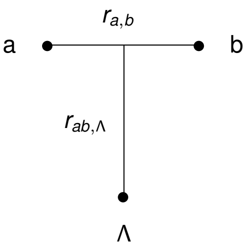
$GL, (GT)$ - orbital (spin-isospin) c.f.p. =
recoupling coeffcients for SU(3) and SU(4) group

with obvious constrains :

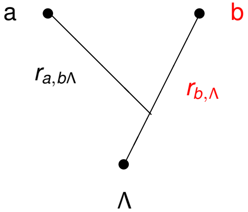
$$\begin{aligned} n' + n'' + \nu &= N \\ [f'] + [f''] &= [f] \\ (\lambda\mu)' + (\lambda\mu)'' + (\nu 0) &= (\lambda\mu) \end{aligned}$$

Hyper FRAGMENTS

T :



Y :



Transformation of Jacobi coordinates:

$$< \varphi_\nu(r_{ab})\, \varphi_0^\Lambda(r_{ab,\Lambda}) \mid \varphi_0(r_{b,\Lambda})\, \varphi_\nu(r_{a,b\Lambda}) >= \left(\frac{m_A+1}{m_A}\, \frac{m_b}{m_b+1}\right)^{\frac{\nu}{2}}$$

Hyperfragments from ${}^{7}_{\Lambda}$He

E th			$[f_c]$
2.82	${}^{6}_{\Lambda}\mathrm{He}$	$+\,n$	[41]
3.08	${}^{5}_{\Lambda}\mathrm{He}$	$+\,2n$	[4]
5.23	${}^{6}\mathrm{He}$	$+\,\Lambda$	[42]
6.95	${}^{5}\mathrm{He}$	$+\,\Lambda n$	[41]
15.49	${}^{4}_{\Lambda}\mathbf{H}$	$+\,t$	[**3**]
21.41	${}^{4}_{\Lambda}\mathrm{He}$	$+\,3n$	[3]
23.66	${}^{3}_{\Lambda}\mathbf{H}$	$+\,t\ n$	[**2**]
23.81	${}^{6}_{\Lambda}\mathbf{H}$	$+\,p$	[**32**]

Hyperfragments from $^{9}_{\Lambda}$Li

E_{thr}	decay			$[f_i][f_k]$	T_1 T_2
3.7	n	+	$^{8}_{\Lambda}$Li	[43][1]	$\frac{1}{2}$ $\frac{1}{2}$
8.5	^{8}Li	+	Λ	[431]	1 0
9.7	t	+	$^{6}_{\Lambda}$He	[3][41]	$\frac{1}{2}$ $\frac{1}{2}$
			$\downarrow$		
9.9	tn	+	$^{5}_{\Lambda}$He	[3][1][4]	$\frac{1}{2}$ $\frac{1}{2}$ 0
11.8	^{5}He	+	$^{4}_{\Lambda}\mathbf{H}$	[41][**3**]	$\frac{1}{2}$ $\frac{1}{2}$
12.2	$2n$	+	$^{7}_{\Lambda}$Li	[2][42]	1 0
13.0	d	+	$^{7}_{\Lambda}$He	[2][42]	0 1
13.8	p	+	$^{8}_{\Lambda}$He	[1][421]	$\frac{1}{2}$ $\frac{3}{2}$
18.2	^{6}He	+	$^{3}_{\Lambda}\mathbf{H}$	[42][**2**]	1 0
31.5	tnn	+	$^{4}_{\Lambda}$He	[3][1][1][3]	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
38.5	^{3}He	+	$^{6}_{\Lambda}\mathbf{H}$	**[3][32]**	$\frac{1}{2}$ $\frac{3}{2}$

SUGGESTIONS & OUTLOOK

Some SUGGESTIONS for next steps:

- see for ${}^{4}_{\Lambda}$H in other *p*-shell targets;
- see for ${}^{3}_{\Lambda}$H in *p*-shell targets;
- ^{7}Li is a source of ${}^{6}_{\Lambda}$H hyperfragment.

OUTLOOK

Calculation of ratio

$$Y({}^{6}_{\Lambda}\mathrm{H}) \; : \; Y({}^{3}_{\Lambda}\mathrm{H}) \; : \; Y({}^{4}_{\Lambda}\mathrm{H})$$

THANK YOU!