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FIELD-THEORETICAL ELECTRON-PROTON SCATTERING AMPLITUDE IN THE COULOMB AND LORENTZ GAUGES

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CONTENT

[•] 3D field-theoretical relativistic equations for the ep system with the on mass shell protons.

[••] Analytic and exact reproduction of the One Photon Exchange leading ep potential through the equal-time canonical commutation relations.

[• • •] Analytic and exact relationship between the 3D field-theoretical relativistic equations in the Coulomb and Lorentz gauges with the same quantization rules.

[• • ••] Haag-Nishijima-Zimmermann quantum field theory of the hadrons as quark bound (composite) states.
Suggested field equations with on mass shell protons for ep scattering with and without quark-gluon degrees of freedom.

[⊙] Conclusion

Standard 3D time-ordered field theoretical $\mathcal{S}$-matrix approach

$$\mathcal{S}_{e'N',eN} =< in; \mathbf{p'}_e, \mathbf{p'}_N|\mathbf{p}_e, \mathbf{p}_N; in >$$
$$+i(2\pi)^4\delta(p_e + p_N - p'_e - p'_N)\mathcal{A}_{e'N',eN}$$

$$\mathcal{A}_{e'N',eN} =< out; \mathbf{p'}_N|\eta_{\mathbf{p'_e}}(0)|\mathbf{p}_e, \mathbf{p}_N; in >$$

Both protons and e are on mass shell in $\mathcal{A}_{e'N',eN}$.

$$\eta_{\mathbf{p'_e}}(x) = \overline{u}(\mathbf{p'}_e)\eta(x)$$

$$b^+_{\mathbf{p}_e}(in) = b^+_{\mathbf{p}_e}(0) - \int d^4x e^{ip'_e x}\theta(-x_o)\overline{\eta}(x)u(\mathbf{p}_e)$$

$$\mathcal{A}_{e'N',eN} =< \mathbf{p'}_N|\{\eta_{\mathbf{p'_e}}(0), b^+_{\mathbf{p_e}}(0)\}|\mathbf{p}_N > -$$
$$i\int d^4x e^{-ip_e x}\theta(-x_o) < \mathbf{p'}_N|\{\eta_{\mathbf{p'_e}}(0), \overline{\eta}_{\mathbf{p_e}}(x))\}|\mathbf{p}_N >$$

$$\mathcal{A}_{e'N',eN} =< \mathbf{p'}_N|\{\eta_{\mathbf{p'_e}}(0), b^+_{\mathbf{p_e}}(0)\}|\mathbf{p}_N > -$$

$$i\int d^4xe^{-ip_ex}\theta(-x_o) < \mathbf{p'}_N|\{\eta_{\mathbf{p'_e}}(0), \overline{\eta}_{\mathbf{p_e}}(x))\}|\mathbf{p}_N >$$

Both protons are on mass shell in $\mathcal{A}_{e'N',eN}$.

$$\sum_n |n; in >< in; n| = 1$$

$$\mathcal{A}_{e'N',eN} =< \mathbf{p'}_N|\{\eta_{\mathbf{p'_e}}(0), b^+_{\mathbf{p_e}}(0)\}|\mathbf{p}_N >$$

$$+ \sum_n \mathcal{A}_{e'N',n} \frac{(2\pi)^3\delta(\mathbf{P_n} - \mathbf{p_e} - \mathbf{p_N})}{E_{\mathbf{p_e}} + E_{\mathbf{p_N}} - P^o_n + i\epsilon} \mathcal{A}^+_{n,eN}$$

$$+ \sum_m \mathcal{A}_{N',em} \frac{(2\pi)^3\delta(\mathbf{P_m} + \mathbf{p_e} - \mathbf{p'_N})}{E_{\mathbf{p_e}} - E_{\mathbf{p'_N}} + P^o_m} \mathcal{A}^+_{N,e'm}$$

$$\mathcal{A}_{e'N',n} =< \mathbf{p'}_N|\eta_{\mathbf{p'_e}}(0)|n; in >; \quad n = H, ep, ep\gamma, ...$$

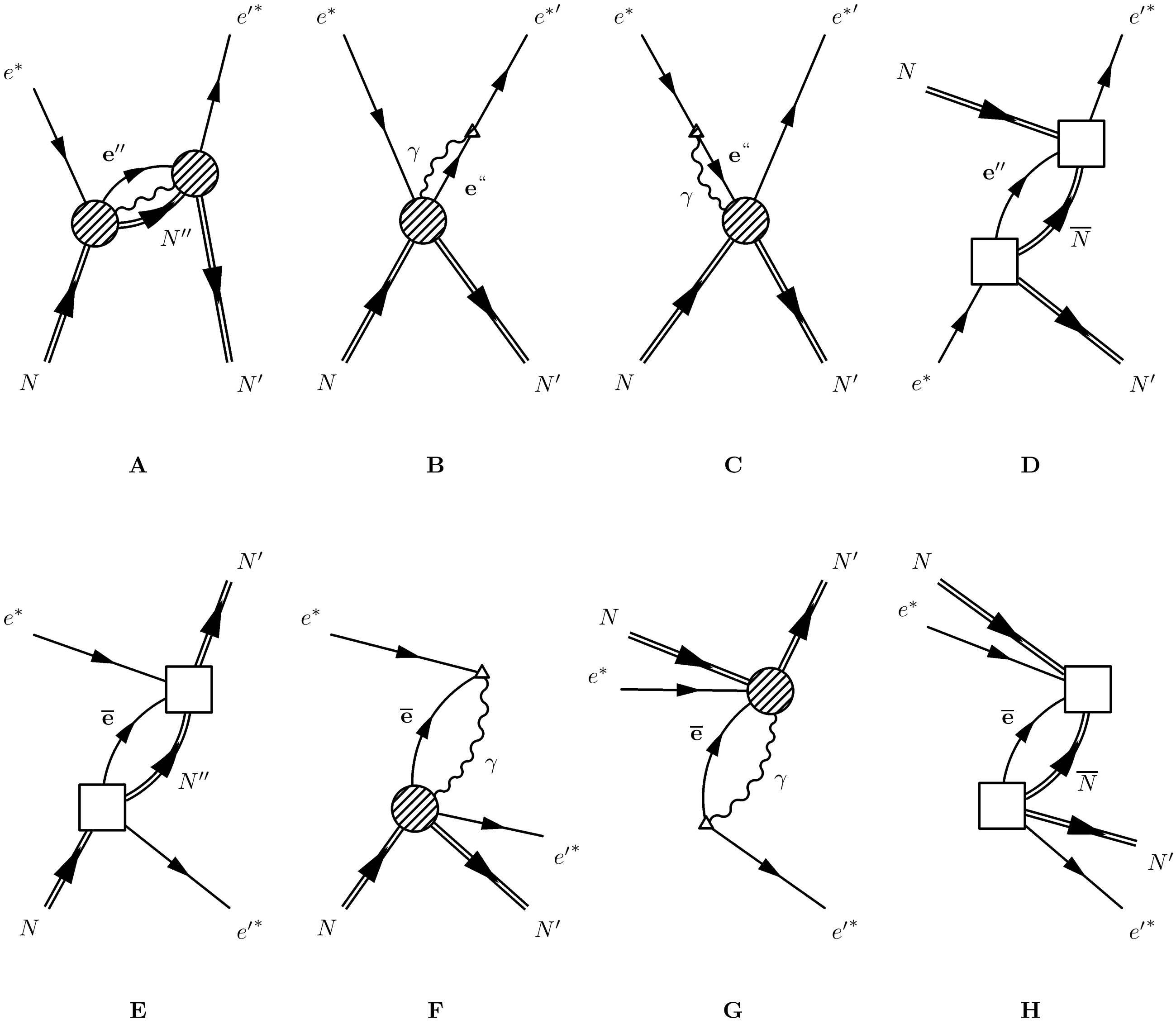
$$\mathcal{A}^+_{N,e'm} =< in; m|\eta_{\mathbf{p'_e}}(0)|\mathbf{p}_N; in >; \quad m = \overline{e}p, \overline{e}\gamma p...$$

$$E_{\mathbf{p'_e}} = \sqrt{m_e^2 + \mathbf{p'}^2_e}; \qquad E_{\mathbf{p'_N}} = \sqrt{M^2 + \mathbf{p'}^2_e}$$

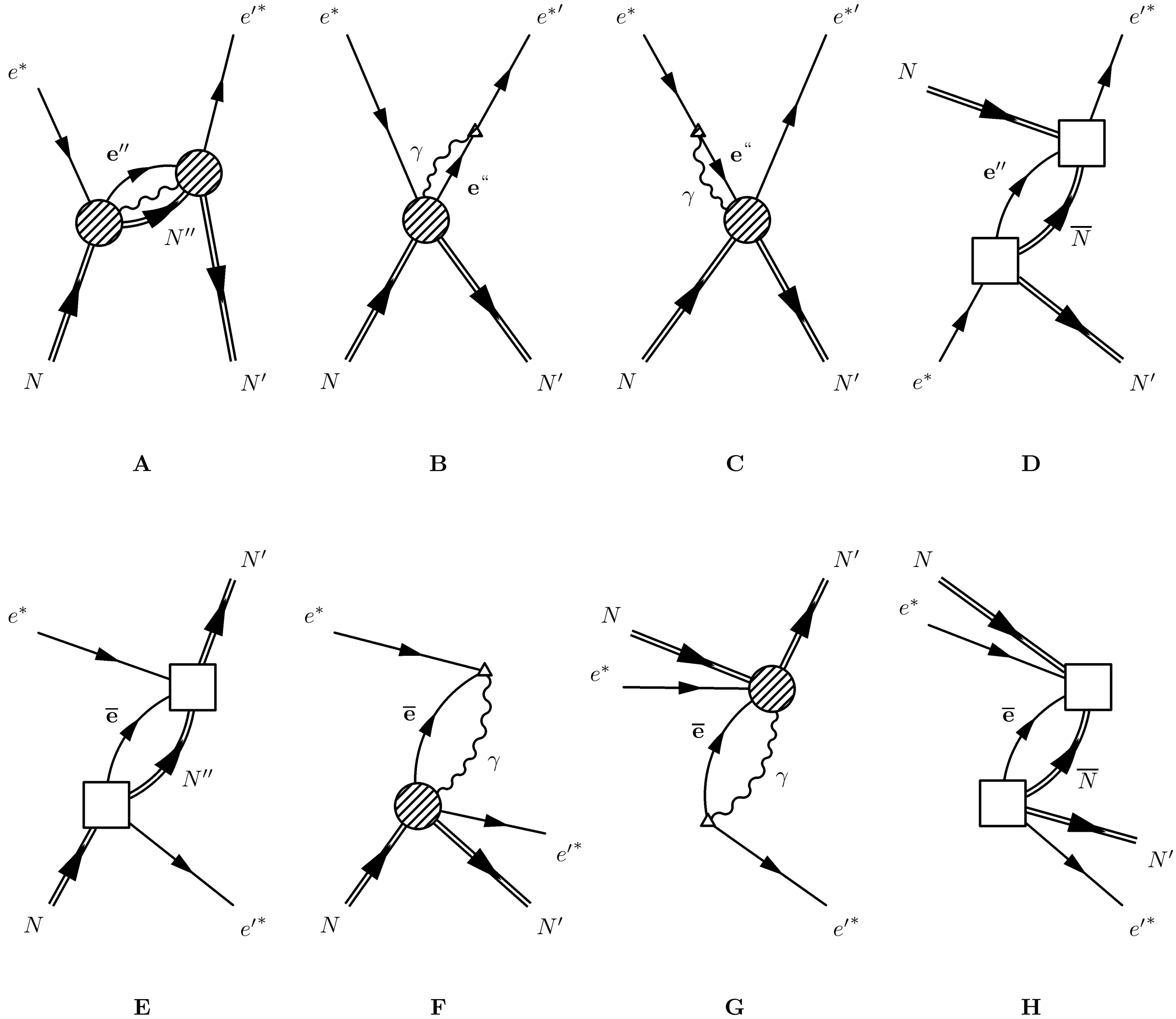
- General field theoretical completeness condition for $ep \to ep$ with any number of the intermediate particles n.

- Generalized Low type equations for the ep scattering

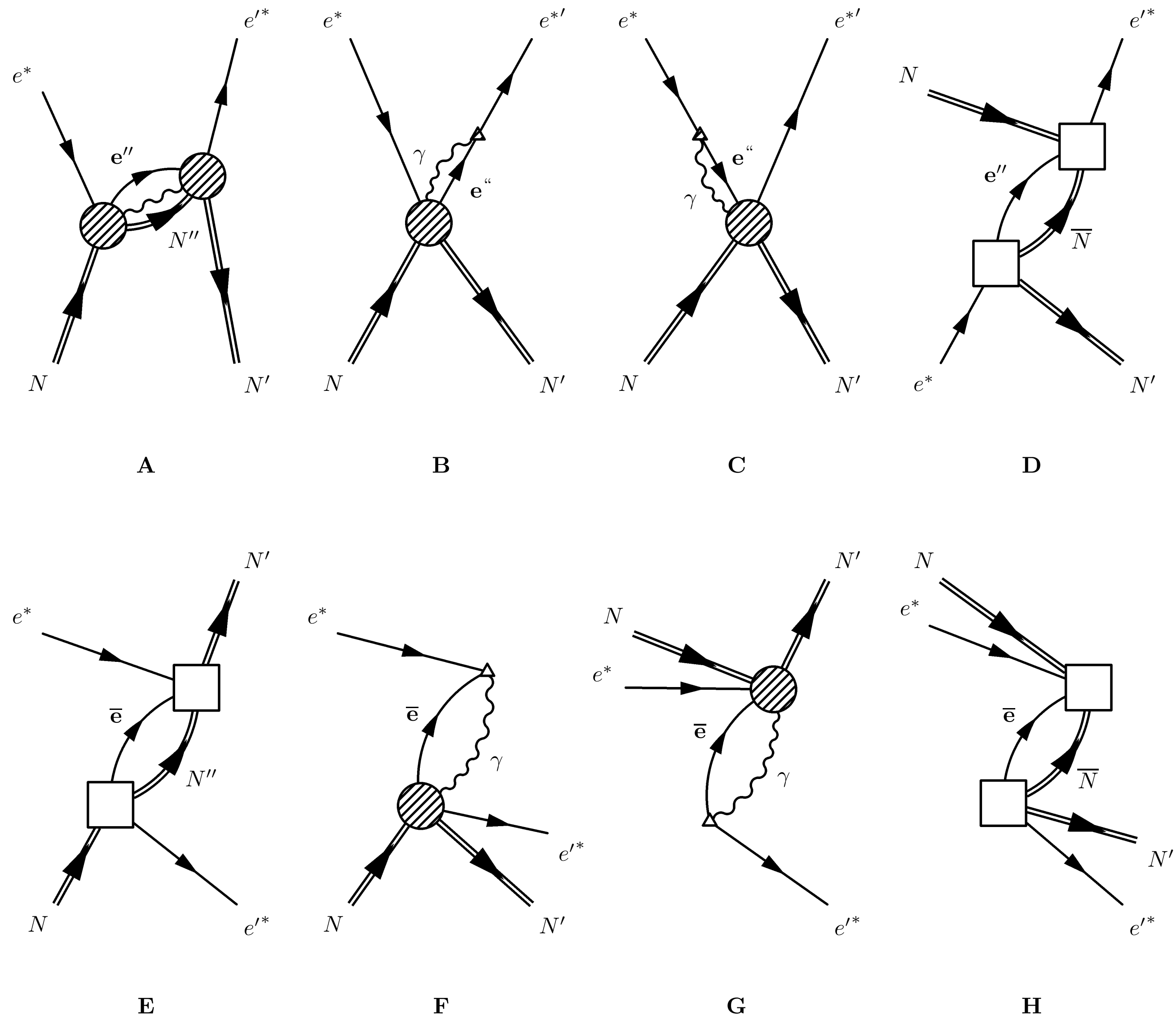
- Matrix representation of the Bogoljubov-Medvedev-Polivanov equations
- Spectral decomposition of $\mathcal{A}_{e+p \to e'+p'}$ over the complete set asymptotic states



Potential $\mathcal{W}_{e+p\to e'+p'}$ with the off mass shell external electrons: (A) s-channel $\gamma e N$ exchange. (B), (C) Parts with the intermediate amplitude $ep - ep\gamma$ (D), (H) Z-diagrams with intermediate antinucleon (E), (F), (G) Anti-electron exchange parts



$\mathcal{W}_{e+p\to e'+p'}$ with the off mass shell external electrons: • nucleons are on mass shell. • Proton vertex correction are included in $\gamma^* pp$ vertex.(Equal-time term). • Self-energy terms do not appear. • electron vertex correction are included in (B) and (C).



Potential $\mathcal{W}_{e+p\to e'+p'}$ with the off mass shell external electrons: the time-ordered 3D diagrams. Complete set of the next of the leading order terms $\sim \alpha^4$.

Linearization:

$$\mathcal{W}_{e'N',eN} = \mathrm{A}_{e'N',eN} + (E_{\mathbf{p'_e}} + E_{\mathbf{p'_N}})\mathrm{B}_{e'N',eN}$$

Linear energy depending potential

$$\mathcal{U}_{e'N',eN}(E) = < \mathbf{p}'_N | \{\eta_{\mathbf{p'_e}}(0), b^+_{\mathbf{p_e}}(0)\} | \mathbf{p}_N > + \mathrm{A}_{e'N',eN} + E\ \mathrm{B}_{e'N',eN}$$

Final 3D relativistic Lippmann-Schwinger-type equation

$$\mathcal{T}_{e'N',eN}(E) = \mathcal{U}_{e'N',eN}(E) + \sum_{e"N"} \mathcal{U}_{e'N',e"N"}(E) g_o(E) \mathcal{T}_{e"N",eN}(E)$$

$$g_o = \frac{(2\pi)^3 \delta(\mathbf{p'_e} + \mathbf{p'_N} - \mathbf{p_e} - \mathbf{p_N})}{E_{\mathbf{p_e}} + E_{\mathbf{p_N}} - E + i\epsilon}$$

$$\mathcal{T}_{e'N',eN}(E = E_{\mathbf{p'_e}} + E_{\mathbf{p'_N}}) = \mathcal{A}_{e'N',eN} = < out; \mathbf{p}'_N | \eta_{\mathbf{p'_e}}(0) | \mathbf{p}_e, \mathbf{p}_N; in >

Equal-time commutators and One Photon Exchange (OPE) Usual canonical quantization

of the four independent photon components

$$Y_{e'N',eN} =< out; \mathbf{p'}_N|\{\eta_{\mathbf{p'_e}}(0), b^+_{\mathbf{p_e}}(0)\}|\mathbf{p}_N; in >$$

$$[\overset{o}{A}_\mu(x_o, \mathbf{x}), A_\nu(x_o, \mathbf{y})] = ig_{\mu\nu}\delta(\mathbf{x} - \mathbf{y})$$

$$\{\psi_\alpha(x_o, \mathbf{x}), \psi^+_\beta(x_o, \mathbf{y})\} = \delta_{\alpha\beta}\delta(\mathbf{x} - \mathbf{y})$$

$$b^+_{\mathbf{p}_e}(x_o) = \int d^3x e^{-ip_e x}\overline{\psi}_e(x)\gamma_o u(\mathbf{p_e})$$

$$\eta_e(x) = e\gamma^\mu\psi_e(x)A_\mu(x)$$

$$Y_{e'N',eN} = e\overline{u}(\mathbf{p'_e})\gamma_\mu u(\mathbf{p_e}) < out; \mathbf{p'}_N|A^\mu(0)|\mathbf{p}_N; in$$

$$J^\mu(x) = \Box A^\mu(x) = e\overline{\psi}_e(x)\gamma^\mu\psi_e(x) + e\overline{\psi}_N(x)\gamma^\mu\psi_N($$

$$Y_{e'N',eN} = e\overline{u}(\mathbf{p'_e})\gamma_\mu u(\mathbf{p_e})\frac{< out; \mathbf{p'}_N|J^\mu(0)|\mathbf{p}_N; in}{t_N}$$

$$Y_{e'N',eN} = e\overline{u}(\mathbf{p'_e})\gamma_\mu u(\mathbf{p_e})\frac{< out; \mathbf{p}'_N|J^\mu(0)|\mathbf{p}_N; in}{t_N}$$

$$t_N = (p^o_N{}' - p^o_N)^2 - (\mathbf{p'_N} - \mathbf{p_N})^2 \text{ OPE term}$$

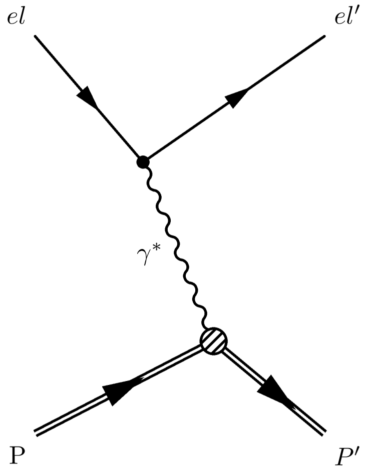


Figure 1: **One photon exchange** $V_{OPE} \equiv Y_{e'N',eN}$.

Gauge Condition and quantization rules are independent.

Coulomb gauge: only transverse components are quantized

Lorentz gauge: USUALLY all four components of $A_\mu(x)$ are independent and quantized

Gupta-Bleuer indefinite metric, additional conditions,....

We shall consider Lorentz gauge + quantization of the transverse part of $A_\mu(x)$

Coulomb gauge:

$$\frac{\partial A_i^C(x)}{\partial x_i} = 0; \qquad i = 1, 2, 3$$

$$(i\gamma^\mu \frac{\partial}{\partial x^\mu} - m_e)\psi_e(x) = \eta(x) = e\gamma^\mu A_\mu^C(x)\psi_e(x)$$

$$\Box_x A^C{}_i(x) = J_i^{tr}(x) = J_i(x) - \frac{\partial}{\partial x^i}\frac{\partial J^k(x)}{\partial x^k}$$

Poisson eq. (Nonlocality *is generated by* Coulomb energy)

$$-\Delta A^C{}_o(x) \equiv -\frac{\partial}{\partial x^i}\frac{\partial A^C{}_o(x)}{\partial x_i} = J_i^o(x)$$

$$A_o^C(x) = \int \frac{d\mathbf{x}' J_o(x_o, \mathbf{x}')}{4\pi|\mathbf{x} - \mathbf{x}'|}$$

$A_o^C(x)$ is defined via $J_o(x) = e\overline{\psi}(x)\gamma_o\psi(x)$

$$[\frac{\partial A_i^C(x_o, \mathbf{x})}{\partial x_o}, A_j^C(x_o, \mathbf{y})] =$$

$$\delta_{ij}\delta(\mathbf{x} - \mathbf{y}) - \frac{1}{\Delta}\frac{\partial^2}{\partial x_i \partial y_j}\frac{1}{|\mathbf{x} - \mathbf{y}|}$$

$$Y_{e'N',eN} =< out; \mathbf{p'}_N | \{\eta_{\mathbf{p'_e}}(0), b^+_{\mathbf{p_e}}(0)\} | \mathbf{p}_N; in >$$

$$Y_{e'N',eN} = Y^C_I + Y^C_{II}$$

OPE in Coulomb gauge:

$$Y^C_I = \frac{e\overline{u}(\mathbf{p'_e})\gamma^o u(\mathbf{p_e})}{-(\mathbf{p'_N} - \mathbf{p_N})^2} < \mathbf{p'}_N | J^{tr}_o(0) | \mathbf{p}_N >$$

$$-e\overline{u}(\mathbf{p'_e})\gamma^i u(\mathbf{p_e}) \frac{1}{t_N} < \mathbf{p'}_N | J^{tr}_i(0) | \mathbf{p}_N >$$

Second nonlocal part is generated by $[A^C_{\mu=o}(0), \psi^+(0$

$$Y^C_{II} = -e\overline{u}(\mathbf{p'_e})\gamma^o \int \frac{d\mathbf{x'}}{4\pi|\mathbf{x'}|}$$

$$< \mathbf{p'}_N | \psi^+_e(0, \mathbf{x'})\psi_e(0,0) | \mathbf{p_N} > u(\mathbf{p_e})$$

Y^C_{II} is generated by the Poisson relation i.e. definition of $A^C_o(x)$ via $J_o(x)$.

This follows from the Electro-static (Coulomb) interaction Nonlocal interaction

Y^C_{II} is next of the leading order over α^2

Lorentz gauge + quantization of the transverse part of $A\mu(x)$

$$\frac{\partial A_\mu^L(x)}{\partial x_\mu} = 0; \qquad \mu = 0, 1, 2, 3$$

$$\Box_x A^{L}{}_i^{tr}(x) = J_i(x) = e\overline{\Psi}_e^L(x)\gamma_i \Psi_e^L(x); \quad i = 1, 2$$

$$A^L{}_3(x) = \Box_x^{-1} J_3(x); \quad J_3(x) = e\overline{\Psi}_e^L(x)\gamma_3\Psi_e^L(x)$$

$$A^L{}_o(x) = \Box_x^{-1} J_o(x); \quad J_o(x) = e\overline{\Psi}_e^L(x)\gamma_o\Psi_e^L(x)$$

$$(A^L)_i^{tr}(x) = (A^L)_i(x) - \frac{\partial}{\partial x^i}\frac{\partial (A^L)^k(x)}{\partial x^k}$$

$$(A^L)_i^l(x) = \frac{\partial}{\partial x^i}\frac{\partial (A^L)^k(x)}{\partial x^k}$$

$$[\frac{\partial (A^L)_i^{tr}(x_o, \mathbf{x})}{\partial x_o}, (A^L)_j^{tr}(x_o, \mathbf{y})] =$$

$$\delta_{ij}\delta(\mathbf{x} - \mathbf{y}) - \frac{1}{\Delta}\frac{\partial^2}{\partial x_i \partial y_j}\frac{1}{|\mathbf{x} - \mathbf{y}|}$$

Relationship between Lorentz and Coulomb gauges

$$(i\gamma^{\mu}\frac{\partial}{\partial x^{\mu}} - m_e)\psi_e^L(x) = e\gamma^{\mu}A_{\mu}^L(x)\psi_e^L(x)$$

$$\updownarrow \qquad\qquad\qquad \updownarrow \qquad\qquad\qquad \updownarrow$$

$$(i\gamma^{\mu}\frac{\partial}{\partial x^{\mu}} - m_e)\psi_e^C(x) == e\gamma^{\mu}A_{\mu}^C(x)\psi_e^C(x)$$

$$\psi_e^C(x) = e^{ie\lambda(x)}\psi_e^L(x)$$

$$A_{\mu}^C(x) = e^{-ie\lambda(x)}A_{\mu}^L(x)e^{ie\lambda(x)} + \frac{\partial\lambda(x)}{\partial x_{\mu}}$$

$$e^{-ie\lambda(x)}A_{\mu}^L(x)e^{ie\lambda(x)} = A_{\mu}^L(x) + ie[A_{\mu}^L, \lambda]$$

$$+ie[ie[A_{\mu}^L, \lambda], \lambda] + ... \equiv A_{\mu}^L(x) + \mathcal{D}(A_{\mu}^L, \lambda)$$

If λ is determined through the relations

$$\mathcal{D}(A_{\mu}^L, \lambda) + \frac{\partial\lambda(x)}{\partial x_{\mu}} = +\frac{1}{\Delta}\frac{\partial}{\partial x_{\mu}}\frac{\partial \mathsf{a}_o(x)}{\partial x_o} = 0$$

then

$$A_\mu^C(x) = {A^L}_\mu(x) - \frac{1}{\Delta}\frac{\partial}{\partial x_\mu}\frac{\partial {A^L}_o(x)}{\partial x_o}$$

$$-\Delta A_o^C = \Box {A^L}_o = J_o(x)$$

$$\frac{\partial A_i^C(x)}{\partial x_i} = 0$$

$A_i^C(x)$ is the photon field in the Coulomb gauge

$$< \mathbf{p}'_N|{A^L}_{\mu=1,2}(0)|\mathbf{p}_N >=< \mathbf{p}'_N|{A^C}_{\mu=1,2}(0)|\mathbf{p}_N >$$

$$< \mathbf{p}'_N|{A^L}_o(0)|\mathbf{p}_N >= \frac{t_N < \mathbf{p}'_N|{A^C}_o(0)|\mathbf{p}_N >}{-(\mathbf{p'_N} - \mathbf{p_N})^2}$$

Nucleon as three quark bound (cluster) state
R. Haag, Phys. Rev. **112** (1958)
K. Nishijima, Phys. Rev. **111** (1958)
W. Zimmermann, Nuovo Cim. 10 (1958)
K. Huang and H. A. Weldon, Phys. Rev. D11 (1975) 257.

Construction of the cluster (bound) state asymptotic crearion annihilation) operator

$$\mathcal{B}^{in(out)}(\mathbf{p}) = \lim_{X^0 \to -\infty(+\infty)}^{weekly} \mathcal{B}_{\mathbf{p}}(X^0),$$

$$\mathcal{B}_{\mathbf{p}}(X^0) = \int d^3\mathbf{X} \exp{(ipX)}\overline{u}(\mathbf{p})\gamma_0\Upsilon_p(X)$$

with canonical quantization of asymptotic fields

$$\{\mathcal{B}^{in(out)^\dagger}(\mathbf{p}), \mathcal{B}^{in(out)}(\mathbf{p}')\} = \delta(\mathbf{p} - \mathbf{p})$$

$$\{\mathcal{B}^\dagger_{\mathbf{p}}(0), \mathcal{B}_{\mathbf{p}'}(0)\} \neq \delta(\mathbf{p} - \mathbf{p}')$$

THIS IS NOT a NONLOCAL QFT

Jacobi four-coordinates

$$\rho_{12} = x_1 - x_2$$

$$\rho_3 = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} - x_3$$

$$X = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$$

,

$$\Upsilon_p(X) = \int d^4\rho_{12} d^4\rho_3 \chi_p^\dagger(X = 0, \rho_{12}, \rho_3)$$
$$T(q_1(x_1) q_2(x_2) q_3(x_3)).$$

$$\chi_p^\dagger(x_1, x_2, x_3) = < \mathbf{p}_N | T(q_1(x_1) q_2(x_2) q_3(x_3)) | 0 >$$

The leading term in the formulations with off shell nucleons and on shell electrons

$$Y_{e'N',eN} =< out; \mathbf{p'}_e | \{ J_{\mathbf{p'_N}}(0), B^+_{\mathbf{p_N}}(0) \} | \mathbf{p}_e; in >$$

$$J_{\mathbf{p'_N}}(X) = (i\gamma_\mu \frac{\partial}{\partial X_\mu} - m_N) \Upsilon_p(X)$$

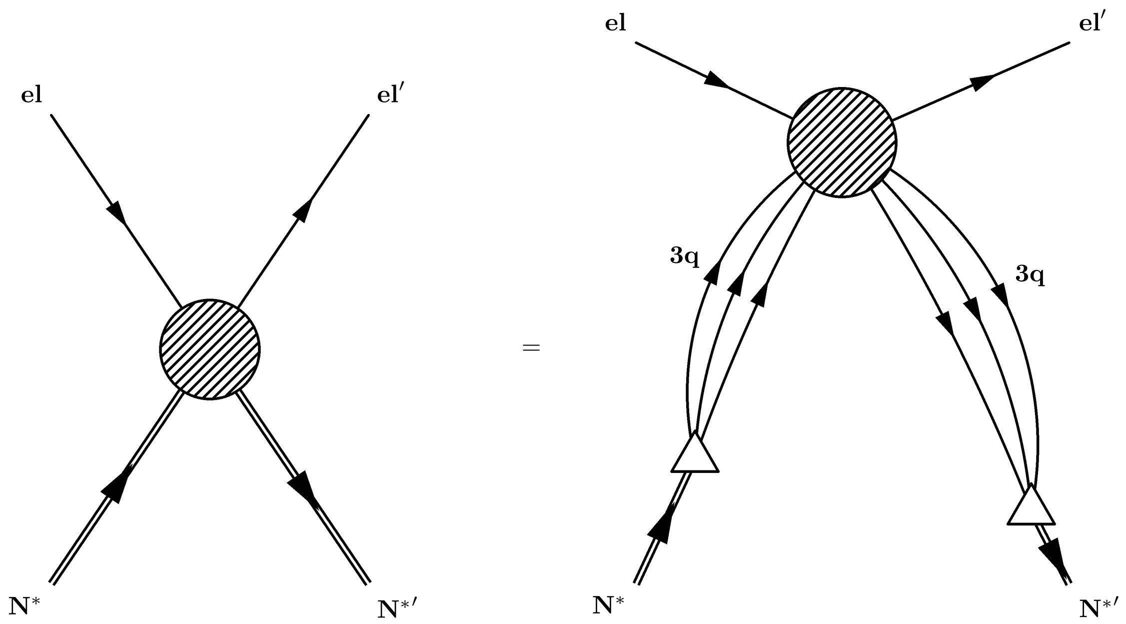


Figure 2: ep **scattering amplitude with off mass shell nucleons and on shell electrons. Other kind set of the completeness condition**

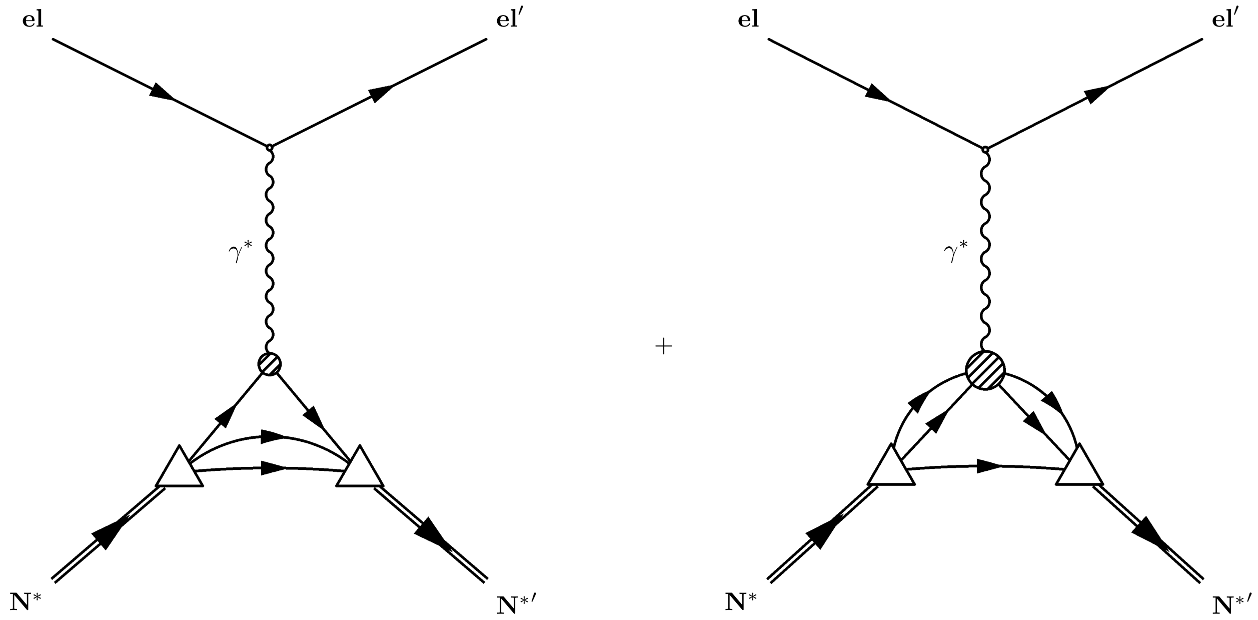


Figure 3: **The leading terms of ep scattering amplitude calculated in the canonical equal-time commutation relations within QCD.**

Quark-gluon degrees of freedom are included in the $p\gamma - p'$ form factors because proton in the present formulation with and without quark degrees of freedom are ON Mass Shell

- **Propagation of the quark and gluons in the intermediate states does not contribute into the completeness and unitarity conditions with hadrons and leptons.**

- **Completeness and unitarity in the hadron sector ensure separation of the quark and hadron degrees of freedom**

- **Unitarity allows to avoid the double-counting**

- **The form of the 3D equations with and without quarks are the same. in equations with and without quark degrees of freedom.**

Conclusion

♣ New three dimensional field theoretical equations for the unified description of the Hydrogen-like systems and the lepton-nucleon scattering is suggested.

♣ The exact coupling between the ep scattering potentials in the Lorentz and Coulomb gauges is obtained using the gauge transformation of the Heisenberg electron fields

$$\psi^{Lorentz}(x) = e^{\lambda(x)}\psi^{Coulomb}(x)$$

♣ It is demonstrated, that the ep potential in the Coulomb gauge is much more transparent, simpler and convenient as in the Lorentz gauge.

♣ Unlike to the Bethe-Salpeter equations and their quasipotential reductions, the potential of the present equation is constructed from the one variable form factors

♣ The leading Born term of these equation is generated by the equal-time canonical commutators, which produces also the non-local next of the leading order terms.

♣ In the present 3D approach are exactly separated the positron degrees of freedom.