Nucleon structure in light-front quark model constrained by AdS/QCD

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based on
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Introduction

- Nucleon structure
  parton distributions (PDFs)
  generalized parton distributions (GPDs)
  form factors (FFs)

- Current and future Experiments
  BNL, CERN, DESY, GSI, JINR, JLab, Electron Ion Collider (EIC) . . .

- Many theoretical approaches
  pQCD, Lattice, QCD sum rules, ChPT, different types of quark models (from naive quark model to Schwinger-Dyson/Bethe-Salpeter approaches)

- AdS/QCD $\equiv$ Holographic QCD (HQCD) – approximation to QCD:
  attempt to model Hadronic Physics in terms of fields/strings living in extra dimensions – anti-de Sitter (AdS) space

- HQCD models reproduce main features of QCD at low and high energies:
  chiral symmetry, confinement, power scaling of hadron form factors

Introduction

- AdS metric   Poincaré form

  $$ds^2 = g_{MN}(z)\, dx^M dx^N = \frac{R^2}{z^2}\left(dx_\mu dx^\mu - dz^2\right)$$ where R – AdS radius

- Metric Tensor $g_{MN}(z) = \epsilon^a_M(z)\epsilon^b_N(z)\eta_{ab}$

- Vielbein $\epsilon^a_M(z) = \frac{R}{z}\,\delta^a_M$ (relates AdS and Lorentz metric)

- Manifestly scale-invariant $x \to \lambda x, \;\; z \to \lambda z$.

- z – extra dimensional (holographic) coordinate; $z = 0$ is UV boundary

- Five Dimensions: L = Length, W = Width, H = Height, T = Time, S = Scale

Introduction

- Action for scalar field

$$S_\Phi = \frac{1}{2} \int d^4x dz \sqrt{g}\, e^{-\varphi(z)} \Big(\partial_M \Phi(x,z) \partial^M \Phi(x,z) - m^2\, \Phi^2(x,z) \Big)$$

- Dilaton field $\varphi(z) = \kappa^2 z^2$ – gravitational background field providing breaking of conformal symmetry and confinement

- $g = |\mathrm{det} g_{MN}|$

- m – 5d mass, $m^2 R^2 = \Delta(\Delta - 4)$, $\Delta = 3$ is conformal dimension

- Kaluza-Klein (KK) expansion $\Phi(x,z) = \sum\limits_n \phi_n(x)\, \Phi_n(z)$

- Tower of KK modes $\phi_n(x)$ dual to 4-dimensional fields describing hadrons

- Bulk profiles $\Phi_n(z)$ dual to hadronic wave functions

Introduction

- Put on mass-shell $-\partial_\mu \partial^\mu \phi_n(x) = M_n^2 \phi_n(x)$
- Substitute
$$\Phi_n(z) = \left(\frac{R}{z}\right)^{-3} \phi_n(z)$$
- Identify $\Delta = \tau = N + L$ (here $N = 2$ – number of partons in meson)
$$\left[-\frac{d^2}{dz^2} + \frac{4L^2-1}{4z^2} + \kappa^4 z^2 - 2\kappa^2 \right] \phi_n(z) = M_n^2 \phi_n(z)$$
- Solutions: $\phi_{nL}(z) = \sqrt{\frac{2\Gamma(n+1)}{\Gamma(n+L+1)}}\, \kappa^{L+1}\, z^{L+1/2}\, e^{-\kappa^2 z^2/2}\, L_n^L(\kappa^2 z^2)$
- $M_{nL}^2 = 4\kappa^2 \left(n + \frac{L}{2}\right)$
- Massless pion $M_\pi^2 = 0$ for $n = L = 0$ Brodsky, Téramond
- Extension to higger spins $M_{nLJ}^2 = 4\kappa^2 \left(n + \frac{L+J}{2}\right) \sim 4\kappa^2\, (n + J)$ at large J
Brodsky, Téramond, Lyubovitskij, Gutsche, Schmidt, Vega

Introduction

- Scattering problem for AdS field gives information about propagation of external field from z to the boundary $z = 0$ — bulk-to-boundary propagator $\Phi_{\rm ext}(q, z)$ [Fourier-trasform of AdS field $\Phi_{\rm ext}(x, z)$]:

$$\Phi_{\rm ext}(q, z) = \int d^d x e^{-iqx} \, \Phi_{\rm ext}(x, z)$$

- Equation of motion $\partial_z \left(\frac{e^{-\varphi(z)}}{z} \partial_z \Phi_{\rm ext}(q, z) \right) + q^2 \, \frac{e^{-\varphi(z)}}{z} \Phi_{\rm ext}(q, z) = 0 \, .$

- Solution in Euclidean space $Q^2 = -q^2$

  $\Phi_{\rm ext}(Q, z) = \Gamma\left(1 + \frac{Q^2}{4\kappa^2}\right) U\left(\frac{Q^2}{4\kappa^2}, 0, \kappa^2 z^2\right)$

- Hadron form factors

  $F_{n\tau}(Q^2) = \int\limits_0^\infty dz \Phi_{\rm ext}(Q, z) \phi^2_{n\tau}(z)$

- Hadron structure is implemented by a nontrivial dependence of AdS fields on 5-th (holographic) coordinate

Introduction

- Master formula for PDFs, GPDs, FFs

  FF $F_\tau(Q^2) = \int\limits_0^1 dx H_\tau(x,Q^2)$

  GPD $H_\tau(x,Q^2) = \underbrace{(1-x)^{\tau-2}}_{=q_\tau(x)} \exp\Big(-\frac{Q^2}{4\kappa^2}\log(1/x)\Big)$

  PDF $q_\tau(x) = (1-x)^{\tau-2}$

- Consistent with quark counting rules
  Matveev-Muradyan-Tavkhelidze-Brodsky-Farrar

  $$F_\tau(Q^2) = \frac{\Gamma(\frac{Q^2}{4\kappa^2}+1)\,\Gamma(\tau)}{\Gamma(\frac{Q^2}{4\kappa^2}+\tau)} \sim \frac{1}{(Q^2)^{\tau-2}} \quad \text{at large } Q^2 \to \infty$$

- $F_{\tau=2}(Q^2) \sim \frac{1}{Q^2}$ for mesons, $F_{\tau=3}(Q^2) \sim \frac{1}{Q^4}$ for baryons

Introduction

- Matching to Light-Front QCD using Drell-Yan-West formula

  $$F_\tau(Q^2) = \int_0^1 dx \int \frac{d^2\mathbf{k}_\perp}{16\pi^3}\, \psi^\dagger_\tau(x, \mathbf{k}'_\perp)\, \psi_\tau(x, \mathbf{k}_\perp)$$

  where $\mathbf{k}'_\perp = \mathbf{k}_\perp + (1-x)\mathbf{q}_\perp$ and $Q^2 = \mathbf{q}^2_\perp$

- Result of matching

  $$\psi_\tau(x, \mathbf{k}_\perp) = N_\tau \, \frac{4\pi}{\kappa} \, \sqrt{\log(1/x)} (1-x)^{\frac{\tau-4}{2}} \, \exp\left[-\frac{\mathbf{k}^2_\perp}{2\kappa^2} \, \frac{\log(1/x)}{(1-x)^2} \right]$$

  where $N_\tau = \sqrt{\tau - 1}$

- For $\tau = 2$ first done by Brodsky, Téramond

- Matching at the initial scale $\mu_0 \sim 1$ GeV (hard and soft evolutions neglected)

- Derived LFWF is not symmetric under the exchange $x \to 1 - x$

  Extracted from matching matrix element of bare electromagnetic current between dressed LFWF in LF QCD and matrix element of the dressed electromagnetic current between hadronic WF in AdS/QCD

Introduction

- Light-front wave functions (LFWFs)

  - Brodsky-Huang-Lepage prescription for two partonic bound state

  $\psi(x, \mathbf{k}_\perp) \sim \exp\left(-\frac{\mathbf{k}_\perp^2}{\lambda^2 x(1-x)}\right)$ where λ is scale parameter

  - Different forms, 3 quark and quark-diquark structures

  - Normally analysis of form factors start from parametrization GDPs

  See e.g.
  Guidal-Polyakov-Radyushkin-Vanderhaegen, PRD 72 (2005) 054013
  Selyugin-Teryaev, PRD 79 (2009) 033003
  Diehl-Kroll, EPJC 73 (2013) 2397

  - Also LF 3-quark/qd models used for calculation of PDFs, TMDs

  See e.g.
  Bacchetta-Conti-Radici, PRD 78 (2008) 074010
  Boffi-Efremov-Pasquini-Schweitzer, PRD 79 (2009) 094012

Pion

- Global analysis of PDFs tells that they are considerably softer

- At large x and initial scale $q_\pi(x, \mu_0) \sim (1-x)^{2.03}$
  Aicher-Schafer-Vogelsang, PRL 105 (2010) 252003

- Performing updated analysis of the E615 data on the cross section of the Drell-Yan process $\pi^- N \to \mu^+ \mu^- X$ including next-to-leading logarithmic threshold resummation effects

- Threshold means $z = Q^2/(Sx_1x_2) \to 1$, where x_1 and x_2 are momentum fractions of partons participating in hard-scattering reaction, $S = (P_1 + P_2)^2$ is hadronic CM energy squared, Q^2 is photon momentum squared

- Threshold means: most of initial partonic energy is used to produce virtual photon

- pion PDF at initial scale $\mu_0 = 0.63$ GeV   Aicher-Schafer-Vogelsang

  $q_\pi(x, \mu_0) = N_\pi x^{\alpha - 1} (1-x)^\beta (1 + \gamma x^\delta)$

  where $\alpha = 0.70$, $\beta = 2.03$, $\gamma = 13.8$, $\delta = 2$, $N_\pi = 0.915$

Pion

- Required pion LFWF reads

$$\psi_\pi(x,\mathbf{k}_\perp) \quad = \quad \frac{4\pi N}{\kappa}\,\frac{\sqrt{\log(1/x)}}{1-x}\,\sqrt{f(x)\,\bar{f}(x)}\,\exp\left[-\frac{\mathbf{k}_\perp^2}{2\kappa^2}\,\frac{\log(1/x)}{(1-x)^2}\,\bar{f}(x)\right]$$

where functions $f(x)$ and $\bar{f}(x)$ are specified as

$$\begin{aligned} f(x) \quad &= \quad x^{\alpha-1}\,(1-x)^{\beta}\,(1+\gamma x^{\delta})\,, \\ \bar{f}(x) \quad &= \quad x^{\bar{\alpha}}\,(1-x)^{\beta}\,(1+\bar{\gamma} x^{\bar{\delta}})\,, \end{aligned}$$

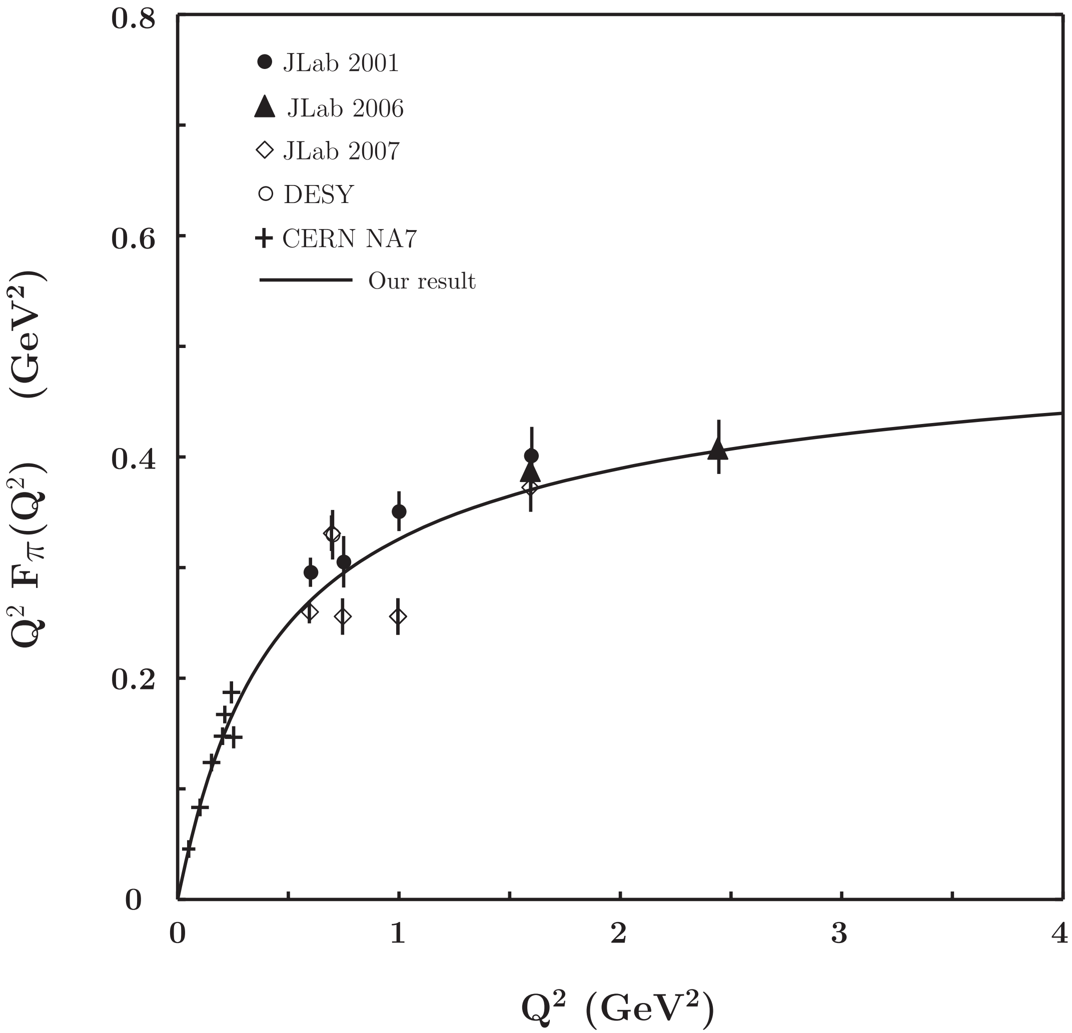
where α, β, γ, δ, $\bar{\alpha}$, $\bar{\gamma}$ and $\bar{\delta}$ are free parameters and N is the normalization factor

$$N^{-2} = B(\alpha, 1+\beta) + \gamma B(\alpha+\delta, 1+\beta)$$

$$\bar{\alpha} = 0.15\,,\ \bar{\gamma} = 2.30\,,\ \bar{\delta} = 1.30$$

Pion

$\langle r_\pi^2 \rangle = 0.424$ fm^2 (data 0.452 fm^2)



Nucleons

- Valence quark decompositions of nucleon FFs
  [see e.g. Radyushkin, PRD 58 (1998) 114008]
  $F_i^{p(n)}(Q^2) = \frac{2}{3} F_i^{u(d)}(Q^2) - \frac{1}{3} F_i^{d(u)}(Q^2)$

- quark GPDs in nucleons

  $F_1^q(Q^2) = \int_0^1 dx\, \mathcal{H}^q(x, Q^2)$
  $F_2^q(Q^2) = \int_0^1 dx\, \mathcal{E}^q(x, Q^2)$.

- PDFs (at $Q^2 = 0$)
  $\mathcal{H}^q(x, 0) = q_v(x), \quad \mathcal{E}^q(x, 0) = \mathcal{E}^q(x)$

- Normalized as

  Number of quarks valence quarks in the proton $n_q \; = \; F_1^q(0) = \int\limits_0^1 dx\, q_v(x)$

  Anomalous magnetic moment $\kappa_q \; = \; F_2^q(0) = \int\limits_0^1 dx\, \mathcal{E}^q(x)$

Nucleons

- Sachs FFs and EM radii

$$G_E^N(Q^2) = F_1^N(Q^2) - \frac{Q^2}{4m_N^2} F_2^N(Q^2)$$

$$G_M^N(Q^2) = F_1^N(Q^2) + F_2^N(Q^2)$$

$$\langle r_E^2 \rangle^N = -6 \frac{dG_E^N(Q^2)}{dQ^2}\bigg|_{Q^2=0}$$

$$\langle r_M^2 \rangle^N = -\frac{6}{G_M^N(0)} \frac{dG_M^N(Q^2)}{dQ^2}\bigg|_{Q^2=0}$$

Nucleons

- LF representation for quark FFs

$$F_1^q(Q^2) = \int_0^1 dx \int \frac{d^2\mathbf{k}_\perp}{16\pi^3} \Big[\psi_{+q}^{+\,*}(x,\mathbf{k}'_\perp)\psi_{+q}^{+}(x,\mathbf{k}_\perp) + \psi_{-q}^{+\,*}(x,\mathbf{k}'_\perp)\psi_{-q}^{+}(x,\mathbf{k}_\perp)\Big]$$

$$F_2^q(Q^2) =$$
$$-\frac{2M_N}{q^1 - iq^2} \int_0^1 dx \int \frac{d^2\mathbf{k}_\perp}{16\pi^3} \Big[\psi_{+q}^{+\,*}(x,\mathbf{k}'_\perp)\psi_{+q}^{-}(x,\mathbf{k}_\perp) + \psi_{-q}^{+\,*}(x,\mathbf{k}'_\perp)\psi_{-q}^{-}(x,\mathbf{k}_\perp)\Big]$$

- $\psi_{\lambda_q q}^{\lambda_N}(x,\mathbf{k}_\perp)$ — LFWFs with specific helicities for the nucleon $\lambda_N = \pm$ and for the struck quark $\lambda_q = \pm$, where plus and minus correspond to $+\frac{1}{2}$ and $-\frac{1}{2}$

- We work in the frame with $q = (0, 0, \mathbf{q}_\perp)$, and therefore the Euclidean momentum squared is $Q^2 = \mathbf{q}_\perp^2$

- As initial scale we choose the value $\mu_0 = 1$ GeV accepted in the Martin-Stirling-Thorne-Watt global fit EPJC 63 (2009) 189

Nucleons

- Scalar quark-diquark model

  $$\psi^{+}_{+q}(x,\mathbf{k}_\perp) = \varphi^{(1)}_q(x,\mathbf{k}_\perp)$$

  $$\psi^{+}_{-q}(x,\mathbf{k}_\perp) = -\frac{k^1+ik^2}{xM_N}\,\varphi^{(2)}_q(x,\mathbf{k}_\perp)$$

  $$\psi^{-}_{+q}(x,\mathbf{k}_\perp) = \frac{k^1-ik^2}{xM_N}\,\varphi^{(2)}_q(x,\mathbf{k}_\perp)$$

  $$\psi^{-}_{-q}(x,\mathbf{k}_\perp) = \varphi^{(1)}_q(x,\mathbf{k}_\perp)$$

  where $\varphi^{(1)}_q(x,\mathbf{k}_\perp)$ and $\varphi^{(2)}_q(x,\mathbf{k}_\perp)$ are the twist-3 LFWFs

- From AdS/QCD — LF QCD matching with a trivial dilaton potential

  $$\varphi^{\mathrm{AdS/QCD}(i)}_q(x,\mathbf{k}_\perp) = N^{(i)}_q\,\frac{4\pi}{\kappa}\,\sqrt{\frac{\log(1/x)}{1-x}}\,\exp\left[-\frac{\mathbf{k}^2_\perp}{2\kappa^2}\,\frac{\log(1/x)}{(1-x)^2}\right]$$

Nucleons

- Generalization of the nucleon LFWF

- $\varphi_q^{(i)}(x, \mathbf{k}_\perp) = N_q^{(i)} \frac{4\pi}{\kappa} \sqrt{\frac{\log(1/x)}{1-x}} \sqrt{f_q^{(i)}(x) \bar{f}_q(x)} \exp\left[-\frac{\mathbf{k}_\perp^2}{2\kappa^2} \frac{\log(1/x)}{(1-x)^2} \bar{f}_q(x) \right]$

- Functions $f_q^{(i)}$ and $\bar{f}_q$ are specified as

$$f_q^{(1)}(x) = x^{\eta_q^{(1)} - 1} (1-x)^{\eta_q^{(2)} - 1} (1 + \epsilon_q \sqrt{x} + \gamma_q x)$$

$$f_q^{(2)}(x) = x^{2+\rho_q} (1-x)^{\sigma_q} (1 + \lambda_q \sqrt{x} + \delta_q x)^2 f_q^{(1)}(x)$$

$$\bar{f}_q(x) = x^{\bar{\eta}_q^{(1)}} (1-x)^{\bar{\eta}_q^{(2)}} (1 + \bar{\epsilon}_q \sqrt{x} + \bar{\gamma}_q x)$$

Nucleons

Parameters specifying functions $f_q^{(i)}(x)$ and $\bar{f}_q(x)$

Parameter	Value	Parameter	Value
$\eta_u^{(1)}$	0.45232	$\eta_d^{(1)}$	0.71978
$\eta_u^{(2)}$	3.0409	$\eta_d^{(2)}$	5.3444
ϵ_u	-2.3737	ϵ_d	-4.3654
γ_u	8.9924	γ_d	7.4730
$\bar{\eta}_u^{(1)}$	0.195	$\bar{\eta}_d^{(1)}$	0.280
$\bar{\eta}_u^{(2)}$	$\frac{\eta_u^{(2)}-1}{2} - 0.54$	$\bar{\eta}_d^{(2)}$	$\frac{\eta_d^{(2)}-1}{2} - 0.60$
$\bar{\epsilon}_u$	-0.71	$\bar{\epsilon}_d$	-0.10
$\bar{\gamma}_u$	0	$\bar{\gamma}_d$	0
ρ_u	0.091	ρ_d	-0.17
σ_u	$(\eta_u^{(2)} - 1) - 0.2409$	σ_d	$(\eta_d^{(2)} - 1) - 2.3444$
λ_u	-2.40	λ_d	-0.35
δ_u	3.18	δ_d	4.26

$\kappa = 350$ MeV remains the same as fixed in soft-wall AdS/QCD

Nucleons

- At large x PDFs scale as

- $q_v(x) \sim (1-x)^{\eta_q^{(2)}}$

  $u_v(x) \sim (1-x)^3\,, \;\; d_v(x) \sim (1-x)^5$

- $\mathcal{E}^q(x) \sim q_v(x)\,(1-x)^{1+\sigma_q/2} \sim (1-x)^{\eta_q^{(2)}+1+\sigma_q/2}$

  $\mathcal{E}^u(x) \sim (1-x)^5\,, \;\; \mathcal{E}^d(x) \sim (1-x)^7$

Nucleons

- Scaling of nucleon FFs

$$F_1^q(Q^2) \sim \int_0^1 dx(1-x)^{\eta_q^{(2)}} \exp\left[-\frac{Q^2}{4\kappa^2}(1-x)^{1+\bar{\eta}_q^{(2)}}\right] \sim \left(\frac{1}{Q^4}\right)^{1+\Delta_q^{(1)}}$$

and

$$F_2^q(Q^2) \sim \int_0^1 dx(1-x)^{1+\eta_q^{(2)}+\sigma_q/2} \exp\left[-\frac{Q^2}{4\kappa^2}(1-x)^{1+\bar{\eta}_q^{(2)}}\right] \sim \left(\frac{1}{Q^6}\right)^{1+\Delta_q^{(2)}}$$

- $\Delta_q^{(1)}$ and $\Delta_q^{(2)}$ are the small corrections encoding a deviation of the Dirac and Pauli quark form factors from power-scaling laws $1/Q^4$ and $1/Q^6$, respectively

$$\Delta_q^{(1)} = \frac{1+\eta_q^{(2)}}{2(1+\bar{\eta}_q^{(2)})} - 1\,, \qquad \Delta_q^{(2)} = \frac{2}{3}\Delta_q^{(1)} + \frac{1}{3}\left(\frac{1+\sigma_q/2}{1+\bar{\eta}_q^{(2)}} - 1\right)$$

Nucleons

- Analytically these corrections vanish when

$$\bar{\eta}_q^{(2)} = \frac{\sigma_q}{2} = \frac{\eta_q^{(2)} - 1}{2}$$

- Also in this limit we are consistent with Drell-Yan-West duality between the large-Q^2 behavior of nucleon electromagnetic form factors and the large-x behavior of the structure functions.

- However fine-tuning fit of electromagnetic form factors requires a deviation of $\Delta_q^{(i)}$ from zero. Numerically they are

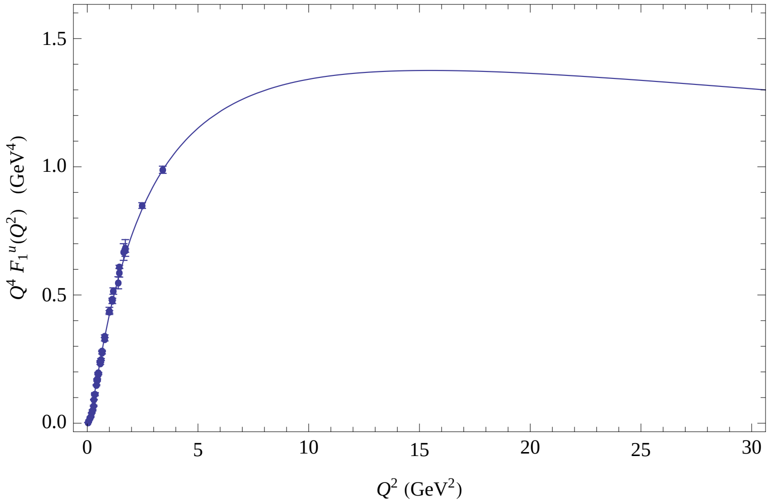
$$\Delta_u^{(1)} = 0.365\,,\ \Delta_d^{(1)} = 0.233\,,$$
$$\Delta_u^{(2)} = 0.338\,,\ \Delta_d^{(2)} = 0.081\,.$$

Nucleons

Electromagnetic properties of nucleons

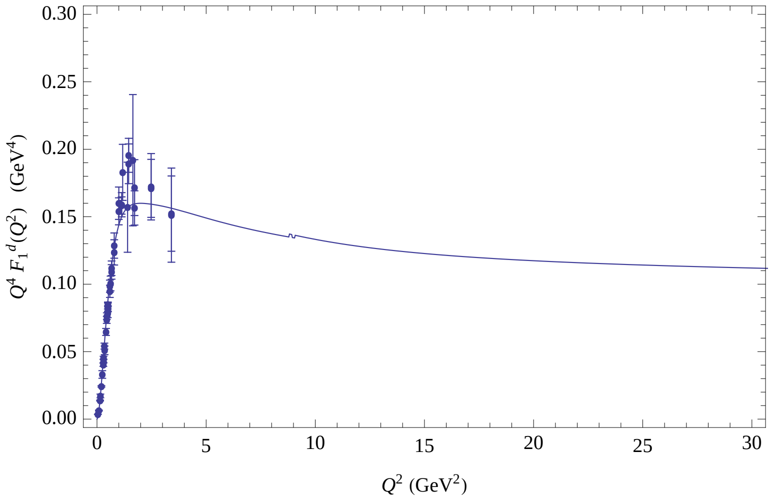
Quantity	Our results	Data
μ_p (in n.m.)	2.793	2.793
μ_n (in n.m.)	-1.913	-1.913
r_E^p (fm)	0.781	0.8768 ± 0.0069
$\langle r_E^2 \rangle^n$ (fm^2)	-0.113	-0.1161 ± 0.0022
r_M^p (fm)	0.717	$0.777 \pm 0.013 \pm 0.010$
r_M^n (fm)	0.694	$0.862^{+0.009}_{-0.008}$

Nucleons



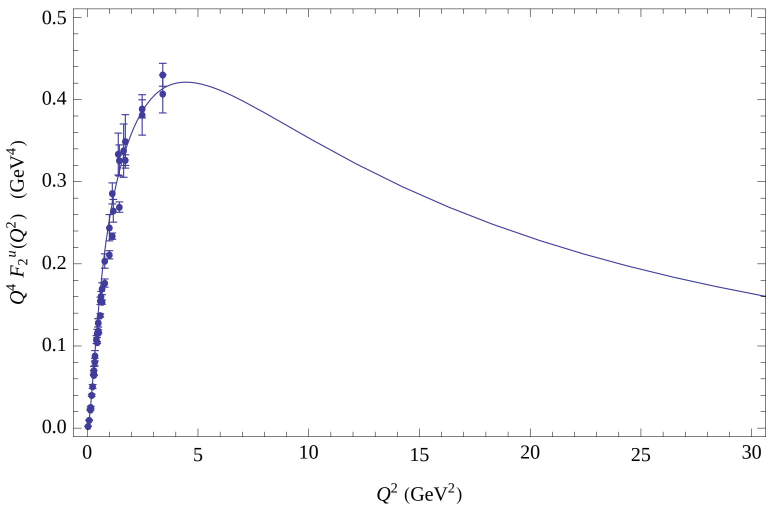
Dirac u quark form factor multiplied by Q^4

Nucleons



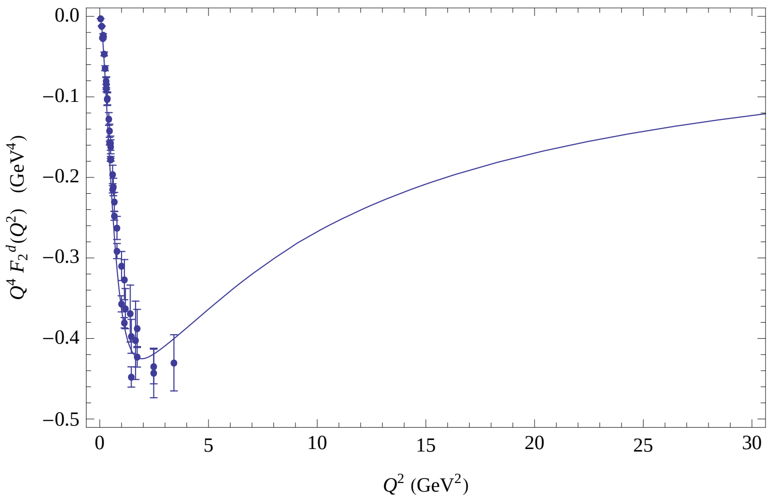
Dirac d quark form factor multiplied by Q^4

Nucleons



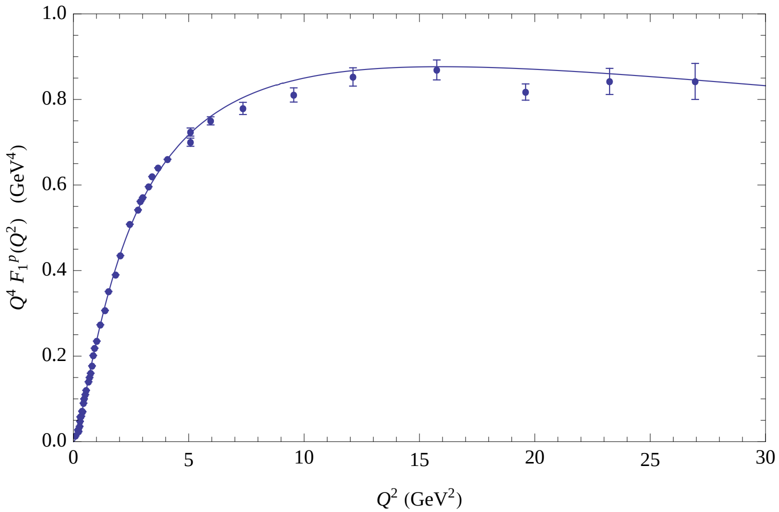
Pauli u quark form factor multiplied by Q^4

Nucleons



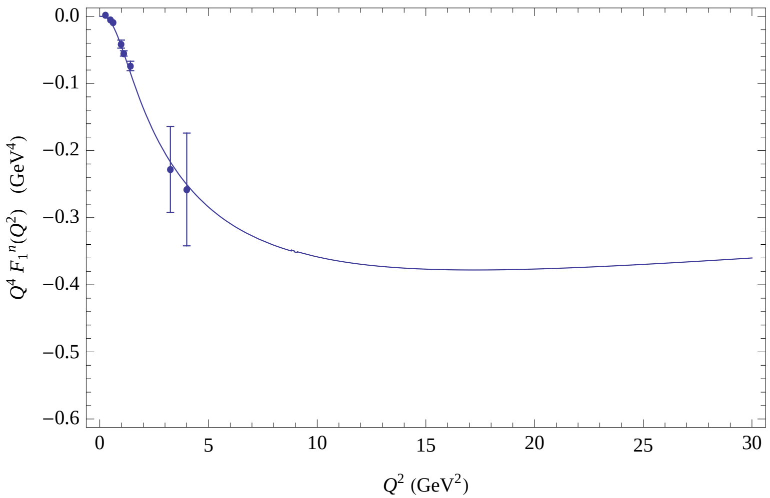
Pauli d quark form factor multiplied by Q^4

Nucleons



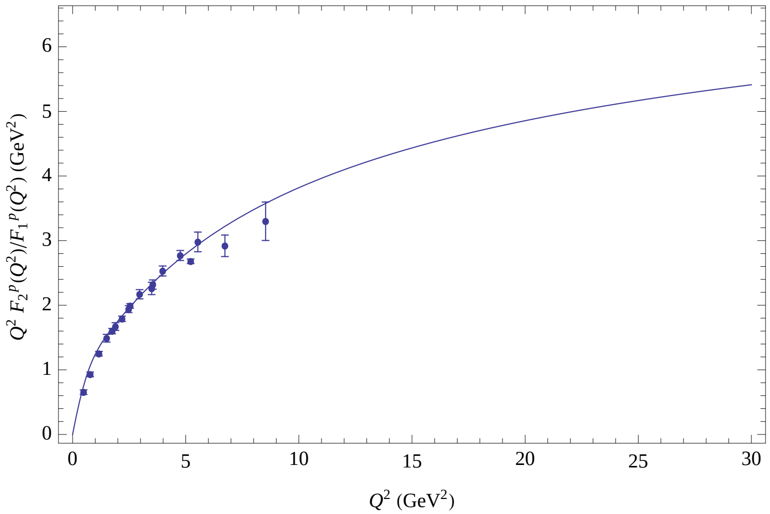
Dirac proton form factor multiplied by Q^4

Nucleons



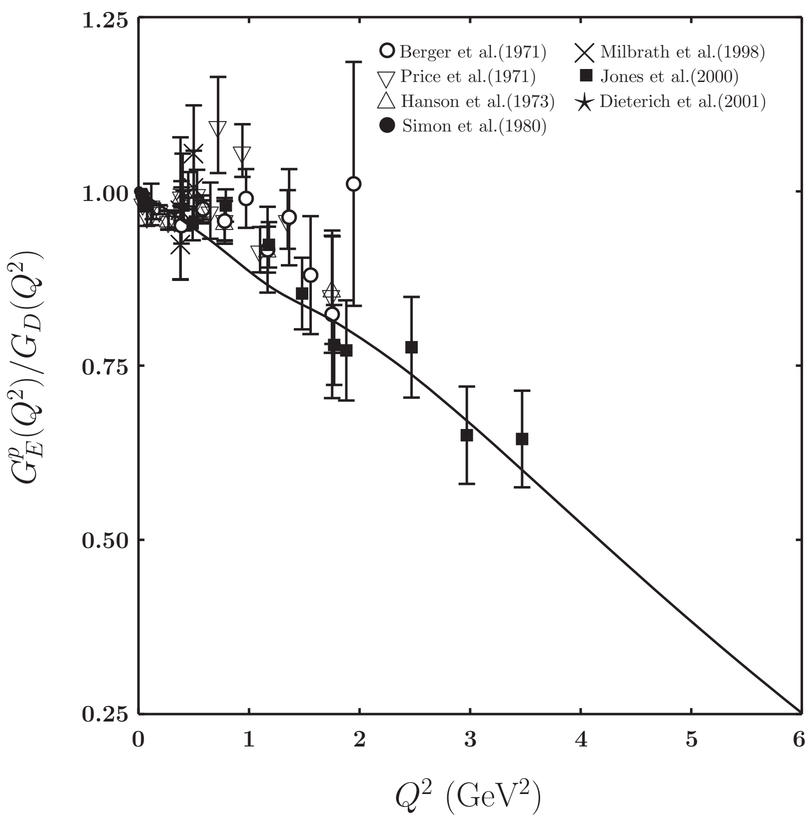
Dirac neutron form factor multiplied by Q^4

Nucleons



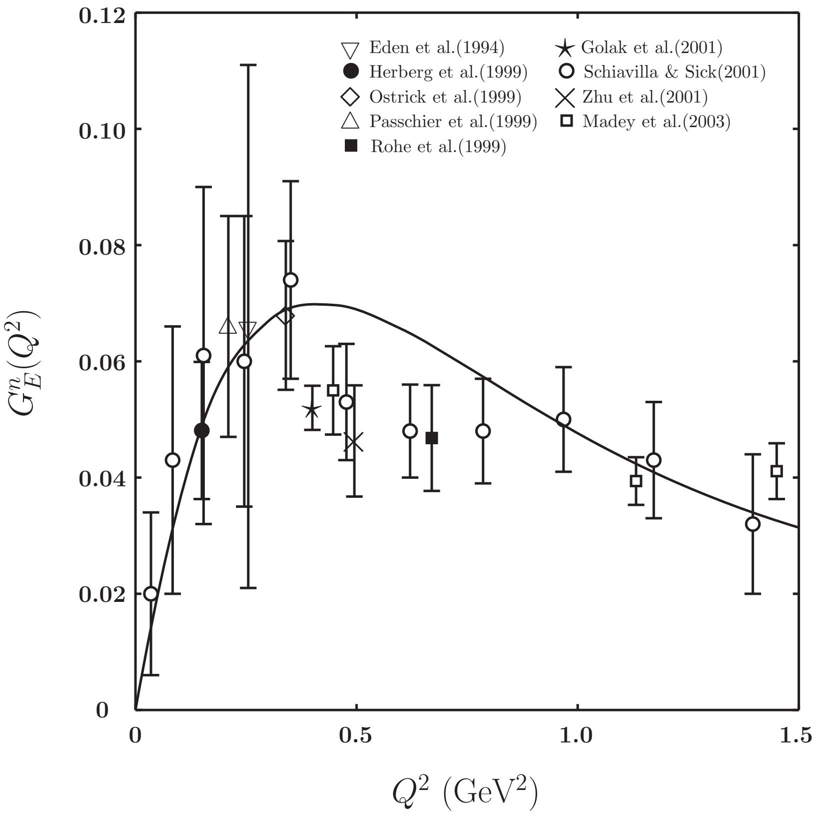
Ratio $Q^2 F_2^p(Q^2)/F_1^p(Q^2)$

Nucleons



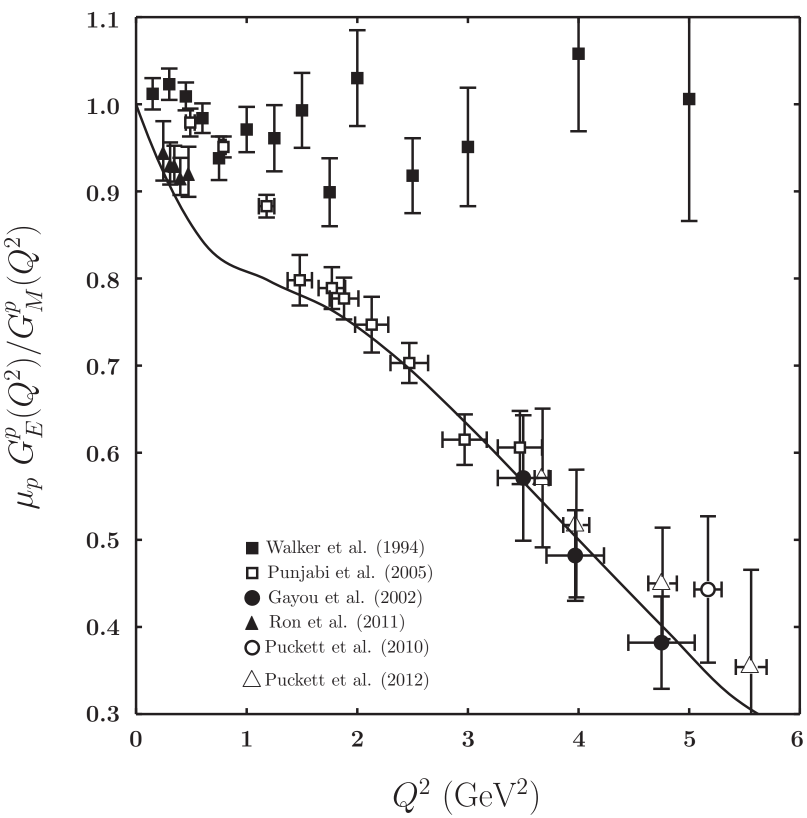
Ratio $G_E^p(Q^2)/G_D(Q^2)$

Nucleons



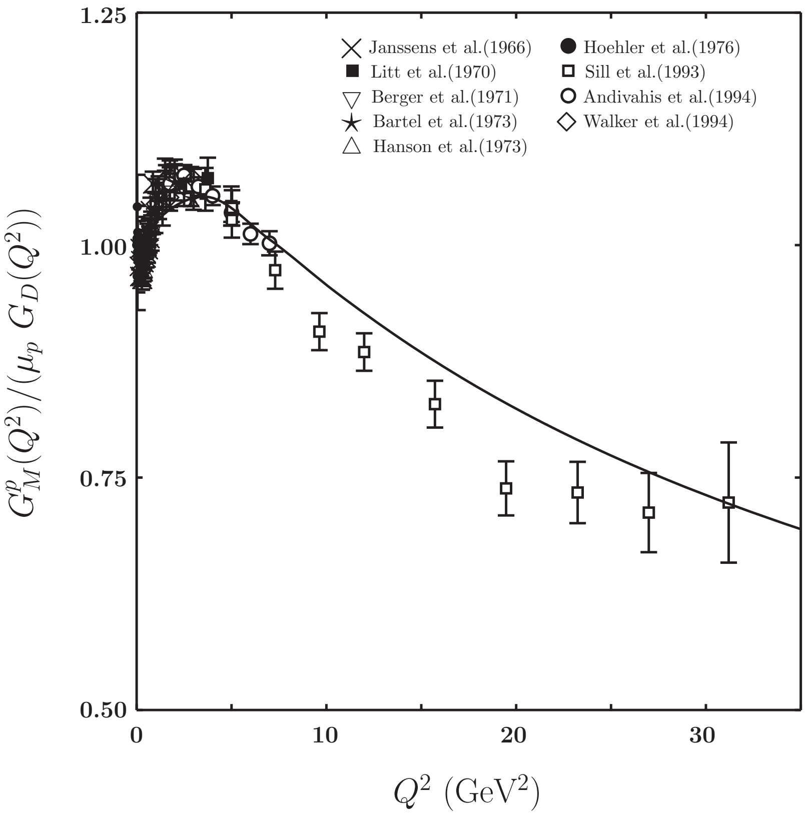
Charge neutron form factor $G_E^n(Q^2)$

Nucleons



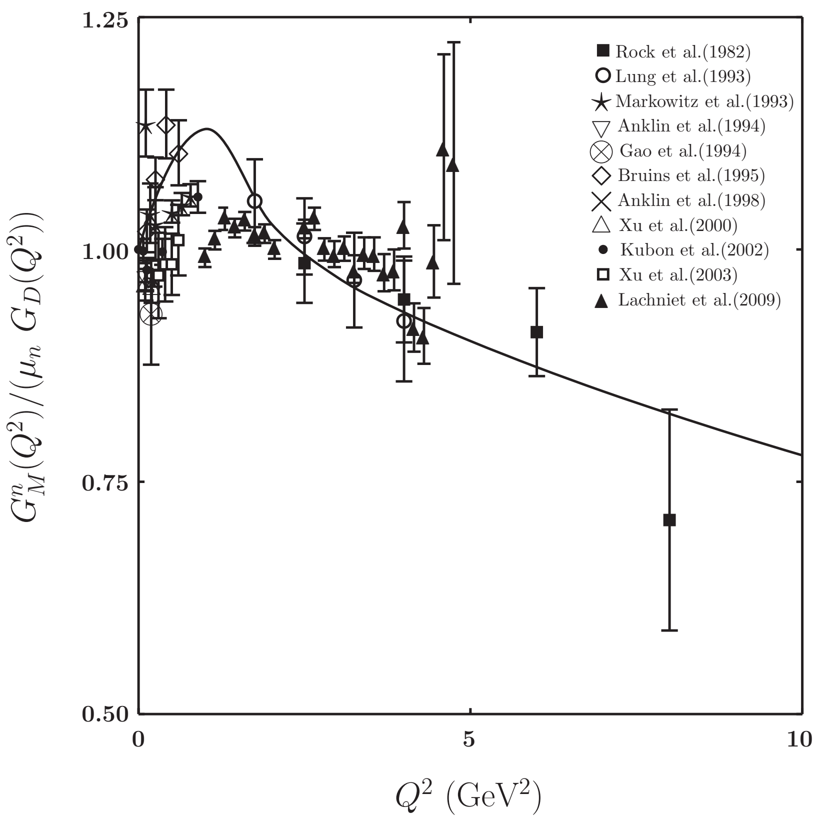
Ratio $\mu_p G_E^p(Q^2)/G_M^p(Q^2)$

Nucleons



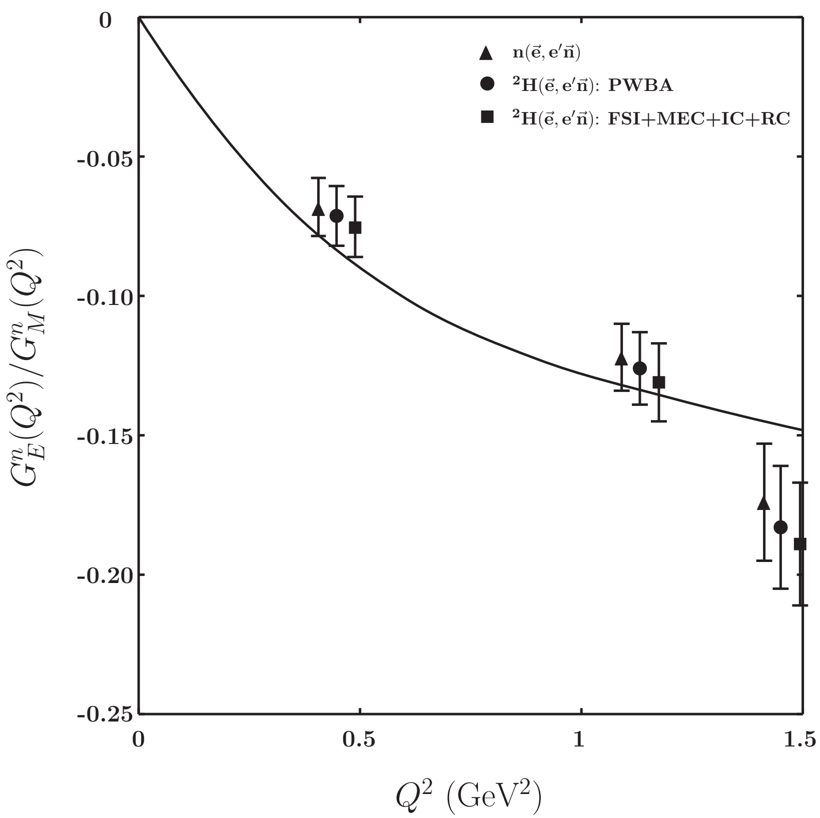
Ratio $G_M^p(Q^2)/(\mu_p G_D(Q^2))$

Nucleons



Ratio $G_M^n(Q^2)/(\mu_n G_D(Q^2))$

Nucleons



Ratio $G_E^n(Q^2)/G_M^n(Q^2)$

Summary

- AdS/QCD $\equiv$ Holographic QCD (HQCD) – approximation to QCD: attempt to model Hadronic Physics in terms of fields/strings living in extra dimensions – anti-de Sitter (AdS) space
- HQCD models reproduce main features of QCD at low and high energies
- Soft–wall holographic approach – covariant and analytic model for hadron structure with confinement at large distances and conformal behavior at short distances
- Including nontrivial dilaton potential could help to describe FFs, PDFs and GPDs from unified point of view (in progress)
- Now: we fit FFs and PDFs using generalized LFWFs (must be derived from AdS/QCD — LF QCD matching)
- Future work: nucleon TMDs