Femtoscopic correlations of two identical particles with nonzero spin in the model of one-particle multipole sources

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1. In the framework of the model of independent sources [Podgoretsky, 1989] with the angular momentum J and the projections of angular momentum onto the coordinate axis z , equaling M and M' , the amplitude of emission of two identical particles with the momentum $\mathbf{p}_1$, helicity λ_1 and momentum $\mathbf{p}_2$, helicity λ_2 has the following structure :

$$A_{M\,M'}(\mathbf{p}_1,\lambda_1;\ \mathbf{p}_2,\lambda_2) = D^{(J)}_{\lambda_1 M}(\mathbf{n}_1)\, D^{(J)}_{\lambda_2 M'}(\mathbf{n}_2)\ e^{i\,p_1 x_1}\ e^{i\,p_2 x_2} + D^{(J)}_{\lambda_2 M}(\mathbf{n}_2)\, D^{(J)}_{\lambda_1 M'}(\mathbf{n}_1)\ e^{i\,p_1 x_2}\ e^{i\,p_2 x_1}, \quad (1)$$

where x_1 and x_2 are the space–time coordinates of two multipole sources,

$$p_1 x_1 = E_1 t_1 - \mathbf{p}_1 \mathbf{x}_1\ ,\ \ p_2 x_2 = E_1 t_1 - \mathbf{p}_2 \mathbf{x}_2\ ,$$

$$D^{(J)}_{\lambda_1 M}(\mathbf{n}_1) = D^{(J)}_{\lambda_1 M}(\,0, \theta_1, \varphi_1) = \left(d_y(\,0, \theta_1, \varphi_1)\;\; e^{\,i\,M\,\varphi_1} \right)_{\lambda_1 M} \;, \tag{2}$$

$$D^{(J)}_{\lambda_2 M'}(\mathbf{n}_2) = D^{(J)}_{\lambda_2 M'}(\,0, \theta_2, \varphi_2) = \left(d_y(\,0, \theta_2, \varphi_2)\;\; e^{\,i\,M'\,\varphi_2} \right)_{\lambda_2 M'}$$

are elements of the finite rotation matrix corresponding to the angular momentum J,

$$\mathbf{n}_1 = \mathbf{p}_1 / |\mathbf{p}_1| \;, \quad \mathbf{n}_2 = \mathbf{p}_2 / |\mathbf{p}_2| \;,$$

θ_1 and θ_2 , φ_1 and φ_2 – polar and azimuthal angles of the momenta $\mathbf{p}_1$ and $\mathbf{p}_2$, respectively .

2. The probability of emission of two identical particles with spin S, respective 4-momenta p_1 , p_2 and helicities λ_1 , λ_2 by two multipole sources with the angular momentum J and projections M , M' of angular momentum onto the axis z amounts to :

$$W_{M\,M'}\,(p_1, \lambda_1;\; p_2, \lambda_2) =$$

$$= |\,D^{(J)}_{\lambda_1 M}(\mathbf{n}_1)\,|^2\,|\,D^{(J)}_{\lambda_2 M'}(\mathbf{n}_2)\,|^2 + |\,D^{(J)}_{\lambda_1 M'}(\mathbf{n}_1)\,|^2\,|\,D^{(J)}_{\lambda_2 M}(\mathbf{n}_2)\,|^2 + \qquad (3)$$

$$+\,2\,(-1)^{2S}\,\mathrm{Re}\left(D^{(J)}_{\lambda_1 M}(\mathbf{n}_1)\; D^{*(J)}_{\lambda_2 M}(\mathbf{n}_2)\; D^{(J)}_{\lambda_1 M'}(\mathbf{n}_1)\; D^{(J)}_{\lambda_2 M'}(\mathbf{n}_2) \right) \cos\,(qx)\,,$$

where $q = p_1 - p_2$ is the difference of 4-momenta of two identical particles and $x = x_1 - x_2$ is the difference of 4-coordinates of two one-particle multipole sources.

Now let us average this expression over the angular momentum projections M , M' and over the space–time dimensions of the emission region . In doing so,

we take into account that, due to the unitarity of the finite rotation matrix, the following relations hold :

$$\sum_{M=-J}^{J} |\, D^{(J)}_{\lambda_1 M}(\mathbf{n}_1)\,|^2 = \sum_{M'=-J}^{J} |\, D^{(J)}_{\lambda_2 M'}(\mathbf{n}_2)\,|^2 =$$

$$= \sum_{M=-J}^{J} |\, D^{(J)}_{\lambda_2 M}(\mathbf{n}_2)\,|^2 = \sum_{M'=-J}^{J} |\, D^{(J)}_{\lambda_1 M'}(\mathbf{n}_1)\,|^2 = 1 \quad . \qquad (4)$$

Let us remark that, without losing generality, we may choose the coordinate axis z as lying in the plane of the momenta $\mathbf{p}_1$ and $\mathbf{p}_2$, with the axis y being perpendicular to this plane . Then the azimuthal angles of the momenta $\mathbf{p}_1$ and $\mathbf{p}_2$ will be equal to zero :

$$\varphi_1 = \varphi_2 = 0 \quad ,$$

and the angle $\beta = \theta_1 - \theta_2$ will have the meaning of angle between the momenta $\mathbf{p}_1$ and $\mathbf{p}_2$. In doing so, once again due to the unitarity of the finite rotation matrix, we obtain :

$$\sum_{M=-J}^{J} D^{(J)}_{\lambda_1 M}(\mathbf{n}_1)\, D^{*(J)}_{M \lambda_2}(\mathbf{n}_2) = \sum_{M=-J}^{J} \left(e^{-i J_y \theta_1} \right)_{\lambda_1 M} \left(e^{i J_y \theta_2} \right)_{M \lambda_2} =$$

$$= \left(e^{-i(\theta_1 - \theta_2) J_y} \right)_{\lambda_1 \lambda_2} = \left(d_y^{(J)}(\beta) \right)_{\lambda_1 \lambda_2} \quad ; \qquad (5)$$

$$\sum_{M'=-J}^{J} D^{(J)}_{\lambda_2 M'}(\mathbf{n}_2)\, D^{*(J)}_{M' \lambda_1}(\mathbf{n}_1) = \sum_{M'=-J}^{J} \left(e^{-i J_y \theta_2} \right)_{\lambda_2 M'} \left(e^{i J_y \theta_1} \right)_{M' \lambda_1} =$$

$$= \left(e^{i(\theta_1 - \theta_2) J_y} \right)_{\lambda_2 \lambda_1} = \left(d_y^{(J)}(-\beta) \right)_{\lambda_2 \lambda_1} \quad ; \qquad (6)$$

There is a well-known symmetry relation [Landau, Lifshitz, 1989]:

$$\left(d_y^{(J)}(\beta) \right)_{\lambda_1 \lambda_2} = \left(d_y^{(J)}(-\beta) \right)_{\lambda_2 \lambda_1} . \qquad (7)$$

As a result, we obtain :

$$\overline{W_{M M'}}(p_1, \lambda_1; p_2, \lambda_2) = \frac{1}{(2J+1)^2} \left(2 + 2 \left(d_{\lambda_1 \lambda_2}^{(J)}(\beta) \right)^2 (-1)^{2S} \langle \cos(q x) \rangle \right) . \qquad (8)$$

Let us emphasize that the quantity

$$r = \left(d_{\lambda_1 \lambda_2}^{(J)}(\beta) \right)^2$$

has the meaning of the degree of non-orthogonality (non-distinguishability) of particle states with different helicities with respect to the momenta with the angle $\beta = \theta_1 - \theta_2$ between them: $\langle \lambda_1 | \lambda_2 \rangle \neq 0$.

3. If the emitted identical particles with the momenta $\mathbf{p}_1$ and $\mathbf{p}_2$ are unpolarized, then – after averaging over all the values of helicity being allowed at spin S – we obtain :

$$\overline{W}(q) = \left(2(2S+1)^2 + (-1)^{2S}\, 2 \sum_{\lambda_1=-S}^{S} \sum_{\lambda_2=-S}^{S} |d^{(J)}_{\lambda_1\lambda_2}(\beta)|^2 \langle \cos(qx) \rangle \right) \frac{1}{(2J+1)^2} \frac{1}{(2S+1)^2} \quad . \tag{9}$$

At sufficiently large momentum differences q the correlation function, normalized by unity, will have the form :

$$R(q) = 1 + \frac{(-1)^{2S}}{(2S+1)^2} \sum_{\lambda_1=-S}^{S} \sum_{\lambda_2=-S}^{S} |d^{(J)}_{\lambda_1\lambda_2}(\beta)|^2 \langle \cos(qx) \rangle \quad . \tag{10}$$

If the angle between the momenta equals zero (or if we deal not with helicities but with projections of spin of both the identical particles onto the same coordinate axis), then, since

$$d^{(J)}_{\lambda_1\lambda_2}(0) = \delta_{\lambda_1\lambda_2} \qquad (11)$$

we obtain :

$$R(q) = 1 + (-1)^{2S} \frac{1}{2S+1} \langle \cos(qx) \rangle \qquad (12)$$

Taking into account the unitarity of the matrix $d^{(J)}_{\lambda_1\lambda_2}(\beta)$, at $J = S$ formula (12) is valid at any angles between the momenta $\mathbf{p}_1$ and $\mathbf{p}_2$. Let us stress that Eq. (12) is related to particles with nonzero mass .

4. In the case of unpolarized photons, when the mass equals zero, spin $S = 1$ and the helicities of each of the photons take only two values -1 and 1, irrespective of the momentum direction, the correlation function for dipole sources takes the form [Lyuboshitz, Podgoretsky, 1995] :

$$R(q) = 1 + \frac{1}{4}\left[\left(d^{(1)}_{11}(\beta) \right)^2 + \left(d^{(1)}_{-1,1}(\beta) \right)^2 + \left(d^{(1)}_{-1,-1}(\beta) \right)^2 + \left(d^{(1)}_{1,-1}(\beta) \right)^2 \right] \langle \cos(qx) \rangle \quad (13)$$

Taking into account the equalities :

$$d^{(1)}_{11}(\beta) = d^{(1)}_{-1,-1}(\beta) = \frac{1+\cos\beta}{2}, \quad d^{(1)}_{1,-1}(\beta) = d^{(1)}_{-1,1}(\beta) = \frac{1-\cos\beta}{2}, \quad (14)$$

we find :

$$R(q) = 1 + \frac{1}{4}\,(1 + \cos^2\beta)\,\langle \cos(qx) \rangle \qquad (15)$$

At $\beta \ll 1$,

$$R(q) = 1 + \frac{1}{2}\,\langle \cos(qx) \rangle \qquad (16)$$

For the case of quadrupole sources ,

$$R(q) = 1 + \frac{1}{4}\left[(d^{(2)}_{11}(\beta))^2 + (d^{(2)}_{-1,1}(\beta))^2 + (d^{(2)}_{-1,-1}(\beta))^2 + (d^{(2)}_{1,-1}(\beta))^2 \right] \langle \cos(qx) \rangle\,. \qquad (17)$$

Taking into account the equalities :

$$d_{11}^{(2)}(\beta) = d_{-1,-1}^{(2)}(\beta) = \frac{1+\cos\beta}{2}\,(2\cos\beta - 1)\,, \tag{18}$$

$$d_{11}^{(2)}(\beta) = d_{-1,-1}^{(2)}(\beta) = \frac{1+\cos\beta}{2}\,(2\cos\beta - 1)\,, \tag{19}$$

we find the correlation function of two unpolarized photons emitted by the quadrupole sources :

$$R(q) = 1 + \frac{1}{4}\,(4\cos^4\beta - 3\cos^2\beta + 1)\,\langle \cos(qx) \rangle \tag{20}$$

At $\beta \approx 0$ we have :

$$R(q) = 1 + \frac{1}{2} \langle \cos (qx) \rangle \qquad (21)$$

i.e. we obtain the standard formula corresponding to two directions of polarization for each of the photons [Lyuboshitz, Podgoretsky, 1995] .

In the case of two "left" neutrinos (two "right" antineutrinos), with helicity taking only one value $\lambda_1 = +\frac{1}{2}$, the correlation function in the model of multipole sources is as follows :

$$R(q) = 1 - (d^{(J)}_{1/2\,1/2}(\beta))^2 \langle \cos (qx) \rangle \quad . \qquad (22)$$

At $J = S = \frac{1}{2}$

$$R(q) = 1 - \cos^2\frac{\beta}{2} \langle \cos(qx) \rangle \quad (23)$$

In the limit $\beta \to 0$ we obtain :

$$R(q) = 1 - \frac{1+\cos\beta}{2} \langle \cos(q\,x) \rangle = 1 - \langle \cos(q\,x) \rangle \quad (24)$$

References

- **M.I. Podgoretsky**, EChAYa, 1989, v. 20, p. 266
- **L.D. Landau, E.M. Lifshitz** . *Quantum Mechanics. Nonrelativistic theory* (Nauka, Moscow, 1989), § 58 .
- **V.L. Lyuboshitz, M.I. Podgoretsky**. Yadernaya Fizika, 1995, v.58, p.33 (Physics of Atomic Nuclei, 1995, v.58, p.30)

Thank you !