Light mesons in a strong magnetic field on the lattice

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Computer simulations of strongly interacting systems

Quantum chromodynamics:

1. spectrum of mesons ($\bar{q}_i q_j$) and barions ($q_i q_j q_k$) (masses, decay constants, excited states);

2. confinement problem (computers can prove confinement "numerically");

3. the structure of QCD phase diagram;

QCD effects which occurs at very big magnetic fields in:

1. noncentral heavy ion collisions;

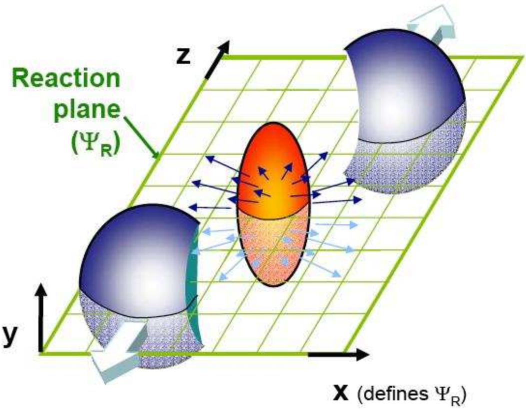
2. early Universe. It is assumed that magnetic fields $\sim 2\ GeV$ existed in the Universe during the electroweak phase transition;

Graphene as quantum field theory

1. monolayer and bilayer graphene (conductivity, dielectric permittivity);

2. graphene in external magnetic field.

$eB \sim \Lambda^2_{QCD}$



$eB \gtrsim 15 m_\pi^2 \sim 0.27\, GeV^2, \quad m_\pi = 135\, MeV$

Experiments: ALICE(LHC), RHIC(BNL), FAIR(GSI), NICA(JINR)

Introduction

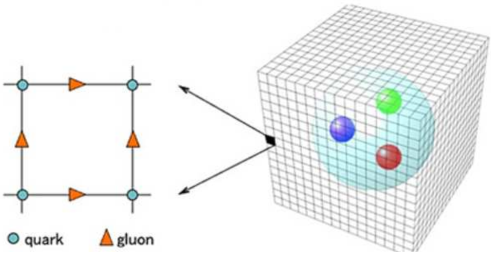
We work in $SU(3)$ lattice gauge theory without dynamical quarks in a constant external magnetic field directed along the third axis z and calculate:

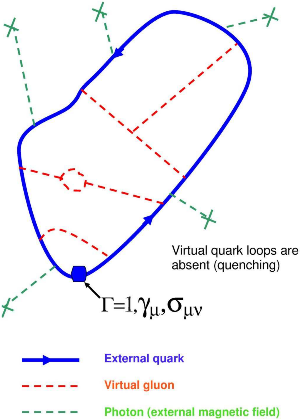
1. explore the dependence of neutral ρ^0 and π^0 meson masses versus the value of the magnetic field;
2. explore the dependence of charged $\rho^{\pm}$ and $\pi^{\pm}$ meson masses as a function of the field.

$\pi^0(J=0),\ \rho^0(J=1)$: $\psi_{\pi^0,\rho^0} = (\psi_u \psi_{\bar{u}} - \psi_d \psi_{\bar{d}})/\sqrt{2}$
$\pi^+(J=0),\ \rho^+(J=1)$: $\psi_{\pi^+,\rho^+} = \psi_u \psi_{\bar{d}}$
$\pi^-(J=0),\ \rho^-(J=1)$: $\psi_{\pi^-,\rho^-} = \psi_d \psi_{\bar{u}}$

Technical details

Model: quenched $SU(3)$ lattice gauge theory.
Generation of A_μ: the tadpole-improved Wilson-Symanzik action, 16^4 and 18^4 lattice volumes, $a = 0.115\ fm,\ 0.125\ fm$ lattice spacing.
Fermion spectrum: chirally-invariant overlap operator, 50 lowest modes for the inversion, different quark masses $am_q^{lat} = 0.15, 0.02, 0.03$.





We calculate correlators of currents $\bar{\psi} O \psi$ (where $O = \gamma_\mu,\ \gamma_5$) in the external $U(1)$ magnetic field and in the presence of vacuum $SU(3)$ nonabelian gluon fields.

Technical details

Solve Dirac equation with the overlap operator numerically:

$$D\psi_k = i\lambda_k\psi_k, \;\; D = \gamma^\mu(\partial_\mu - iA_\mu).$$

Calculate the propagators:

$$D^{-1}(x,y) = \sum_{k<M} \frac{\psi_k(x)\psi_k^\dagger(y)}{i\lambda_k + m}.$$

Calculate the correlators on the lattice:

$$\langle\bar\psi O_1\psi\bar\psi O_2\psi\rangle_A = -tr[O_1 D^{-1}(x,y)O_2 D^{-1}(y,x)] +$$

$$+tr[O_1 D^{-1}(x,x)]tr[O_2 D^{-1}(y,y)], \;\; x = (\mathbf{n}a, n_t a), \;\; y = (\mathbf{n}'a, n_t' a)$$

$$\mathbf{n}, \mathbf{n}' \in \Lambda_3 = \{(n_1, n_2, n_3) | n_i = 0, 1, ..., N-1\}$$

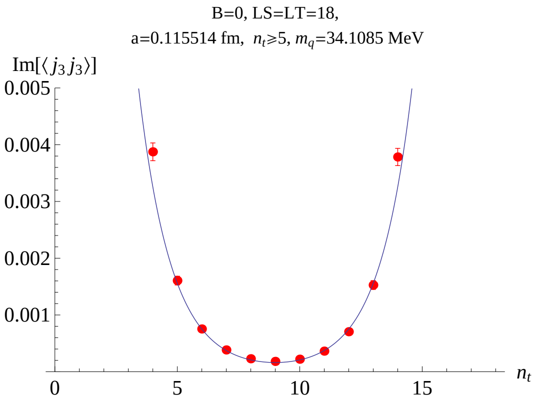
Correlators

$$G(\vec{p},\tau) = \frac{1}{N^{3/2}} \sum_{\mathbf{n}\in\Lambda_3} \langle j_\mu(\mathbf{n}, n_t) j_\mu^\dagger(\mathbf{0}, 0)\rangle e^{-ia\mathbf{np}}$$

$p_i = 2\pi k_i/(aN), \; k_i = -N/2+1, ..., N/2.$
We obtain the masses from the correlator of currents

$$\langle \psi^\dagger(\vec{0}, n_t) O_1 \psi(\vec{0}, n_t) \psi^\dagger(\vec{0}, 0) O_2 \psi(\vec{0}, 0)\rangle_A = \sum_k \langle 0|O_1|k\rangle\langle k|O^\dagger|0\rangle e^{-n_t E_k}.$$

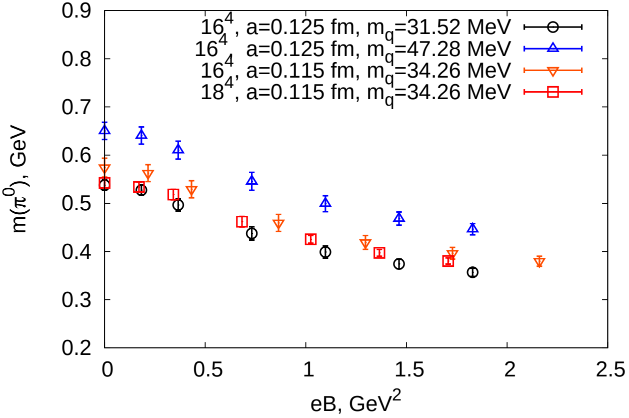
The main contributions comes from $\langle 0|O_1|k\rangle\langle k|O^\dagger|0\rangle e^{-n_t E_0}$, We set $\langle \mathbf{p}\rangle = 0$. So $E_0 = m_0$ because $E^2 - \mathbf{p}^2 = m^2$.



We fit the correlator by the function
$\tilde{C}_{fit}(n_t) = A_0 e^{-n_t a E_0} + A_0 e^{-(N_T - n_t) a E_0} =$
$2 A_0 e^{-N_T a E_0 / 2} \cosh((N_T - n_t) a E_0)$ at $5 \leq n_t \leq N_T - 5$

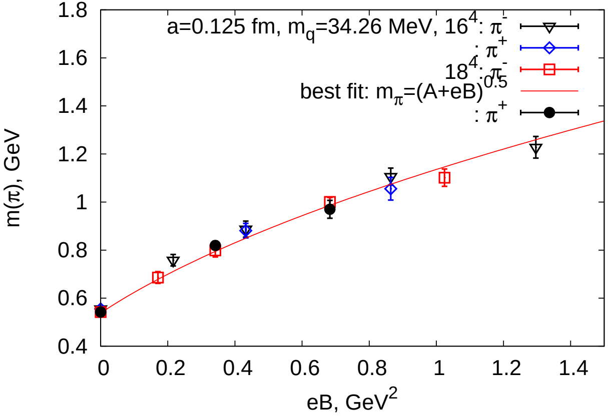
Mass of π^0 meson vs. the magnetic field

$$C^{PSPS} = \langle \bar{\psi}(\vec{0}, n_t)\gamma_5 \psi(\vec{0}, n_t)\bar{\psi}(\vec{0}, 0)\gamma_5 \psi(\vec{0}, 0)\rangle$$



Mass of $\pi^{\pm}$ meson vs. the magnetic field

$$C^{PSPS} = \langle \bar{\psi}_u(\vec{0}, n_t)\gamma_5\psi_u(\vec{0}, n_t)\bar{\psi}_d(\vec{0}, 0)\gamma_5\psi_d(\vec{0}, 0)\rangle$$



Lowest Landau level for the charged pointlike particle:

$$m_{\pi^\pm}(B) = \sqrt{m^2_{\pi^\pm}(B=0) + eB}, \quad r_L = 1/\sqrt{eB}$$

Correlators of vector currents in the directions perpendicular to the magnetic field

$$C_{xx}^{VV} = \langle\bar{\psi}(\vec{0},n_t)\gamma_1\psi(\vec{0},n_t)\bar{\psi}(\vec{0},0)\gamma_1\psi(\vec{0},0)\rangle$$

$$C_{yy}^{VV} = \langle\bar{\psi}(\vec{0},n_t)\gamma_2\psi(\vec{0},n_t)\bar{\psi}(\vec{0},0)\gamma_2\psi(\vec{0},0)\rangle$$

parallel to the magnetic field

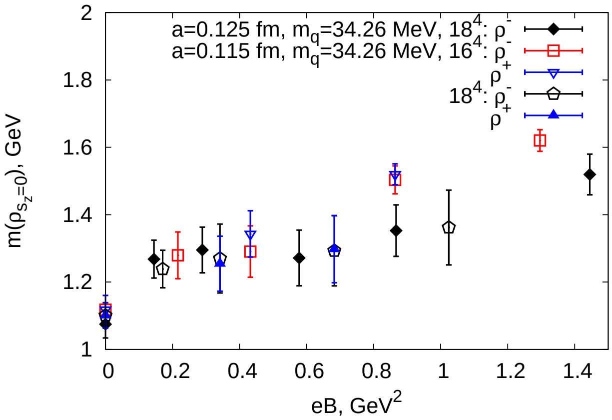
$$C_{zz}^{VV} = \langle\bar{\psi}(\vec{0},n_t)\gamma_3\psi(\vec{0},n_t)\bar{\psi}(\vec{0},0)\gamma_3\psi(\vec{0},0)\rangle$$

We obtain masses with $s_z = \pm 1$ from the combinations

$$C^{VV}(s_z = \pm 1) = C_{xx}^{VV} + C_{yy}^{VV} \pm i(C_{xy}^{VV} - C_{yx}^{VV})$$

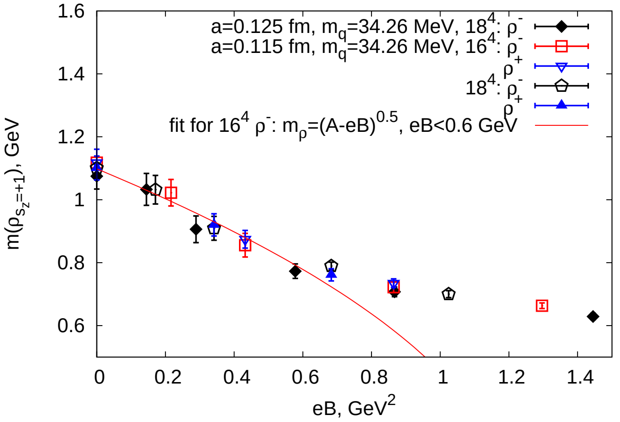
The masses with $s_z = 0$ from $C^{VV}(s_z = 0) = C_{zz}^{VV}$

Charged $\rho^{\pm}$ meson with $s_z = 0$



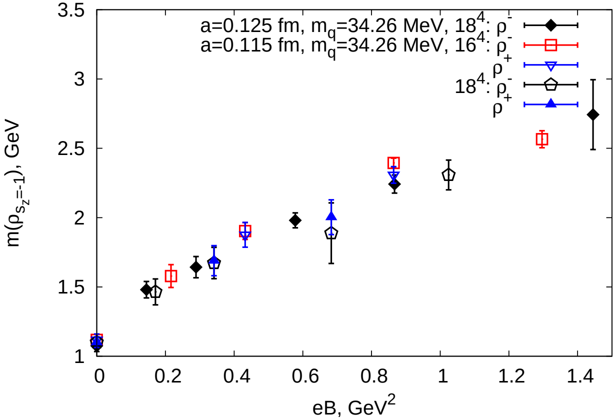
The $\rho^{\pm}(s_z = 0)$ meson mass with zero spin projection to the direction of the magnetic field.

The mass of charged $\rho^{\pm}$ meson with $s_z = +1$ vs. the magnetic field



Maxim Chernodub, 2010 : $m_{\rho^{\pm}}(B) = \sqrt{m^2_{\rho^{\pm}}(B=0) - eB}$.

The mass of charged $\rho^{\pm}$ meson with $s_z = -1$ vs. the magnetic field

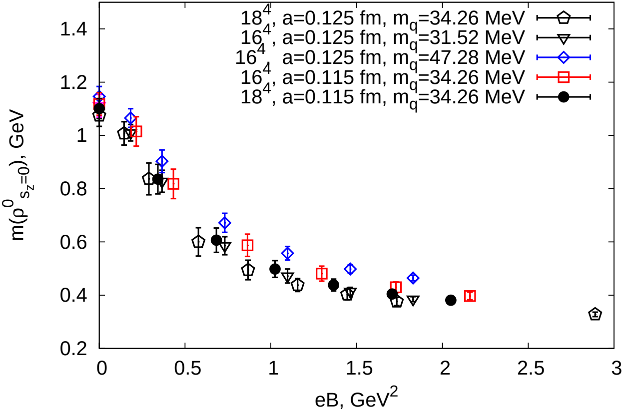


g-factor of ρ meson (preliminary)

$$g = \frac{E(0)(E(H) - E(-H))}{H} \quad at \;\; H \to 0$$

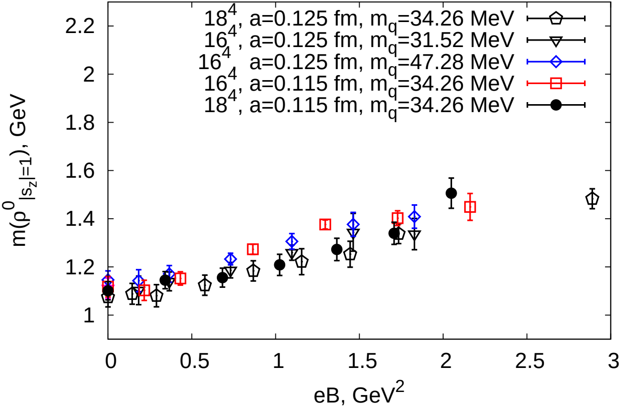
Our data: $g = 2.5 \pm 0.2$ (in collaboration with O.V.Teryaev)
Preliminary, because we need to make extrapolations to $m_q = 3\, MeV$.
Previous results:
$g \simeq 2.4$ F. X. Lee et.al., Phys. Rev. D 78, 094502 (2008).
$g \simeq 2.3$ J. N. Hedditch et. al. Phys. Rev. D 75, 094504 (2007).
Relativistic quark model: $g \simeq 2.37$ Ya.A. Simonov, arXiv: 1211.4349.
QCD sum rules: $g = 2.4 \pm 0.4$ T. M. Aliev et. al., Phys. Lett. B 678, 470 (2009).

Neutral ρ^0 meson with zero spin projection to B



The $\rho^0(s_z = 0)$ meson mass versus the magnetic field.

Neutral ρ^0 meson with nonzero spin projection to B



The $\rho^0(s_z = \pm 1)$ meson mass as a function of the magnetic field.

Conclusions

1. In $SU(3)$ gluodynamics the masses of charged vector mesons $\rho^{\pm}$ with spin $s_z = 0$ and $s_z = -1$ increase with the field.
2. The masses of $\rho^{\pm}$ with $s_z = 1$ decrease with the field in accordance with [M. Chernodub '10].
3. The masses of neutral vector ρ^0 mesons with spin projection $s_z = 0$ to the direction of the magnetic field decrease with the value of the field.
4. The masses of ρ^0 with spin projection $s_z = \pm 1$ increase with the magnetic field.