Loop mixing of the opposite parity fermion fields and its manifestation in πN scattering

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Outline

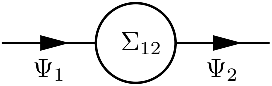
- Introduction
- Opposite parity fermion (OPF) fields mixing and K-matrix
- Descripting of PWA results for S_{11} and P_{11} waves
- Conclusions

Mixing of states (fields) is a well-known phenomenon existing in the systems of neutrinos, quarks and hadrons. As for theoretical description of mixing phenomena, a general tendency with time and development of experiment consists in transition from a simplified quantum-mechanical description to the quantum field theory methods.

Mixing of fermion fields has some specifics as compared with boson case. Firstly, there exists γ-matrix structure in a propagator. Secondly, fermion and antifermion have the opposite P-parity, so fermion propagator contains contributions of different parities. As a result, besides a standard mixing of fields with the same quantum numbers, for fermions there exists a mixing of fields with opposite parities (OPF-mixing) at loop level, even if the parity is conserved in Lagrangian.

Below we say about non-standard effect of OPF-mixing and its manifestation in systems of baryon resonances.

First of all, look at the non-diagonal self-energy:



Let parity is conserved in Lagrangian.
Mixing of fields with the same quantum numbers:

$$\begin{aligned}\Sigma_{12} &= A(p^2) + \hat{p}B(p^2) = \\ &= \Lambda^+\left[A(W^2) + WB(W^2)\right] + \Lambda^-\left[A(W^2) - WB(W^2)\right]\end{aligned}$$

Mixing of fields with opposite parities:

$$\begin{aligned}\Sigma_{12} &= \gamma^5 C(p^2) + \hat{p}\gamma^5 D(p^2) = \\ &= \Lambda^+\gamma^5\left[C(W^2) + WD(W^2)\right] + \Lambda^-\gamma^5\left[C(W^2) - WD(W^2)\right]\end{aligned}$$

Main statement: $\Sigma_{12} \neq 0$ for mixing of opposite parities fields. Fermion specifics!

Where OPF-mixing can be seen?

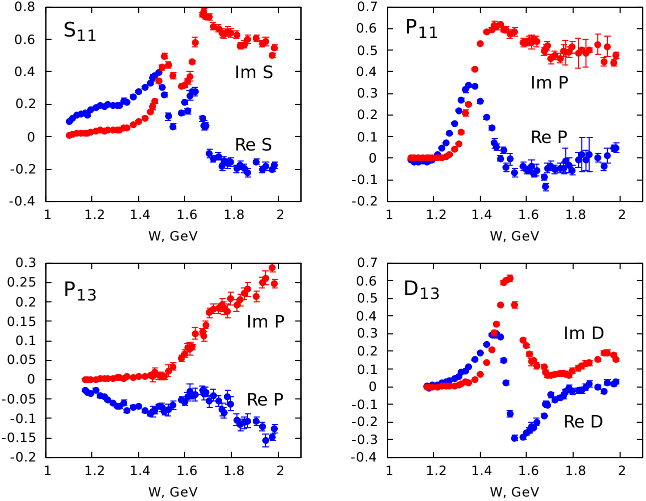
Below we will discuss manifestation of OPF-mixing in πN scattering. There are two places, where we can identify this effect:

1. Simplest one is the pair of partial waves P_{13}, D_{13}, where baryons $3/2^{\pm}$ are produced. It was discussed in: **A. Kaloshin, E. Kobeleva and V. Lomov, Int. J. Mod. Phys. A26 (2011) 2307** on the base of the matrix propagator.
2. OPF-mixing in another pair: S_{11}, P_{11} ($J^P = 1/2^{\pm}$) is subject of paper: **A. Kaloshin, E. Kobeleva and V. Lomov, Mod. Phys. Lett. A28 (2013) 1350156**. This required to develop a variant of K-matrix, which includes this effect.

We will say mainly about last item: OPF-mixing in partial waves S_{11}, P_{11}.

Partial wave analysis (PWA) of $\pi N \to \pi N$ with $I = 1/2$

R. A. Arndt et al. Phys. Rev. C74 (2006) 045205; (gwdac.phys.gwu.edu)



The pair of partial waves P_{13}, D_{13} looks as simplest case for identification of the discussed OPF-mixing effect.

We need to discuss the effect of OPF-mixing in amplitudes of πN scattering and its implementation in framework of K-matrix description. For a first step one may restrict oneself by a simplified case: two resonance states and two channels.

Effective Lagrangians $\pi N N'$ without derivatives and conserving the parity:

$$\mathscr{L}_{\text{int}} = g_1 \bar{N}_1(x) N(x) \phi(x) + \text{h.c.}, \quad \text{for } J^P(N_1) = 1/2^-, \tag{1}$$

$$\mathscr{L}_{\text{int}} = \imath g_2 \bar{N}_2(x) \gamma^5 N(x) \phi(x) + \text{h.c.}, \quad \text{for } J^P(N_2) = 1/2^+. \tag{2}$$

Let us consider two baryon states of opposite parities with masses m_1 ($J^P = 1/2^-$), m_2 ($J^P = 1/2^+$) and two intermediate states πN, ηN. Using the effective Lagrangians we can calculate contributions of states N_1, N_2 to partial waves at tree level:

s-wave amplitudes:

$$f_{s,+}^{\text{tree}}(\pi N \to \pi N) = -\frac{(E_N^{(\pi)} + m_N)}{8\pi W}\left(\frac{g_{1,\pi}^2}{W - m_1} + \frac{g_{2,\pi}^2}{W + m_2}\right),$$
$$f_{s,+}^{\text{tree}}(\pi N \to \eta N) = -\frac{\sqrt{(E_N^{(\pi)} + m_N)(E_N^{(\eta)} + m_N)}}{8\pi W}\left(\frac{g_{1,\pi}g_{1,\eta}}{W - m_1} + \frac{g_{2,\pi}g_{2,\eta}}{W + m_2}\right),$$
$$f_{s,+}^{\text{tree}}(\eta N \to \eta N) = -\frac{(E_N^{(\eta)} + m_N)}{8\pi W}\left(\frac{g_{1,\eta}^2}{W - m_1} + \frac{g_{2,\eta}^2}{W + m_2}\right) \tag{3}$$

and p-wave amplitudes:

$$f_{p,-}^{\text{tree}}(\pi N \to \pi N) = \frac{(E_N^{(\pi)} - m_N)}{8\pi W}\left(\frac{g_{1,\pi}^2}{-W - m_1} + \frac{g_{2,\pi}^2}{-W + m_2}\right),$$
$$f_{p,-}^{\text{tree}}(\pi N \to \eta N) = \frac{\sqrt{(E_N^{(\pi)} - m_N)(E_N^{(\eta)} - m_N)}}{8\pi W}\left(\frac{g_{1,\pi}g_{1,\eta}}{-W - m_1} + \frac{g_{2,\pi}g_{2,\eta}}{-W + m_2}\right),$$
$$f_{p,-}^{\text{tree}}(\eta N \to \eta N) = \frac{(E_N^{(\eta)} - m_N)}{8\pi W}\left(\frac{g_{1,\eta}^2}{-W - m_1} + \frac{g_{2,\eta}^2}{-W + m_2}\right). \tag{4}$$

Here $W = \sqrt{s}$ is the total CMS energy and $E_N^{(\pi)}$ ($E_N^{(\eta)}$) is nucleon CMS energy of system πN (ηN)

$$E_N^{(\pi)} = \frac{W^2 + m_N^2 - m_\pi^2}{2W}. \tag{5}$$

Short notations for coupling constants, e.g. $g_{1,\pi} = g_{N_1 \pi N}$.
The tree amplitudes (3)–(4) contain poles with both positive and negative energy, originated from propagators of N_1 and N_2 fields of opposite parities.
Accounting the loop transitions results in dressing of states and also in mixing of these two fields.
Note that $W \to -W$ replacement gives

$$E_N^{(\pi)} + m_N \to -\big(E_N^{(\pi)} - m_N\big), \tag{6}$$

so tree amplitudes (3)–(4) exhibit the MacDowell symmetry property (**S. W. MacDowell, Phys. Rev. 116 (1959) 774**)

$$f_{p,-}(W) = -f_{s,+}(-W). \tag{7}$$

In K-matrix representation for partial amplitudes

$$f = K\big(1 - \imath PK\big)^{-1}, \qquad (8)$$

diagonal matrix $\imath P$, constructed from CMS momenta, originates from imaginary part of a loop. Therefore, K-matrix here is simply a matrix of tree amplitudes that should be identified with amplitudes (3), (4).
As a result we come to representation of partial amplitudes for s- and p-waves

$$f_s(W) = K_s(W)\big(1 - \imath PK_s(W)\big)^{-1}, \quad f_p(W) = K_p(W)\big(1 - \imath PK_p(W)\big)^{-1}, \qquad (9)$$

where the matrices K_s, K_p (i.e. tree amplitudes (3), (4)), may be written in factorized form

$$K_s = -\frac{1}{8\pi}\rho_s\hat{K}_s\rho_s, \quad K_p = \frac{1}{8\pi}\rho_p\hat{K}_p\rho_p. \qquad (10)$$

OPF-mixing and K-matrix

Here ρ_s, ρ_p are

$$\rho_s(W) = \begin{pmatrix} \sqrt{\dfrac{E_N^{(\pi)} + m_N}{W}}, & 0 \\ 0, & \sqrt{\dfrac{E_N^{(\eta)} + m_N}{W}} \end{pmatrix}, \qquad (11)$$

$$\rho_p(W) = \begin{pmatrix} \sqrt{\dfrac{E_N^{(\pi)} - m_N}{W}}, & 0 \\ 0, & \sqrt{\dfrac{E_N^{(\eta)} - m_N}{W}} \end{pmatrix}, \qquad (12)$$

and matrix P consists of CMS momenta as analytic functions of W. In this case "primitive" K-matrices contain poles with both positive and negative energies

$$\hat{K}_s(W) = \begin{pmatrix} \frac{g_{1,\pi}^2}{W - m_1} + \frac{g_{2,\pi}^2}{W + m_2}, & \frac{g_{1,\pi}g_{2,\eta}}{W - m_1} + \frac{g_{2,\pi}g_{2,\eta}}{W + m_2} \\ \frac{g_{1,\pi}g_{2,\eta}}{W - m_1} + \frac{g_{2,\pi}g_{2,\eta}}{W + m_2}, & \frac{g_{1,\eta}^2}{W - m_1} + \frac{g_{2,\eta}^2}{W + m_2} \end{pmatrix}, \tag{13}$$

$$\hat{K}_p(W) = \hat{K}_s(-W) = \begin{pmatrix} \frac{g_{1,\pi}^2}{-W - m_1} + \frac{g_{2,\pi}^2}{-W + m_2}, & \frac{g_{1,\pi}g_{2,\eta}}{-W - m_1} + \frac{g_{2,\pi}g_{2,\eta}}{-W + m_2} \\ \frac{g_{1,\pi}g_{2,\eta}}{-W - m_1} + \frac{g_{2,\pi}g_{2,\eta}}{-W + m_2}, & \frac{g_{1,\eta}^2}{-W - m_1} + \frac{g_{2,\eta}^2}{-W + m_2} \end{pmatrix}. \tag{14}$$

Recall that m_1 is mass of $J^P = 1/2^-$ state and m_2 is mass of $J^P = 1/2^+$ one. Generalization of this construction for the case of more channels and states is obvious.

Since CMS momenta have the property $P(-W) = -P(W)$, the MacDowell symmetry property (7) is extended from tree amplitudes to unitarized K-matrix ones (9).

Naive expectations

Look again at tree partial amplitudes:

$$f_{s,+}^{\text{tree}}(\pi N \to \pi N) = -\frac{(E_N^{(\pi)} + m_N)}{8\pi W}\left(\frac{g_{1,\pi}^2}{W - m_1} + \frac{g_{2,\pi}^2}{W + m_2}\right),$$

$$f_{p,-}^{\text{tree}}(\pi N \to \pi N) = \frac{(E_N^{(\pi)} - m_N)}{8\pi W}\left(\frac{g_{1,\pi}^2}{-W - m_1} + \frac{g_{2,\pi}^2}{-W + m_2}\right).$$

From a common sense one can expect that negative energy pole should give a negligible effect in physical energy region. However, this is not the case if corresponding coupling constant is large $|g_{2,\pi}| \gg |g_{1,\pi}|$. One can compare decay width of s- and p-states

$$\Gamma(N_1 \to \pi N) = g_{N_1 \pi N}^2 \Phi_s, \quad \Gamma(N_2 \to \pi N) = g_{N_2 \pi N}^2 \Phi_p, \tag{15}$$

where Φ_s, Φ_p are corresponding phase volumes. For resonance states not far from threshold, with masses, e.g. 1.5–1.7 GeV, phase volumes differ greatly, $\Phi_s \gg \Phi_p$. If both resonances have typical hadronic width $\Gamma \sim 100$ MeV, then coupling constants differ dramatically too, $|g_{N_2\pi N}| \gg |g_{N_1 \pi N}|$.

Above we use the simplest effective Lagrangians (1)–(2) to derive tree amplitudes. However, it is well-known, that spontaneous breaking of chiral symmetry requires pion field to appear in Lagrangian only through derivatives

$$\mathscr{L}_{\text{int}} = f_2 \bar{N}_2(x)\gamma^5\gamma^\mu N(x)\partial_\mu\phi(x) + \text{h.c.},\ J^P = 1/2^+,\ f_2 = \frac{g_2}{m_2 + m_N}. \quad (16)$$

It is not difficult to understand how inclusion of derivative changes tree amplitudes and, hence K-matrix. Pole contribution $\pi(k_1)N(p_1) \to N_2(p) \to \pi(k_2)N(p_2)$ in that case takes the form:

$$T = f_2^2 \bar{u}(p_2)\gamma^5\hat{k}_2\frac{1}{\hat{p} - M}\gamma^5\hat{k}_1 u(p_1). \quad (17)$$

With the use of equations of motion, we see that inclusion of derivative at vertex leads to the following modification of resonance contribution

$$g_2^2\frac{1}{\hat{p} - M} \to f_2^2(\hat{p} + m_N)\frac{1}{\hat{p} - M}(\hat{p} + m_N). \quad (18)$$

Separation of the positive and negative energy poles is performed with the off-shell projector operators $\Lambda^{\pm} = 1/2\big(1 \pm \hat{p}/W\big)$

$$f_2^2(\hat{p}+m_N)\frac{1}{\hat{p}-m_N}(\hat{p}+m_N) = \Lambda^+\frac{f_2^2(W+m_N)^2}{W-M} + \Lambda^-\frac{f_2^2(W-m_N)^2}{-W-M}, \quad (19)$$

where the first term gives contribution to p-wave and second one to s-wave. Modification of the pole contributions in "primitive" K-matrices (13)–(14) is evident

$$g_2^2 \to f_2^2(W-m_N)^2, \quad \text{for } s\text{-wave}, \quad (20)$$

$$g_2^2 \to f_2^2(W+m_N)^2, \quad \text{for } p\text{-wave}. \quad (21)$$

One can expect that the inclusion of derivatives most strongly affects on threshold properties of s-wave due to dumping factor $(W-m_N)^2$.

Fit of P_{11}

First of all, let us try to describe S_{11} and P_{11} waves separately. p-wave is described rather well by our formulas with derivative in vertex (20)– (21), see Fig. 1. In this case the s-wave states are missing in amplitudes, the p-wave K-matrix has two positive energy poles.

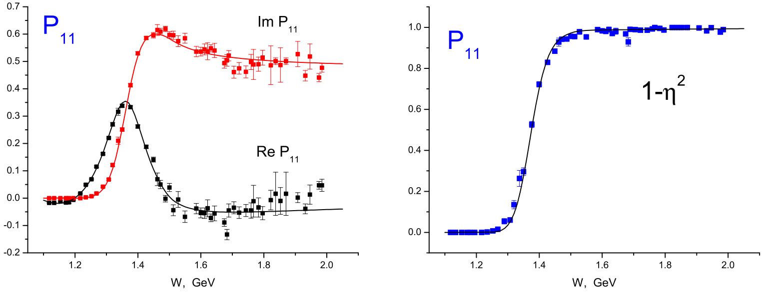


Figure 1: The results of fitting of P_{11}-wave of πN scattering. Dots are results of PWA (**R. A. Arndt et al. Phys. Rev. C76 (2006) 045205**), solid lines represent our amplitudes (9)–(14) in the presence of derivative in vertex (20)– (21). K-matrix has only p-wave states. Partial wave normalization corresponds to **R. A. Arndt et al.**: $\text{Im}\, f = |f|^2 + (1 - \eta^2)/4$.

Quality of description is defined by:

$$\chi^2/\text{DOF} = 273/95. \tag{22}$$

The use of vertices without derivative leads to impairment of quality of description: $\chi^2 > 350$, again we need two poles with close masses. Both variants give a negative background contribution to S_{11} wave, comparable in magnitude with other contributions, as it seen on Fig. 2. Variant without derivative in vertex gives a larger background contribution, rapidly changing near thresholds. It seems that description of P_{11} partial wave without derivative in vertices contradicts to data on S_{11}. On Fig. 2 some typical curves are shown, there exist different variants with sharp behavior near thresholds. The presence of derivative in a vertex suppresses the threshold region in background contribution due to factor $(W - m_N)^2$, but in resonance region this is rather large contribution, see Fig. 2.

Background in s-wave

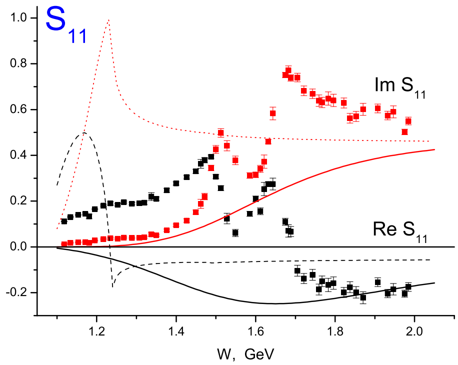


Figure 2: Background contribution to s-wave, generated by p-wave states, i.e. in this case K-matrix for s-wave (13) has only negative energy poles. Solid lines represent variant with derivative in vertex (corresponding to curves on Fig. 1), dashed lines – variant without derivative in vertex.

Attempt to describe S_{11} without background has no success: it doesn't allow to reach even qualitative agreement with PWA.

As a next step, let us add the background contribution, arising from p-wave states (solid lines on Fig. 1) with fixed parameters of p-wave.

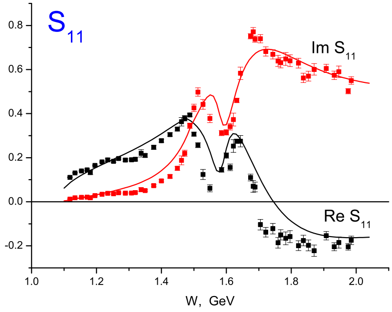


Figure 3: Results of s-wave fitting with fixed parameters for p-wave states. Parameters of p-wave correspond to curves on Fig. 1, s-wave contains two states with K-matrix masses 1.55 and 1.75 GeV.

Fit of S_{11}

One can see from Fig. 3 that quality of description is unsatisfactory in this case but double-peak behavior is arisen in partial wave for the first time. It means that to describe S_{11} wave a background contribution is necessary and its value is close to solid line curves at Fig. 1

Let's perform the joint analysis of S_{11} and P_{11} amplitudes, when resonance states in one wave generate background in other and vice versa. In this case K-matrices (13)–(14) have poles with both positive and negative energies: we use two s-wave and two p-wave poles. This leads to noticeable improvement of description, as can be seen from Fig. 4; in this case χ^2/DOF $= 850/190$.

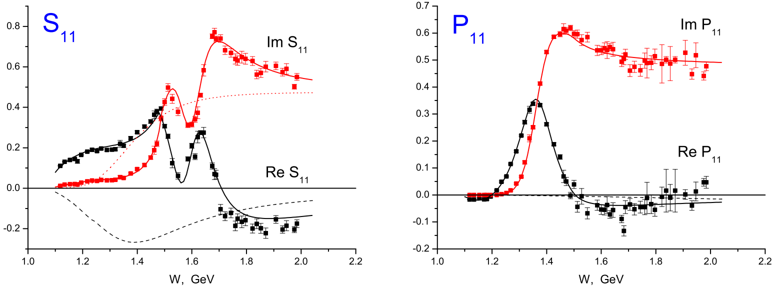


Figure 4: Result of joint fitting of S_{11} and P_{11}-waves of πN scattering. Dashed lines show real and imaginary parts of (unitarized) background contribution.

Joint fit of S_{11} and P_{11}

At last, background can be generated not only by negative energy poles but by other terms. We accounted it by adding to elastic amplitudes $\pi N \to \pi N$ a smooth contributions of the form:

$$\hat{K}_s^B = A + B(W - m_N)^2, \quad \hat{K}_p^B = A + B(W + m_N)^2, \tag{23}$$

which do not violate the MacDowell symmetry property. Note that we have quite good description $\chi^2/\text{DOF} = 584/187$ and background contribution in S_{11} is close to simplest variant of Fig. 2.

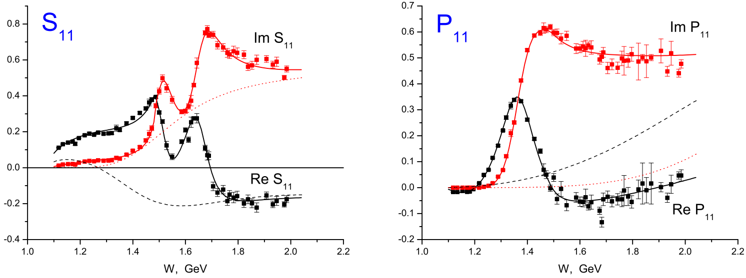


Figure 5: Result of joint fitting of S_{11} and P_{11} waves of πN scattering.

In Table 1 we present the pole masses and widths obtained by continuation of our amplitudes to complex W plane. As a whole, we see that our values for m_p, Γ_p are rather close to previously obtained. The only hint for disagreement is appearance at some sheets of a stable pole $1/2^+$ with $m_p \approx 1500$ MeV instead of generally accepted mass $m_p \approx 1365$ MeV.

Partial wave, PDG values	This work	Some other works
S_{11}, $1/2^-$ N(1535) (1510, 70) N(1650) (1655, 165)	(1507, 87) (1659, 149)	(1502, 95), (1648, 80)† (1519, 129), (1669, 136)††
P_{11}, $1/2^+$ N(1440) (1365, 190)	(1365, 194) (1500, 160)	(1359, 162)† (1385, 164)* (1387, 147)††

Table 1: Pole masses and widths (M_R, Γ_R) extracted from poles position in the complex plane W: $W_0 = M_R - \imath\Gamma_R/2$.

† **R. Arndt et al. Phys. Rev. C74 (2006) 045205.**
†† **M. Doring et al. Nucl. Phys. A829 (2009) 170.**
* **G. Hohler πN Newslett. (1993) 108.**

Conclusions

- Effect of mixing of fermion fields with opposite parity can be readily realized in the framework of K-matrix approach. It leads to well-known MacDowell symmetry

$$f_{l,+}(W) = -f_{l+1,-}(-W),$$

  connecting two partial waves.
  BUT: Taking OPF-mixing into account, MacDowell symmetry leads to practical consequences: resonance in one partial wave gives rise to background contribution in another and vice versa.
- This connection, as in case of $3/2^{\pm}$ resonances, works mainly in one direction: it generates large negative background in a wave with lower orbital momentum.
- As for practical use: we suppose that this connection may be of interest as a source of additional information about wave with higher orbital momentum (in our case about P_{11} and baryons $1/2^{+}$)

Thank you for your attention!

Few remark after fit

- We used simplified description of πN partial waves (σN is some "quasi-channel") to recognize the effect of OPF-mixing in system of baryons $1/2^{\pm}$. Rather unexpectedly we obtained a good quality of description $\chi^2/\text{DOF} = 584/187$, which is comparable with much more comprehensive analyses up to 6 channels.
- It seems that OPF-mixing may be introduced into dynamical models used for baryon physics, e.g. **H. Kamano, S. Nakamura, T.-S. Lee and T. Sato, Phys. Rev. C81 (2010) 065207**. Besides theoretical constrains it can have also some practical meaning.

We will use **off-shell** projection operators $\Lambda^{\pm}$:

$$\Lambda^{\pm} = \frac{1}{2}\Big(1 \pm \frac{\hat{p}}{W}\Big), \quad W = \sqrt{p^2},$$

where W is the rest-frame energy.
Main properties of projection operators are:

$$\Lambda^{\pm}\Lambda^{\pm} = \Lambda^{\pm}, \quad \Lambda^{\pm}\Lambda^{\mp} = 0, \quad \Lambda^{\pm}\gamma^5 = \gamma^5\Lambda^{\mp},$$

$$\Lambda^{+} + \Lambda^{-} = 1, \quad \Lambda^{+} - \Lambda^{-} = \frac{\hat{p}}{W}.$$

Dyson–Schwinger equation for dressed propagator $G(p)$:

$$G(p) = G_0 + G\Sigma G_0, \tag{24}$$

where G_0 is a bare propagator and Σ is a self-energy.
We can expand all elements in (24) in the basis of projection operators:

$$G = \sum_{M=1}^{2} \mathcal{P}_M G_M, \quad \mathcal{P}_1 = \Lambda^+, \ \mathcal{P}_2 = \Lambda^-. \tag{25}$$

After it Dyson–Schwinger equation is reduced to equations on scalar functions:

$$G_M = G_{0,M} + G_M \Sigma_M G_{0,M}, \quad M = 1, 2, \tag{26}$$

or

$$\left(G^{-1}\right)_M = \left(G_0^{-1}\right)_M - \Sigma_M. \tag{27}$$

Decomposition of inverse dressed propagator:

$$G^{-1} = \mathcal{P}_1(W - m - \Sigma_1) + \mathcal{P}_2(-W - m - \Sigma_2). \tag{28}$$

Usual form of the self-energy is

$$\Sigma(p) = A(p^2) + \hat{p}B(p^2), \tag{29}$$

and its decomposition in projection basis:

$$\Sigma_1 = A(W^2) + WB(W^2), \quad \Sigma_2 = A(W^2) - WB(W^2). \tag{30}$$

Note the property of coefficients in the projection basis:

$$\Sigma_2(W) = \Sigma_1(-W).$$

Dressed propagator has a form:

$$G = \mathcal{P}_1 \frac{1}{W - m - \Sigma_1} + \mathcal{P}_2 \frac{1}{-W - m - \Sigma_2}. \tag{31}$$

When we have two fermion fields Ψ_i, the inclusion of interaction leads also to mixing of these fields. In this case the Dyson–Schwinger equation (24) acquires matrix indices:

$$G_{ij} = (G_0)_{ij} + G_{ik}\Sigma_{kl}(G_0)_{lj}, \quad i,j,k,l = 1,2. \tag{32}$$

Therefore we have the same equation, but all factor are matrices 2×2

$$G(p) = G_0 + G\Sigma G_0. \tag{33}$$

The simplest variant is when the fermion fields Ψ_i have the same quantum numbers and the parity is conserved in the Lagrangian. Inverse propagator in this case:

$$\begin{gathered} G^{-1} = \mathcal{P}_1 S_1(W) + \mathcal{P}_2 S_2(W) = \\ \mathcal{P}_1 \begin{pmatrix} W - m_1 - \Sigma^1_{11} & -\Sigma^1_{12} \\ -\Sigma^1_{21} & W - m_2 - \Sigma^1_{22} \end{pmatrix} + \mathcal{P}_2 S_1(-W). \end{gathered} \tag{34}$$

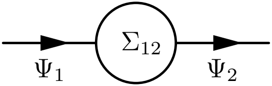
The matrix coefficients as before have the symmetry property $S_2(W) = S_1(-W)$. To obtain the matrix dressed propagator $G(p)$ one should reverse the matrix coefficients:

$$G(p) = \mathcal{P}_1\big(S_1(W)\big)^{-1} + \mathcal{P}_2\big(S_2(W)\big)^{-1}. \tag{35}$$

We see that with use of projection basis the problem of fermion mixing is reduced to studying of the same mixing matrix as for bosons besides the obvious replacement $s - m^2 \to W - m$.

Appendix: Off-shell projection operators and fermion dressing

First of all, look at the non-diagonal self-energy:



Let parity is conserved in Lagrangian.
Mixing of fields with the same quantum numbers:

$$\Sigma_{12} = A(p^2) + \hat{p}B(p^2) =$$
$$= \Lambda^+ \left[A(W^2) + WB(W^2)\right] + \Lambda^- \left[A(W^2) - WB(W^2)\right]$$

Mixing of fields with opposite parities:

$$\Sigma_{12} = \gamma^5 C(p^2) + \hat{p}\gamma^5 D(p^2) =$$
$$= \Lambda^+ \gamma^5 \left[C(W^2) + WD(W^2)\right] + \Lambda^- \gamma^5 \left[C(W^2) - WD(W^2)\right]$$

Main statement: $\Sigma_{12} \neq 0$ for mixing of opposite parities fields. Fermion specifics!

Let us consider the joint dressing of two fermion fields of opposite parities provided that the parity is conserved in a vertex. In this case the diagonal transition loops Σ_{ii} contain only I and $\hat{p}$ matrices, while the off-diagonal ones Σ_{12}, Σ_{21} must contain γ^5.
Projection basis should be supplemented by elements containing γ^5, it is convenient to choose the γ-matrix basis as:

$$\mathcal{P}_1 = \Lambda^+,\ \mathcal{P}_2 = \Lambda^-,\ \mathcal{P}_3 = \Lambda^+\gamma^5,\ \mathcal{P}_4 = \Lambda^-\gamma^5. \tag{36}$$

In this case the γ-matrix decomposition has four terms:

$$S = \sum_{M=1}^{4} \mathcal{P}_M S_M, \tag{37}$$

where the coefficients S_M are matrices and have the obvious symmetry properties $S_2(W) = S_1(-W)$, $S_4(W) = S_3(-W)$.

Inverse propagator in this basis looks as:

$$S(p) = \mathcal{P}_1 \begin{pmatrix} W - m_1 - \Sigma^1_{11} & 0 \\ 0 & W - m_2 - \Sigma^1_{22} \end{pmatrix} + \\ + \mathcal{P}_2 \begin{pmatrix} -W - m_1 - \Sigma^2_{11} & 0 \\ 0 & -W - m_2 - \Sigma^2_{22} \end{pmatrix} + \\ + \mathcal{P}_3 \begin{pmatrix} 0 & -\Sigma^3_{12} \\ -\Sigma^3_{21} & 0 \end{pmatrix} + \mathcal{P}_4 \begin{pmatrix} 0 & -\Sigma^4_{12} \\ -\Sigma^4_{21} & 0 \end{pmatrix}, \tag{38}$$

where the indices $i, j = 1, 2$ in the self-energy Σ^M_{ij} numerate dressing fermion fields and the indices $M = 1, \ldots, 4$ are refered to the γ-matrix decomposition (37).

Reversing of (38) gives the matrix dressed propagator:

$$G = \mathcal{P}_1 \begin{pmatrix} \frac{-W - m_2 - \Sigma^2_{22}}{\Delta_1} & 0 \\ 0 & \frac{-W - m_1 - \Sigma^2_{11}}{\Delta_2} \end{pmatrix} + \\ + \mathcal{P}_2 \begin{pmatrix} \frac{W - m_2 - \Sigma^1_{22}}{\Delta_2} & 0 \\ 0 & \frac{W - m_1 - \Sigma^1_{11}}{\Delta_1} \end{pmatrix} + \\ + \mathcal{P}_3 \begin{pmatrix} 0 & \frac{\Sigma^3_{12}}{\Delta_1} \\ \frac{\Sigma^3_{21}}{\Delta_2} & 0 \end{pmatrix} + + \mathcal{P}_4 \begin{pmatrix} 0 & \frac{\Sigma^4_{12}}{\Delta_2} \\ \frac{\Sigma^4_{21}}{\Delta_1} & 0 \end{pmatrix}. \tag{39}$$

$$\Delta_1 = (W - m_1 - \Sigma^1_{11})(-W - m_2 - \Sigma^2_{22}) - \Sigma^3_{12}\Sigma^4_{21},$$
$$\Delta_2 = (-W - m_1 - \Sigma^2_{11})(W - m_2 - \Sigma^1_{22}) - \Sigma^4_{12}\Sigma^3_{21} = \Delta_1(W \to -W).$$

Note that our K-matrix differs from one used by other authors (e.g. **R. A. Arndt et al. Phys. Rev. C74 (2006) 045205**) by:

- Another form of phase-space factor (QFT calculations);
- Presence of the negative energy poles in $\hat{K}$.

These two points together lead to MacDowell symmetry.
We will use our K-matrix for description of partial waves S_{11} and P_{11} of πN scattering in the energy region $W < 2\,\mathrm{GeV}$. Following to idea of **M. Batinic et al. Phys. Rev. C51 (1995) 2310**, we will use three channels of reaction: πN, ηN and σN, where the last is "effective" channel, imitating different $\pi\pi N$ states.
"Primitive" $\hat{K}$-matrices have a form (13)–(14) but can contain several $J^P = 1/2^+$ and $J^P = 1/2^-$ states.

Appendix: Comparison with R. Arndt el al.

Note that our K-matrix amplitudes (9) may be rewritten in other form, close to the one used in **R. A. Arndt et al. Phys. Rev. D32 (1985) 1085**

$$
\begin{aligned}
f_s(W) &= -\frac{1}{8\pi}\rho_s \hat{K}_s \left[1 + \imath \rho_s P \rho_s \hat{K}_s(W)/(8\pi)\right]^{-1} \rho_s, \\
f_p(W) &= \quad \frac{1}{8\pi}\rho_p \hat{K}_p \left[1 - \imath \rho_p P \rho_p \hat{K}_p(W)/(8\pi)\right]^{-1} \rho_p.
\end{aligned}
\tag{40}
$$