Quantum Chromodynamics with massive gluons

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September, 2014

Introduction

- Since the discovery of asymptotic freedom
  D.J. Gross, F. Wilczek 1973
  H.D. Politzer 1973
  G. 't Hooft 1972
  QCD is concidered as the theory of strong interactions.
- The gauge bosons of the theory, the gluons A^a_μ, are considered to be massless to have gauge invariance and renormalizability.
- The QCD Lagrangian should be modified by the adding gluon masses to ensure that QCD does not contradict to experiments.
- On mass-shell renormalizability of the resulting theory is discussed.

The Lagrangian of QCD in the covariant gauge is well known

$$L_{QCD} = -\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + i\overline{\psi}_f \gamma_\mu D_\mu \psi_f - m_f \overline{\psi}_f \psi_f \qquad (1)$$

$$-\frac{1}{\xi}(\partial^\mu A^a_\mu)^2 + \partial^\mu \overline{c}^a (\partial_\mu c^a - g f^{abc} c^b A^c_\mu) + counterterms,$$

$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$ is the gluon field strength tensor,
$D_\mu = \partial_\mu - igA^a_\mu T^a$ is the covariant derivative,
quark fields ψ_f transform as the fundamental representation of the colour group $SU(3)$, $f = u, d, s, c, b, t$
c^a - ghost fields, ξ is the gauge parameter.

Let us consider the vacuum polarization function $\Pi(q^2)$

$$(-q^2 g_{\mu\nu} + q_\mu q_\nu)\Pi(q^2) = i \int \mathrm{d}x e^{iqx} \langle 0|\ T\, j_\mu(x) j_\nu(0)\ |0\rangle. \qquad (2)$$

where $j_\mu = \sum_f q_f \overline{\psi}_f \gamma_\mu \psi_f$ is the electromagnetic quark current,
$q_f = 2/3, -1/3, ...$

According to general principles of local quantum field theory the function $\Pi(q^2)$ satisfies the Källen-Lehmann spectral representation

$$\Pi(q^2) = \frac{1}{12\pi^2} \int_{4m_\pi^2}^{\infty} ds \; \frac{R(s)}{s - q^2 - i0}, \tag{3}$$

where $R(s) = \sigma_{tot}(e^+e^- \to hadrons)/\sigma(e^+e^- \to \mu^+\mu^-)$,

$\Pi(q^2)$ is an analytic function in the complex q^2-plane with the cut starting from the two-pion threshold $q^2 = 4m_\pi^2$. The discontinuity on the cut

$$\Delta\Pi(q^2) \equiv \Pi(q^2+i0) - \Pi(q^2-i0) = \begin{cases} i\ R(q^2)/(6\pi) & at \quad s > 4m_\pi^2 \\ 0 & at \quad s < 4m_\pi^2. \end{cases} \tag{4}$$

Perturbative QCD gives

$$\Delta\Pi(q^2)_{pQCD} = \theta(q^2)\ \rho_{gluon}(q^2) + \theta(q^2 - 4M_u^2)\ \rho_{quark}(q^2). \tag{5}$$

The gluon spectral density $\rho_{gluon}(q^2)$ contributes for $q^2 > 0$. This is the known zero threshold which arises from the Cutcosky cuts crossing only gluon propagators of diagrams.

Thus within pQCD

$$\Delta\Pi(q^2) \neq 0 \quad \textit{for } 0 < q^2 < 4m_\pi^2$$

Non-perturbative contributions:

$$e^{-1/a_s} = 0 \cdot a_s + 0 \cdot a_s^2 + ...$$

They can not exactly cancel perturbative contributions for $0 < q^2 < 4m_\pi^2$
One should move perturbative gluon and quark thresholds above $q^2 = 4m_\pi^2$.
Hence restrictions on (perturbative pole) masses of gluons and quarks

$$(3M_{gl})^2 > 4m_\pi^2, \tag{6}$$

$$4M_u^2 > 4m_\pi^2.$$

The first (naive) objection is that nobody trusts perturbation theory below the two-pion threshold.
But only the existence of the pertubative series is important here (independently on the question of its convergence) .

On mass-shell renormalizable theory of massive gluons without color scalars

Let us add a scalar part to the massless QCD Lagrangian

$$L_{QCD+scalars} = -\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + i\overline{\psi}_f\gamma_\mu D_\mu\psi_f - m_f\overline{\psi}_f\psi_f +$$

$$(D_\mu\Phi)^+ D_\mu\Phi + (D_\mu\Sigma)^+ D_\mu\Sigma - \lambda_1\left(\Phi^+\Phi - v_1^2\right)^2 - \lambda_2\left(\Sigma^+\Sigma - v_2^2\right)^2$$

$$-\lambda_3\left(\Phi^+\Phi + \Sigma^+\Sigma - v_1^2 - v_2^2\right)^2 - \lambda_4\left(\Phi^+\Sigma\right)\left(\Sigma^+\Phi\right)$$

$$+L_{gf} + L_{gc} + counterterms,$$

with two scalar triplets $\Phi(x)$ and $\Sigma(x)$ to get all gluon massive.
The mechanism of spontaneous symmetry breaking:

$$\Phi(x) = \begin{pmatrix} \phi_1(x) + i\phi_2(x) + v_1 \\ \phi_3(x) + i\phi_4(x) \\ \phi_5(x) + i\phi_6(x) \end{pmatrix}, \quad \Sigma(x) = \begin{pmatrix} \sigma_1(x) + i\sigma_2(x) \\ \sigma_3(x) + i\sigma_4(x) + v_2 \\ \sigma_5(x) + i\sigma_6(x) \end{pmatrix}.$$

Massive terms for gluons in the Lagrangian

$$L_M = M_{gl}^2 \left[(A^1)^2 + (A^2)^2 + (A^3)^2 + \frac{1}{2}(A^4)^2 + \right.$$

$$\left. \frac{1}{2}(A^5)^2 + \frac{1}{2}(A^6)^2 + \frac{1}{2}(A^7)^2 + \frac{1}{3}(A^8)^2 \right],$$

where $M_{gl}^2 \equiv g^2 v^2$, $(v = v_1 = v_2)$ is the gluon mass parameter of the theory.

After the shift four combinations of scalar fields

$$\phi_1 + \frac{\lambda_3}{\lambda_1 + \lambda_3}\sigma_3, \quad \sigma_3, \quad \sigma_1 + \phi_3, \quad \sigma_2 - \phi_4$$

become massive Higgs particles.

The following eight combinations

$$\sigma_1 - \phi_3, \;\; \phi_4 + \sigma_2, \;\; \phi_2 - \sigma_4, \;\; \phi_2 + \sigma_4, \;\; \phi_5, \;\; \phi_6, \;\; \sigma_5, \;\; \sigma_6$$

become massless Goldstone ghosts.

Transition to the unitary gauge.
Ghost fields disappear from the Lagrangian.
One can remove in the unitary gauge all Higgs fields from the Lagrangian preserving on mass-shell renormalizability of the theory.
As the simplified case - the $SU(2)$-invariant Lagrangian

$$L = -\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \left(D_\mu \Phi\right)^+ D_\mu \Phi - \lambda \left(\Phi^+ \Phi - v^2\right)^2 \tag{7}$$

one makes the shift of the scalar field

$$\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} i\phi_1(x) + \phi_2(x) \\ \sqrt{2}v + \chi(x) - i\phi_3(x) \end{pmatrix}$$

fixes the gauge and adds ultraviolet counterterms.

In the R_ξ-gauge one gets the theory

$$L_{R_\xi} = -\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + \frac{m^2}{2}W^a_\mu W^a_\mu - mW^a_\mu\partial_\mu\phi^a + \frac{1}{2}\partial_\mu\phi^a\partial_\mu\phi^a + \frac{1}{2}\partial_\mu\chi\partial_\mu\chi \quad (8)$$

$$-\frac{M^2}{2}\chi^2 + \frac{g}{2}W^a_\mu(\phi^a\partial_\mu\chi - \chi\partial_\mu\phi^a + \epsilon^{abc}\phi^b\partial_\mu\phi^c) + \frac{mg}{2}\chi W^a_\mu W^a_\mu$$

$$+\frac{g^2}{8}(\chi^2 + \phi^a\phi^a)W^2_\mu - \frac{gM^2}{4m}\chi(\chi^2 + \phi^a\phi^a) - \frac{g^2M^2}{32m^2}(\chi^2 + \phi^a\phi^a)^2$$

$$-\frac{1}{2\xi}(\partial_\mu W^a_\mu + \xi m\phi^a)^2$$

$$+\partial_\mu\overline{c}^a(\partial_\mu c^a - g\epsilon^{abc}c^bW^c_\mu) - \xi m^2\overline{c}^ac^a - \frac{g}{2}\xi m\chi\overline{c}^ac^a + \frac{g}{2}\xi m\epsilon^{abc}\overline{c}^ac^b\phi^c$$

$$+counterterms$$

The corresponding propagators in momentum space are

$$< T(W^a_\mu W^b_\nu) >= -i\delta^{ab}\left(\frac{g_{\mu\nu} - k_\mu k_\nu/m^2}{k^2 - m^2} + \frac{k_\mu k_\nu/m^2}{k^2 - \xi m^2}\right) \qquad (9)$$

$$< T(\phi^a \phi^b) >= -i\delta^{ab}\frac{1}{k^2 - \xi m^2}$$

$$< T(\bar{c}^a c^b) >= -i\delta^{ab}\frac{1}{k^2 - \xi m^2}$$

$$< T(\chi\chi) >= -i\frac{1}{k^2 - M^2}$$

In the unitary gauge defined by the gauge condition $\phi^a = 0$ one has the Lagrangian

$$L_U = -\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + \frac{m^2}{2}W^a_\mu W^a_\mu + \frac{1}{2}\partial_\mu\chi\partial_\mu\chi - \frac{M^2}{2}\chi^2 \qquad (10)$$

$$+\frac{mg}{2}\chi W^a_\mu W^a_\mu + \frac{g^2}{8}\chi^2 W^a_\mu W^a_\mu - \frac{gM^2}{4m}\chi^3 - \frac{g^2 M^2}{32m^2}\chi^4 + counterterms$$

The propagators in the unitary gauge are obtained from those of the R_ξ-gauge by taking the limit $\xi \to \infty$.

To consider renormalization for our purpose it is convenient to use the Bogoliubov-Parasiuk-Hepp subtraction scheme within Dimensional regularization.
Counterterms of mass dependent diagrams are also mass dependent.
Subtractions should respect Slavnov-Taylor identities.
Within the large-M expansion diagrams with χ-propagators contain either terms with integer negative powers of M^2

$$\frac{1}{M^{2n}}, \quad n = 1, 2, 3, ...$$

or terms with non-integer powers of M^2 (non-integer powers contain ϵ, $D = 4 - 2\epsilon$)

$$\frac{1}{M^{2(k+l\epsilon)}}, \quad k - integer, \quad l - positive\ integer$$

Diagrams with ϕ-propagators can have polynomial in M terms. But this polynomial terms cancel in S-matrix elements.
After renormalization the M-dependent terms are finite at $\epsilon \to 0$ separately from M-independent terms. Thus if one removes all M-dependent terms one is left with a finite expression.

On the Lagrangian level it means in the unitary gauge that one removes from the lagrangian all terms containing the Higgs field χ and also all M-dependent terms in the counterterms. The resulting theory is on mass-shell finite.

This is the massive Yang-Mills theory.

$$L_{YM} = -\frac{1}{4}(\partial_\mu W^a_\nu - \partial_\nu W^a_\mu + gf^{abc} W^b_\mu W^c_\nu)^2 + m^2 W^a_\mu W^a_\mu + counterterms$$

The resulting Lagrangian is

$$L_{massive\ QCD} = L_M - \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + i\overline{\psi}_f \gamma_\mu D_\mu \psi_f - m_f \overline{\psi}_f \psi_f + counterterms.$$

$$L_M = M^2_{gl} \left[(A^1)^2 + (A^2)^2 + (A^3)^2 + \frac{1}{2}(A^4)^2 + \frac{1}{2}(A^5)^2 + \frac{1}{2}(A^6)^2 + \frac{1}{2}(A^7)^2 + \frac{1}{3}(A^8)^2 \right],$$

The one-loop β-function in this theory for a massless renormalization scheme (i.e. a scheme where renormalization group functions do not depend on masses):

$$\beta(a_s) = \mu^2 \frac{\partial a_s}{\partial \mu^2} = \sum_{i \geq 0} \beta_i a_s^{i+2}, \quad \beta_0 = -\frac{7}{2} C_A + \frac{4}{3} T_F n_f,$$

here $C_A = 3$ is the Casimir operator of the adjoint representation of the $SU(3)$ color group, $T_F = 1/2$ is the trace normalization of the fundamental representation, n_f is the number of active quark flavors, $a_s = \frac{g^2}{16\pi^2}$.
Thus asymptotic freedom remains valid in the considered theory with massive gluons.