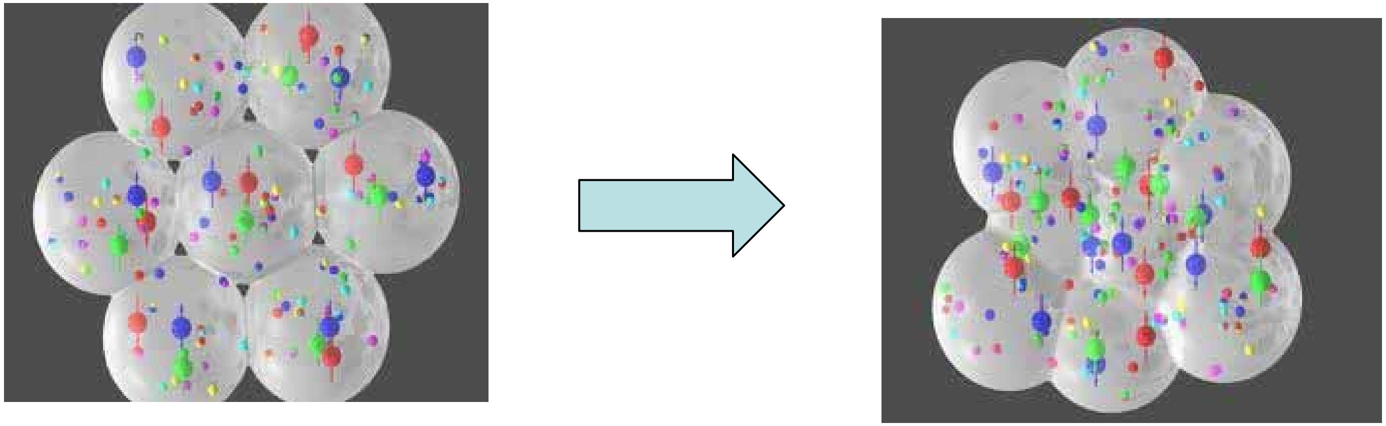
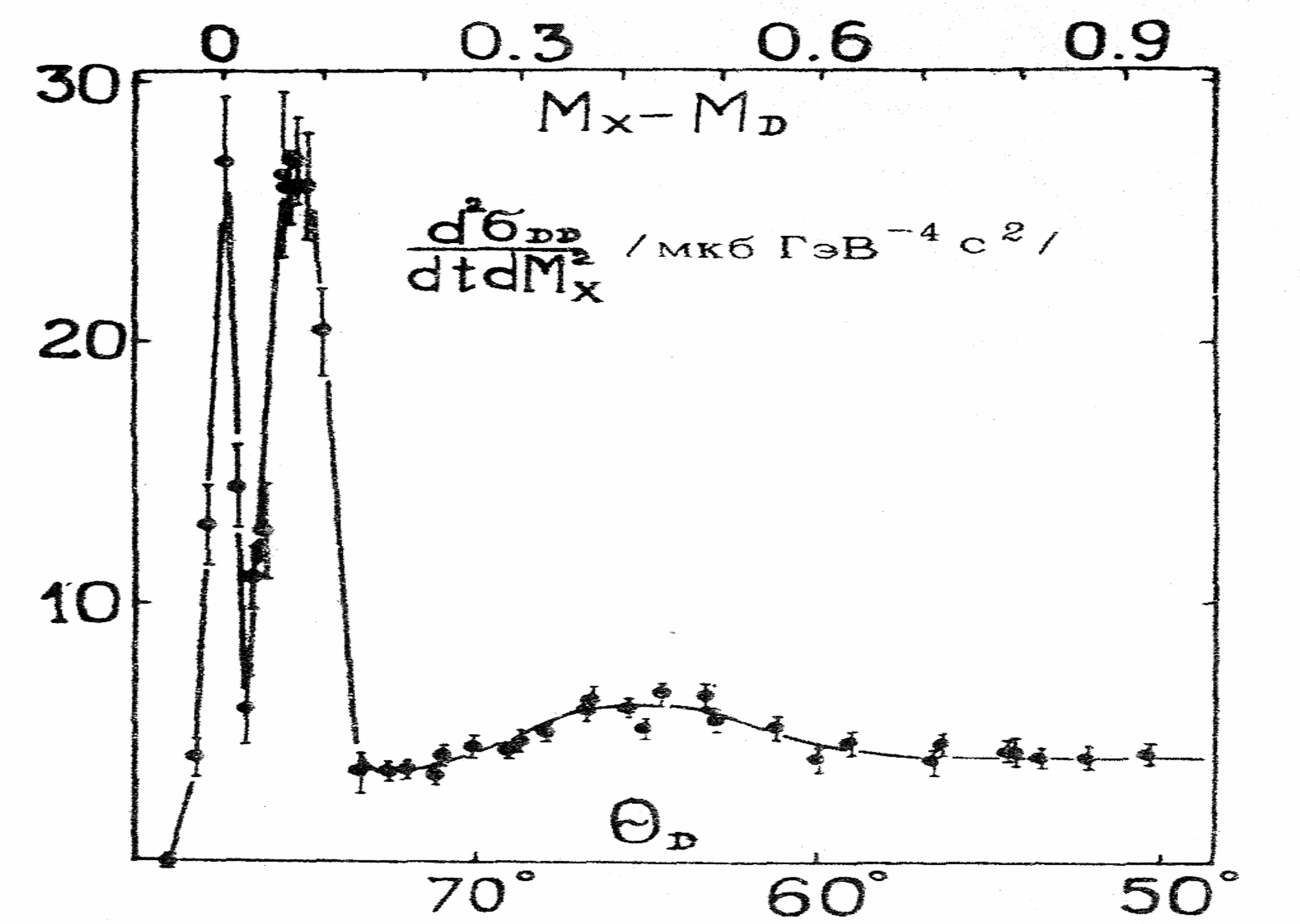
POSSIBLE OBSERVATION OF PHASE TRANSITIONS IN N-N SYSTEMS AT JINR SYNCHROPHASOTRON

B.F. Kostenko, J. Pribis



Baldin ISHEPP XXII, 19.09.2014

Experiment: *A.M. Baldin et al. Differential Elastic Proton-Proton,Nucleon-Deuteron and Deuteron-Deuteron Scatterings at Big Transfer Momenta*, JINR Communication,1-12397, 1979

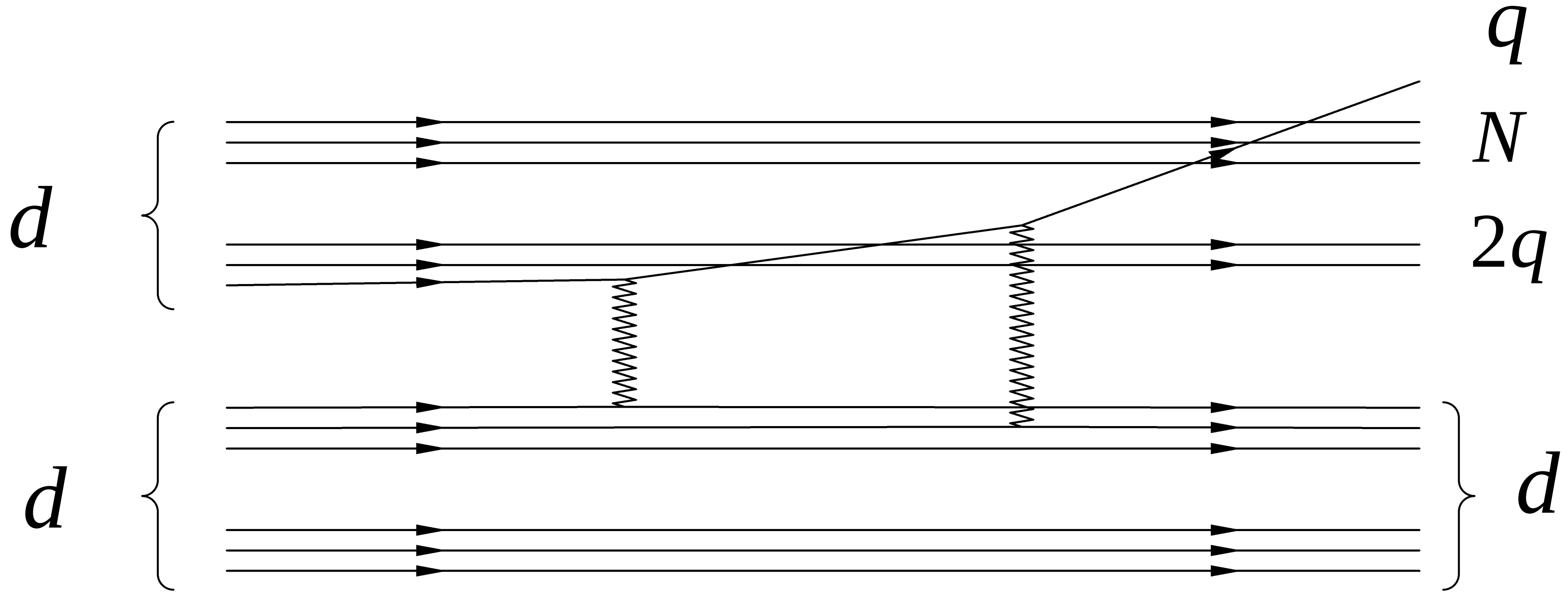


$d+d \rightarrow X+d$

P = 8.9 GeV

t= - 0.5 GeV^2

Peak III. Model $P_X = X \cdot P1$, $M_X = X \cdot M_d$



cos θ = 0.423

M_X = 0.311

M_q = 0.318

V.B. Jovanovic Phys. Rev. **D 82** (2010) 117501

Peak III. Model N(1440)+N(1520)+ N(1535)

Partial-wave analysis: $\vec{J} = \vec{L} + \vec{S},\ P$

L=0 or 1 (SP-approximation)

$$P_i = P_N P_d (-1)^L = (-1)^L, \quad P_f = P_{N^*} P_d (-1)^L = P_{N^*} (-1)^L$$

1) $J = 1/2,\ P = 1,\quad L = 0,\ S = 1/2,$

2) $J = 1/2,\ P = -1,\ L = 1,\ S = 1/2 \text{ or } 3/2,$

3) $J = 3/2,\ P = 1,\quad L = 0,\ S = 3/2,$

4) $J = 3/2,\ P = -1,\ L = 1,\ S = 1/2 \text{ or } 3/2,$

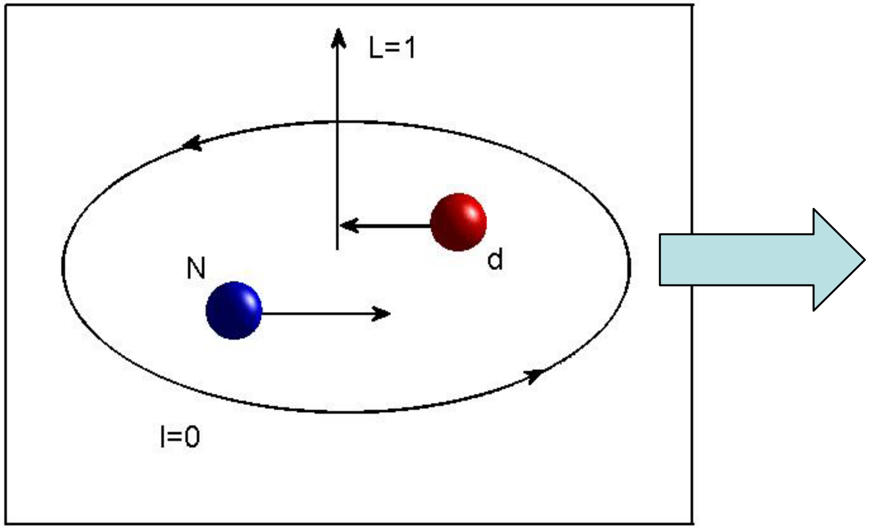
5) $J = 5/2,\ P = -1,\ L = 1,\ S = 3/2$

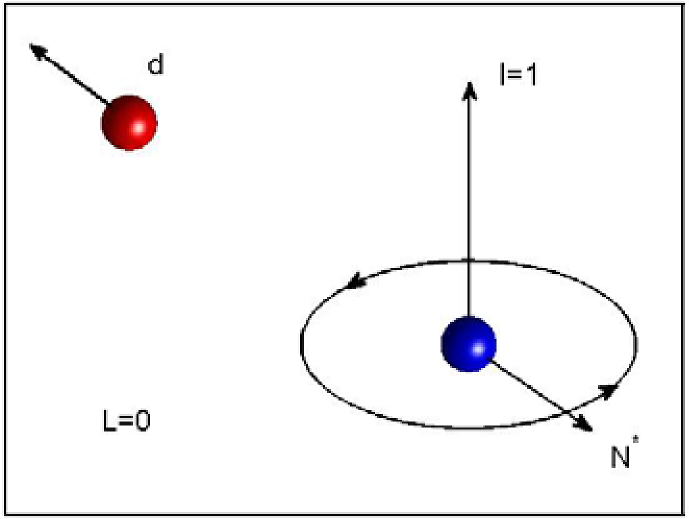
$$SU(6) \times O(3)$$

quark model of baryon excitation

(RH Dalitz, RR Horgan, M Jones, LJ Reiners)

$$\vec{j}_{N^*} = \vec{S}_N + \vec{l}$$



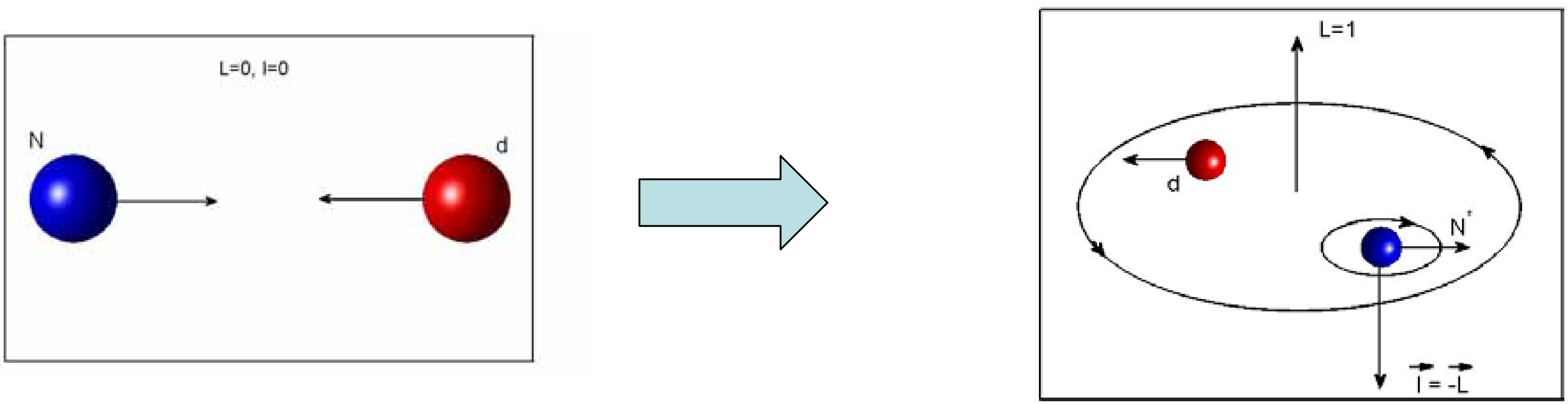


Transfer of external orbital momentum into nucleon

$SU(6) \times O(3)$

quark model of baryon excitation

$$\vec{j}_{N^*} = \vec{S}_N + \vec{l}$$



Simultaneous excitations of *L* and *l*

Partial-wave analysis based on $SU(6)\times O(3)$ quark model

$$\vec{j}_{N^*} = \vec{S}_N + \vec{l},$$

$$N(1440):\ \vec{j}_{N^*} = \vec{S}_N,\quad P = P_N = 1,$$

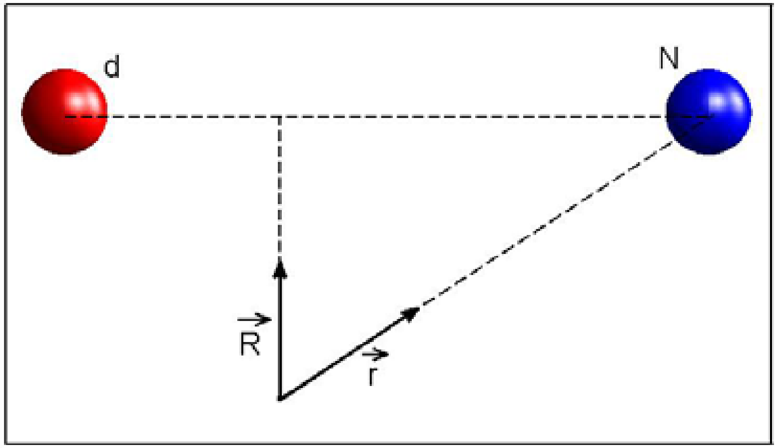
$$N(1520):\ \vec{j}_{N^*} = \vec{S}_N + \vec{1} = \vec{3}/2,\ P = -1,$$

$$N(1535):\ \vec{j}_{N^*} = \vec{S}_N + \vec{1} = \vec{1}/2,\ P = -1.$$

Conservation of total orbital momentum and total spin: $\vec{L}+\vec{l}$ and $\vec{S}_N+\vec{S}_d$

Amplitude N+d --> N*+d

$$T(N+d \to N^* + d) = F_1(S_N, S_d) + (\vec{R}\cdot\vec{r})F_2(S_N, S_d)$$



$$\left| L=0, l=0 \right\rangle \overset{(\vec{R}\cdot\vec{r})}{\to} \left| L=1, l=1 \right\rangle$$ Amplitude =1/3

$$\left| L=1, l=0 \right\rangle \overset{(\vec{R}\cdot\vec{r})}{\to} \left| L=0, l=1 \right\rangle$$ Amplitude =1

Choice of $F_i(S_N, S_d)$

$$T(\vec{S}_N, \vec{S}_d) = C_1 + C_2(\vec{S}_N + \vec{S}_d)\cdot\vec{v} + C_3(\vec{S}_N - \vec{S}_d)\cdot\vec{v} + C_4(\vec{S}_N\cdot\vec{v})(\vec{S}_d\cdot\vec{v}) +$$

$$+C_5(\vec{S}_N\cdot\vec{\lambda})(\vec{S}_d\cdot\vec{\lambda}) + C_6(\vec{S}_N\cdot\vec{\mu})(\vec{S}_d\cdot\vec{\mu})$$

$$T_{N^*}(\vec{S}_N, \vec{S}_d) = C_1 + C_2(\vec{S}_N + \vec{S}_d)\cdot\vec{v}$$

$$\vec{v} = \frac{\vec{p}\times\vec{p}\,'}{\left|\vec{p}\times\vec{p}\,'\right|}, \qquad \vec{\lambda} = \frac{\vec{p}-\vec{p}\,'}{\left|\vec{p}-\vec{p}\,'\right|}, \qquad \vec{\mu} = \frac{\vec{p}+\vec{p}\,'}{\left|\vec{p}+\vec{p}\,'\right|}$$

$$T(N + d \to N^* + d) = A + B(\vec{S}_N + \vec{S}_d)\cdot\vec{v} + (\vec{R}\cdot\vec{r})\left[C + D(\vec{S}_N + \vec{S}_d)\cdot\vec{v}\right]$$

A step to observable particles

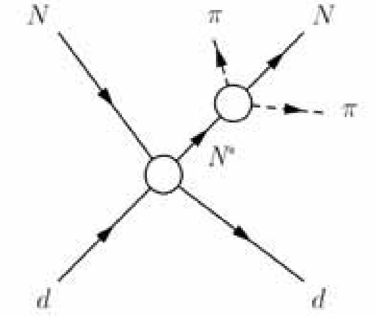
$$A + B(\vec{S}_N + \vec{S}_d)\cdot\vec{v} \rightarrow \frac{A + B(\vec{S}_N + \vec{S}_d)\cdot\vec{v}}{M^2_{N(1440)} - s - iM_{N(1440)}\Gamma_{N(1440)}}$$

$$(\vec{R}\cdot\vec{r})\left[C + D(\vec{S}_N + \vec{S}_d)\cdot\vec{v}\right] \rightarrow f(S_i, S_{N^*})\frac{C + D(\vec{S}_N + \vec{S}_d)\cdot\vec{v}}{M^2_{N^*} - s - iM_{N^*}\Gamma_{N^*}}$$

$$f(S_i, S_{N^*}) = \sum_{\sigma_1=\pm 1/2}\sum_{\sigma_2=0,\pm 1}\sum_{m=0,\pm 1}\left\langle \frac{1}{2}\sigma_1 1\sigma_2 \middle| S_i, \sigma_1+\sigma_2\right\rangle\left\langle \frac{1}{2}\sigma_1 1 m \middle| S_{N^*}, \sigma_1+m\right\rangle$$

$$f\left(\frac{1}{2},\frac{1}{2}\right) = 2+\sqrt{2}, \qquad f\left(\frac{3}{2},\frac{1}{2}\right) = \frac{2}{3},$$

$$f\left(\frac{1}{2},\frac{3}{2}\right) = 0, \qquad f\left(\frac{3}{2},\frac{3}{2}\right) = \frac{4}{3}\left(\sqrt{2}+\sqrt{3}+\sqrt{6}\right)$$



The final expression for amplitude

$$\frac{A + B(\vec{S}_N + \vec{S}_d) \cdot \vec{v}}{M^2_{N(1440)} - s - iM_{N(1440)}\Gamma_{N(1440)}} T(N(1440) \to N + \pi) +$$

$$\left[\frac{f(S,3/2)}{M^2_{N(1440)} - s - iM_{N(1440)}\Gamma_{N(1440)}} T(N(1520) \to N + \pi) \right] \left(C + D(\vec{S}_N + \vec{S}_d) \cdot \vec{v} \right) +$$

$$\left[\frac{f(S,1/2)}{M^2_{N(1440)} - s - iM_{N(1440)}\Gamma_{N(1440)}} T(N(1535) \to N + \pi) \right] \left(C + D(\vec{S}_N + \vec{S}_d) \cdot \vec{v} \right)$$

Three types of macroscopically distinguishable states:

$$N\pi,\ N\pi\pi,\ N\eta$$

Observable cross-section

$$\frac{d^2\sigma}{dt\cdot dM_X^2} = \frac{\pi}{\lambda^{1/2}(s,m_N^2,m_d^2)}\frac{1}{(2S_i+1)}\sum_{S_f}\int dLips(5,6,\ldots)\,Tr(\hat{T}^\dagger\hat{T})$$

$$\lambda(s,m_N^2,m_d^2) = (s+m_N^2-m_d^2)^2 - 4s\,m_N^2$$

$$\vec{S}_i = \vec{S}_N + \vec{S}_d$$

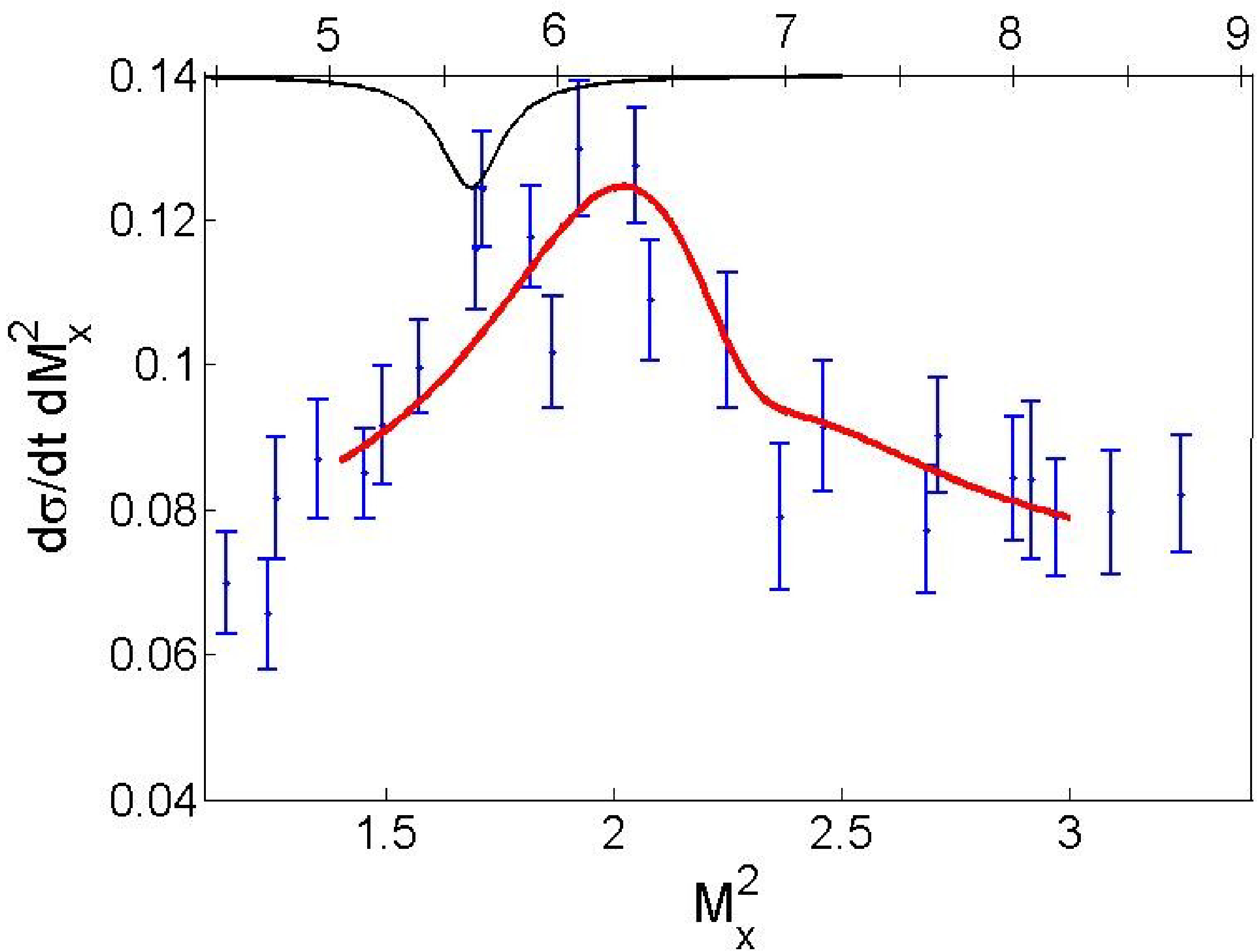
$$Tr(\hat{1}) = \begin{cases} 2, & S_i = 1/2, \\ 4, & S_i = 3/2 \end{cases}$$

$$Tr(\vec{S}_i\cdot\vec{v}) = 0$$

$$Tr(\vec{S}_i\cdot\vec{v})^2 = \begin{cases} 1/2, & S_i = 1/2, \\ 5, & S_i = 3/2 \end{cases}$$

$$2\,\Gamma_{N^*}\,M_{N^*} = \frac{1}{2S_{N^*}+1}\sum_{\sigma_i,\sigma_f}\left|T_{if}(\sigma_i,\sigma_f)\right|^2 dLips(5,6,\ldots)$$

Result



Solid line – results of WASA-at-COSY,
M=2.37 GeV, Γ=0.07 GeV

EXPERIMENTAL PROPOSAL

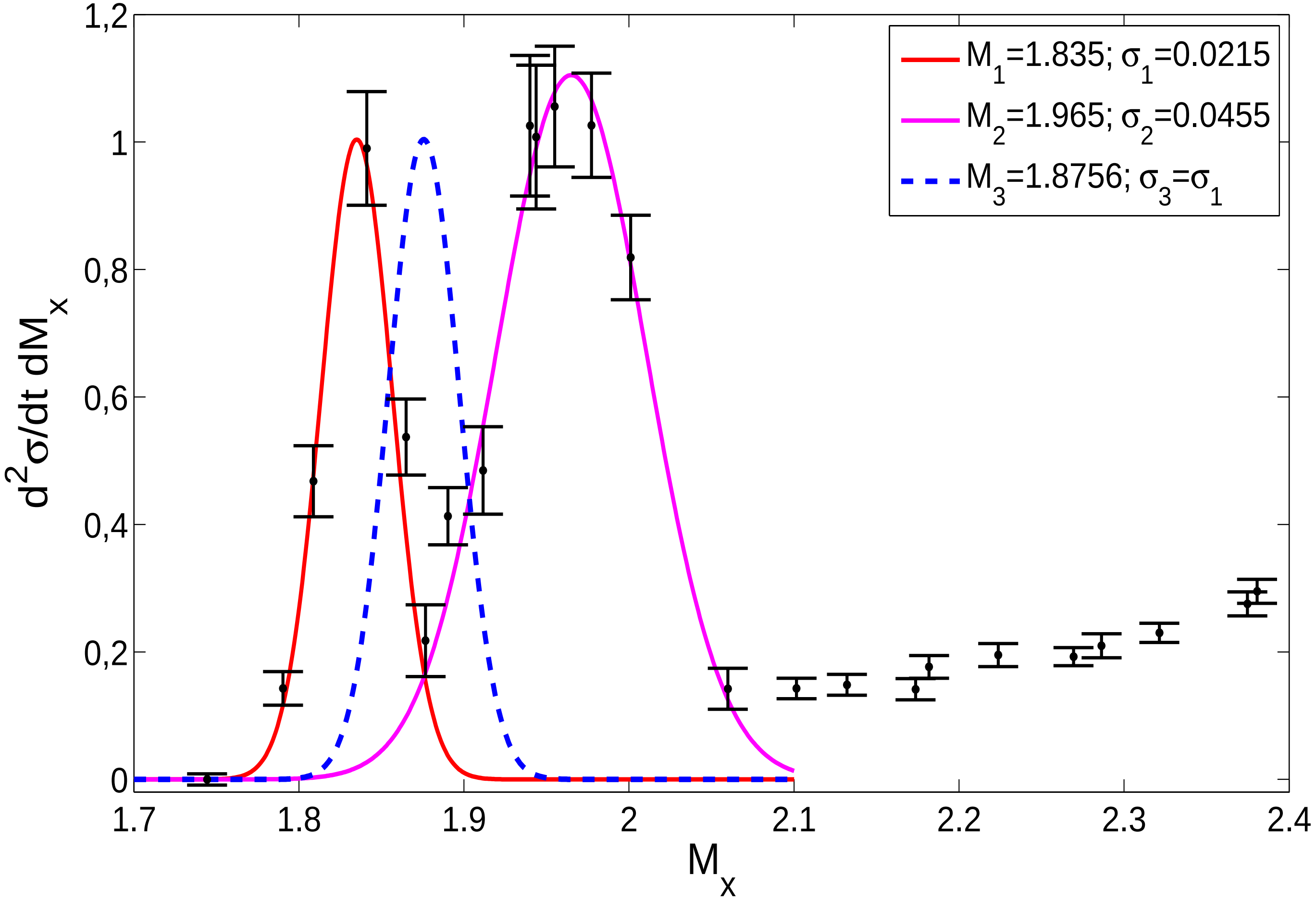
To repeat experiment

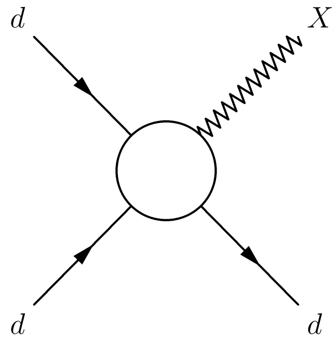
АМ Балдин, ВК Бондарев, АН Манятовский, НС Мороз, ЮА Панебратцев, АА Повторейко, СВ Рихвицкий, ВС Ставинский, АН Хренов. Дифференциальные упругие протон-протонные, нуклон-дейтронные рассеяния при больших переданных импульсах, Сообщение ОИЯИ 1-12397, 1979

with increased statistics

The first peak: model d+d→X+d

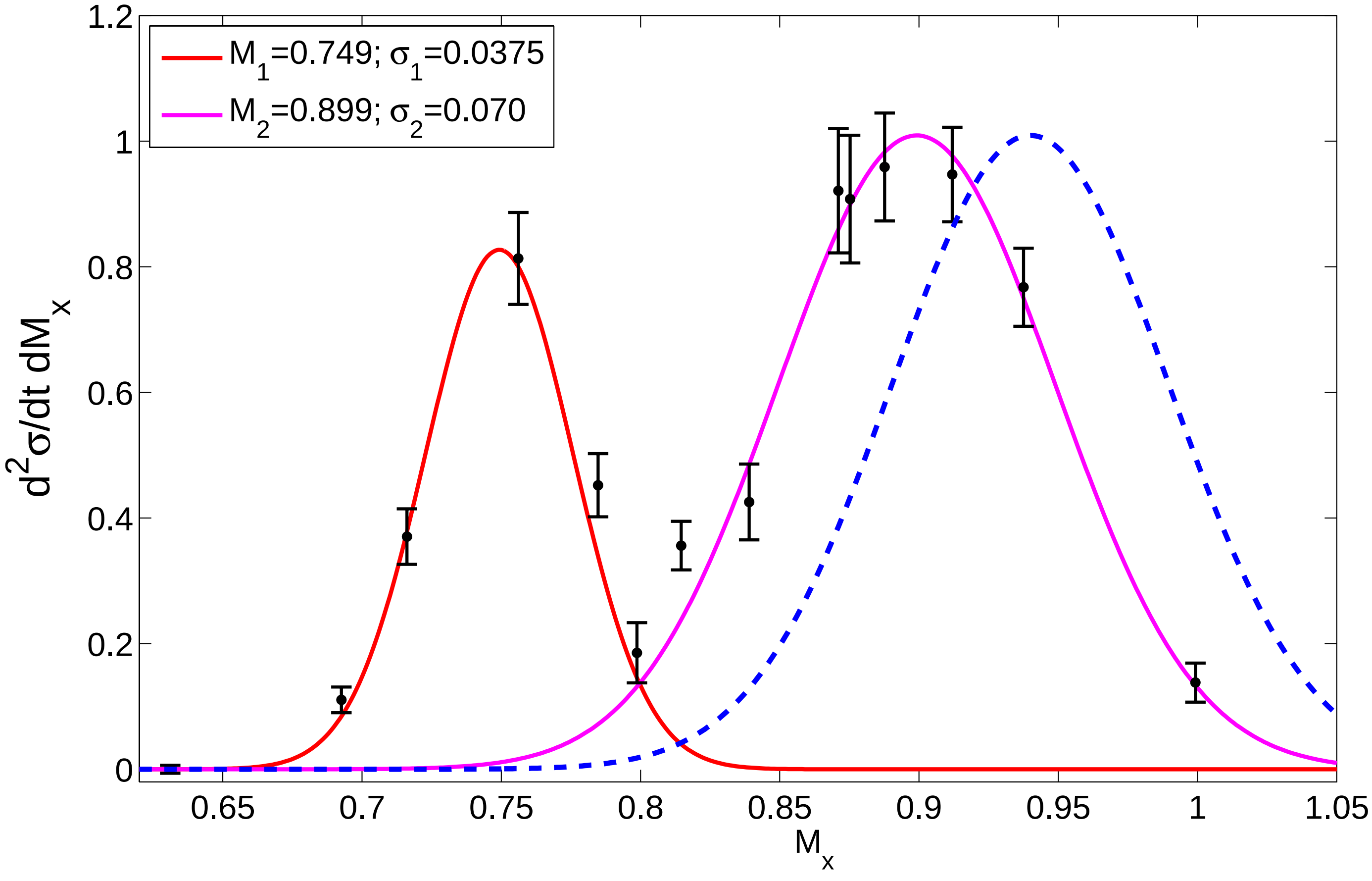
$$M_X^2 = M_d^2 + t + \frac{E_1 t + P_1\sqrt{t(-4M_d^2 + t)}\cos\theta}{M_d}$$

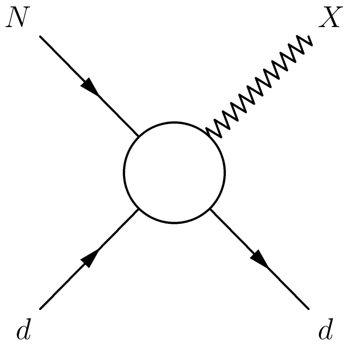




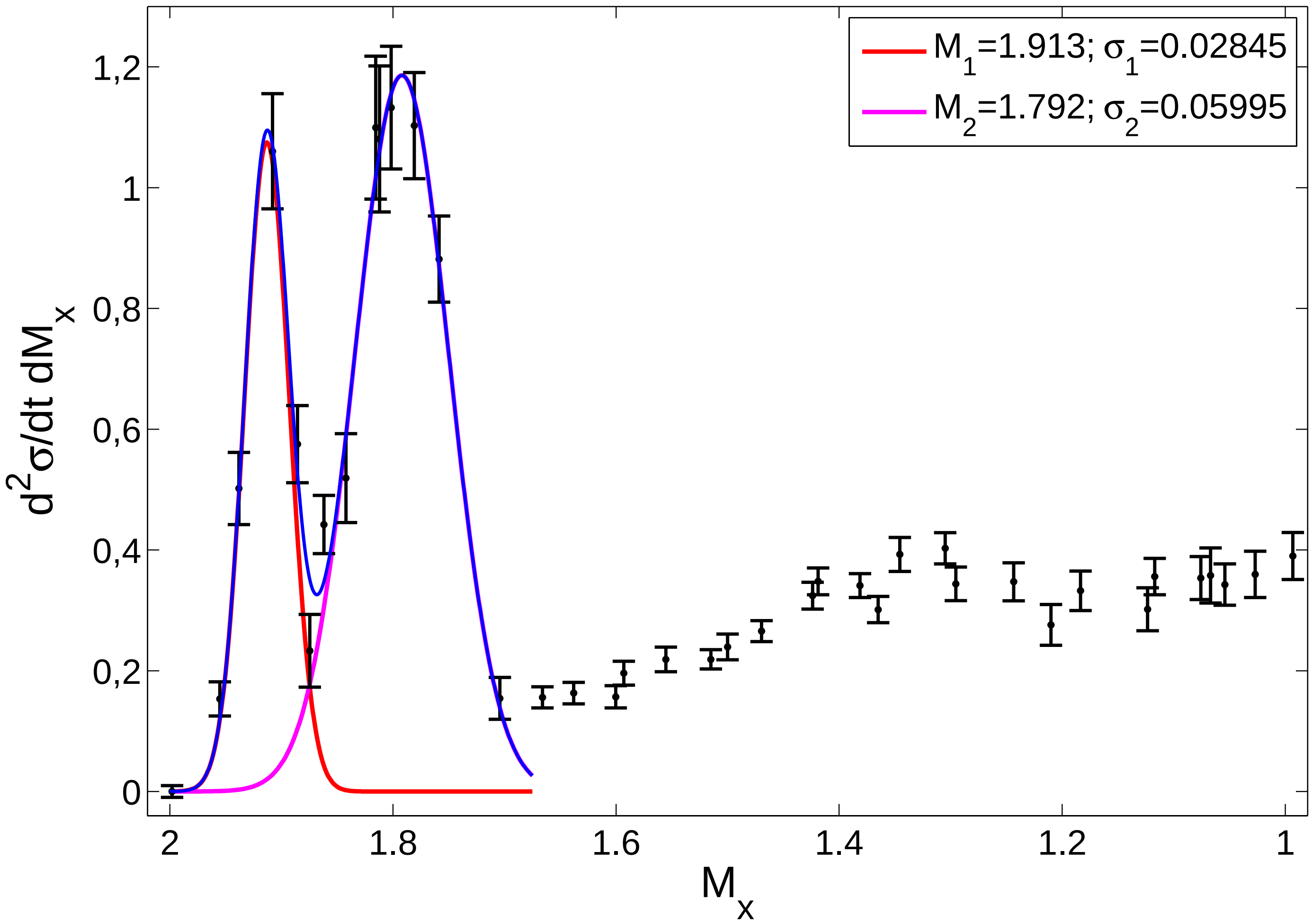
The second peak: model N+d → X+d

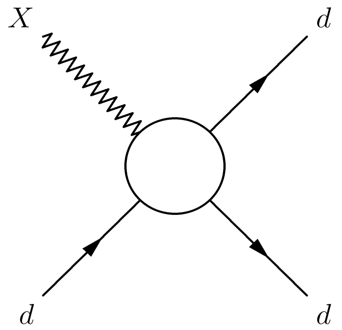
$$M_X^2 = M_N^2 + t + \frac{\sqrt{P_1^2 + 4M_N^2 t} + P_1\sqrt{t(-4M_d^2 + t)}\cos\theta}{2M_d}$$





Peak I. Model X+d → d+d, $P_X = P_1 \cdot M_X / M_d$





Yu. A. Troyan, PEPAN **24** (1993) 683 $M = 1916 \pm 2, \quad \Gamma = 9 \pm 4$

A difference between d and pp

Masses:

$2\ M_p = 1876.6$ MeV

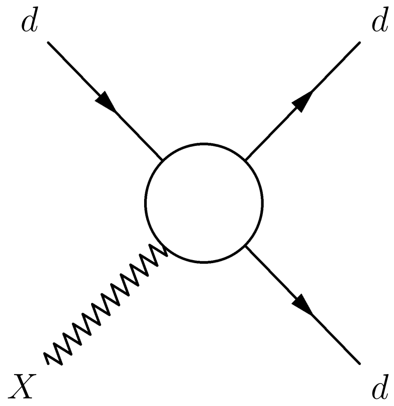
$M_d = 1875.6$ MeV

$2\ M_p - M_d = 1$ MeV

Isospins:

I(pp)=2, I(d)=0

Peak I. Model d+X $\rightarrow$ d+d



$$0 = M_X^2 E_1 + M_X t - E_1(M_d^2 - t) +$$

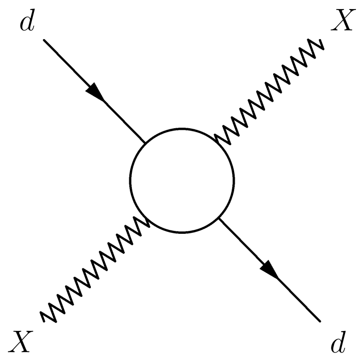
$$P_1\sqrt{M_X^4 - 2M_d^2 M_X^2 - 2M_X^2 t + M_d^4 - 2M_d^2 t + t^2}\cos\theta$$

M_X=1.884

Yu. A. Troyan, PEPAN **24** (1993) 683 :

$$M_X = 1886 \pm 1, \quad \Gamma = 4 \pm 1$$

Peak I. Model d+X $\rightarrow$ X+d



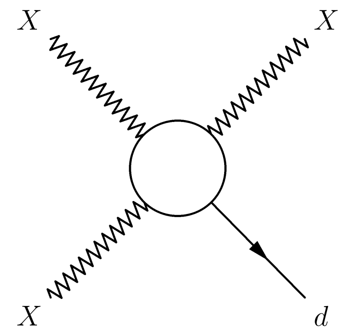
$$M_X^3 - E_1 M_X^2 - (M_d^2 + t)M_X + E_1(M_d^2 - t) -$$
$$P_1\sqrt{M_X^4 - 2M_X^2(M_d^2 + t) + M_d^4 - 2M_d^2 t + t^2}\cos\theta = 0$$

cos θ = 0.2154, $\underline{M_X = 1.886\ \text{GeV}}$

Yu.A. Troyan: M=1886 ±1, Γ= 4± 1 MeV

Peak I. Model X+X $\rightarrow$ X+d

$$M_X^2 = \frac{-E_1^2(M_d^2 - t) + E_1 M_d t + (M_d^2 + t)P_1^2 \cos^2\theta}{-E_1^2 + P_1^2\cos^2\theta} +$$



$$\frac{\sqrt{4E_1 M_d t^2 P_1^2 - 4M_d^2 P_1^2 E_1^2 t + 4M_d^2 P_1^4 t\cos^2\theta + 4E_1^2 t^2 P_1^2 + P_1^2 M_d^2 t^2}}{-E_1^2 + P_1^2\cos^2\theta}\cos\theta$$

Model: M_X=1.884 GeV

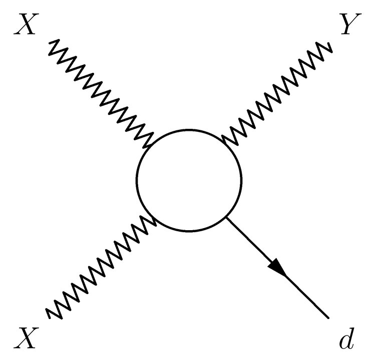
M=1886 $\pm$ 1, Γ= 4 $\pm$1 MeV,

Yu. A. Troyan, PEPAN **24** (1993) 683

Peak I. Model X+X → Y+d

$$M_Y^2 = M_d^2 - \frac{E_1(M_X^2 + M_d^2 - t)}{M_d} + \frac{P_1\sqrt{(M_X^2 + M_d^2 - t)^2 - 4M_d^2M_X^2}\cos\theta}{M_d} +$$

$$2M_X\left(\frac{M_X E_1}{M_d} + M_X - \frac{M_X^2 + M_d^2 - t}{2M_X}\right)$$



cos θ =0.2154

M_X=1.886 (Yu.A.T: I dibaryon):

M_Y=1.898 (Yu.A.T : II dibaryon, M_2=1898 ±1)

Peak I. Model X+d → Y+d

1.916 → 1.884, 1.965 → 1.937,

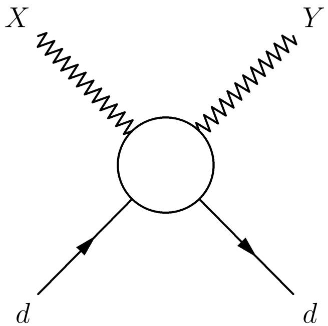
1.980 → 1.953, 2.106 → 2.086.

Yu.A.T.: 1916 ± 2, 1886 ± 1, 1937 ± 2,

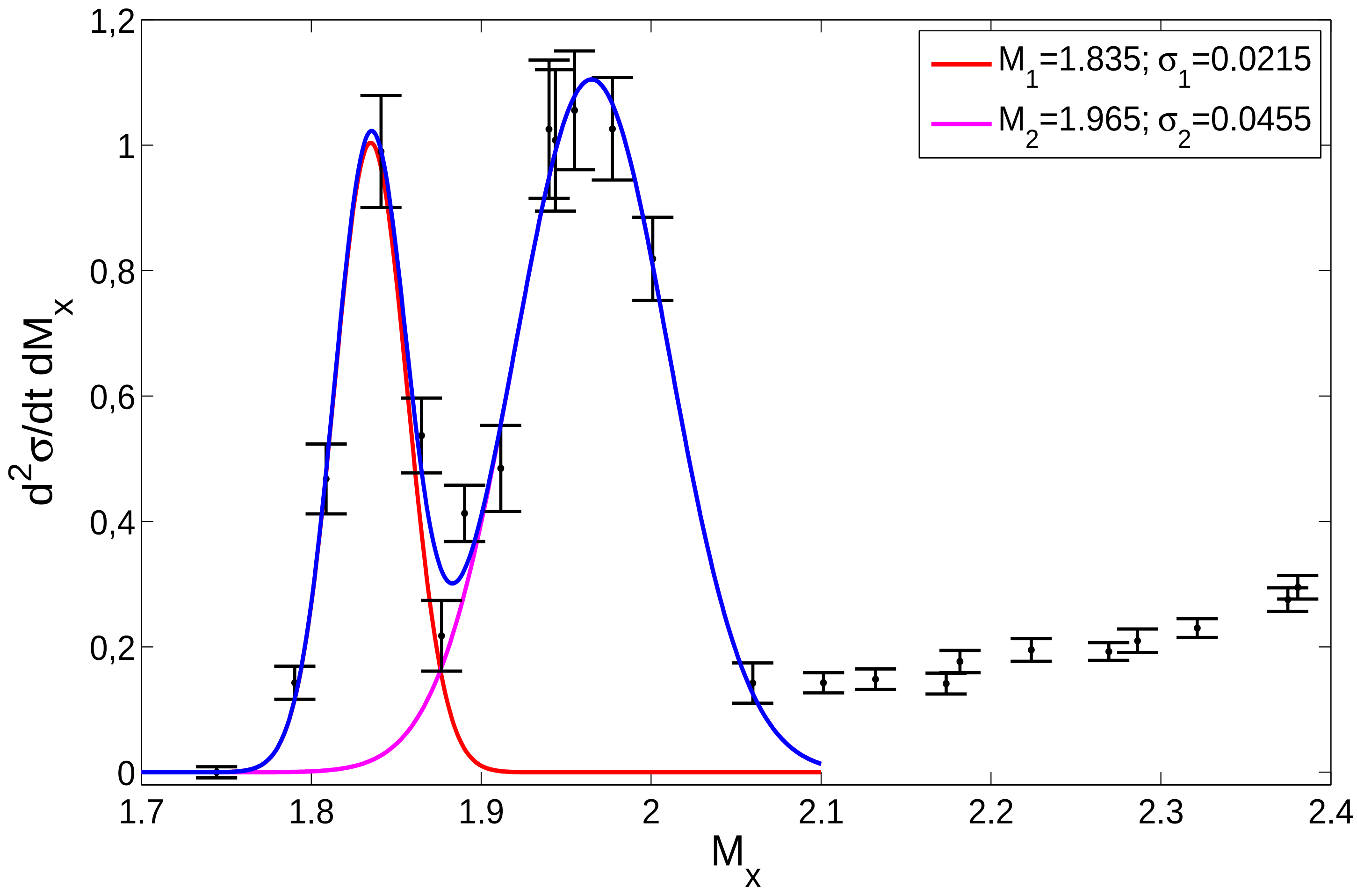
1955 ± 2, 1965 ± 2, 1980 ± 2, 2087 ± 3,

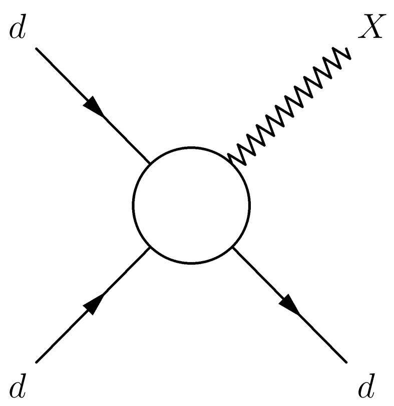
2106 ± 2

$$M_Y^2 = M_X^2 + t + M_X P_1 \frac{\sqrt{t(-4M_d^2 + t)}}{M_d^2} \cos\theta + \frac{M_X E_1 t}{M_d^2}$$



Peak II. Model d+d → X+d



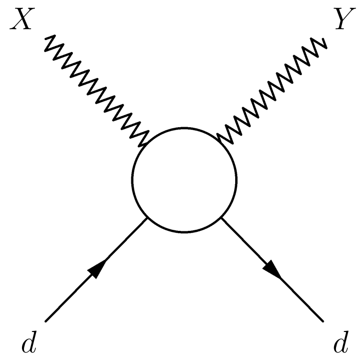


M=1965 ± 2 , Γ= 6 ± 2 MeV,

Yu. A. Troyan, PEPAN **24** (1993) 683

Peak II. Model X+d → Y+d

$$M_Y^2 = M_X^2 + t + M_X P_1 \frac{\sqrt{t(-4M_d^2 + t)}}{M_d^2} \cos\theta + \frac{M_X E_1 t}{M_d^2}$$



1.886 → 1.966, 1.898 → 1.979, 1.916 → 1.998,

1.937 → 2.020, 1.999 → 2.086, 2.017 → 2.105

Yu.A.T. : 1886 ± 1, 1898 ± 1, 1965 ± 2,

1980 ± 2, 1916 ± 2, 1937 ± 2, 1999 ± 2,

2017 ± 3, 2087 ± 3, 2106 ± 2

«ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА»
1993, ТОМ 24, ВЫП.3

УДК 539.17

УЗКИЕ ДИПРОТОННЫЕ РЕЗОНАНСЫ

Ю.А.Троян

Объединенный институт ядерных исследований, Дубна

Сделан критический обзор экспериментальных данных из работ, посвященных поиску и исследованию узких дипротонных резонансов, и опубликованных в 1986—1992 годах. Рассмотрена область эффективных масс двух протонов до 2300 МэВ/c^2. Обсуждается возможность существования 15 резонансных состояний.

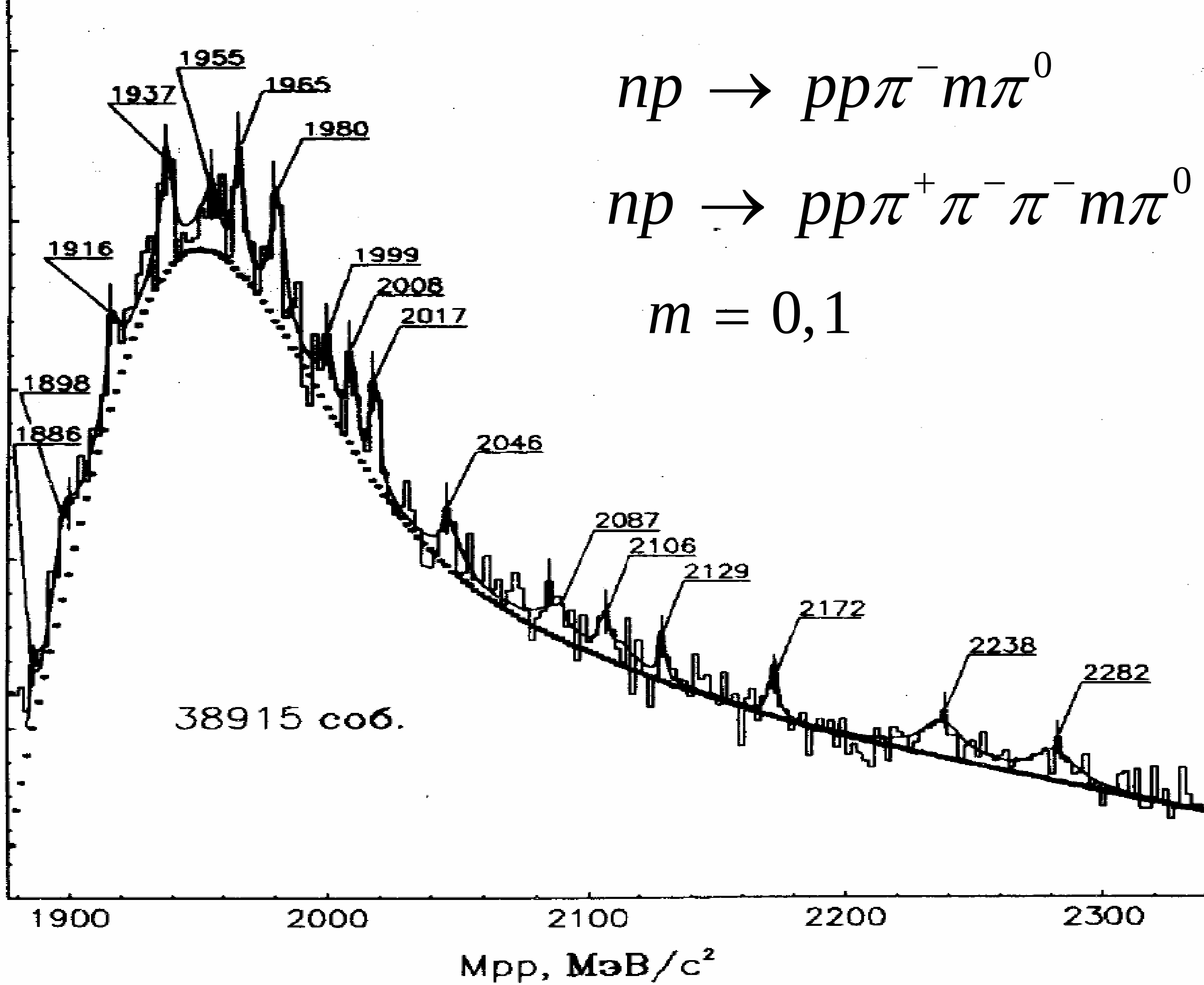
Работа [6] является итогом длительных исследований различных реакций *np*-взаимодействий, информация о которых получена с однометровой водородной пузырьковой камеры ЛВЭ ОИЯИ, экспонированной в пучках монохроматических нейтронов различных энергий. Хорошо сформированный пучок нейтронов ($\Delta P_n/P_n \approx 3\%$, $\Delta\Omega_{кан} \approx 10^{-7}$ ср) и высокая точность измерения параметров треков ($\Delta P/P \approx 1{,}5\%$, $\Delta\theta \approx 20'$) позволяют практически без примесей выделить различные каналы реакций *np*-взаимодействий и получить высокое разрешение по эффективной массе ($\Gamma_{уст}$ меняется от ≈ 1 МэВ/с^2 при массах вблизи суммы масс двух протонов до ≈ 16 МэВ/с^2 при массах в районе 2300 МэВ/с^2).

Таблица 1

$M_{exp} \pm \Delta M_{exp}$, МэВ/с^2	$\Gamma_{exp} \pm \Delta\Gamma_{exp}$, МэВ/с^2	$\Gamma_R \pm \Delta\Gamma_R$, МэВ/с^2	S.D.	P
1886±1	4±1	4±1	5,5	$5\cdot10^{-6}$
1898±1	11±3	11±3	5,6	$2\cdot10^{-6}$
1916±2	7±2	5±2	4,1	$2\cdot10^{-3}$
1937±2	7±2	5±2	5,5	$5\cdot10^{-6}$
1955±2	9±4	7±4	3,5	$2\cdot10^{-2}$
1965±2	6±2	0+2	4,5	$6\cdot10^{-4}$

Table 1 (continuation)

1980±2	9±2	6±2	5,7	$2\cdot10^{-6}$
1999±2	9±4	5±4	3,5	$2\cdot10^{-2}$
2008±3	4±2	0+2	3,7	$2\cdot10^{-2}$
2017±3	4±2	0+2	5,0	$1\cdot10^{-4}$
2046±3	10±5	4^{+5}_{-4}	3,8	$7\cdot10^{-3}$
2087±3	12±7	4^{+7}_{-4}	3,4	$2\cdot10^{-2}$
2106±2	11±5	0+5	3,4	$5\cdot10^{-2}$

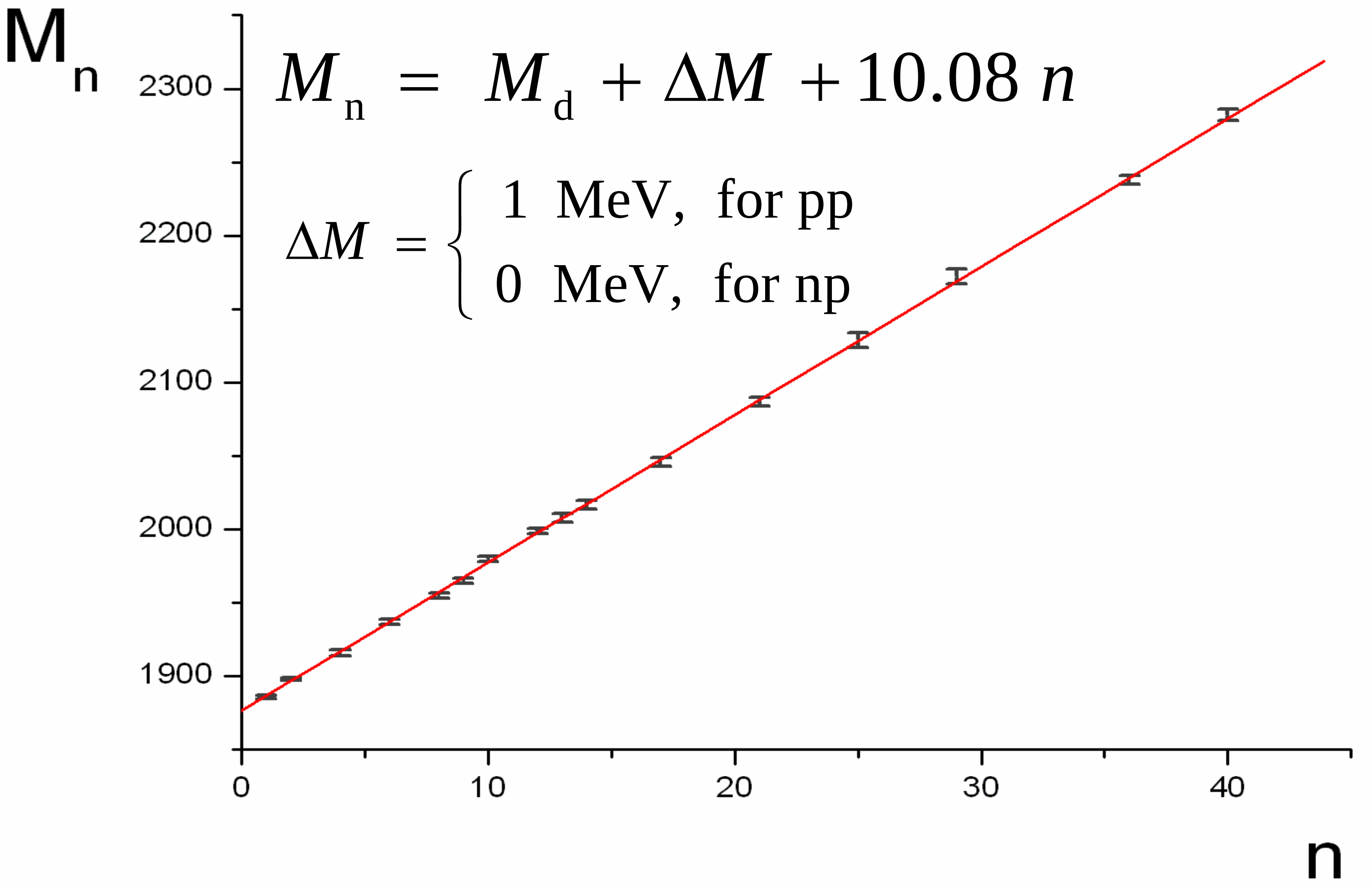


$$np \to pp\pi^{-} m\pi^{0}$$

$$np \to pp\pi^{+}\pi^{-}\pi^{-} m\pi^{0}$$

$$m = 0,1$$

Mass spectrum



Spectrum assumption

Reaction	KAM	dibaryon masses
X+D→Y+D	1916→1884	1916, 1886
	1926→1895	1926, 1896
	1936→1905	1936, 1906
	1946→1916	1946, 1916
	1956→1927	1956, 1926
	1966→1938	1966, 1936
	1976→1948	1976, 1946
	1986→1959	1986, 1956
	2047→2024	2047, 2027
	2057→2034	2057, 2037
	2067→2045	2067, 2047
	2077→2056	2077, 2057
	2087→2066	2087, 2067
	2097→2078	2097, 2077
	2107→2087	2107, 2087
	2118→2099	2118, 2097
	2128→2109	2128, 2107
	2138→2120	2138, 2118
	2148→2131	2148, 2128
	2158→2141	2158, 2138

The first peak

KAM – kinematically allowed masses,
dibaryon masses – according to

$$M_n = M_d + 10.08\, n$$

Spectrum assumption

Reaction	KAM	dibaryon masses
X+D→Y+D	1886→1966	1886, 1966
	1896→1977	1896, 1976
	1916→1998	1916, 1997
	1926→2009	1926, 2007
	1936→2019	1936, 2017
	1946→2030	1946, 2027
	1997→2084	1997, 2087
	2007→2095	2007, 2097
	2017→2105	2017, 2107
	2027→2116	2027, 2118
	2037→2127	2037, 2128
	2047→2137	2047, 2138
	2057→2148	2057, 2148
	2067→2158	2067, 2158
	2077→2169	2077, 2168
	2087→2179	2087, 2178
	2097→2190	2097, 2188
	2107→2200	2107, 2198

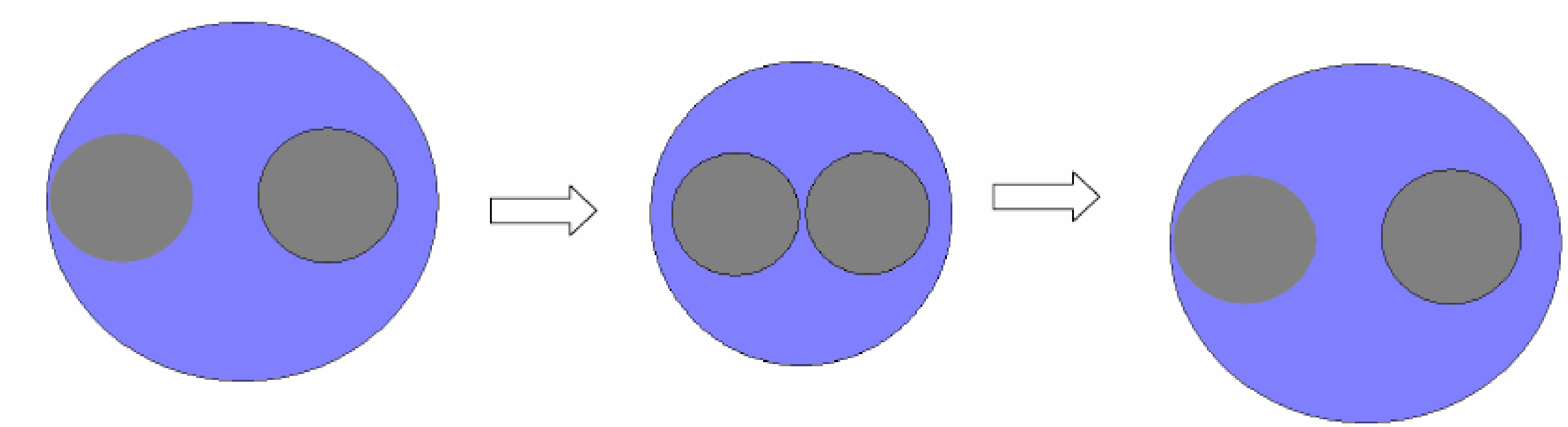
The second peak

All dibaryons in the range from 1886 to 2198 MeV/c^2 may be met in deuteron

Possible explanation

Dynamical Casimir effect

Reported at MMCP2013
arXiv:1310.2874v2



Pions are emitted into the bound state E=5.04 MeV during change of the potential well in close analogy with emission of electromagnetic waves due to motion of resonator's walls

Bound state of meson field

$$(\Box + m^2 + U(r))\varphi(\vec{r},t) = 0$$

$$\varphi(\vec{r},t) = e^{-iEt}\varphi_E(r)$$

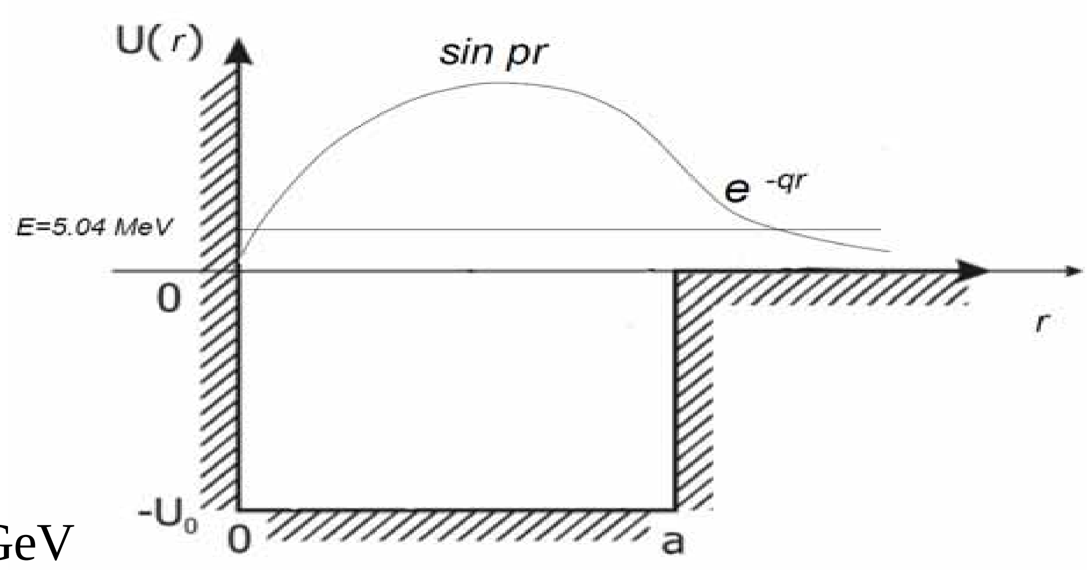
$$\varphi_E(r) = \frac{R_E(r)}{r}$$

$$U(r) = \begin{cases} -U_0, 0 \le r \le a \\ 0, r > a \end{cases}$$

$$E = \sqrt{p^2 + m^2 - U_0}$$

$$r < a : R_E(r) = A\sin pr, R_E(0) = 0$$

— energy of pions unside the potential well



Depths of the well:

$$p \simeq 0.528 \text{ GeV/c}, \sqrt{U_0} \simeq 0.546 \text{ GeV}$$

Generalized coherent states

$$\left|\psi_{coherent}\right\rangle = e^{\xi K_+ - \bar{\xi} K_-}\left|0\right\rangle \leftrightarrow \left|\psi_{Glauber}\right\rangle = e^{\alpha a^+ - \bar{\alpha} a}\left|0\right\rangle$$

$$\left|\psi_{Glauber}\right\rangle = e^{-|\alpha|^2/2}\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}\left|n\right\rangle,$$

$$\pi^0\pi^0 \text{ system: } K_+ = \frac{(a^+)^2}{2}, \quad K_- = \frac{a^2}{2}, \quad K_0 = \frac{aa^+ + a^+a}{4}$$

$$\pi^+\pi^- \text{ system: } K_+ = a_+^+ a_-^+, \quad K_- = a_+ a_-, \quad K_0 = \frac{1}{2}(a_+^+ a_+ + a_-^+ a_- + 1)$$

$$\left|\psi_{coherent}\right\rangle = (1-|\zeta|^2)^k \sum_{m=0}^{\infty}\left(\frac{\Gamma(m+2k)}{m!\,\Gamma(2k)}\right)^{1/2}\zeta^m\left|k, k+m\right\rangle$$

m – number of pion pairs, *k* – defines of SU(1,1) representation,

$$\zeta = \sqrt{\rho}\, e^{i\varphi}$$

Multiplicity distributions

$$\pi^0\pi^0 \ \text{system:} \ w_n = \left|\langle n | \psi_{coherent} \rangle\right|^2 = \sqrt{1-\rho}\,\frac{n!}{2^n\left[\left(\frac{n}{2}\right)!\right]^2}\,\rho^{\frac{n}{2}}, \quad n = 2m$$

$$\pi^+\pi^- \ \text{system:} \ w_n = \left|\langle n | \psi_{coherent} \rangle\right|^2 = (1-\rho)\rho^{\frac{n}{2}}$$

$$\rho = \frac{|v|^2}{|u|^2}$$

v, u – coefficients of the Bogolyubov transformation

Experimental effects

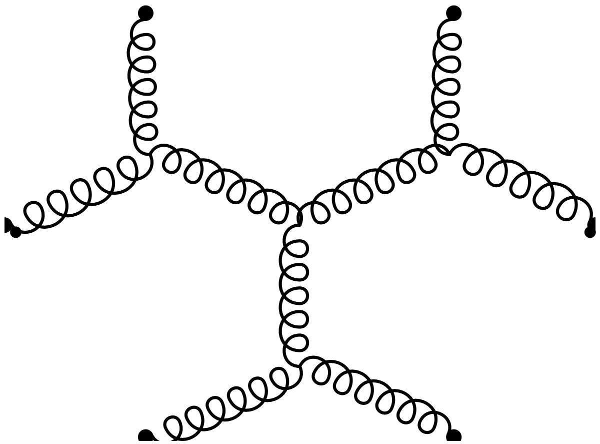
- Production probabilities for different dibaryons decaying into 2N. Observation of dibaryons up to m=40 means that $\rho \approx 1$, or

  $\varepsilon^2 \ll \sin^2(5.04 / \Gamma)$.
- Narrow multibarions decaying into several nucleons without secondary pions may be found. They corresponds to Bose-Einstein condensate of pions in compressed few nucleon systems.
- Pion condensate may be responsible for collective flows at the final stage of high-energy nuclear collisions

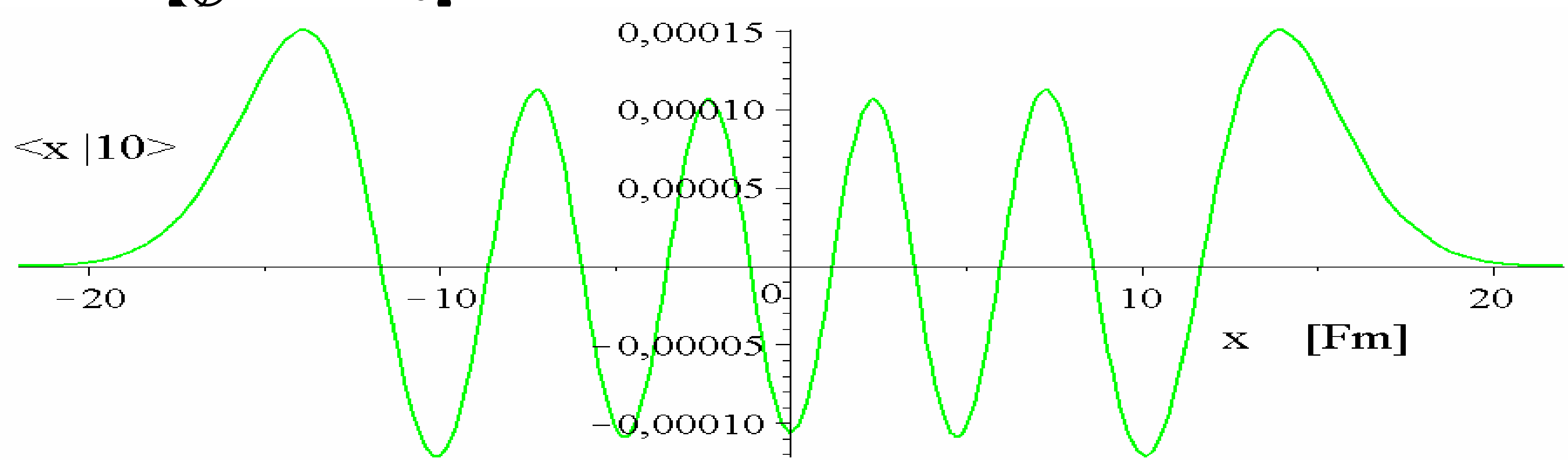
Thank you for attention!

Explanation of light Troyan's dibaryon

- Quark oscillator? NO!



$M_q \approx M_N/3$, $\Delta = 10.08$ MeV/c2



Energy level E=5.04 MeV

$$\varphi(\vec{r},t) = e^{-iEt}\varphi_E(r)$$

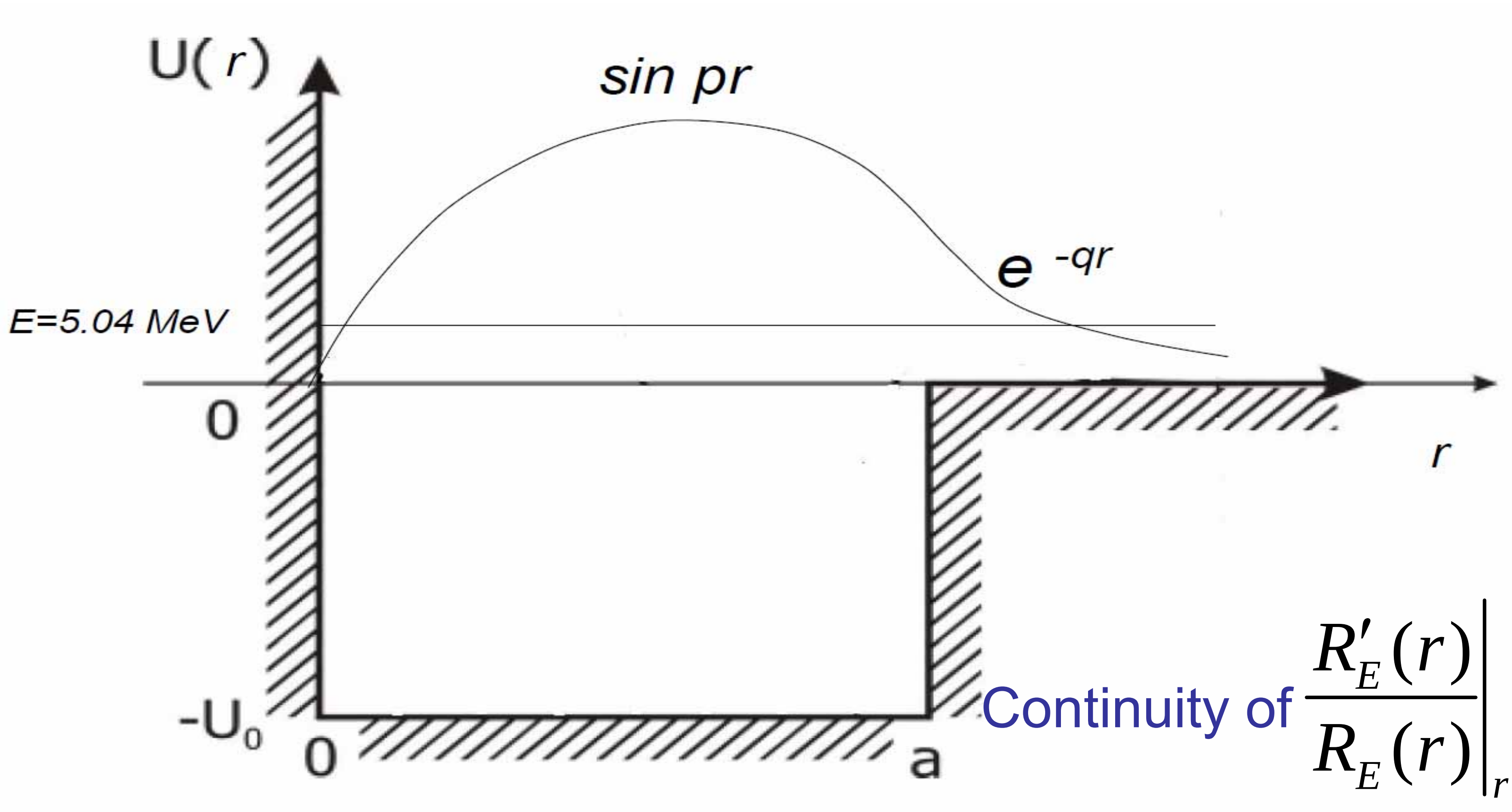
$$\frac{1}{r^2}\frac{d}{dr}r^2\frac{d\varphi_E(r)}{dr} + (E^2 - m^2 - U(r))\varphi_E(r) = 0$$

$$\varphi_E(r) = \frac{R_E(r)}{r}$$

$$\frac{d^2R_E(r)}{dr^2} + (E^2 - m^2 - U(r))R_E(r) = 0$$

Solution

$$r < a : R_E(r) = A\sin pr, R_E(0) = 0$$



Continuity of $\left.\frac{R_E'(r)}{R_E(r)}\right|_{r=a}$

Estimation of physical values

Boundary condition
at r=a

$$\Rightarrow \; p \ \mathrm{ctg}\, pa = \sqrt{m^2 - E^2}$$

$$t \simeq -0.5 \ \frac{\mathrm{GeV}^2}{\mathrm{c}^2} \ \simeq -|\vec{q}|^2$$

$$a = \langle r^2 \rangle^{1/2} \simeq \sqrt{6} \ \hbar / |\vec{q}| = 0.684 \ \mathrm{Fm}$$

$$\mathrm{m{=}0.14\ GeV/c^2},\ E = 5.04 \cdot 10^{-3} \ \mathrm{GeV}$$

$$\mathrm{p} \simeq 0.528 \ \mathrm{GeV/c}, \ \sqrt{U_0} \simeq 0.546 \ \mathrm{GeV}$$

— Depths of the well

Parity and isospin conservation

$$P\psi_n = (-1)^n \psi_n, \quad P\psi_0 = \psi_0 \quad \Rightarrow \psi_0 \to \psi_{2m}$$

$\Rightarrow$ Pions may be produced only in pairs

$$\Psi_{2\pi} = \frac{1}{\sqrt{3}}(\pi_a^+ \pi_b^- + \pi_a^- \pi_b^+ - \pi_a^0 \pi_b^0), \quad I_{2\pi} = 0, \; I_{2\pi}{}^3 = 0$$

$$I_\pi = 1, \; I_{\pi-}{}^3 = -1, \; I_{\pi 0}{}^3 = 0, \; I_{\pi+}{}^3 = +1$$

$\Rightarrow$ Pions' state should have zero quantum numbers

$\Rightarrow$ Conservation of deutron quantum numbers

Field evolution during motion of the well

$$\left[\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial r^2} + m^2 - U_0(t)\right]\psi(r,t) = 0$$

$$\psi(r,t) = \sin pr \cdot \chi(t)$$

$$\frac{\partial^2 \chi(t)}{\partial t^2} + (p^2 + m^2 - U_0(t))\chi(t) = 0, \quad \omega^2(t) = p^2 + m^2 - U_0(t) \equiv E^2$$

We have an oscillator excited by changing its parameters.

Hamiltonian formalism

$$H = \frac{1}{2}\left(\pi_\omega^2 + \omega^2(t)\chi_\omega^2\right) = \omega(t)\left(a_\omega^+(t)a_\omega(t) + \frac{1}{2}\right)$$

$$\frac{\partial H}{\partial \pi_\omega} = \dot{\chi}_\omega, \quad -\frac{\partial H}{\partial \chi_\omega} = \dot{\pi}_\omega$$

$$\chi_\omega = \frac{a_\omega + a_\omega^+}{\sqrt{2\omega}}, \quad \pi_\omega = \frac{a_\omega - a_\omega^+}{\sqrt{2\omega}}$$

Field quantization

$$\hat{\psi}(r,t) = const \cdot \left(\frac{\hat{a}^{+}_{\omega}(t) + \hat{a}_{\omega}(t)}{\sqrt{2\omega_1}} \right) \sin pr,$$

$$\chi_{\omega}(t) = \frac{\hat{a}^{+}_{\omega}(t) + \hat{a}_{\omega}(t)}{\sqrt{2\omega_1}}, \quad \omega_1 \equiv \omega(t_1) = \sqrt{p^2(t_1) + m^2 - U_0(t_1)}$$

Bogolyubov's transformation

$$\begin{pmatrix} \hat{a}(t_1 + dt) \\ \hat{a}^+(t_1 + dt) \end{pmatrix} = \begin{pmatrix} u(dt) & v(dt) \\ \overline{v}(dt) & \overline{u}(dt) \end{pmatrix} \begin{pmatrix} \hat{a}(t_1) \\ \hat{a}^+(t_1) \end{pmatrix} \equiv S(dt) \begin{pmatrix} \hat{a}(t_1) \\ \hat{a}^+(t_1) \end{pmatrix}$$

$$S(dt_1 + dt_2 + \ldots + dt_n) = S(dt_n)\ldots S(dt_2)S(dt_1)$$

$$|u|^2 - |v|^2 = 1 \Rightarrow S(t) \in SU(1,1)$$

Matrixes of the Bogolyubov transformation generate SU(1,1) group at multiplication .

Lie algebra of SU(1,1) and elements of SU(1,1) group expressed through generators of the group

$$[K_0, K_\pm] = \pm K_\pm, \quad [K_-, K_+] = 2K_0$$

$$[K_1, K_2] = -iK_0, \quad [K_2, K_0] = iK_1, \quad [K_0, K_1] = iK_2$$

$$K_\pm = \pm i(K_1 \pm iK_2)$$

$$S(dt) = e^{(\beta K_+ - \bar{\beta} K_- - i\gamma K_0)dt} = e^{-iHdt}$$

— Hamiltonian of the field should be a linear combination of SU(1,1) generators

Expression of *H* through generators

$$H = i(\beta K_+ - \bar{\beta} K_- - i\gamma K_0) = \frac{1}{2}\left(\pi_\omega^2 + \omega^2(t)\chi_\omega^2\right)$$

$$\Rightarrow K_0 = \frac{1}{4}\left(\frac{\pi_\omega^2}{\omega_0} \pm \omega_0 \chi_\omega^2\right), \quad K_2 = \frac{1}{4}\left(\pi_\omega \chi_\omega + \chi_\omega \pi_\omega\right),$$

1

$$\beta = \frac{i\omega_0}{2}\left[\left(\frac{\omega(t)}{\omega_0}\right)^2 - 1\right], \qquad \gamma = \omega_0\left[\left(\frac{\omega(t)}{\omega_0}\right)^2 + 1\right]$$

Glauber coherent states

$$\left|\psi_{coherent}\right\rangle = e^{\xi K_+ - \bar{\xi} K_-}\left|0\right\rangle \leftrightarrow \left|\psi_{Glauber}\right\rangle = e^{\alpha a^+ - \bar{\alpha} a}\left|0\right\rangle$$

$$\left|\psi_{Glauber}\right\rangle = e^{-|\alpha|^2/2}\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}\left|n\right\rangle,$$

$$w_n = \left|\left\langle n|\alpha\right\rangle\right|^2 = e^{-|\alpha|^2}\frac{|\alpha|^{2n}}{n!}, \quad \left\langle n\right\rangle = |\alpha|^2$$

Generalized coherent states

$$\pi^0\pi^0 \text{ system: } K_+ = \frac{(a^+)^2}{2}, \quad K_- = \frac{a^2}{2}, \quad K_0 = \frac{aa^+ + a^+a}{4}$$

$$C_2 \equiv K_0^2 - K_1^2 - K_2^2 \equiv k(k-1)I = -\frac{3}{16}I, \quad k = \begin{cases} 1/4 \quad , \text{ states } |2n\rangle \\ 3/4, \text{ states } |2n+1\rangle \end{cases}$$

$$\pi^+\pi^- \text{ system: } K_+ = a_+^+ a_-^+, \quad K_- = a_+ a_-, \quad K_0 = \frac{1}{2}(a_+^+ a_+ + a_-^+ a_- + 1)$$

$$C_2 = -\frac{1}{4} + \frac{1}{4}(a_+^+ a_+ - a_-^+ a_-)^2, \quad k = \frac{1}{2}$$

$$|\psi_{coherent}\rangle = (1 - |\zeta|^2)^k \sum_{m=0}^{\infty} \left(\frac{\Gamma(m+2k)}{m!\Gamma(2k)} \right)^{1/2} \zeta^m |k, k+m\rangle$$

m – number of pion pairs, *k* – defines of SU(1,1) representation

$$\zeta = \sqrt{\rho}\, e^{i\varphi}$$

Calculation of ρ

$$\frac{\partial^2 f(t)}{\partial t^2} + \omega^2(t) f(t) = 0, \quad \omega^2(t) = p^2 + m^2 - U_0(t) \equiv E^2$$

$$f(t) = \begin{cases} v(t) \\ u(t) \end{cases}$$

$$u(t_1) = e^{-i\omega_1 t_1}, \quad v(t_1) = 0$$

$$u(t_2) = c_1 e^{-i\omega_1 t_2}, \quad v(t_2) = c_2 e^{-i\omega_1 t_2}, \quad \omega(t_1) = \omega(t_2) = \omega_1$$

$$\rho = \frac{|v|^2}{|u|^2} = \frac{|c_2|^2}{|c_1|^2} \quad |c_1|^2 - |c_2|^2 = 1 \Rightarrow |c_1| = \frac{1}{\sqrt{1-\rho}}, \quad |c_2| = \sqrt{\frac{\rho}{1-\rho}}$$

Reduction to a scattering problem

New variable: $w(t) = (u(t) + \overline{v}(t)) / c_1$

$$w(t_1) = e^{-i\omega_1 t_1} / c_1, \quad w(t_2) = e^{-i\omega_1 t_2} + \frac{c_2}{c_1} e^{i\omega_1 t_2}$$

$$\rho = R = \frac{|c_2|^2}{|c_1|^2}, \quad D = \frac{1}{|c_1|^2}$$

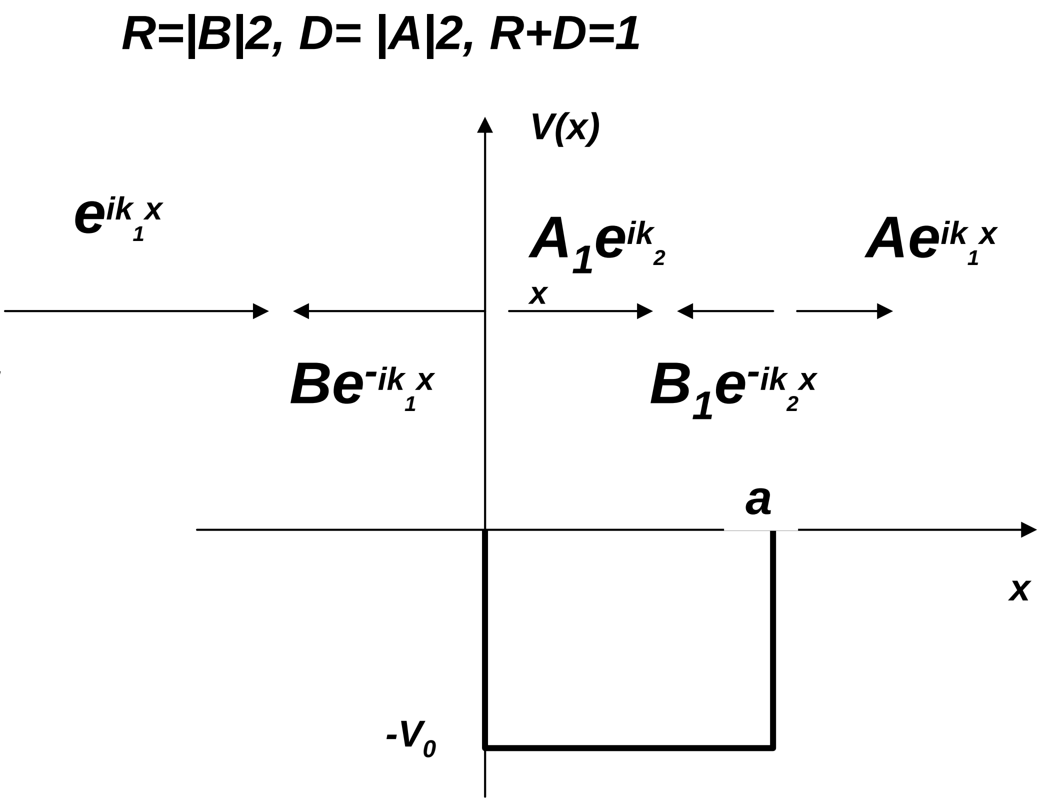
$$\frac{\partial^2 w(t)}{\partial t^2} + \omega^2(t) w(t) = 0, \quad \omega^2(t) = p^2 + m_\pi^2 - U_0,$$

$$\updownarrow$$

$$\frac{\partial^2 \psi(x)}{\partial x^2} + \left(\frac{k^2}{2m} - V(x) \right) \psi(x) = 0,$$

$$\omega^2(t) \leftrightarrow \left(\frac{k^2}{2m} - V(x) \right), \quad 2m = 1$$

Solution of the scattering problem



Correspondence:

$R \to \rho$, $-x \to t$, $a \to$,

$k^2(-x) - V(-x) \to E^2(t) - V(t)$,

$V(t) \leftrightarrow$ boundary condition

$k_1 = k(-x_1) \to E(t_1) = \varepsilon$,

$V(x) = k_1^2 - k^2(-x) \to$

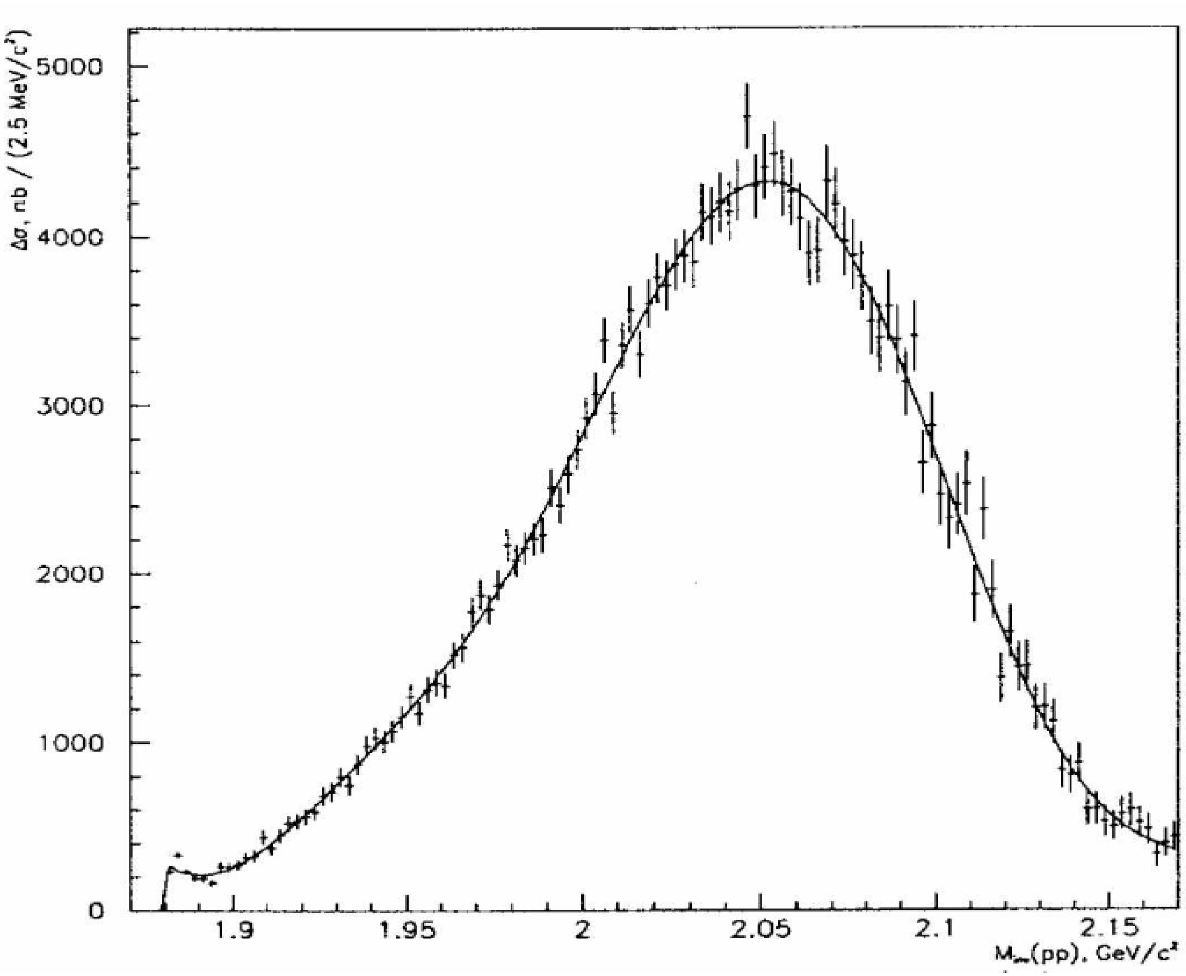
$\to \varepsilon^2 - E^2(t)$,

$-V_0 \to \varepsilon^2 - (5.04\ MeV)^2$

$$\rho = \frac{1}{1+\delta^2}, \quad \delta = \frac{2\varepsilon E}{V_0 \sin E\tau}, \quad E = 5.04\ \mathrm{MeV}, \ \tau \sim \frac{1}{\Gamma}$$

B.M.Abramov et al paper

Z.Phys. C **69** (1996) 409-413.



Only reactions pn ->pp π^-.
Insufficient cooling.

Optimal approximation of the experimental invariant mass spectrum, no methods for background subtraction.

Small statistics (less than in Troyan's experiment) - Insufficient resolution of mass measured.
Necessary: $\Delta M_{pp} \approx$ 1VeV