QUARTET OF TAU-LEPTONS AND POSSIBILITY OF THEIR OSCILLATIONS

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LEPTON SECTOR TAKES KEY POSITIONS IN MODERN PARTICLE PHYSICS

Exclusive role of the leptons is connected with their participation in most cases of nuclear and subnuclear processes. Moreover they are present as a finite product at the decay of every hadrons.
Such reason make high demands of lepton sector description. Most important among them are:

- FUNDAMENTAL INITIAL ASSUMPTIONS
- STRICT CONCLUSIONS
- INNER SELF-CONSISTENT DESCRIPTION OF SECTOR AS A WHOLE

Throughout last years we elaborated holistic description of lepton sector in combination with complete individuality of every lepton type

MASSIVE AND MASSLESS
CHARGED AND NEUTRAL
STABLE AND UNSTABLE

LEPTONS

HOLISTIC DESCRIPTION

— it means unique algorithmic approach; completeness and closure of obtained results.
It is guarantee of self-consistent description for the sector as a whole.

INDIVIDUALITY

— it means presence of own individual structure of every lepton equation.
The structure is carrier of individual properties and quantum numbers.

STUDY OF UNSTABLE LEPTONS IS NECESSARY FOR VERIFICATION AND TO ENSURE SELF-CONSISTENT LEPTON SECTOR DESCRIPTION

Equations for unstable leptons were obtained by extending groups of massive stable leptons.
To extend group — means to add one more fifth generator Γ_5.
If to extend Dirac's group by the generator such as $\Gamma_5^2 = I$, we obtain group Δ_1 and respectively equation for $\mu^{\pm}$.
If to extend Dirac's group by the generator such as $\Gamma_5^2 = -I$, we obtain group Δ_3 and respectively equation for $\tau^{\pm}$.
If to extend Majorana's group by the generator such as $\Gamma_5^2 = -I$, we obtain group Δ_2 and respectively equation for heavy unstable neutrino ν.

Group Δ_3 has the following defining relations:

$$\begin{array}{ll} \Gamma_s\Gamma_t + \Gamma_t\Gamma_s = 2\delta_{st}, & (s,t = 1,2,3,4), \\ \Gamma_s\Gamma_5 + \Gamma_5\Gamma_s = 0, & (s = 1,2,3,4), \\ \Gamma_5^2 = -I. & \end{array}$$

It follows from here such structure of Δ_3:

$$\boxed{\Delta_3\{D_\gamma(II), \quad D_\gamma(I), \quad D_\gamma(III)\}} \qquad In[\Delta_3] = 0.$$

Common properties of all three groups are:

1. Order od every group equal to 64.
2. Every group has two nonequivalent irreducible representations of fourth dimension.
3. Every group has center containing four elements. It looks for Δ_1 and Δ_2 as following $\{I; -I; I; -I\ \}$. Center of Δ_3 looks otherwise $\{I; -I; iI; -iI\ \}$

Main distinctions between groups are they have different structural invariants

- $In[\Delta_1] = -1$
- $In[\Delta_2] = 1$
- $In[\Delta_3] = 0$

STRUCTURAL DISTINCTIONS LEAD TO SIGNIFICANT PHYSICAL CONSEQUENCES

In the case of group Δ_1 and Δ_2 we obtain separate description particle and antiparticle for different nonequivalent representations.

IN THE Δ_3-GROUP EVERY NONEQUIVALENT FOUR-DIMENSION REPRESENTATION IS RELATED WITH DOUBLET EQUATION. EACH OF THEM DESCRIBES PARTICLE AND ANTIPARTICLE LIKE DIRAC EQUATION

By virtue of nonequivalent representations these two pairs of particles are

not identical.
They form the quartet state.

IN THAT WAY WE HAVE HERE TWO PAIRS OF PARTICLE

$\tau^{\pm}$ **AND** $(\tau^{*})^{\pm}$

WAVE-CORPUSCULAR DUALITY IS FUNDAMENTAL PRINCIPLE OF PARTICLE PHYSICS

Most evident and successful demonstration of this principle is Dirac's decomposition of Klein-Fock-Gordon (KFG) equation on two factors linear in first order derivatives. Dirac's algorithm makes it possible to construct lepton wave equations omitting decomposition of KFG equation on two factors

IF PARTICLE POSSESS WAVE PROPERTIES, THEN ITS WAVE EQUATION MUST REDUCE TO SOME TYPE OF KFG EQUATION

In this way we get additional possibilities for description of different wave properties of lepton from unique point of view. They are

interference
annihilation
oscillations

THE PRESENCE OF STRUCTURAL INDIVIDUALITY OF LEPTONS IN COMMON WITH GENERALITY OF THEIR GROUP BASIS CREATE PREREQUISITES FOR FULL DESCRIPTION LEPTON WAVE PHENOMENA

The wave-corpuscular duality in Dirac's method has got simple and clear design. Schematic picture of Dirac's decomposition looks as following

$$(KFG) = (...e(+p_0)...)(...e(-p_0)...) \tag{1}$$

It is well known that interaction of electron waves with coinciding phases leads to interference

$$(...e^{-}...)(...e^{-}...) \Rightarrow interference \tag{2}$$

In the frame work of doublet equation one can prove following relation

$$(M_T)(...e^{-}...)(...e^{+}...) = (M_T)(KFG) \Rightarrow annihilation \tag{3}$$

Here common factor (M_T) is matrix. It appearance is connected with two different initial spin states of the system $(e^{-}e^{+})$.

THE WAVE EQUATIONS FOR τ AND τ^* ARE VERY SIMILAR

$$(\Gamma_5 p_0 - \Gamma_1 p_1 - \Gamma_2 p_2 - \Gamma_3 p_3 - \Gamma_4 m)\Psi = 0 \tag{4}$$

and

$$(\Gamma_5^{'} p_0 - \Gamma_1 p_1 - \Gamma_2 p_2 - \Gamma_3 p_3 - \Gamma_4 m^*)\Psi = 0 \tag{5}$$

$$m = m^*$$

We obtain apart from usual terms of KFG equation mixed elements

$$2\Gamma_5\Gamma_1\partial^2/\partial x\partial t + 2\Gamma_5\Gamma_2\partial^2/\partial y\partial t + 2\Gamma_5\Gamma_3\partial^2/\partial z\partial t \tag{6}$$

$$m \neq m^*$$

Here we obtain in addition elements of following type

$$(m - m^*)\Gamma_1\Gamma_4\partial/\partial x + (m - m^*)\Gamma_2\Gamma_4\partial/\partial y + (m - m^*)\Gamma_3\Gamma_4\partial/\partial z \tag{7}$$

THE STRUCTURE IS TANGLED, BUT IT EXIST
REGARDLESS OF MASS DIFFERENCE

SUMMARY

TAKING INTO ACCOUNT STRUCTURES OF THE LEPTON EQUATIONS WE CONCLUDE EXISTENCE OF DOUBLES FOR

$\tau^{\pm}$ **ID EST** $(\tau^{*})^{\pm}$

THE STRUCTURES OF LEPTON EQUATIONS OPEN NEW POSSIBILITIES FOR MORE COMPLETE DESCRIPTION OF WAVE PHENOMENA

Tau-leptons are very informative part of the lepton sector. They serve natural and real link between leptons and hadrons due to their different channels for decays.

APPENDICES

A not excessive set of

INITIAL REQUIREMENTS ,

underlying Dirac algorithm, is following.

1. The equations must be invariant and covariant corresponding to the homogeneous Lorentz transformations taking into account all the four connected components;
2. The equations must be formulated on the basis of irreducible representations of the groups determining every lepton equation;
3. Conservation of four-vector of the probability current must be fulfilled and the fourth component of the current must be positively defined;
4. The lepton spin is supposed to be equal to 1/2;
5. Every lepton equation must be reduced to the Klein-Gordon equation.

ALL LEPTONS ARE DIVIDED INTO THREE CLASSES DEPENDING ON THE COMPOSITION OF THE MAXIMAL ORDER SUBGROUPS.

$\boxed{\text{SINGLETS}}$

$\boxed{\text{DOUBLETS}}$

$\boxed{\text{QUARTETS}}$

SINGLETS $\Rightarrow$ They do not have antiparticles.

DOUBLETS $\Rightarrow$ Equation describes only one pair particle-antiparticle.

QUARTETS $\Rightarrow$ Equation describes the two pairs of subgroups associated attitude particle-antiparticle.

Differentiation between the concepts of particle and antiparticle plays an important role for classification of leptons.

MAXIMAL SUBGROUPS FOR STABLE LEPTONS ARE FOUR CONNECTED COMPONENTS OF THE HOMOGENEOUS LORENTZ GROUP

These groups are:

1. **group $\mathbf{d}_\gamma$ $\rightarrow$ proper orthochronous representation;**
2. **group $\mathbf{f}_\gamma$ $\rightarrow$ improper orthochronous, $\langle P \rangle$- conjugate representation**;
3. **group $\mathbf{b}_\gamma$ $\rightarrow$ proper antichronous, $\langle T \rangle$- conjugate representation;**
4. **group $\mathbf{c}_\gamma$ $\rightarrow$ improper antichronous, $\langle PT \rangle$- conjugate representation.**

All they are connected among themselves by discrete transformations $\langle T \rangle$, $\langle P \rangle$, $\langle PT \rangle$.

Structures of the stable lepton groups.

- **The Dirac equation** — $D_\gamma(II)$: $\boxed{\mathbf{d}_\gamma,\ \mathbf{b}_\gamma,\ \mathbf{f}_\gamma,}$ structural invariant $In[D_\gamma(II)] = -1$.
- **The equation for a doublet of massive neutrinos** — $D_\gamma(I)$: $\boxed{\mathbf{d}_\gamma,\ \mathbf{c}_\gamma,\ \mathbf{f}_\gamma}$, structural invariant $In[D_\gamma(I)] = 1$.
- **The equation for a quartet of massless neutrinos** — $D_\gamma(III)$: $\boxed{\mathbf{d}_\gamma,\ \mathbf{b}_\gamma,\ \mathbf{c}_\gamma,\ \mathbf{f}_\gamma}$, structural invariant $In[D_\gamma(III)] = 0$.
- **The equation for a massless T-singlet** — $D_\gamma(IV)$: $\boxed{\mathbf{b}_\gamma}$, structural invariant $In[D_\gamma(IV)] = -1$.
- **The equation for a massless (PT)-singlet** — $D_\gamma(V)$: $\boxed{\mathbf{c}_\gamma}$, structural invariant $In[D_\gamma(V)] = 1$.

Every group related with corresponding equation has

nonrecurrent composition.

Structures of the unstable lepton groups.

Group Δ_1 has the following defining relations:

$$\Gamma_\mu \Gamma_\nu + \Gamma_\nu \Gamma_\mu = 2\delta_{\mu\nu}, \quad (\mu, \nu = 1, 2, 3, 4, 5) \tag{8}$$

As a result we obtain the following composition:

$$\boxed{\Delta_1\{D_\gamma(II), \quad D_\gamma(III), \quad D_\gamma(IV)\}} \qquad In[\Delta_1] = -1.$$

Group Δ_3 has the following defining relations:

$$\begin{array}{ll} \Gamma_s \Gamma_t + \Gamma_t \Gamma_s = 2\delta_{st}, & (s, t = 1, 2, 3, 4), \\ \Gamma_s \Gamma_5 + \Gamma_5 \Gamma_s = 0, & (s = 1, 2, 3, 4), \\ \Gamma_5^2 = -I. & \end{array}$$

It follows from here:

$$\boxed{\Delta_3\{D_\gamma(II), \quad D_\gamma(I), \quad D_\gamma(III)\}} \qquad In[\Delta_3] = 0.$$

Structures of the unstable lepton groups.

Group Δ_2 has the following defining relations:

$$\begin{array}{ll} \Gamma_s\Gamma_t + \Gamma_t\Gamma_s = 2\delta_{st}, & (s,t = 1,2,3), \\ \Gamma_s\Gamma_4 + \Gamma_4\Gamma_s = 0, & (s = 1,2,3), \\ \Gamma_4^2 = -I. & \\ \Gamma_u\Gamma_5 + \Gamma_5\Gamma_u = 0, & (u = 1,2,3,4), \\ \Gamma_5^2 = -I. & \end{array}$$

We obtain in this case:

$$\boxed{\Delta_2\{D_\gamma(I), \quad D_\gamma(III), \quad D_\gamma(V)\}} \qquad In[\Delta_2] = 1.$$

All three groups have its own structures.

We see that four conjugate components of Lorentz group allowed to describe different leptons due to complication of structural constituents.

An effective tool for analysis and constructing lepton equations was used, i.e. numerical characteristic of irreducible matrix group.

Theorem. If $D_\gamma = \{\gamma_1, ..., \gamma_n\}$ is an irreducible matrix group, then

$$\mathbf{In}[D_\gamma] = \frac{1}{n} \sum_{i=1}^{n} Sp(\gamma_i^2) = \left\{ \begin{array}{c} 1, \\ -1, \\ 0. \end{array} \right. \tag{9}$$

Here n - is order of the group, $Sp(\gamma_i^2)$ - is a trace of i-matrix squared. $\mathbf{In}[\mathbf{D}_\gamma]$ — will be called **structural invariant** of D_γ-group.