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RADIATIVE CORRECTIONS TO POLARIZATION OBSERVABLES OF ELASTIC ELECTRON-PROTON SCATTERING

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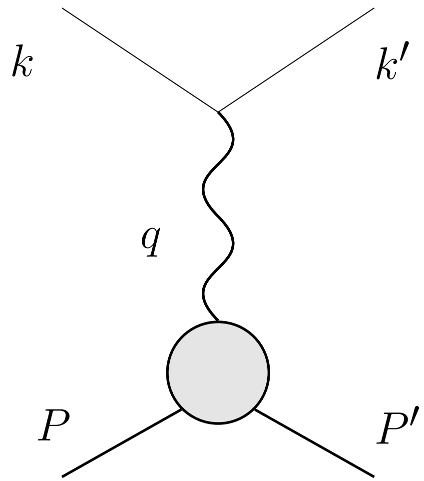
THE NUCLEON FORM FACTORS (NFFs)
ARE
FUNDAMENTAL OBSERVABLES

- **NFFs give information about structure of the nucleon.**
  - **Size of the nucleon** $Q^2 \equiv -q^2 \ll m_N^2$
  - **Quark counting rules, pQCD, etc.** $Q^2 \equiv -q^2 \gg m_N^2$

- **NFFs are used in calculations of the e.m. properties of more complicated objects (the deuteron, ^{3}He, ^{4}He, etc.**

Observables are simply connected with FFs in the framework of One Photon Exchange



$$\alpha \approx \frac{1}{137}$$

$$\Gamma_\mu = 2M(G_E - G_M)\frac{P_\mu + P'_\mu}{(P+P')^2} + G_M\gamma_\mu$$

$G_E \equiv G_E(q^2)$ and $G_M \equiv G_M(q^2)$ - form factors

$$\boxed{\frac{S_\parallel}{S_\perp} = \frac{G_M}{G_E}\sqrt{\tau\,\frac{1+\varepsilon}{2\varepsilon}}}$$

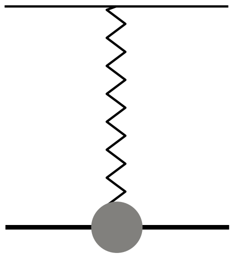
$$\boxed{d\sigma = \frac{2\pi\alpha^2 dQ^2}{E^2Q^2}\frac{1}{1-\varepsilon}\left(\varepsilon G_E^2(Q^2) + \tau G_M^2(Q^2)\right)}$$

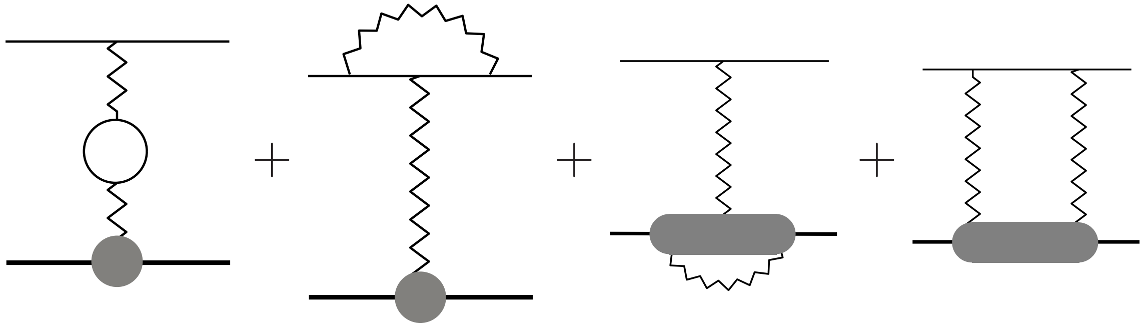
The precision level of present-day electron-proton scattering experiments makes it necessary to take into account effects beyond Born approximation

The aim of my talk is to give estimations of Radiative Corrections to $\frac{S_\parallel}{S_\perp}$

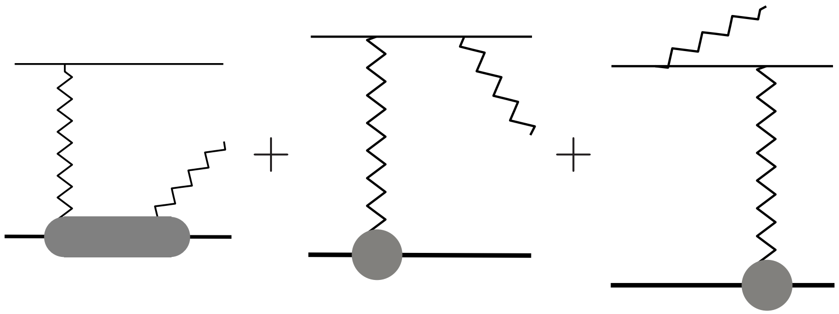
NEXT ORDER PERTURBATION TERMS

(i) corrections to elastic scattering

$\mathcal{M}_1 =$ 

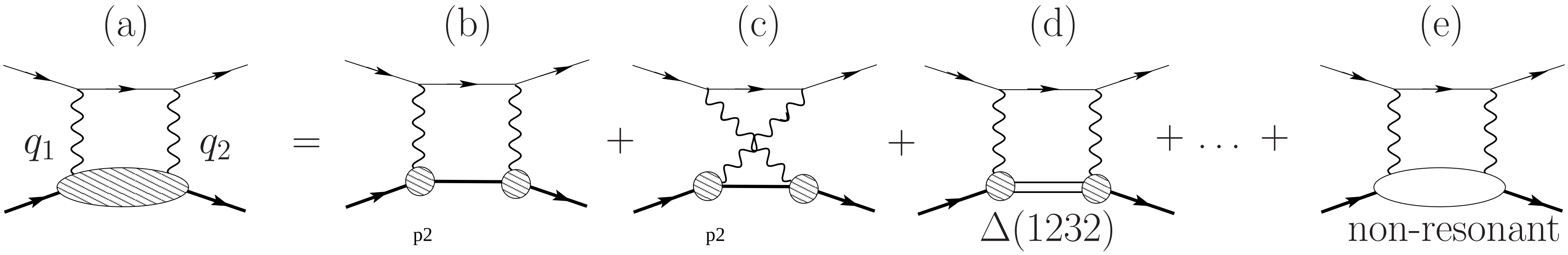
$\mathcal{M}_2 =$ 

(ii) radiation of undetected (soft) photon, bremsstrahlung (BS)

$\mathcal{M}_{2\gamma} =$ 

$$\sigma(\Delta E) \sim |\mathcal{M}_1 + \mathcal{M}_2|^2 + \int |M_{2\gamma}|^2 d\Omega_\gamma$$

TWO PHOTON EXCHANGE



General structure of the amplitude

$$\mathcal{M} = \frac{4\pi\alpha}{q^2}\bar{u}'\gamma_\mu u \cdot \bar{U}'\left(\gamma^\mu \tilde{F}_1 - \frac{1}{4M}[\gamma^\mu, \hat{q}]\tilde{F}_2 + \frac{P^\mu}{M^2}\hat{K}\tilde{F}_3\right)U$$

Generalized form factors

$$\mathcal{G}_E = \tilde{F}_1 - \tau\tilde{F}_2 + \frac{\nu}{4M^2}\tilde{F}_3, \qquad \mathcal{G}_M = \tilde{F}_1 + \tilde{F}_2 + \varepsilon\frac{\nu}{4M^2}\tilde{F}_3, \qquad \mathcal{G}_3 = \nu\tilde{F}_3/4M^2,$$

Generalized FFs are complex functions of two variables t, ε

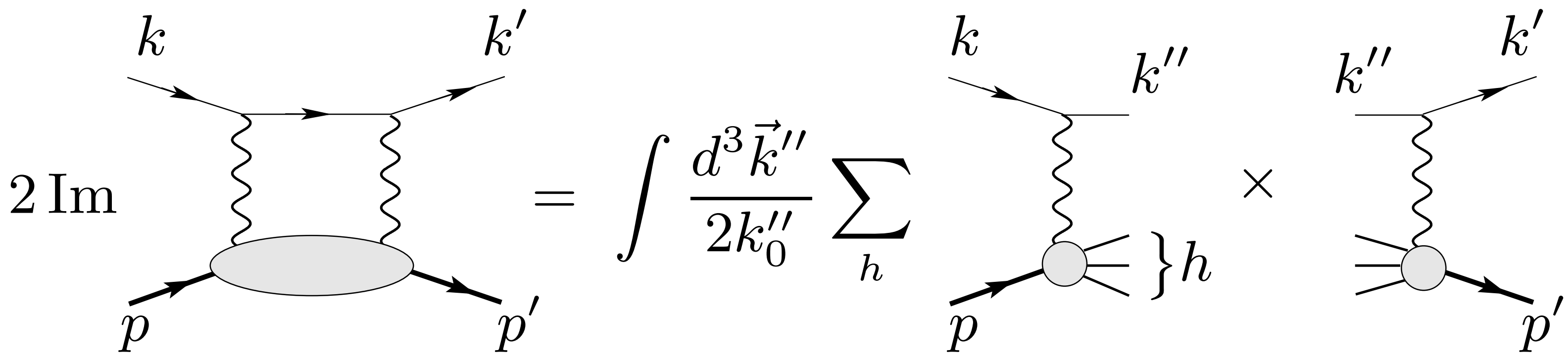
Two mainstream approaches to the theoretical evaluation of the TPE amplitude: "quark" and "hadronic" ones.

- **In the "quark" approach the nucleon is viewed as an ensemble of quarks (partons), interacting according to QCD.**
  - **The applicability of this approach is limited to the high-Q^2 region.**
  - **The serious drawback is that it is hard to calculate the TPE correction to the electric form factor G_E in this approach, while this is surely needed for the correct interpretation of G_E/G_M measurements.**

- **In the "hadronic" approach the TPE is mediated by the production of virtual hadrons and/or hadronic resonances. The TPE amplitudes are broken into different contributions according to the intermediate state involved.**

DISPERSION APPROUCH

Eq. for imagine part



Real part can be reconstructed from fixed t dispersion relation:

$$\pi\delta\mathcal{G}_n(\nu) = \int\limits_{\nu_{th}}^{\infty} \frac{\mathrm{Im}\ \delta\mathcal{G}_n(\nu' + i0)}{\nu' - \nu} d\nu' - \int\limits_{-\infty}^{-\nu_{th}} \frac{\mathrm{Im}\ \delta\mathcal{G}_n(\nu' - i0)}{\nu' - \nu} d\nu'$$

ν_{th} **is the threshold ν value, corresponding to $s = (M+m)^2$.**

Elastic contribution: $pp\gamma$ vertex is well defined.

Main problems are connected with inelastic contribution.

INELASTIC CONTRIBUTION

$\Delta(1232)$ as example

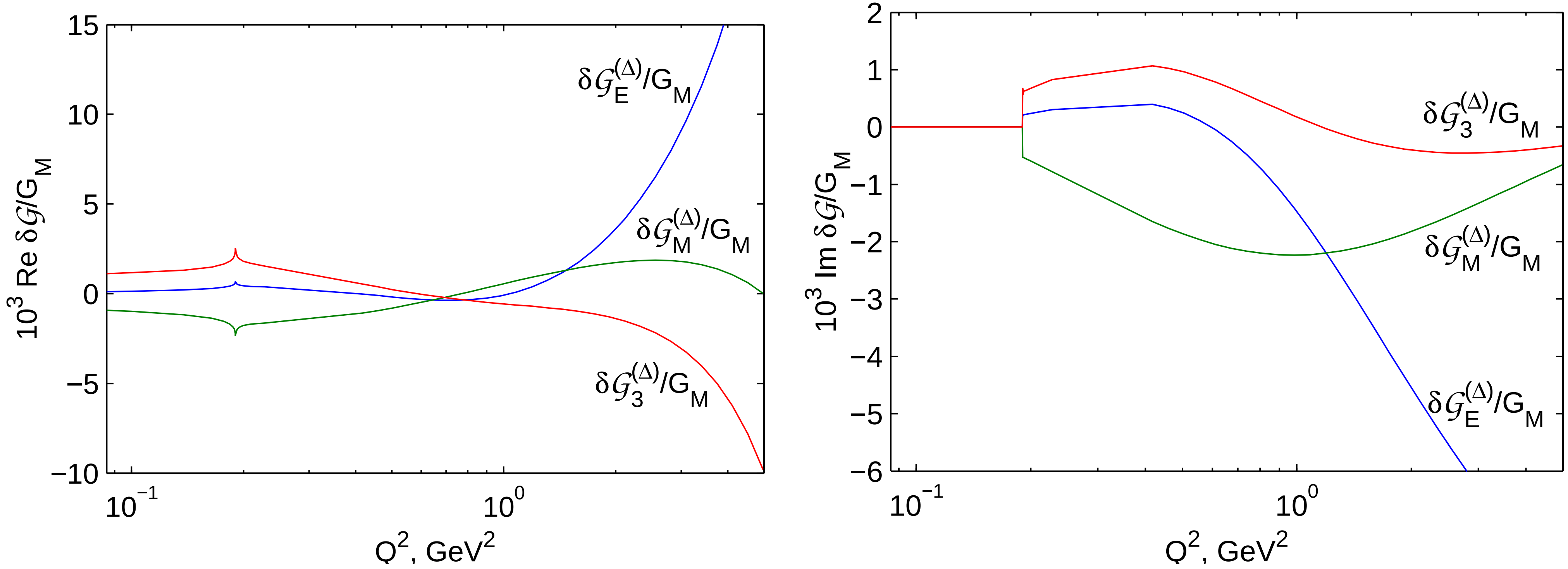
Approximation of zero width

$\Delta N\gamma^*$ has FFs 3 (Magn., Elec., Coulomb)

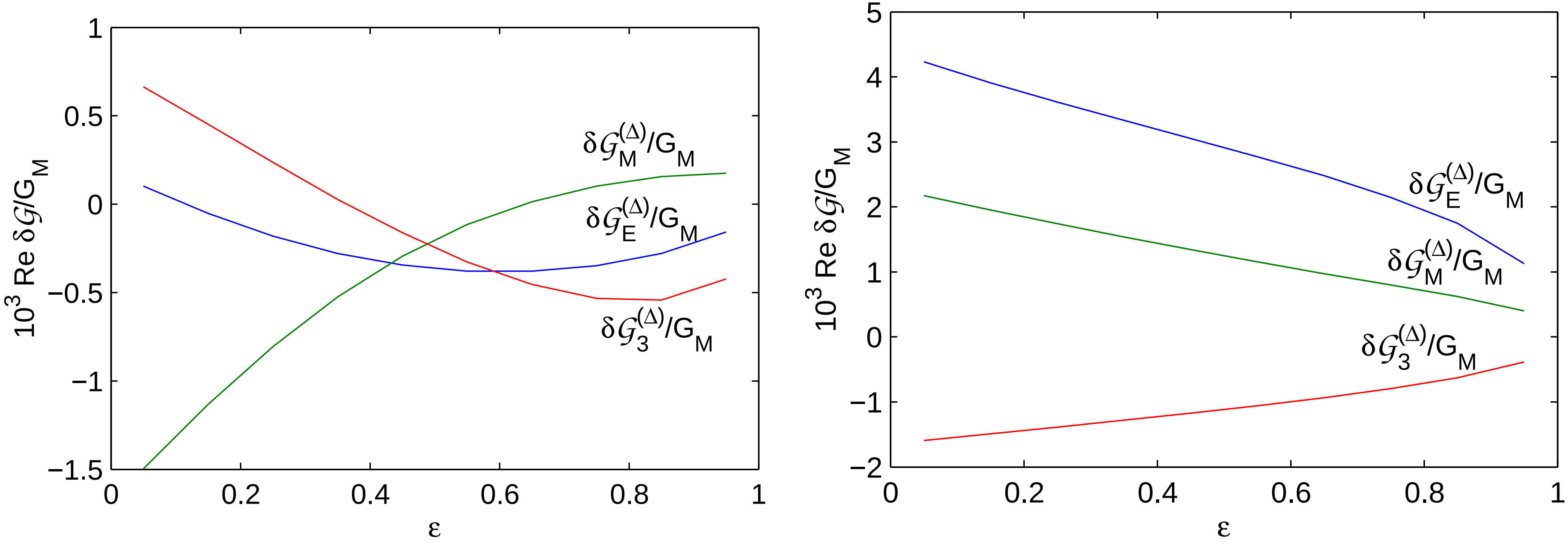
The mostly important is Magnetic FF

$$\mathcal{M}_{\Delta N\gamma} = \sqrt{4\pi\alpha}\,(g_{\mu\alpha}q_\nu - g_{\mu\nu}q_\alpha)\,\bar{U}\,(\hat{p}\gamma_\nu - p_\nu)\gamma_5 V_\alpha\,\frac{F_\Delta(q^2)}{2M^2}$$

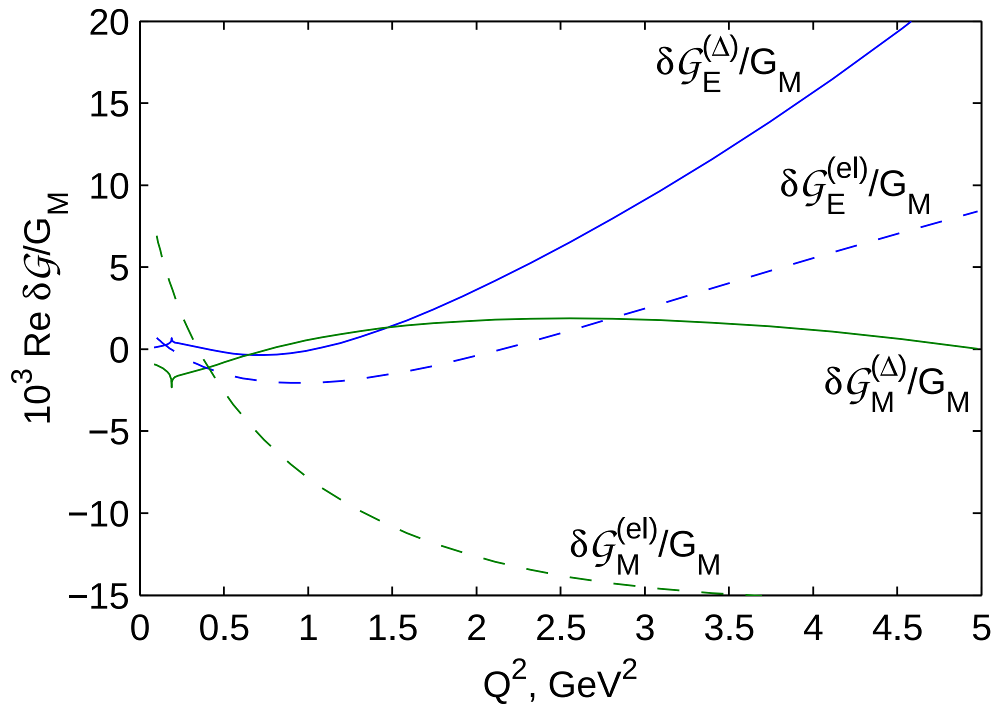
$$F_\Delta(q^2) = \sum_{i=1}^{5} \frac{c_i q^2}{q^2 - m_i^2}$$



Contribution of Delta intermediate state in the generalized FFs at $\varepsilon = 0.25$



TPE amplitudes at $Q^2 = 0.5$ GeV^2 (left) and 2 GeV^2 (right)



Comparison of Delta (solid) and elastic (dashed) contributions at $\varepsilon = 0.25$.

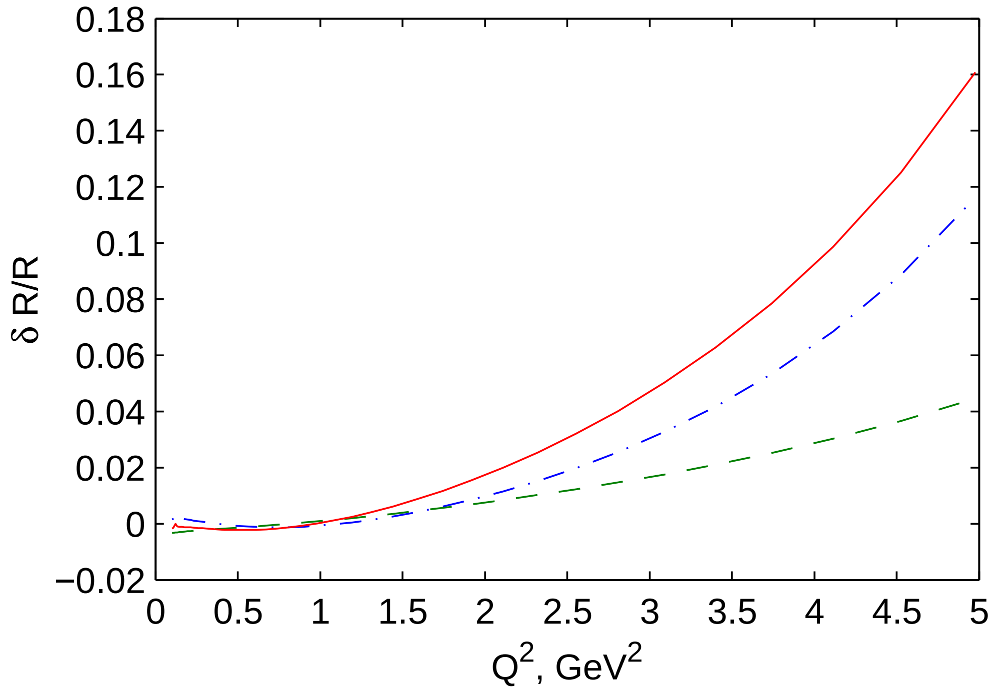


Figure 1: TPE correction to measured form factor ratio at $\varepsilon = 0.5$. Proton contribution (dashed), Delta contribution (dash-dotted) and total (solid)

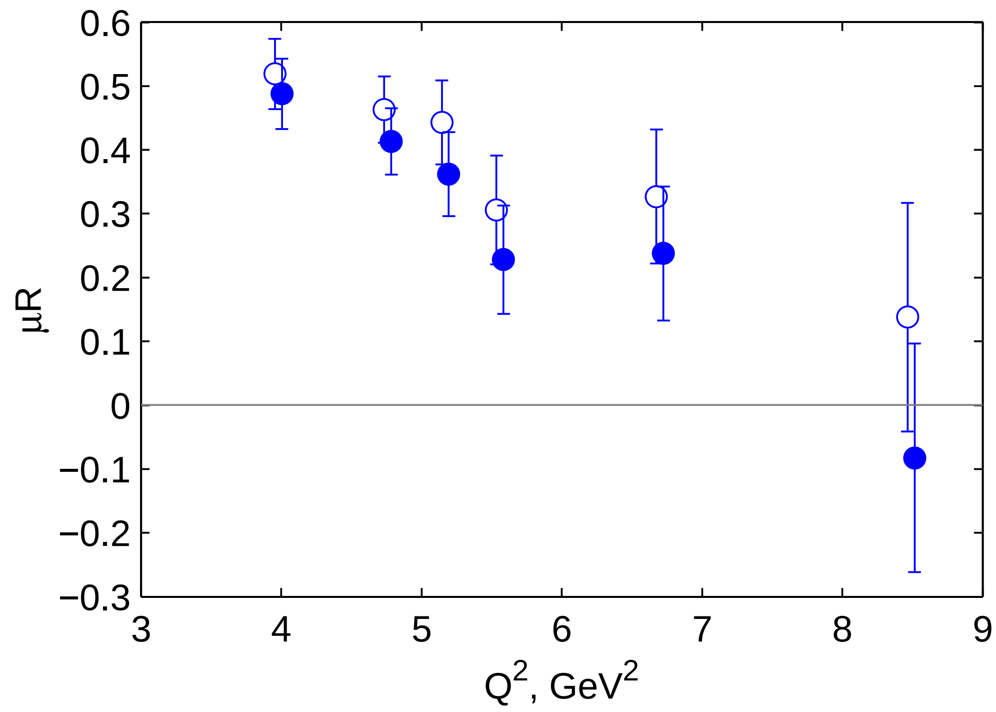


Figure 2: Results of PT experiments, with (solid symbols) and without (hollow symbols) TPE correction. Points are slightly offset in Q^2 for clarity.

“PION APPROACH

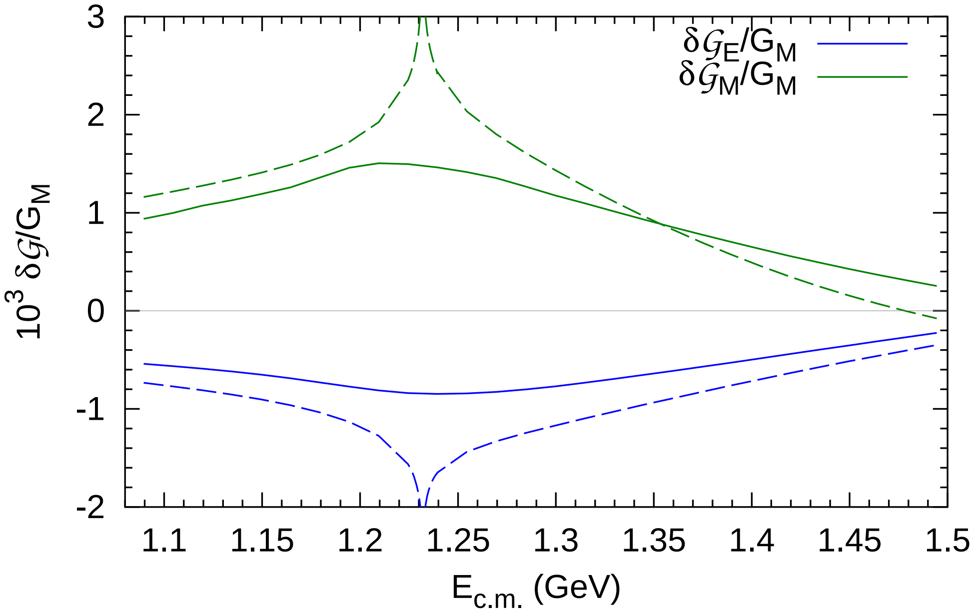
TPE is determined from πN intermediate contribution with quantum numbers of appropriate resonance, e.g. P33 with appropriate invariant mass of πN

πN amplitudes are taken from the unitary isobar model MAID2007.

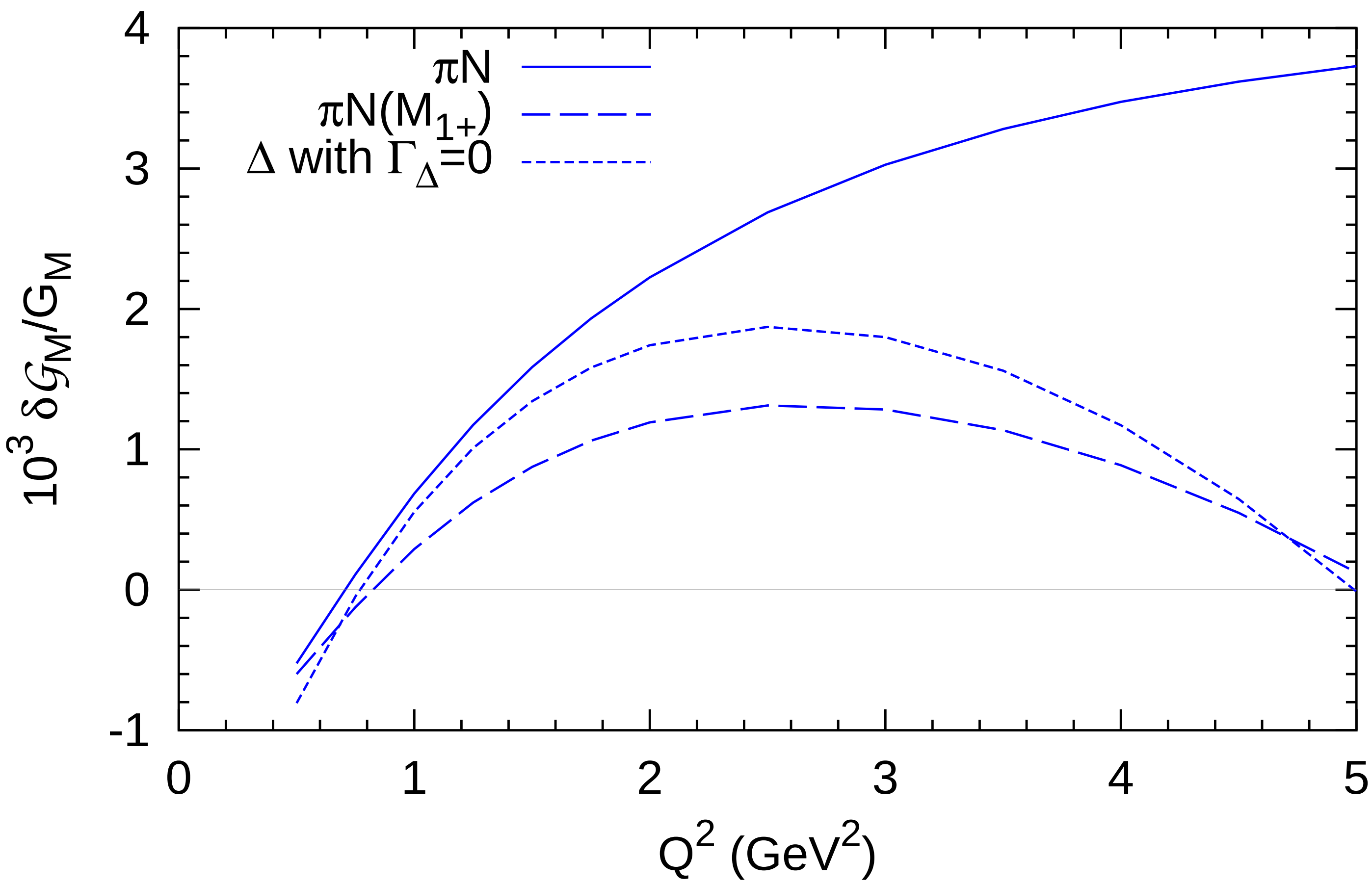
The final result is integrated over the invariant mass of πN

The advantages of our approach are

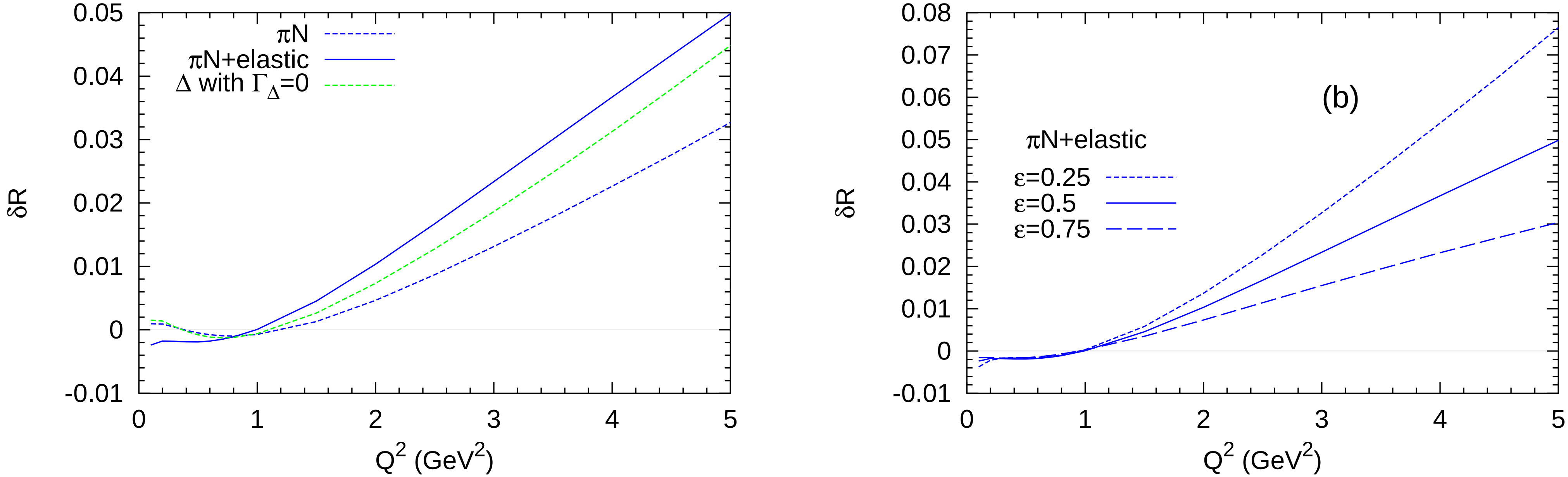
- **automatically having correct resonance width**
- **automatically having correct resonance shape**
- **including not only resonances but background as well**



The TPE amplitudes near the Δ resonance, $\theta_{\rm c.m.} = 90°$, πN contribution from this work (solid), zero-width Δ (dashed).



The TPE amplitude $\delta\mathcal{G}_M/G_M$, at fixed $\varepsilon = 0.25$, calculated in different approximations.



The TPE correction to the proton FF ratio $R = \mu G_E/G_M$, as measured in polarization experiments, various contributions at fixed $\varepsilon = 0.5$ (a) and total at different values of ε (b).

- **At low Q^2 the πN contributions are small (w.r.t. the elastic contribution).**
- **The TPE amplitudes have a smooth bump at the resonance position ($E_{\text{c.m.}} \approx M_\Delta$)**
- **At high Q^2 the main correction comes to the generalized electric form factor.**
- **This correction (and, consequently, the correction to the polarization ratio R) is relatively large and grows with Q^2.**

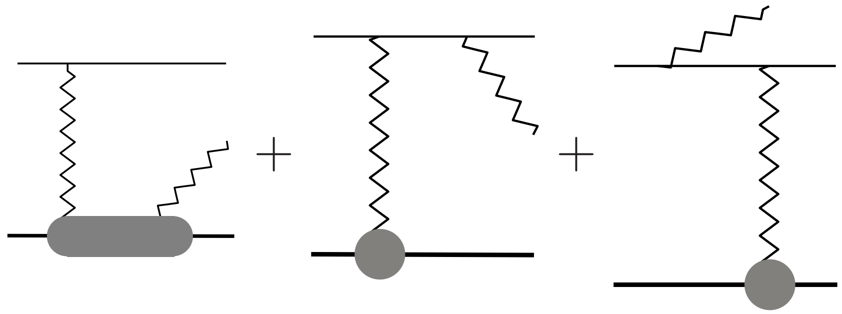
IN FUTURE POLARIZATION EXPERIMENTS AT HIGH $Q^2 > 5$ GeV^2 INELASTIC CORRECTION SHOULD EXTREMELY IMPORTANT IN EXTRACTION OF THE PROTON FF RATIO $R = \mu G_E/G_M$ FROM MEASURED POLARIZATION RATIO $\frac{S_\parallel}{S_\perp}$

The questions which remain open are:

- **whether the contributions of other partial waves (other than P_{33}) are small or not, how many of them should be taken into account,**
- **how large is the error, resulting from leaving out partial waves with higher spins.**

BREMSSTRAHLUNG

$$\mathcal{M}_{2\gamma} =$$



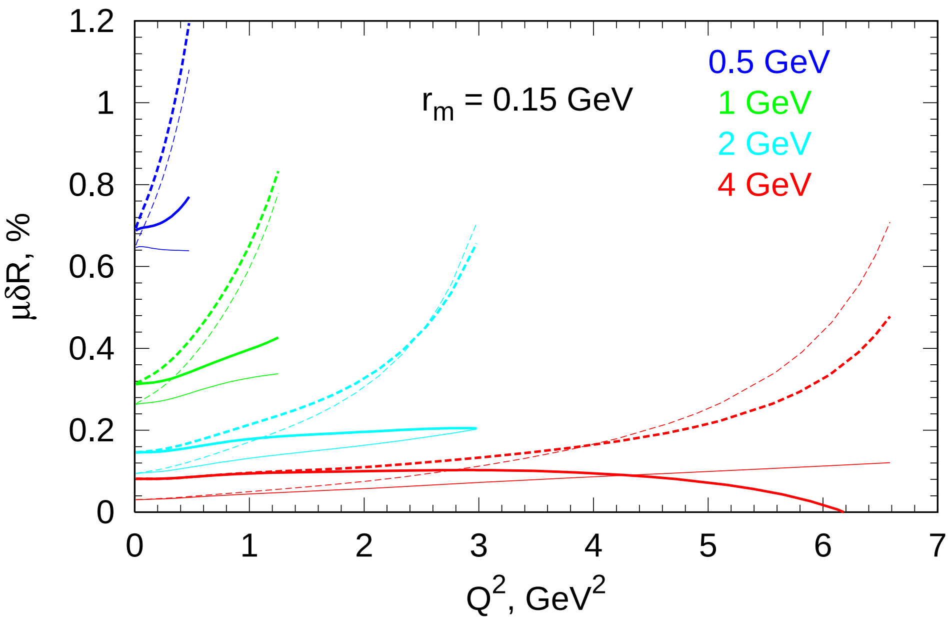


Figure 3: Bremsstrahlung correction to $\mu G_E/G_M$ ratio vs. Q^2 at different beam energies, as labeled on the plot. Solid — missing energy cut-off, dashed — missing mass cut-off; thick — full radiation, thin — electron only.

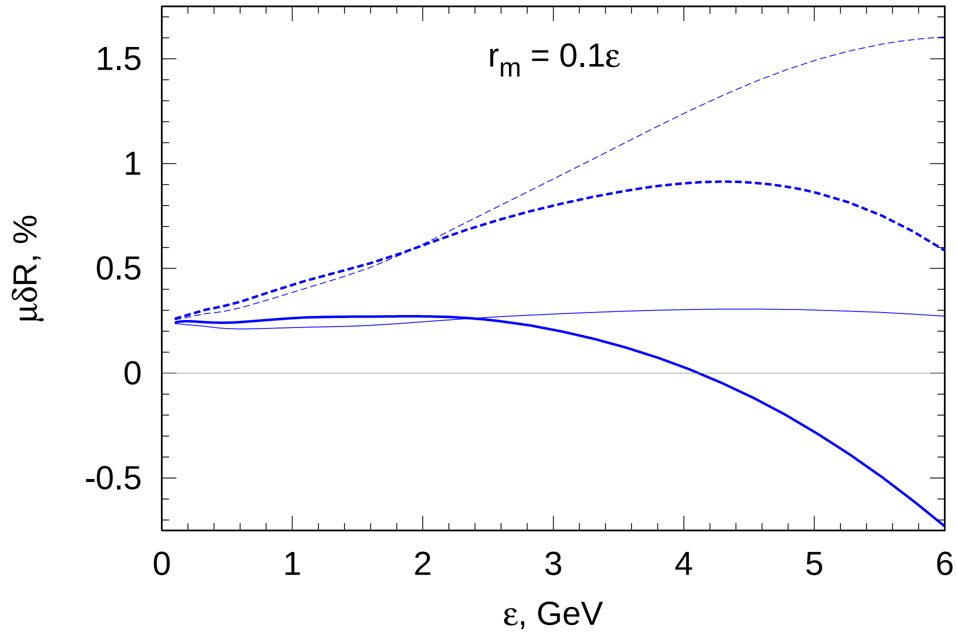


Figure 4: Bremsstrahlung correction to $\mu G_E/G_M$ ratio vs. beam energy at fixed scattering angle 90^o. Curve types are the same as in Fig. 3.

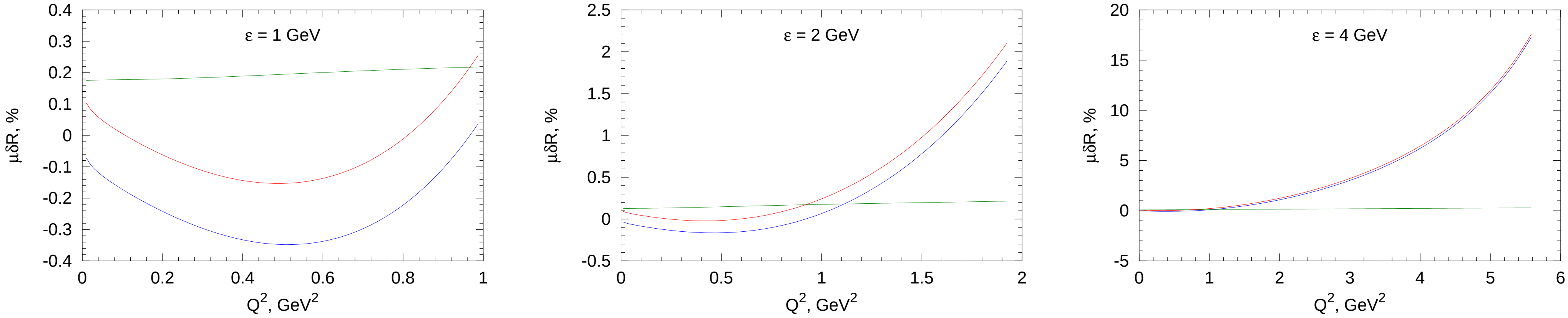


Figure 5: Radiative corrections to $\mu G_E/G_M$ ratio vs. Q^2, bremsstrahlung (green), TPE(blue), and total (red). Missing energy cut-off $r_m = 0.1\varepsilon$.

1. **The proton radiation yields a significant part of BS correction at $E > M$ in both "missing energy" and "missing mass" approaches.**
2. **In "missing mass" approach the correction strongly grows at large angles, whereas in "missing energy" approach it does not.**
3. **The BS correction is small at high energies ($E > M$), where the TPE correction is much larger. However there is no final reliable estimate of TPE amplitude in this region; this is an important open problem. The significance of BS correction at low energies depends of the experimental details; thus it should be checked separately for each case.**