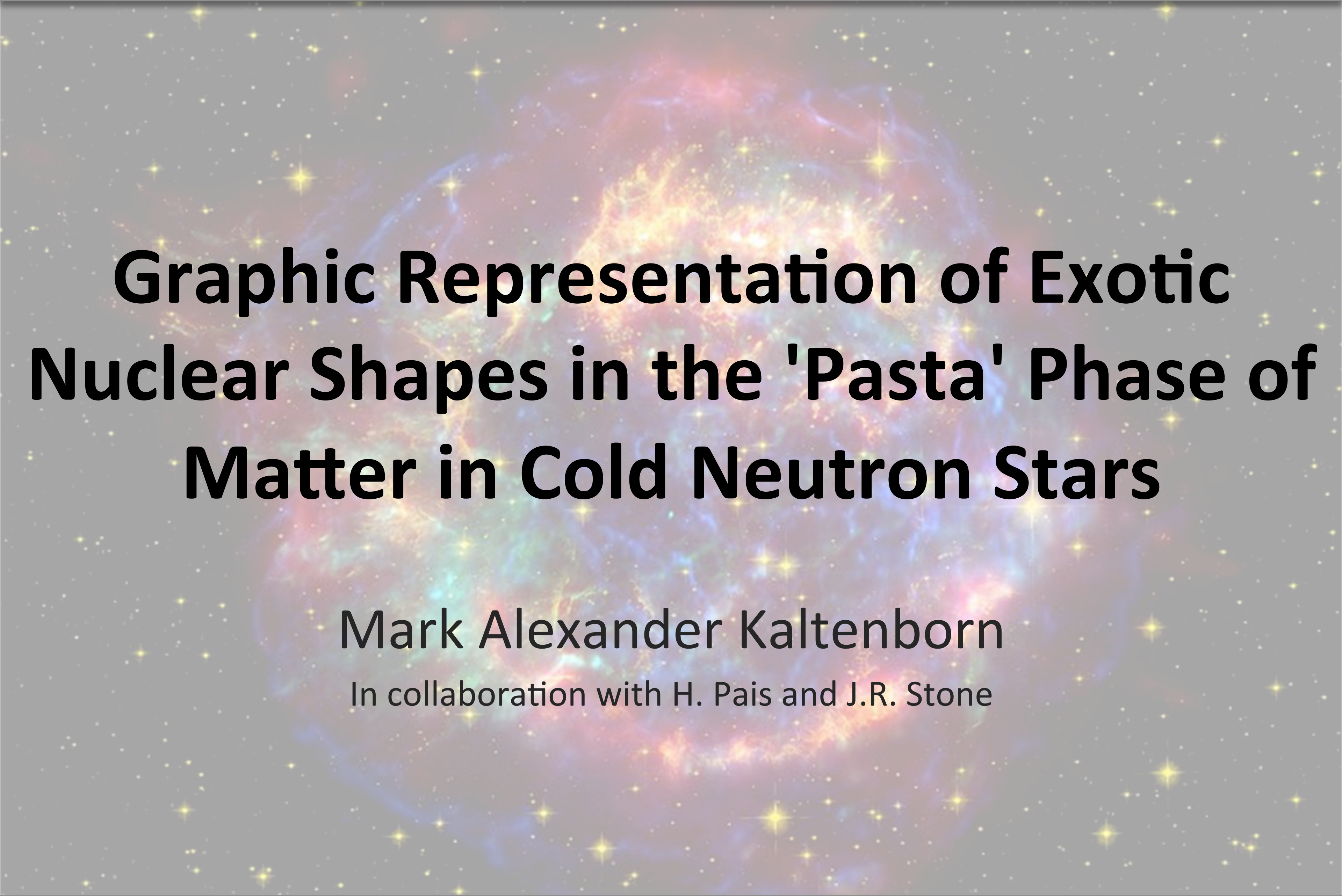

Graphic Representation of Exotic Nuclear Shapes in the 'Pasta' Phase of Matter in Cold Neutron Stars

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In collaboration with H. Pais and J.R. Stone

Neutron Stars

Stellar remnants resulting from the gravitational collapse of massive stars during supernovae

Comprised almost entirely of neutrons

Most dense objects (aside from Black Holes)

Manhattan

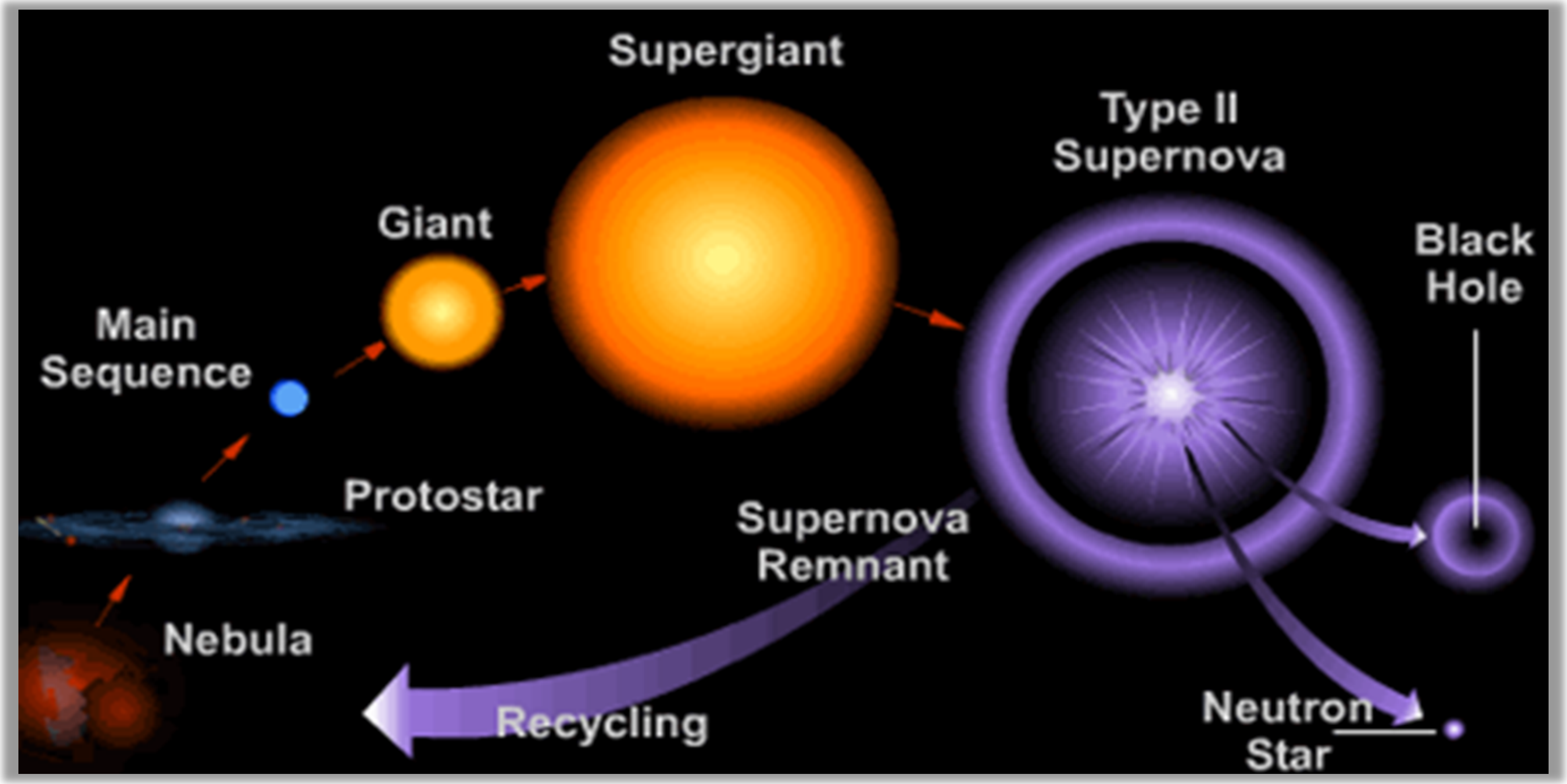
(spaceimaging.com)

$M = 1.5\ M_{sun}$

$R \approx 10$ km

$V_{esc} \approx 0.7c$

Massive Star: Life



Massive Star: Death

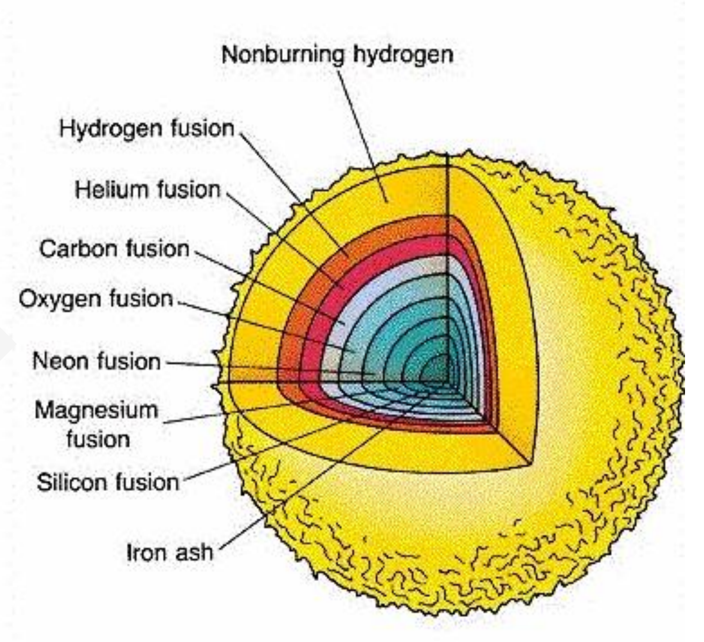
Fusion burns from H to ^{56}Fe

Develops a growing iron core radius

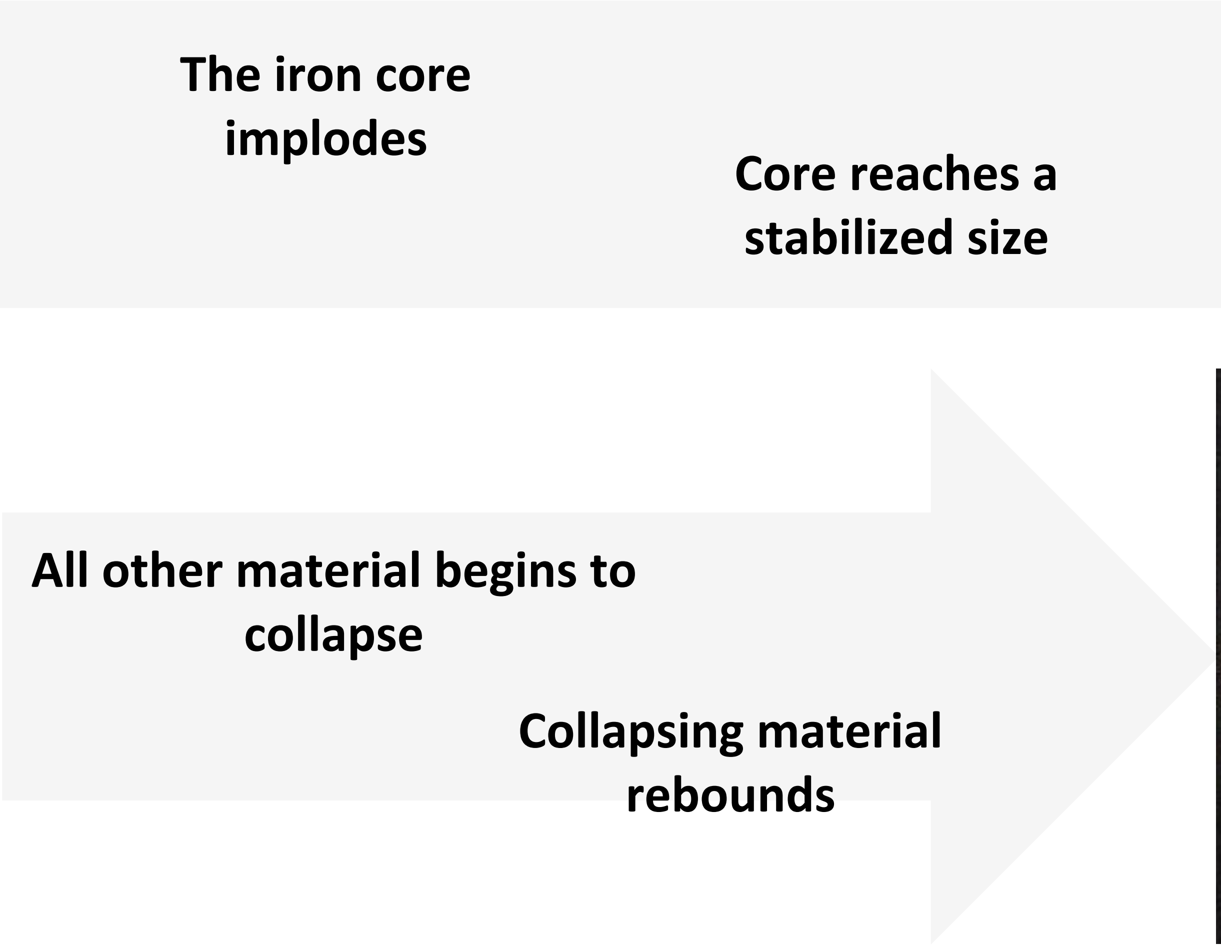
Supported from collapse by electron degeneracy pressure

Density increases and the electrons become relativistic

Maximum mass of the core is the Chandrasekhar mass



Massive Star: Collapse



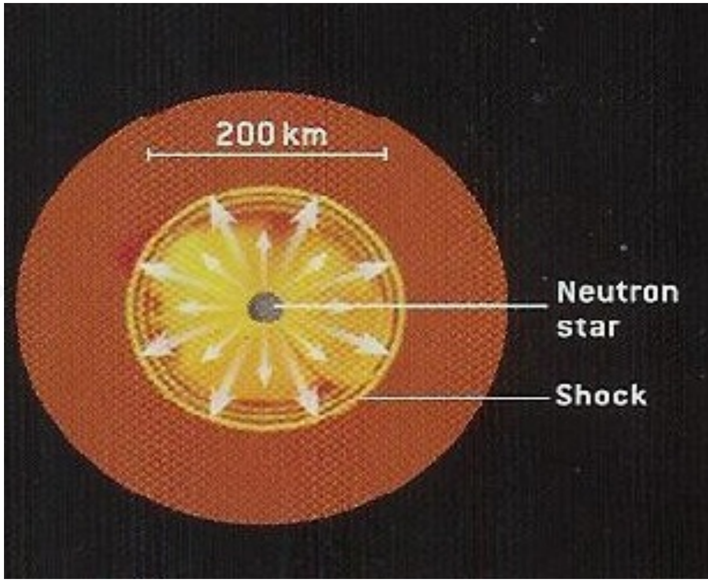
The iron core implodes

Core reaches a stabilized size

Core is now a proto-NS

All other material begins to collapse

Collapsing material rebounds



Neutrino-Powered Convection

Interacts heavily with the material within the star

Begins a convective process that propels the shock-front outward

Shockwave pushes through the entire star producing supernovae

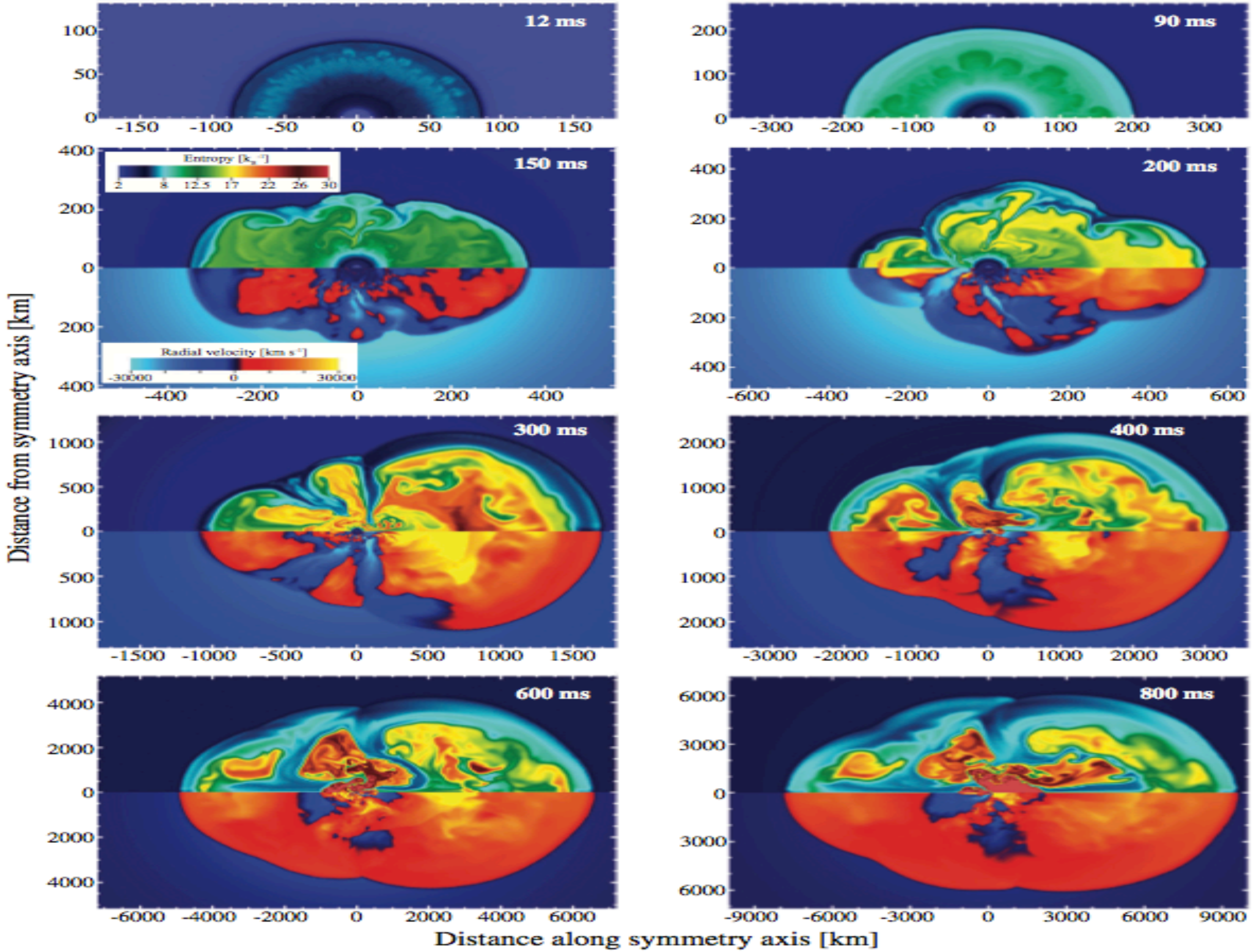
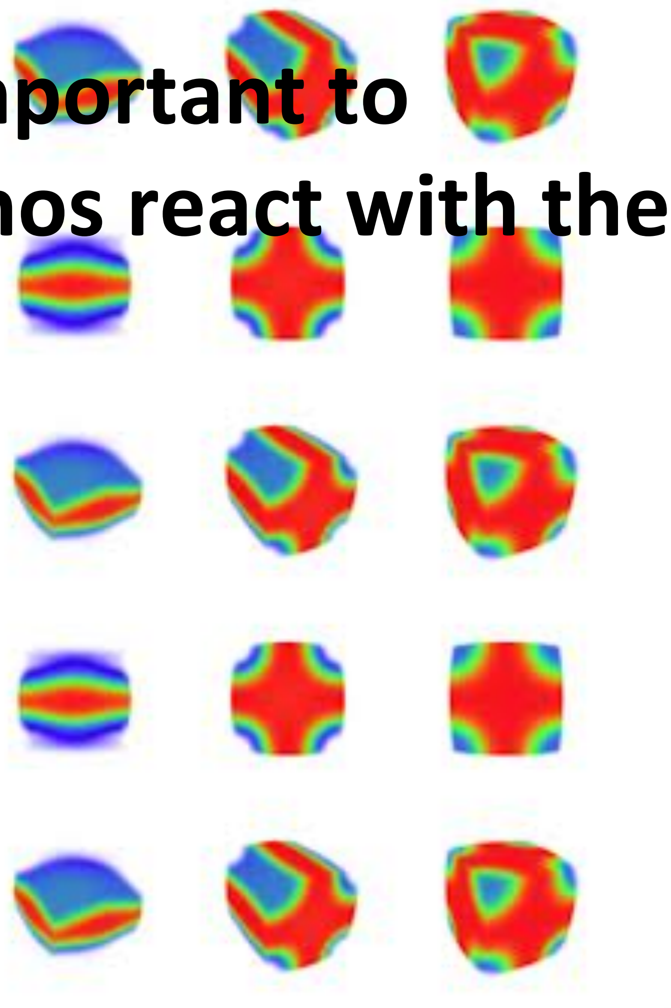


Figure 1. Evolution of the entropy (upper half) and radial velocity (lower half) for B12-WH07, with snapshots at t_{pb} = 12, 90, 150, 200, 300, 400, 600, and 800 ms. The scale grows in time to capture the expansion of the supernova shockwave, but the color maps remain constant. The radial velocity portion is omitted for the first two snapshots.

(An animation of this figure is available in the online journal.)

What Does This Mean?

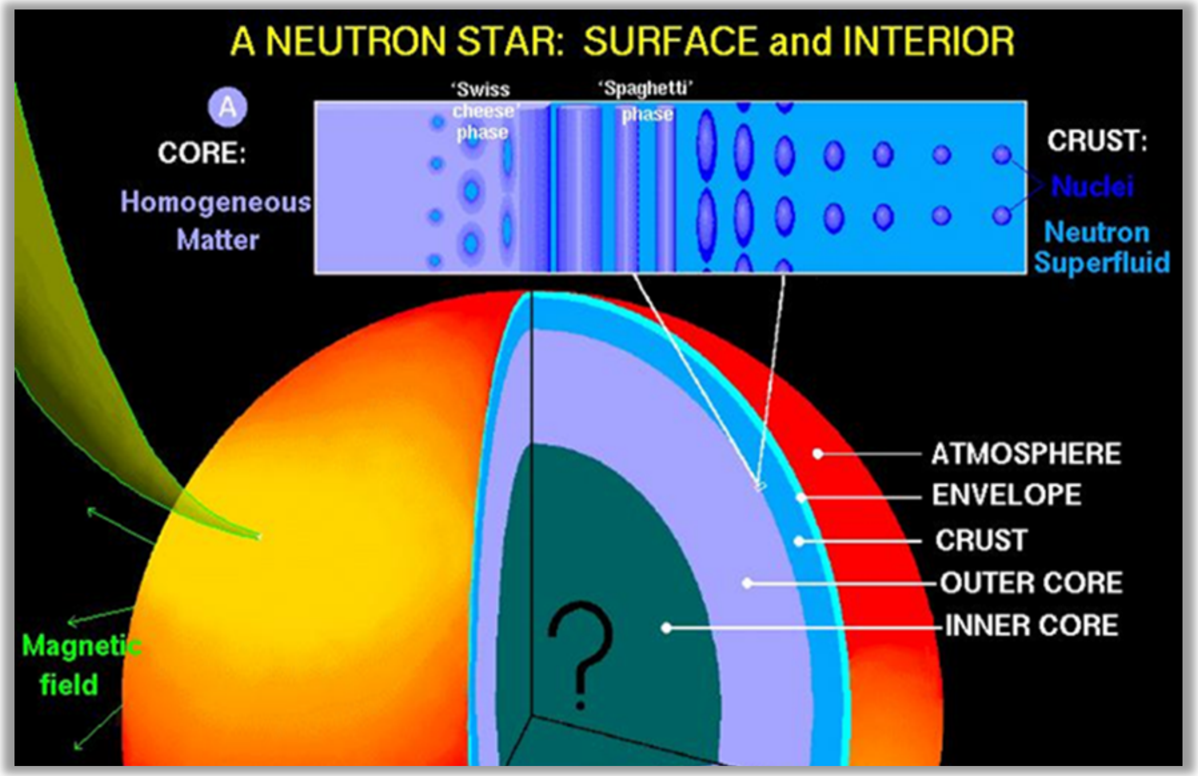
NS matter phases are important to understanding how the neutrinos react with the shock-front



The main interest is in the pasta phase of CCSN is and the neutrino opacity of the core

Anatomy of a Neuron Star

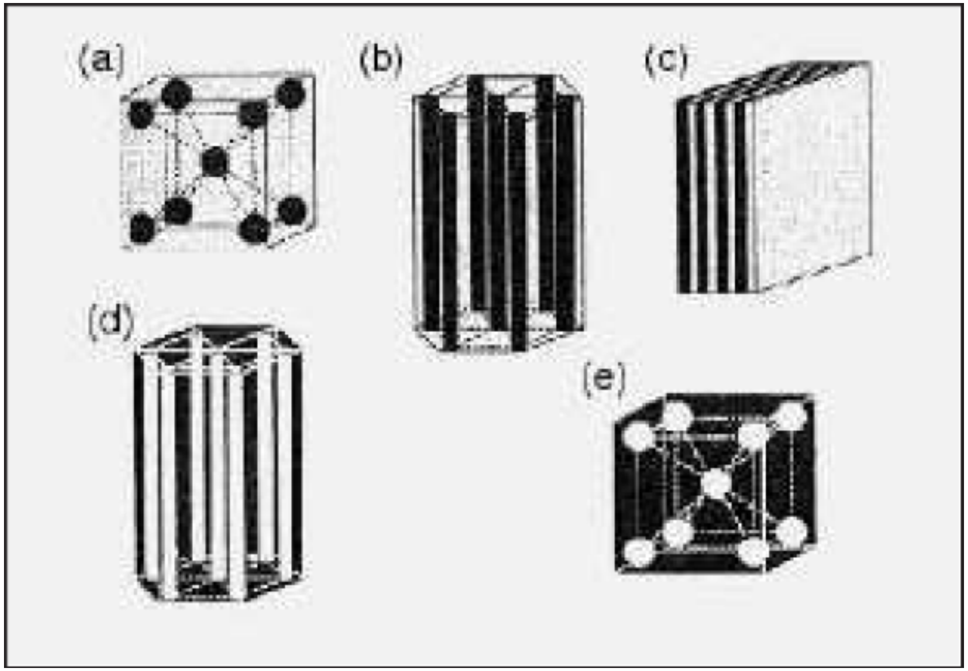
The section of the NS that interests us is the outer core



Exotic matter forms "pasta" shapes

Pasta Phase

The result of a frustrated system where, at low densities, the strong and electromagnetic interactions compete for dominance



Nuclear Pasta! (a) spherical (gnocchi) → (b) rod (spaghetti) → (c) slab (lasagna) → (d) tube (penne) → (e) bubble (swiss cheese?) → uniform matter

Accounts for up to 20% mass of collapsing stellar core; up to 50% mass and radius of NS inner crust

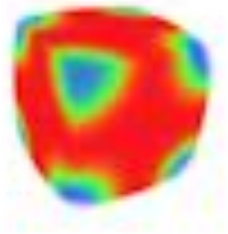
How We Calculate Pasta

Hartree-Fock (H-F): a 3D non-relativistic mean field model in H-F approximation

Generates a set of H-F equations and solves them iteratively

Uses a set of wave functions to mathematically describe the location of particles

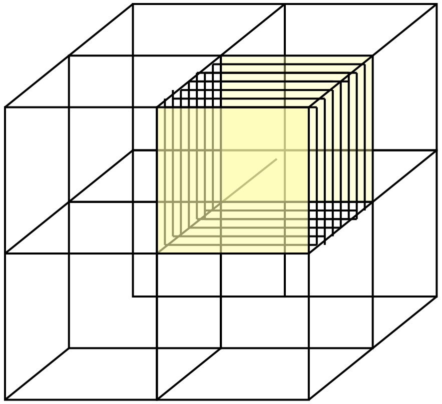
Particles are used to calculate the density at each point of the cube that we use to model particle density



Computational Method

- 3D Hartree-Fock approximation with phenomenological **Skyrme model** for the nuclear force (**SKM* and SLy4)**
- Assume (local) **unit cubic cells** of matter at a given density and temperature, calculate one unit cell containing A nucleons (**A up to 3000)**
- **Periodic boundary conditions** enforced by using FTs to take derivatives and obtain Coulomb potential

$$\phi(x,y,z) = \phi(x+L,y+L,z+L)$$



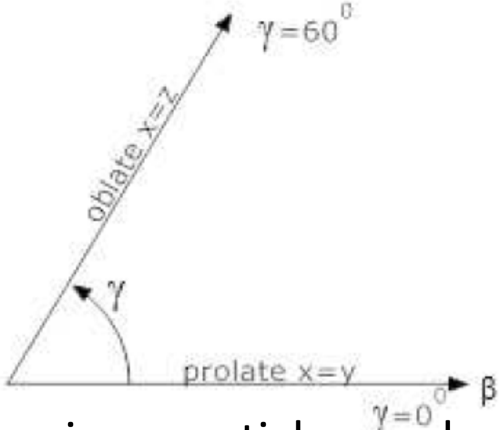
- In progress: **general Bloch boundary conditions**

$$\phi(x,y,z) = e^{ikr}\,\phi(x+L,y+L,z+L)$$

- Impose **parity conservation** in the three dimensions: tri-axial shapes allowed, but not asymmetric ones. Solution only in one octant of cell
- Currently spin-orbit is omitted to speed up computation
- **BCS pairing** (Constant gap)

Computational Method Cont…

- **Quadrupole constraint** placed on neutron density
  > self consistently explore deformation space
- **Parameterized by β,γ;** β is the magnitude of the deformation;
  γ is the direction of the deformation



- Free parameters at a given particle number density and temperature
  - **A (**number of nucleons in the cell**) /cell size,**
  - (proton fraction y_p)
  - **neutron quadrupole moments β,γ**
- **Minimize (free) energy** density w.r.t. free parameters

Computational Method Cont…

- **Initial Wavefunctions:**

  Gaussian x Polynomial (GP) or

  Plane wave (FD)

  - < 0.01% difference between choices of initial wavefunctions

- **Dependence on grid spacing:**
  - Single particle energies differ by 0.01% when increasing grid spacing from 1fm to 1.1fm at T = 0MeV
  - Differences decrease with grid spacing (smaller spacing = smaller difference)
  - Differences increase with temperature (larger no. of wavefunctions required)
  - Optimal grid spacing: 1fm up to T = 5MeV

Effects of Box Sizes

A=1400

A=700

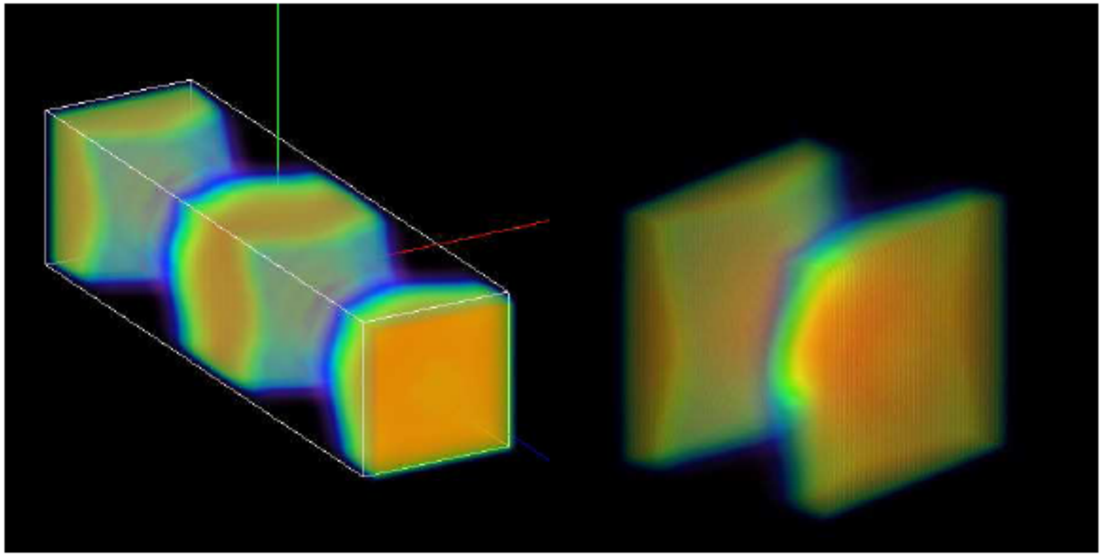
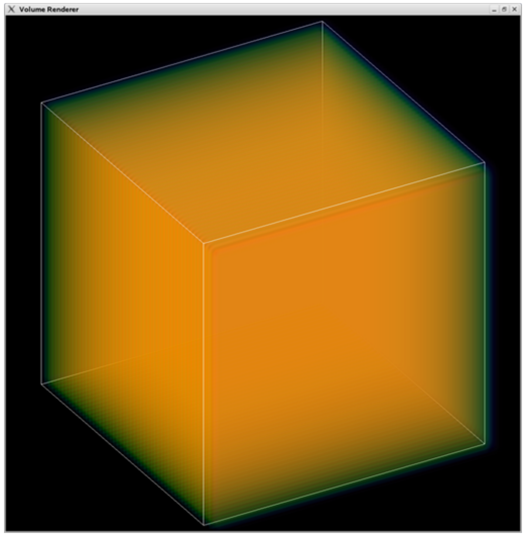


FIG. 2: Obtaining a double nuclear shape. The right picture shows the nuclear configuration obtained at $n_{\rm b} = 0.08{\rm fm}^{-3}$, $y_{\rm p} = 0.3$, $T = 0$ MeV, $A = 700$, and $(\beta, \gamma) = (1.0, 0^{o})$. The left picture was obtained at the same parameter values but double the box size in the z direction - i.e. at $A =$ 1400. Blue indicates the lowest densities and red the highest

Effects of Box Size Cont…

T=2.5MeV
n_b=0.11fm^{-3}

Spurious shell effects from discretization of neutron continuum



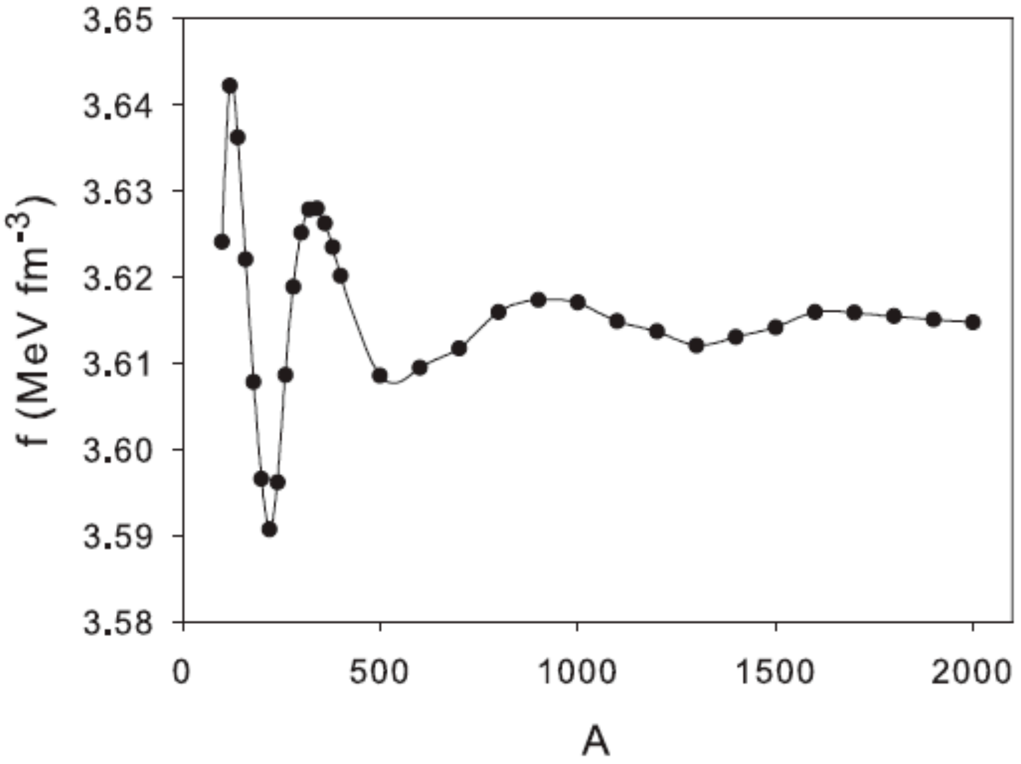
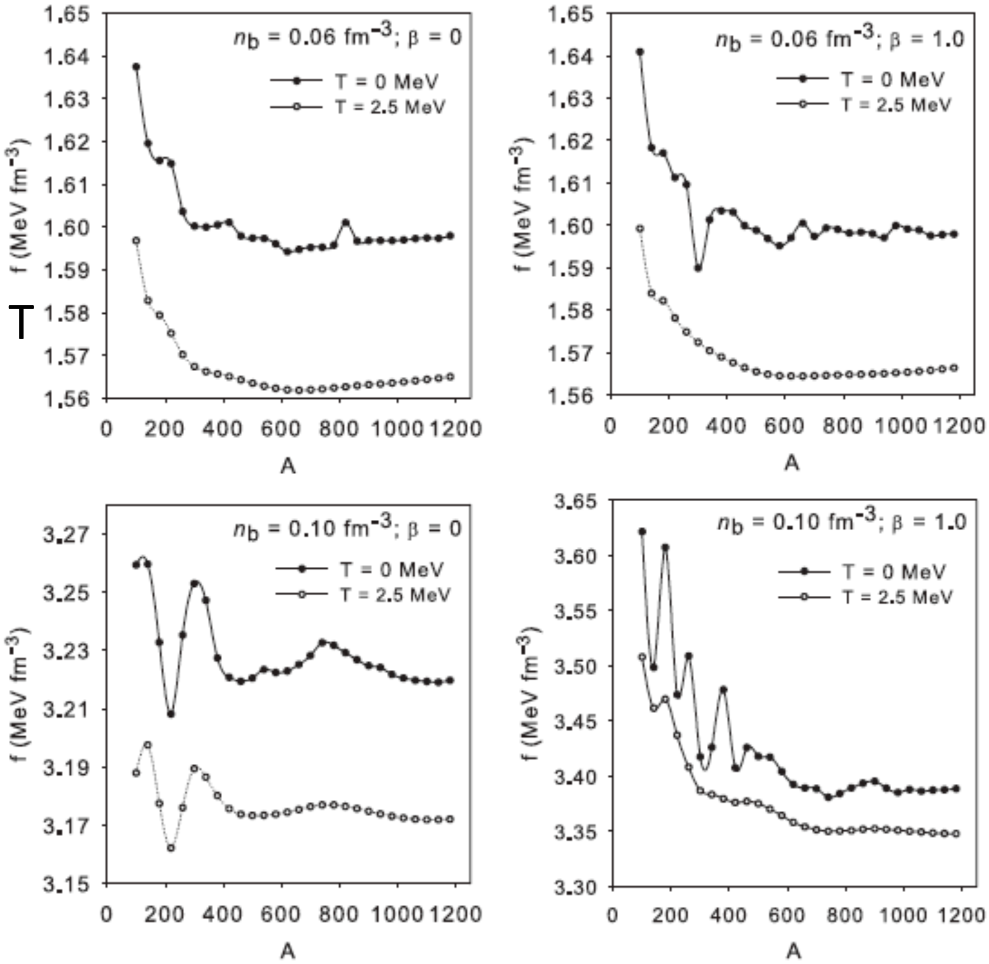


FIG. 2. Free energy density f versus nucleon number A at $n_b = 0.11\ \text{fm}^{-3}$, $(\beta, \gamma) = (0, 0^\circ)$ and $T = 2.5$ MeV. The form of the curve is dominated by a spurious shell energy caused by the finite box discretization of the continuum neutron energy spectrum.

Two Types of Shell Effects

- Spurious:
  - Box effect—Rapid oscillation with A, low densities, and low T
    - Decreases with increase T
- Physical:
  - Combination of shell energies of bound nucleons and unbound neutrons scattered by the bound nucleons
    - Slow oscillation with A



So, What Do We Do?

Goal: To find the pasta shapes to see where they begin appearing and disappearing in order to see what the distribution of matter is between the crust and the core

Examples of Results for Supernova Matter for SQMC700 Interaction

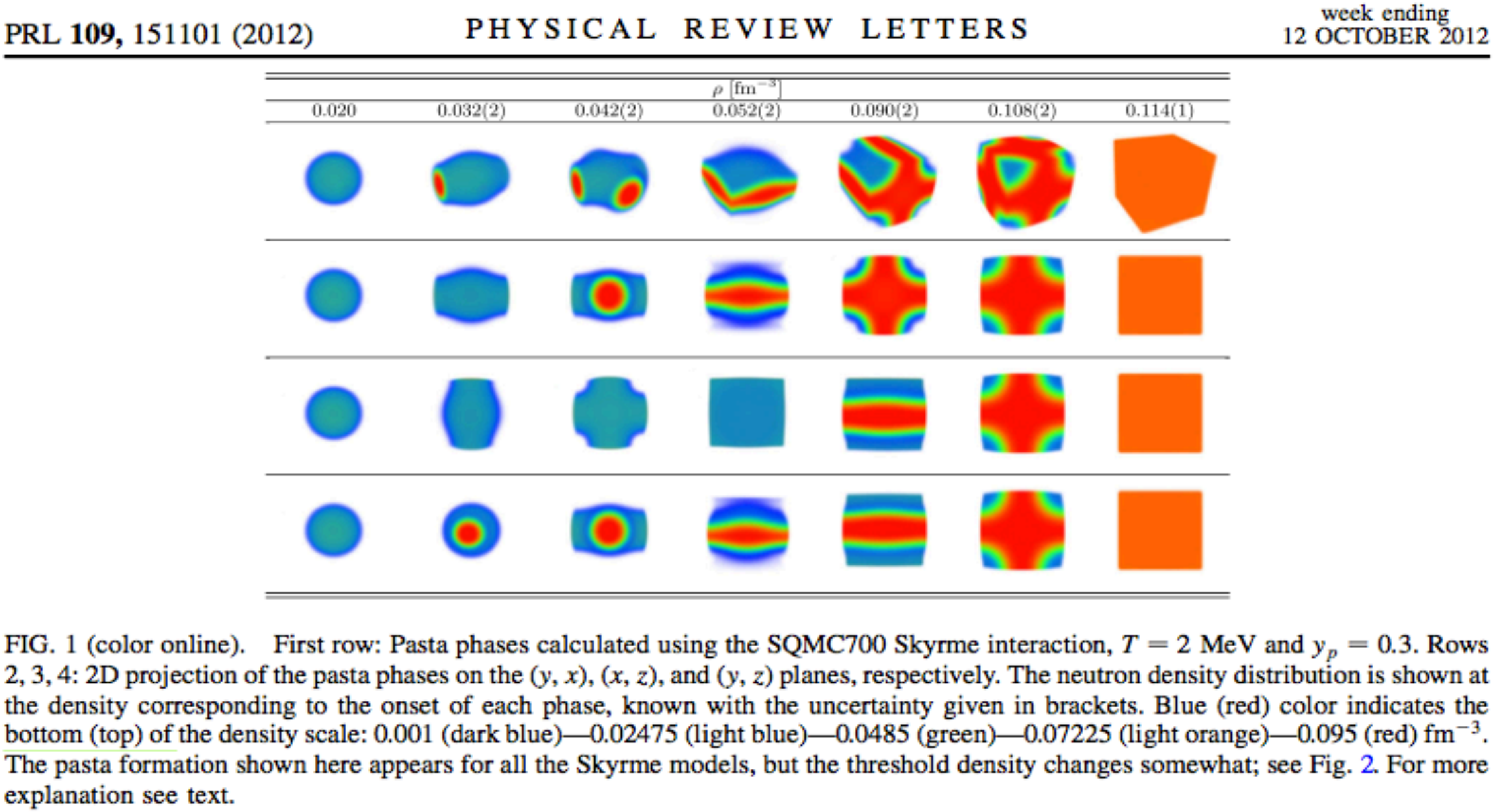


FIG. 1 (color online). First row: Pasta phases calculated using the SQMC700 Skyrme interaction, $T = 2$ MeV and $y_p = 0.3$. Rows 2, 3, 4: 2D projection of the pasta phases on the (y, x), (x, z), and (y, z) planes, respectively. The neutron density distribution is shown at the density corresponding to the onset of each phase, known with the uncertainty given in brackets. Blue (red) color indicates the bottom (top) of the density scale: 0.001 (dark blue)—0.02475 (light blue)—0.0485 (green)—0.07225 (light orange)—0.095 (red) fm^{-3}. The pasta formation shown here appears for all the Skyrme models, but the threshold density changes somewhat; see Fig. 2. For more explanation see text.

Examples of Results for Supernova Matter for All Interactions

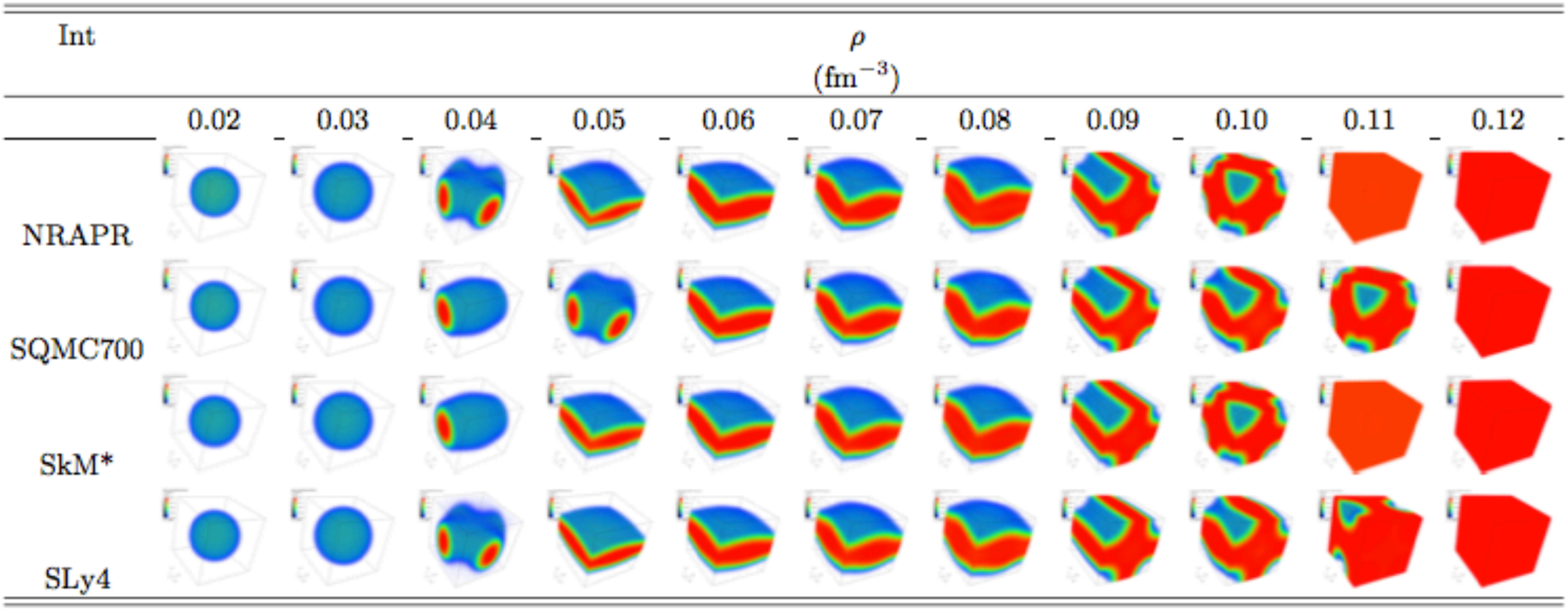


FIG. 1: (Color online) Evolution of the neutron density distribution ρ_N for $y_p = 0.3$ and $T = 2$ MeV with increasing total particle number density ρ for all the Skyrme interactions. Blue indicates low densities and red the high densities.

Examples of Results for Supernova Matter for All Interactions

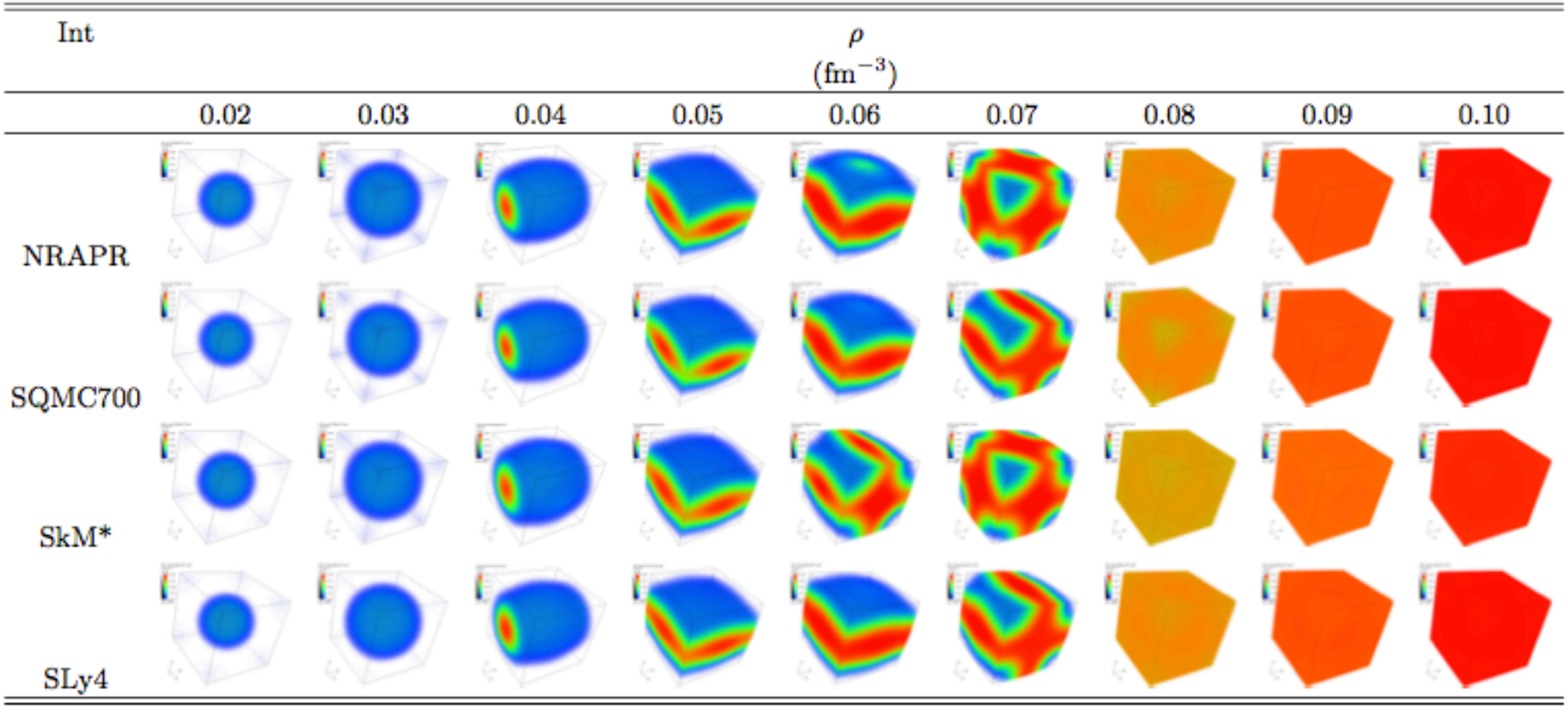


FIG. 2: (Color online) The same as Fig.1 but for $T = 10$ MeV

The Focus

- Low yp’s: 0.05, 0.10, 0.15 p-n ratio
- Temperature: 0 MeV
- Full range of densities: 0.0100-0.1200 fm^{-3}
- These corresponded to cold neutron stars

Results

- Collected images for the four interactions: NRAPR, QMC700, SkMs, SLy4
- The results are the first pasta phases seen in cold neutron stars calculated in a fully self-consistent 3D Hartree-Fock model
- We notice the difference between the pasta at yp = 0.3 and T=2 MeV (supernova matter) and the one in cold stars.

yp0.05

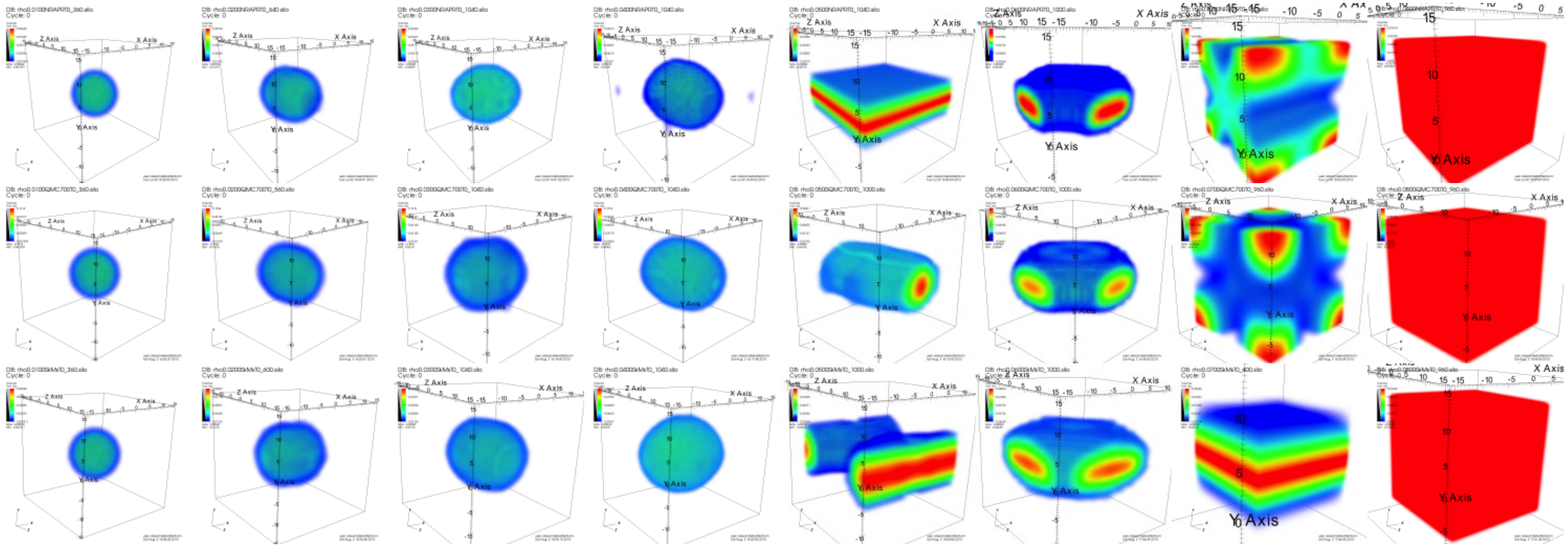
ρ

0.0100 0.0200 0.0300 0.0400 0.0500 0.0600 0.0700 0.0800

NRAPR

QMC700

SkMs



yp0.10

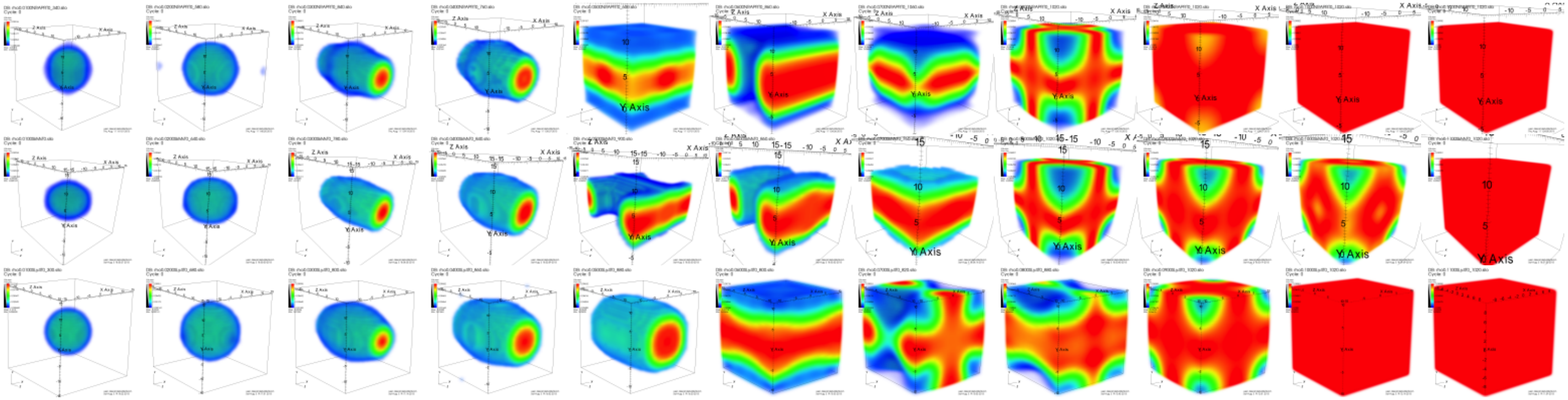
ρ

0.0100 0.0200 0.0300 0.0400 0.0500 0.0600 0.0700 0.0800 0.0900 0.1000 0.1100

NRAPR

SkMs

SLy4



yp0.15

ρ

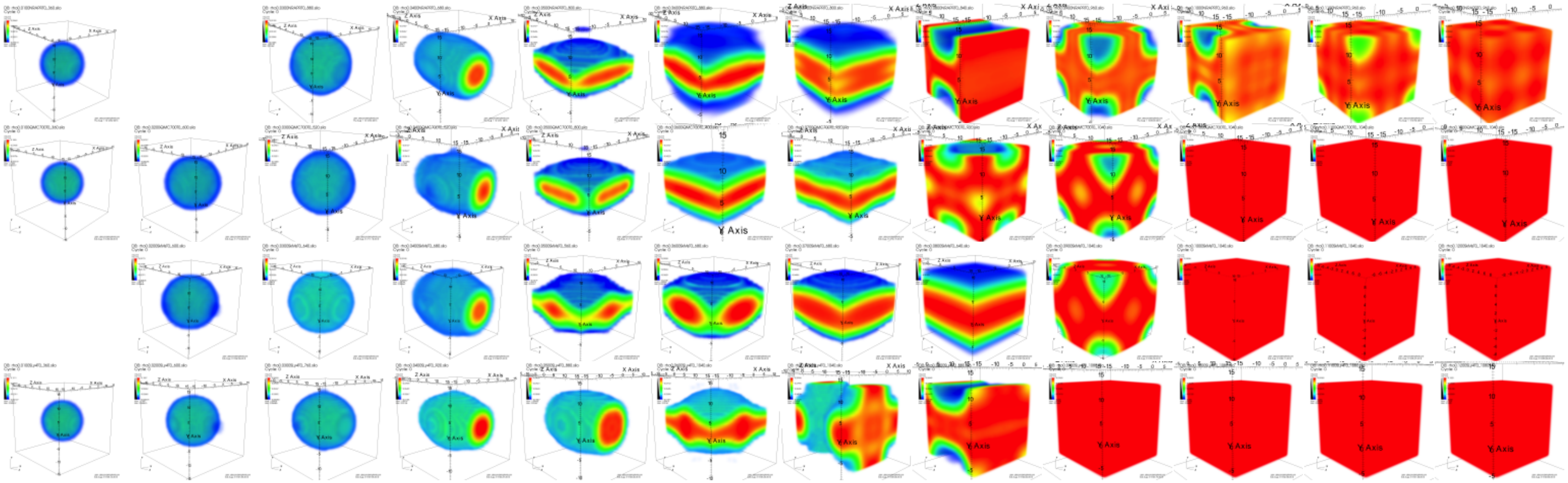
0.0100 0.0200 0.0300 0.0400 0.0500 0.0600 0.0700 0.0800 0.0900 0.1000 0.1100 0.1200

NRAPR

QMC700

SkMs

SLy4



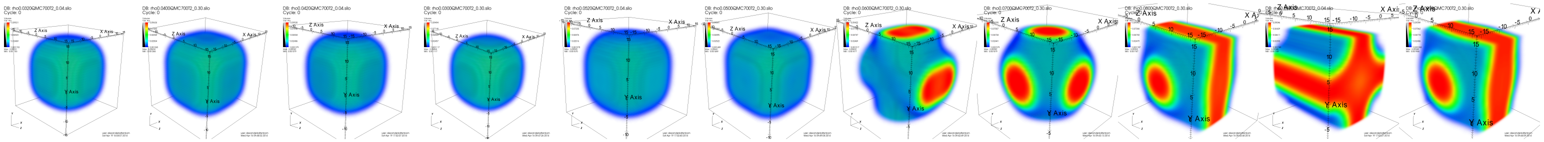
Does it work without constraints

ρ

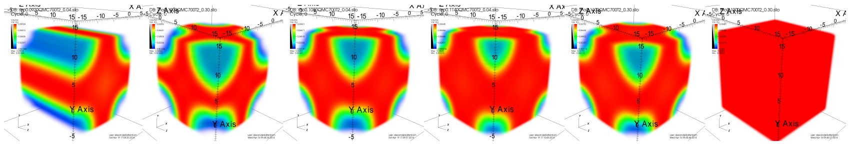
T = 2

0.0300 0.0320 0.040 0.0420 0.0500 0.0520 0.0600 0.0700 0.0800 0.0880 0.0900

QMC700



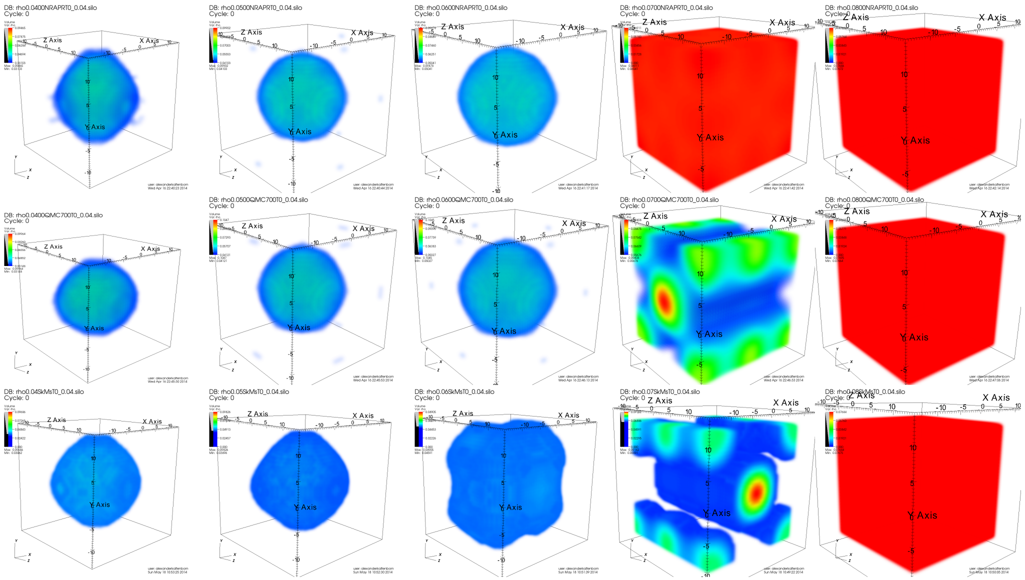
0.0920 0.1000 0.1080 0.1100 0.1140 0.1200



yp0.04 without constraint

ρ

0.0300 0.0400 0.0500 0.0600 0.0700



NRAPR

QMC700

SkMs

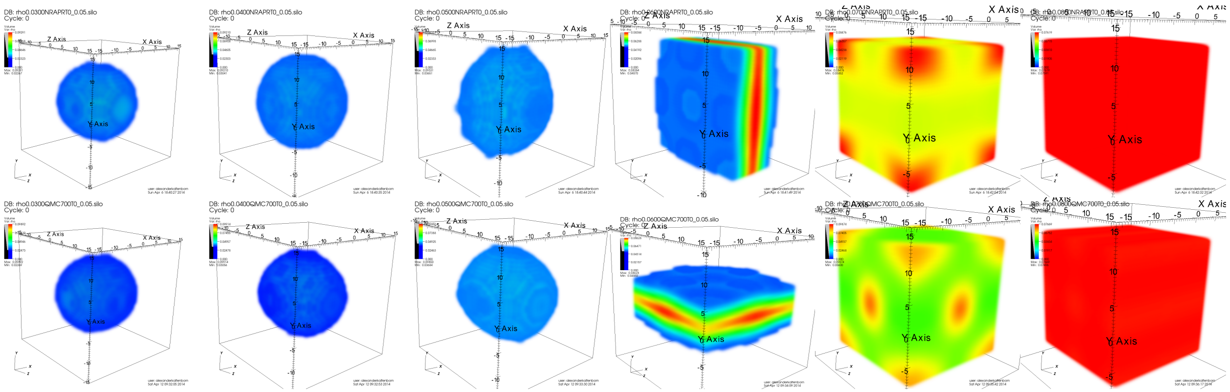
yp0.05 without constraint

ρ

0.0300 0.0400 0.0500 0.0600 0.0700 0.0800

NRAPR

QMC700



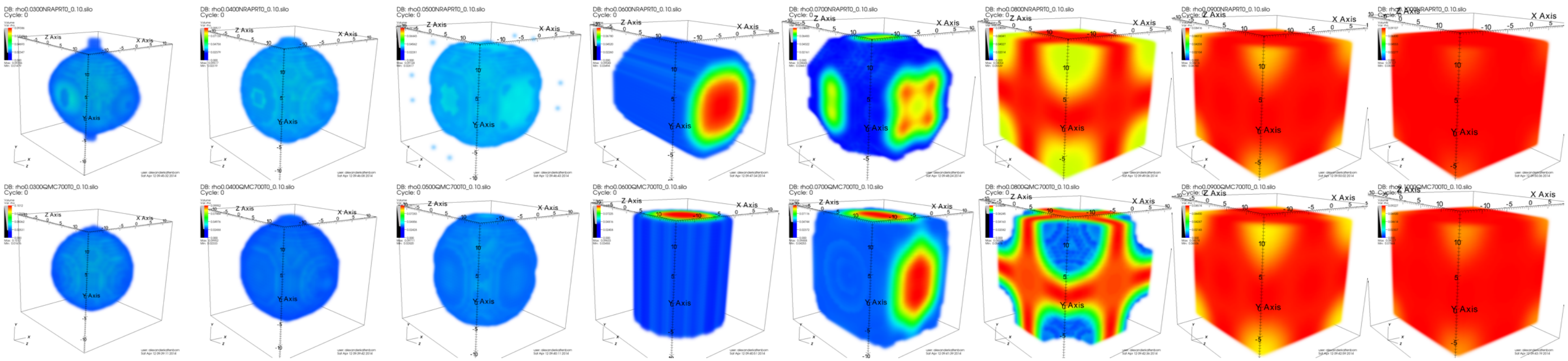
yp0.10 without constraint

ρ

0.0300 0.0400 0.0500 0.0600 0.0700 0.0800 0.0900 0.1000

NRAPR

QMC700

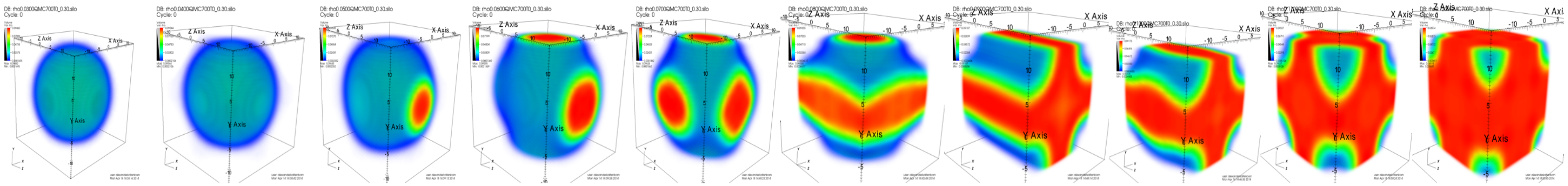


yp0.35 without constraint

ρ

0.0300 0.0400 0.0500 0.0600 0.0700 0.0800 0.0900 0.1000 0.1100 0.1200

QMC700



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