THE STRUCTURE OF TWO LOWEST MASS SCALAR MESON MULTIPLETS DERIVED BY USING A KIND OF THE "INVERSE PROBLEM" APPROACH

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Preliminaries

Motivations: (1)The new data, (2)New challenges
1.The UKQCD Collab., new **unquenched** LQCD-calculation of the $J^{PC} = 0^{++}$ glueball

$$M_G{}^{unquen} = 1795(60) vs M_G{}^{quen} = 1730(50)\ MeV$$

(E. Gregory *et al.*,JHEP, 2010 (1012),170.)
2.Data on $B \to J/psi\ \ f_0$ - decays are accumulated to discern the $q\bar{q}$ or $q^2\bar{q}^2$ nature of light scalar mesons:
(Belle, CDF, D0, LHCb)

Preliminaries

The existence of glueballs is a consequence of the self-interaction of gluons in QCD. Their continued nonobservation would be more serious than a mere embarrassment for QCD, while an unambiguous signal of their observation would mean a remarkable success for the theory.
Efforts applied are impressive: **1312 cit.** in SPIRES for the titles including "glueball".

Preliminaries

Theoretically, it is generally accepted that the lowest mass glueball is the scalar 0^{++} -state. The scalar meson states above (below) 1 GeV, listed by PDG are:
$f_0(1370), f_0(1500), f_0(1720), K^*_H(1430), A_H(1450)$;
$f_0(500)/\sigma, f_0(980), k^*_L(800)/\kappa, a_L(980)$.
The inspection of the mixing within the 3 isoscalar f_0's and on the base of **different** TH- models has lead to **numerous** mixing schemes giving the glueball mass, mainly, either around 1500 MeV or around 1700 MeV.

Preliminaries

Schematically, these approaches may be defined as follows:

TH-model parameters $\Longrightarrow$ **Data**

Our approach will, in fact, be "antiparallel":

Data $\Longrightarrow$ **Model parameters** $\Longrightarrow$ **Mixing angles**, etc.

That is the **direct** spectral problem is replaced by the **inverse** one.

Preliminaries

First we concentrate on the mass region $1.3 \div 1.7$ GeV occupied by the spin-zero 0^{++} mesons. In this group of mesons there are three isoscalar mesons with similar masses which, in the presence of the nearly lying isotriplet and isodublet ones, suggest on the overpopulated nonet where a possible glueball is hidden within structures of the three isoscalar states.

Preliminaries

Whether this idea is right or wrong one should to deduce from the solution of the formulated problem as well as from the relations between branching ratios of the resonance decays. With this in mind, we present results of a simple approach enabling to discuss an acute problem of the existence and properties of glueballs with quantum numbers $I^G J^{PC} = 0^+0^{++}$.

Underlying relations

We define the 3×3 mass-matrix $\hat{V(i)}$ as acting on the basis vectors N, S, G to transform them into one of three vectors of the physical meson states, $f_0(i)$

$$(f_0(i)) = \hat{V(i)} \cdot \begin{pmatrix} N \\ S \\ G \end{pmatrix} \qquad (1)$$

where in simple case

$$N = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}), \qquad S = s\bar{s},$$

and G is the glueball.

Mass formulas

We include the "bare" scalar glueball mass and non-diagonal glueball-quarkonium transition-mass terms into the following 3×3 meson mass-matrix. In the **"quark-flavor"** basis $N = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}), S = s\bar{s}$ our symmetric mass-matrix acquires the following form

$$\hat{M}^2 = \begin{pmatrix} M_N^2 + 2A_Q & \sqrt{2}A_G & \sqrt{2}A_Q \\ \sqrt{2}A_G & M_G^2 & A_G \\ \sqrt{2}A_Q & A_G & M_S^2 + A_Q \end{pmatrix}$$

After reducing it to the diagonal form we should get the matrix of the eigenvalues $\hat{M}^2{}_{ph}$:

The isoscalar "sub-multiplets" in scalar sector

We start the treating of mass relations with the higher-mass, scalar 0^{++}-sector:

$$M_{a_0} = 1474 \pm 19, M_{K^*{}_0} = 1425 \pm 50$$
$$M_{f_0}(1) = 1370 \pm 50, M_{f_0}(2) = 1505 \pm 6,$$
$$M_{f_0}(3) = 1724 \pm 7$$

where all values are in MeV.
We define the "bare" mass values M_N and M_S devoid of the strong annihilation contributions via

$$M_N = M_{a_0}, {M_S}^2 = 2{M_{K^*{}_0}}^2 - {M_{a_0}}^2$$

The isoscalar "sub-multiplets" in scalar sector

The intermediate gluon-matter originated terms A_Q and A_G and the gluon mass M_G are unknown variables which have to be found by solution of the system of **three equations** representing the equalities of **three invariants** of the diagonalizing process: the trace, the determinant and the sum of main minors of the matrices under consideration.

The isoscalar "sub-multiplets" in scalar sector

Successively excluding unknown variables A_Q and A_G in favor of M_G, we solve numerically the last equation by varying remaining unknown M_G under constraint ${A_G}^2 \geq 0$. There are no solutions for M_G for $A_G \geq 0$ and one for $A_G \simeq 0$. We have to accept as physically acceptable the value of the decoupled physical glueball mass

$$M_G(ph) \simeq 1730 \ MeV \text{ vis-a-vis } M_{f_0}(3) = 1720 \pm 7 \ MeV$$

The state vectors of the $f_0(1506)$ and $f_0(1370)$ are obtained by the diagonalizing the rest 2×2 matrix:

$$|f_0(1506) >= 0.868|N > +(-)0.496|S >$$

Isoscalar f_0 -resonance production and decays in processes including the lowest mass charmonia

The sensitive check of our results can also provide the radiative and hadronic decays of the J/Psi-resonance. In the radiative transitions, it is natural to accept the dominance of diagrams of the annihilation. of bound $c\bar{c}$)-quarks to photon and the pair of intermediate gluon followed by the hadronization process. We turn to several processes with participation of the vector $\phi(1.02)$- and $\omega(.783)$-mesons that serving to be very good flavor "filters" for the state vectors of scalars participating in a particular reaction. The matrix elements of the process $(J/\Psi \to V + f_0(1720))$ include a series of virtual transitions $J/\Psi \to 3g \to V + gg \to V + f_0(1720)$

Isoscalar f_0 -resonance production and decays in processes including the lowest mass charmonia

As the intermediate gluons are "hard" vector quanta we replace approximately the ratios of the full ω- and $\phi-$ form-factors by the respective ratios of their radial "functions-at-zero-distance", entering into the ratios of the widths of $V \to e^- e^+$ - decays. Thus,

$$R_{\omega\phi}(f_0(1720) = \frac{|\vec{k}_{\omega f_0(1720)}|}{|\vec{k}_{\phi f_0(1720)}|} \cdot (tan\theta_V)^{-2} \cdot \frac{R^2_\omega(0)}{R^2{}_\phi(0)} \simeq 1.0(1.33 \pm .34)$$

Isoscalar f_0 -resonance production and decays in processes including the lowest mass charmonia

The $SU(3)$-octet dominance of the $f_0(1370)$ filters as well octet parts of the vector meson states and we obtain

$$R_{\omega\phi}(f_0(1370)) = \frac{|\vec{k}_{\omega f_0(1370)}|}{|\vec{k}_{\phi f_0(1370)}|} \cdot (tan\theta_V)^2 \leq .72$$

Isoscalar f_0 -resonance production and decays in processes including the lowest mass charmonia

The color-averaged, gluon-exchange contributions operating in transitions $c\bar{c} \to gg \to gg|_{bound}$ are proportional to the 3-gluon couplings and are $3/(4/3)$ times larger than in the corresponding $c\bar{c} \to gg \to q\bar{q}|_{bound}$ -transitions. Furthermore, the factor of the "wave-function-at-zero" squared which is natural in the transition of the gluon-gluon S-wave state, has further advantage over the derivative of "wave-function-at-zero" squared connected with the transition matrix element of two-quark P-wave state.

A model for $f_0(q\bar{q})$-resonances radiative production and decays

Hence the ratio $BR(J/\Psi \to \gamma + f_0(1506)) = (1.01 \pm .32) \cdot 10^{-4}$ is markedly smaller than $BR(J/\Psi \to \gamma + f_0(1720) \to \gamma K\bar{K}) = (8.5^{+1.2}{}_{-.9}) \cdot 10^{-4}$ (PDG12) and especially than the ratio $BR(J/\Psi \to \gamma + f_0(1720) \to \gamma\bar{K}K)/BR(f_0(1710) \to K\bar{K}) \geq (8.5 \pm 1.2) \cdot 10^{-4}$

The light-scalar "sub-multiplet" in scalar sector

The states belonging to already mentioned higher-mass and lower-mass)scalar sub-multiplets are seen to explicate specific mass hierarchy sequence. We accept the viewpoint that light scalars make also a full SU(3) flavor nonet. Their mass spectrum, with the peculiar inversion of the κ and f_0 or a_0 mass ordering, speaks against the naive $q\bar{q}$ picture. The most popular and natural explanation for such complete multiplet with inverted mass spectrum is that these mesons are diquark–antidiquark (4q) bound states.

The light-scalar "sub-multiplet" in scalar sector

Diquark–antidiquark bound states (tetraquarks, for short) naturally reproduce the SU(3) nonet structure with the correct mass ordering, as indicated by the explicit quark composition:

$\sigma^{[0]} = [ud][\bar{u}\bar{d}]$

$\kappa = [su][\bar{u}\bar{d}];\ [sd][\bar{u}\bar{d}]$ (+ conjugate doublet)

$f_0^{[0]} = \frac{[su][\bar{s}\bar{u}]+[sd][\bar{s}\bar{d}]}{\sqrt{2}}$

$a_0 = [su][\bar{s}\bar{d}];\ \frac{[su][\bar{s}\bar{u}]-[sd][\bar{s}\bar{d}]}{\sqrt{2}};\ [sd][\bar{s}\bar{u}]$

The above expressions for the isoscalar states are valid for the "ideal" mixing in the quark-flavor basis.

The light-scalar "sub-multiplet" in scalar sector

We turn now to specifics of the 2q-4q's mixing mass-matrices

$$\hat{M}^2 = \begin{pmatrix} a_{0L}^2 + g_4 & G_{2-4} \\ G_{2-4} & A_{0H} \end{pmatrix} \quad (2)$$

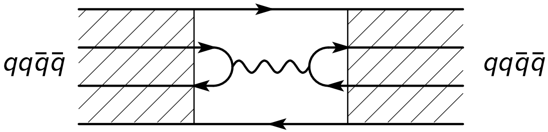
The light-scalar "sub-multiplet" in scalar sector

After reducing it to the diagonal form we should get the eigenvalues-the physical masses of the isovector mesons $m_L(a_0) \simeq .980 GeV$ and $M_H(A_0) \simeq 1.470 GeV$.
Analogously, the relation between isospinor mesons $m_L(\kappa) \simeq .700 - -.800 GeV$ and $M_H(K) \simeq 1.430 GeV$ will serve as the definition of otherwise unknown "non-disturbed" a_{0L} and κ_{0L}.
The important vertex G_{24} parametrize the universal "non-diagonal-mass" (so-called tHooft-6q -constant).

The light-scalar "sub-multiplet" in scalar sector

The diagonal elements of the light isoscalar mass-matrices include the "bare" combinations $(4m_\kappa^2 - m_a^2)/3$ and $(2m_\kappa^2 + m_a^2)/3$ for the $SU(3)$-octet and $SU(3)$-singlet parts and with evident replacements in the heavier mass sub-multiplet, respectively. Virtual gluon - exchange contributions are parametrised by constants g_4 and g_2. The first correspods to the virtual sequence of transition $2q2\bar{q} \rightarrow Gq\bar{q} \rightarrow 2q2\bar{q}$ and the second to the transitions $q\bar{q} \rightarrow 2G \rightarrow q\bar{q}$. The unknown quantities of our approach g_2, g_4, G_{24} and experimentally poorly known mass of the $\sigma(400-1000)$-meson should be obtained by reconstruction of the 4×4-mass matrix, that is equivalent to the solution of the system

The light-scalar "sub-multiplet" in scalar sector



The approximate solutions for combined scalar sectors

From the general 4×4-mass matrix we separate out the following four 2×2-mass matrices:$(4q)_8 \times (2q)_(8)$,$(4q)_0 \times (2q)_(0)$, $(4q)_{0\oplus 8} \times (4q)_(0 \oplus 8)$,$(2q)_{0\oplus 8} \times (2q)_(0 \oplus 8)$. With available information on masses in both scalar sub-multiplets we obtain the first approximation for parameters G_{24},g_4 and g_2,mass of the $\sigma(400 - 1000)$-meson and their interrelations. The such obtained "intermediate" values of all parameters are then combined into full 4×4-mass matrix which has been diagonalized and the obtained "intermediate" eigenvalues by comparison with available experimental values of masses provide needed corrections to firstly obtained "intermediate" values of earlier obtained parameters.

The approximate solutions for combined scalar sectors

The such first order iteration scheme provide values:

$$G_{24} \simeq .39 GeV^2, m(\sigma) \simeq .61 GeV^2, g_2 \simeq .11 GeV^2, g_4 \simeq -.17 GeV^2$$

.

The mixing angle of the leading $q^2\ \bar{q}^2$ or $q\bar{q}$ and in the "admixed" configurations,respectively,$q\bar{q}$ or $q^2\ \bar{q}^2$ is about 20^o. It looks qualitatively interesting that g_2 and g_4 have the opposite signs and it most probably could be understandable without detailed dynamical structure of the $q\bar{q}$- and $q^2\ \bar{q}^2$- scalar mesons.

Concluding citation

Far better an approximate answer to the right question, which is often vague, than an exact answer to the wrong question, which can always be made precise.(by J.W.Tukey)

"Gone with the Wind"? (Problem and role of the $f_0(1370)$ for previous parts of this talk)

Like in many schemes with $1.5 \div 1.7$ GeV glueball, the crucial element of our numerical results is the existence of $f_0(1370$. The PDG review propose the rather wide ranges for mass and width: $M = 1200 \div 1500\ MeV$ and $\Gamma = 200 \div 500\ MeV$. The available data of **many decay modes of $f_0(1370)$ quoted as "seen" do not present a convincing direct signal**,neither a peak above low background nor a clear resonance effect such as were observed from the nearby $f_0(1500)$. (W.Ochs,hep-ph/1001.4486, hep-ph/1304.7634)

"Gone with the Wind"? (Problem and role of the $f_0(1370)$ for previous parts of this talk)

M. Ablikim, et al.(BES Collaboration),"Partial Wave Analysis of $\chi_{c0} \to \pi^+\pi^- K^+ K^-$", Phys.Rev., **D 72** 2005, 092002.
A partial wave analysis of $\chi_{c0} \to \pi^+\pi^- K^+ K^-$ in $\psi(2S) \to \gamma\chi_{c0}$ decay is presented. From the fit, significant contributions to χ_{c0} decays from the channels $f_0(980) f_0(980)$, $f_0(1370) f_0(1710)$, etc.

"Gone with the Wind"? (Problem and role of the $f_0(1370)$ for previous parts of this talk)

$Br(\chi_{c0} \to f_0(1370) f_0(1710)) \cdot 10^4 = 7.12 \pm 1.85 \pm 3.28(1.68); (6.5\sigma).$

$Br(\chi_{c0} \to f_0(1500) f_0(1710)) \cdot 10^4 \leq 0.7$ (PDG)

$Br(J\Psi \to \gamma f_0(1710) \to \gamma K\bar{K}) \cdot 10^4 = 8.5 \pm 1.2(0.8).$

$Br(J\Psi \to \gamma f_0(1500)) \cdot 10^4 = 1.01 \pm 0.32$

$Br(J\Psi \to \gamma f_0(1370)) =$ (?!) - eagerly wanted to be measured!

"Gone with the Wind"? (Problem and role of the $f_0(1370)$ for previous parts of this talk)

There is seemingly strong **mismatch** between the **flavour** $SU(3)$-symmetry and **chiral** symmetry at sufficiently high internal virtual momenta as in the decays of heavy hadrons into the states composed of light quarks. In particular, the idea of the suppression of the scalar coupling to (almost) massless $u(d)$- quarks was advanced in the paper: M.Chanowitz, "Chiral suppression of scalar glueball decay", Phys.Rev.Lett. **95**, 2005, 172001.

Concluding appeal

Consider peculiarities, they solely have the significance.(by Gaston Julia)

Outlook

(•) BES 3: Improved statistics on $J/Psi \to \gamma + \pi\pi, K\bar{K}, \eta\eta$ is expected.

(•) PANDA;J-PARK: $p\bar{p} \to lightresonances,\ J/Psi,$

(•) LHC : heavy hadron weak decays $\to X$ + meson resonances..

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