Meson mass spectrum and
the Fermi coupling
in the Covariant Confined Quark Model

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Motivation

- One of the puzzles of hadron physics is the origin of the hadron masses. The Standard Model (particularly, QCD) operates only with fundamental particles (quarks, leptons, neutrinos), gauge bosons and the Higgs.

- It is not yet clear how to explain an appearance of the numerous number of the observed hadrons and elucidate the generation of their masses.

- Therefore, the calculation of hadron mass characteristics comparable to the precision of experimental data still remains one of the major problems in QCD and involving of more phenomenological inputs are needed.

- CCQM has been successfully applied for calculation of the leptonic decay constants, basic electromagnetic decay widths and form factors needed for semi-leptonic, non-leptonic and rare decays of *B*-meson and Lambda-*b*-baryons.

Goals

- Understanding of the meson mass origin in the framework of the CCQM.

- To study the behavior of the Fermi coupling G as a function of the hadron mass.

CCQ Model (Short Review)

Basic Assumptions:

T. Branz et al., Phys. Rev. **D81**, 034010 (2010).

- Hadrons are considered elementary particles.
- Hadrons are composite fields consisting of constituent quarks.
- Hadrons interact by means of quark exchanges.

- The interaction **Lagrangian** provides full description of the transition between hadrons and its constituents (quarks).

$$L_{\text{int}} = g_H H(x) J_H(x)$$

- The matrix element between the **physical state** and the corresponding **bare state** is determined by the **renormalization constant**.

$$Z_H = \left\langle H_{bare} \mid H_{phys} \right\rangle^2$$

- The renormalized hadron-quark coupling is determined (and eliminated from the consideration) by the **compositeness condition.**

$$Z_H = 1 - g_{ren}^2 \tilde{\Pi}'_H(M_H^2) = 0$$

Compositeness Condition

- **Yukawa-type model**

$$L_Y = \bar{q}\,(i\hat{\partial} - m)\,q + \phi_0\,(-\partial^\mu \partial_\mu - M_0^2)\,\phi_0 + g_0 \phi_0\,(\bar{q}\,\Gamma\,q)$$

Consider Generating functional and take explicit integrations over quark fields

$$Z_Y = \int \delta\phi_0 \int \delta\bar{q} \int \delta q \exp\left\{ i \int dx\, L_Y(x) \right\}$$

$$\Pi(x-y) \equiv i < T\left\{ (\bar{q}\Gamma q)_x (\bar{q}\Gamma q)_y \right\} >= -i \cdot tr\left\{ \Gamma\, S(x-y)\,\Gamma\, S(y-x) \right\}$$

$$\tilde{\Pi}(p^2) = \tilde{\Pi}(M^2) + (p^2 - M^2)\,\tilde{\Pi}'(M^2) + \tilde{\Pi}^{ren}(p^2), \qquad \tilde{\Pi}'(p^2) = \frac{d}{dp^2}\tilde{\Pi}(p^2)$$

Renormalization

$$M^2 \equiv M_0^2 - g_0^2 \tilde{\Pi}(M^2)\,; \qquad \phi_r \equiv Z^{-1/2}\phi_0\,; \qquad g_r \equiv Z^{1/2} g_0\,;$$

$$\mathrm{Z} \equiv [1 - g_0^2 \tilde{\Pi}'(M^2)]^{-1} = 1 - g_r^2 \tilde{\Pi}'(M^2)$$

$$Z_Y^{ren} = \int \delta\phi_r \exp\left\{ \frac{i}{2}\Big(\phi_r (\Box - M^2)\phi_r \Big) + \frac{i\, g_r^2}{2}\Big(\phi_r \tilde{\Pi}^{ren} \phi_r \Big) \right\}$$

$$\cdot \exp\left\{ -\sum_{n=3}^{\infty} \frac{i^n g_r^n}{n} \int dx_1 \ldots \int dx_n\, \phi_r(x_1) \ldots \phi_r(x_n) \cdot \mathrm{tr}\left\{ \Gamma\, S(x_1 - x_2) \ldots \Gamma\, S(x_n - x_1) \right\} \right\}$$

• Fermi-type model

$$L_F = \bar{q}\,(i\hat{\partial} - m)\,q + \frac{G}{2}(\bar{q}\,\Gamma\,q)^2$$

Gaussian representation:

$$\exp\left\{ i\,\frac{G}{2}(\bar{q}\,\Gamma\,q)^2 \right\} = \int \delta\phi \exp\left\{ -i\,\frac{1}{2G}(\phi\,\phi) + i\left(\phi(\bar{q}\,\Gamma\,q)\right) \right\}$$

$$Z_F = \int \delta\bar{q} \int \delta q \exp\left\{ i \int dx\, L_F(x) \right\}$$

$$= \int \delta\phi \exp\left\{ -i\,\frac{1}{2G}(\phi\,\phi) - \sum_{n=2}^{\infty} \frac{i^n}{n} \int dx_1 ... \int dx_n\, \phi(x_1)...\phi(x_n)\cdot tr\left\{ \Gamma\,S(x_1 - x_2)...\Gamma\,S(x_n - x_1) \right\} \right\}$$

Bi-linear terms in boson fields:

$$L_F^{(2)} = \frac{1}{2}\left(\phi \left[-\frac{1}{G} + \Pi(M^2) + (\Box - M^2)\Pi'(M^2) \right] \phi \right) + \frac{1}{2}\left(\phi\,\Pi^{ren}\phi \right)$$

Condition and renormalization

$$-\frac{1}{G} + \tilde{\Pi}(M^2) = 0$$

$$\phi_{ren} = \left[\tilde{\Pi}'(M^2) \right]^{-1/2} \phi$$

$$Z_F^{ren} = \int \delta\phi_{ren} \exp\left\{ \frac{i}{2}\left(\phi_{ren}(\Box - M^2)\phi_{ren} \right) + \frac{i}{2}\,\frac{1}{\tilde{\Pi}'(M^2)}\left(\phi_{ren}\tilde{\Pi}^{ren}\phi_{ren} \right) \right\}$$

$$\cdot \exp\left\{ -\sum_{n=3}^{\infty} \frac{i^n}{n} \left[\frac{1}{\tilde{\Pi}'(M^2)} \right]^{n/2} \int dx_1 ... \int dx_n\, \phi_{ren}(x_1)...\phi_{ren}(x_n)\cdot \mathrm{tr}\left\{ \Gamma\,S(x_1 - x_2)...\Gamma\,S(x_n - x_1) \right\} \right\}$$

$$g_r \equiv \left[\tilde{\Pi}'(M^2)\right]^{-1/2}$$

$$-\frac{1}{G} + \tilde{\Pi}(M^2) = 0$$

$$Z_Y^{ren} \Leftrightarrow Z_F^{ren}$$

$$L_F^{\rm int} = \frac{G}{2} J_H^2(x) \quad \Leftrightarrow \quad L_Y^{\rm int} = g_H H(x) J_H(x)$$

$$\mathrm{Z} \equiv 1 - g_r^2 \cdot \tilde{\Pi}'(M^2) = 0$$

Compositeness Condition

$$\phi_0 \equiv Z^{1/2}\phi_r = 0$$

The vanishing of the wave function renormalization constant (Z=0) in the Yukawa theory may be interpreted as the condition that the bare (unrenormalized) field vanishes for a composite boson.

A. Salam, Nuovo Cim. 25, 224 (1962)
S. Weinberg, Phys. Rev. 130, 776 (1963)

Jouvet condition [B.Jouvet, Nuovo Cim. 3, 1133 (1956)]:

$$Z \to 0: \qquad \mu^2 \to \infty; \quad g^2 \to \infty; \quad \lim_{\substack{g\to 0\\ \mu\to 0}} \frac{g^2}{\mu^2 - M^2} = G < \infty$$

Meson-Quark Interaction

Lagrangian: $$L_{\text{int}} = g_H H(x) J_H(x)$$

Quark currents (for mesons): $$J_H(x) = \int dx_1 \int dx_2\, F_H(x; x_1, x_2)\, \bar{q}(x_2) \Gamma_H q(x_1)$$

$$\Gamma_P = i\gamma^5; \quad \Gamma_V = \gamma^\mu$$

Vertex function (trans. inv.) $$F_H(x; x_1, x_2) = \delta(x - \omega_1 x_1 - \omega_2 x_2) \cdot \Phi_H(|x_1 - x_2|^2)$$

$$\omega_j = m_j / (m_1 + m_2); \quad \omega_1 + \omega_2 = 1$$

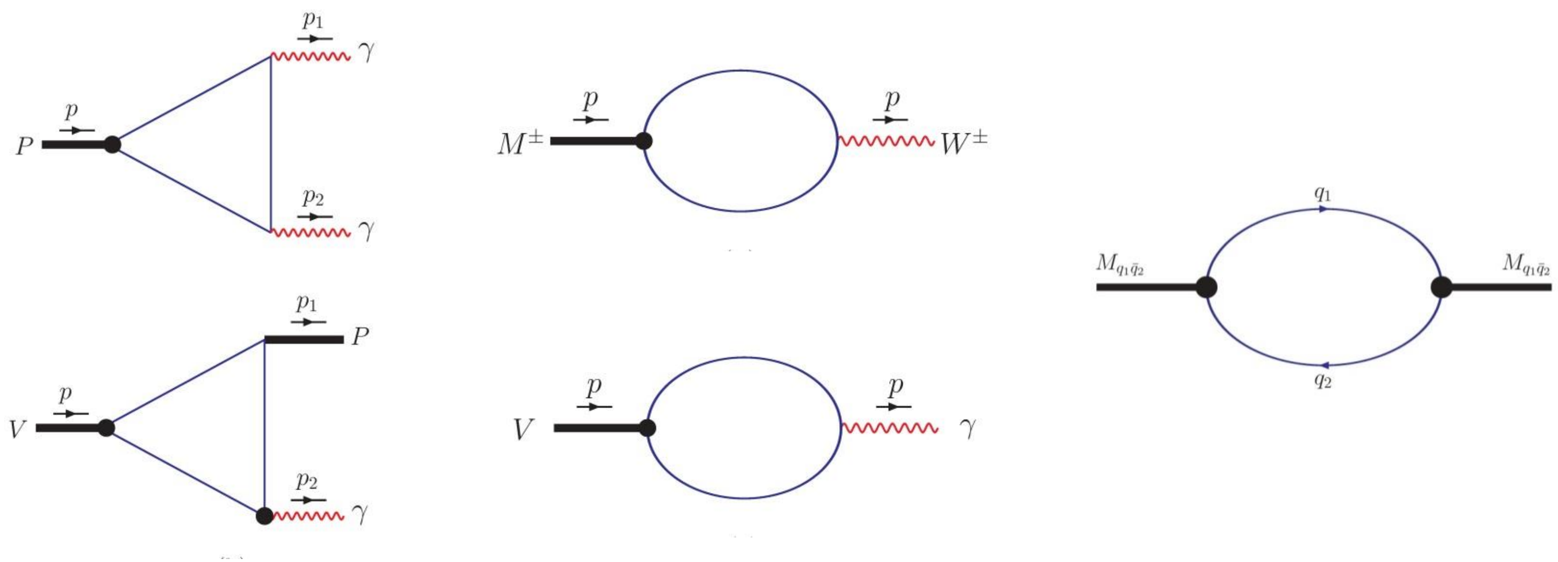
Gaussian form (Fourier) of the vertex function $$\tilde{\Phi}_H(-p^2) = \exp\left(\frac{p^2}{\Lambda_H^2} \right)$$

Schwinger representation of the quark propagator

$$\tilde{S}_{m1}(\hat{p}) = \frac{m_1 + \hat{p}}{m_1^2 - p^2} = \left(m_1 + \hat{p}\right) \cdot \int_0^\infty ds_1 \exp\left[-s\left(m_1^2 - p^2\right) \right]$$

Matrix Elements

Matrix elements for meson (electromagnetic and leptonic) decays and self energy:



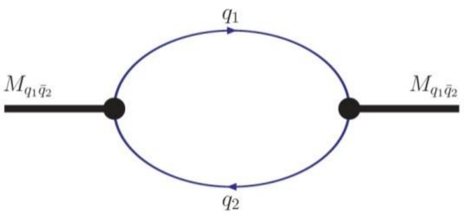
Loop integrals are taken in Euclidean variables:

$$k^0 \to ik_4; \qquad p^0 \to ip_4; \qquad -k^2 \to k_E^2 \geq 0; \qquad -p^2 \to p_E^2 \geq 0$$

Getting the simplex by introducing the identity:

$$1 = \int_0^\infty dt \cdot \delta\left(t - s_1 - s_2\right)$$

Meson Mass Function



$$\Pi_{PP}(x-y) = +i\left\langle T\left\{J_P(x)J_P(y)\right\}\right\rangle_0$$

$$\Pi_{VV}^{\mu\nu}(x-y) = -i\left\langle T\left\{J_V^{\mu}(x)J_V^{\nu}(y)\right\}\right\rangle_0$$

$$\tilde{\Pi}_{PP}(p^2) = \mathrm{N}_c \int \frac{d^4k}{(2\pi)^4 i}\tilde{\Phi}_P^2(-k^2)\,\mathrm{tr}\left\{\gamma^5 \tilde{S}_{m1}(k_1+p\omega_1)\gamma^5 \tilde{S}_{m2}(k_2-p\omega_2)\right\}$$

$$\tilde{\Pi}_{VV}^{\mu\nu}(p^2) = \mathrm{N}_c \int \frac{d^4k}{(2\pi)^4 i}\tilde{\Phi}_V^2(-k^2)\,\mathrm{tr}\left\{\gamma^\mu \tilde{S}_{m1}(k_1+p\omega_1)\gamma^\nu \tilde{S}_{m2}(k_2-p\omega_2)\right\}$$

$$\tilde{\Pi}_H(p^2) = \frac{3}{4\pi^2}\int_0^\infty \frac{dt\cdot t}{a_H^2}\int_0^1 ds\,\exp\left(-t\cdot z_0 + z_H\right)\cdot\left[\frac{n_H}{a_H} + m_1 m_2 + p^2\left(\omega_1 - \frac{b}{a_H}\right)\left(\omega_2 + \frac{b}{a_H}\right)\right]$$

$$z_0 = s\,m_1^2 + (1-s)\,m_2^2 - s(1-s)\,p^2\,; \qquad z_H = \frac{2t}{2+t\cdot\Lambda_H^2}\left(s-\omega_2\right)^2 p^2\,;$$

$$a_H = t + \frac{2}{\Lambda_H^2}\,; \qquad b = t\left(s-\omega_2\right); \qquad n_P = 2; \qquad n_V = 1$$

A branching appears at $p^2 = (m_1+m_2)^2$ because $z_0 = 0$ at $s = \omega_2$ $\Rightarrow \int_0^\infty \frac{dt\,t}{a_H^2}\ldots = \text{diverges!}$

Infrared Confinement (Cut-off)

Cut off the upper bound of t-integral (infrared cut-off in terms of k-integral)

for $\lambda > 0$: no threshold singularity: $\int_0^{1/\lambda^2} \frac{dt\, t}{a_H^2} \ldots =$ converges!

T. Branz et al., Phys. Rev. **D81**, 034010 (2010).

$$\tilde{\Pi}_H(p^2) = \frac{3}{4\pi^2} \int_0^{1/\lambda^2} \frac{dt \cdot t}{a_H^2} \int_0^1 ds \exp\left(-t \cdot z_0 + z_H\right) \cdot \left[\frac{n_H}{a_H} + m_1 m_2 + p^2 \left(\omega_1 - \frac{b}{a_H} \right) \left(\omega_2 + \frac{b}{a_H} \right) \right]$$

A meson in the interaction Lagrangian is characterized by parameters

- the coupling constant (g_H)
- the size parameter (Λ_H)
- two constituent quark masses (m_1 & m_2)
- the infrared confinement parameter λ universal for all hadrons.

Hereby, the Yukawa couplings g_H for all mesons H are removed by $Z = 1 - g_H^2 \tilde{\Pi}'(M^2) = 0$

- **Model parameters**: constituent quark masses, hadron size parameters, a universal infrared cut-off (totally **4+N+1** parameters for N hadrons → **1+5/N** per hadron)

Numerical results for decay constants and widths

Input: fix model parameters by fitting the ***electromagnetic decay widths*** and ***leptonic decay constants.***

CC removes the coupling !

M. A. Ivanov et al, Phys. Rev. D **85**, 034004 (2012).

Process	Fit Values	Data [24]
$\pi^0 \to \gamma\gamma$	5.07×10^{-3}	$(7.7 \pm 0.4) \times 10^{-3}$
$\eta_c \to \gamma\gamma$	3.47	5.0 ± 0.4
$\rho^\pm \to \pi^\pm\gamma$	76.3	67 ± 7
$\omega \to \pi^0\gamma$	687	703 ± 25
$K^{*\pm} \to K^\pm\gamma$	57.7	50 ± 5
$K^{*0} \to K^0\gamma$	129	116 ± 10
$D^{*\pm} \to D^\pm\gamma$	0.59	1.5 ± 0.5
$J/\psi \to \eta_c\gamma$	1.90	1.58 ± 0.37

	Fit Values	Data
f_π	128.4	130.4 ± 0.2
f_K	156.0	156.1 ± 0.8
f_D	206.7	206.7 ± 8.9
f_{D_s}	257.5	257.5 ± 6.1
f_B	189.7	192.8 ± 9.9
f_{B_s}	235.3	238.8 ± 9.5
f_{η_c}	386.6	438 ± 8
f_{B_c}	445.6	489 ± 5
f_{η_b}	609.1	801 ± 9

f_ρ	221.2	221 ± 1
f_ω	204.2	198 ± 2
f_ϕ	228.2	227 ± 2
$f_{J/\psi}$	415.0	415 ± 7
f_{K^*}	215.0	217 ± 7
f_{D^*}	223.0	245 ± 20
$f_{D_s^*}$	272.0	272 ± 26
f_{B^*}	196.0	196 ± 44
$f_{B_s^*}$	229.0	229 ± 46
f_Υ	661.3	715 ± 5

Fixed model parameters:

$$\lambda = 0.181\,\text{GeV},\quad m_{ud} = 0.235\,\text{GeV},\quad m_s = 0.442\,\text{GeV},\quad m_c = 1.61\,\text{GeV},\quad m_b = 5.07\,\text{GeV}$$

TABLE III: The fitted values of the size parameters Λ_H in GeV.

π	K	D	D_s	B	B_s	B_c	η_c	η_b	
0.87	1.02	1.71	1.81	1.90	1.94	2.50	2.06	2.95	
ρ	ω	ϕ	J/ψ	K^*	D^*	D_s^*	B^*	B_s^*	Υ
0.61	0.50	0.91	1.93	0.75	1.51	1.71	1.76	1.71	2.96

+ Agreement between our fit values and the PDG data is quite satisfactory
+ The constituent quark masses and the values of the size parameter Λ_H fall into the expected range.
+ The size parameters follow the expected pattern (meson “size” ~1/Λ_H shrinks as its mass grows).

Numerical results for Fermi coupling G

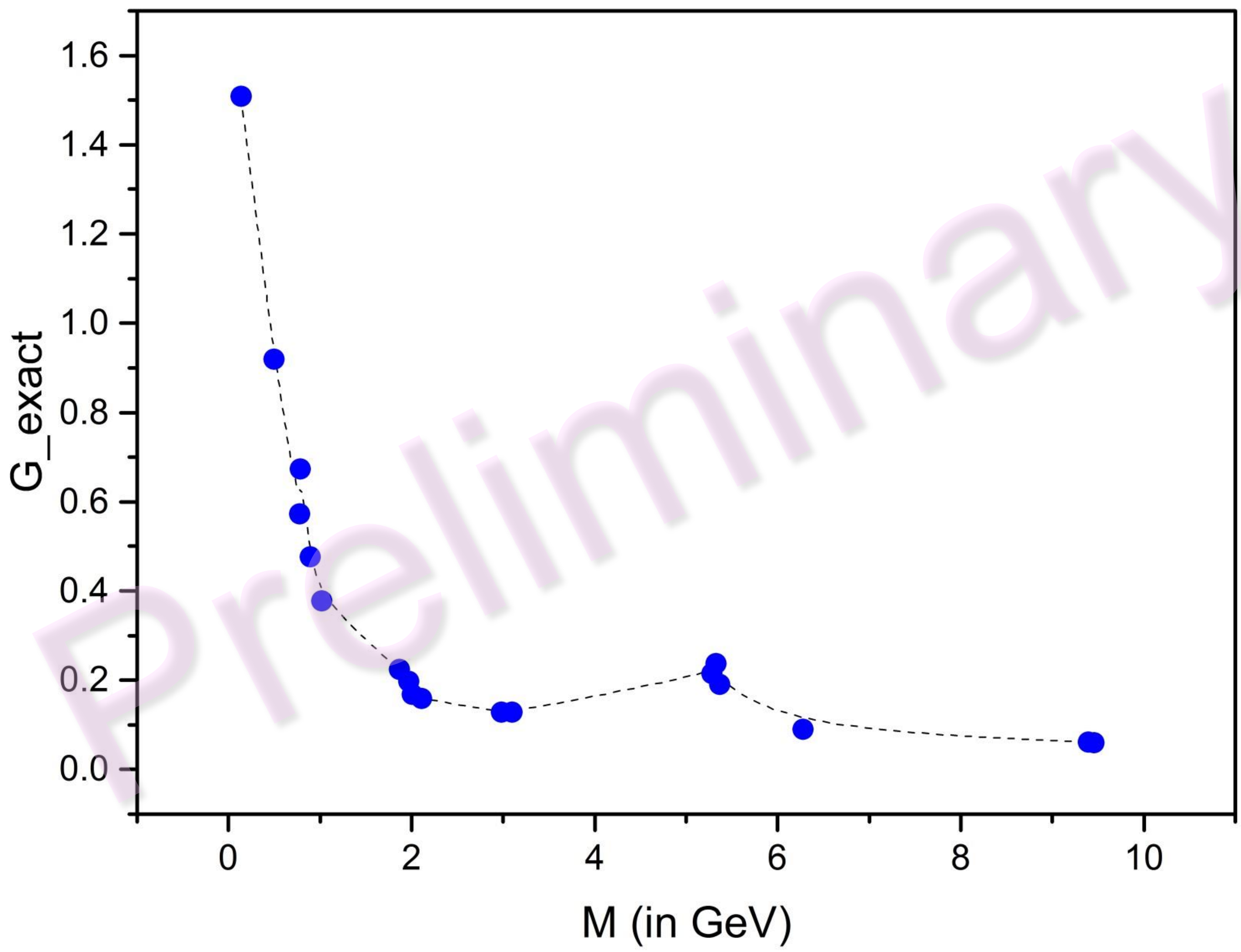
Equation for the Fermi coupling:

$$G = 1/\tilde{\Pi}_H(M_{\exp}^2)$$

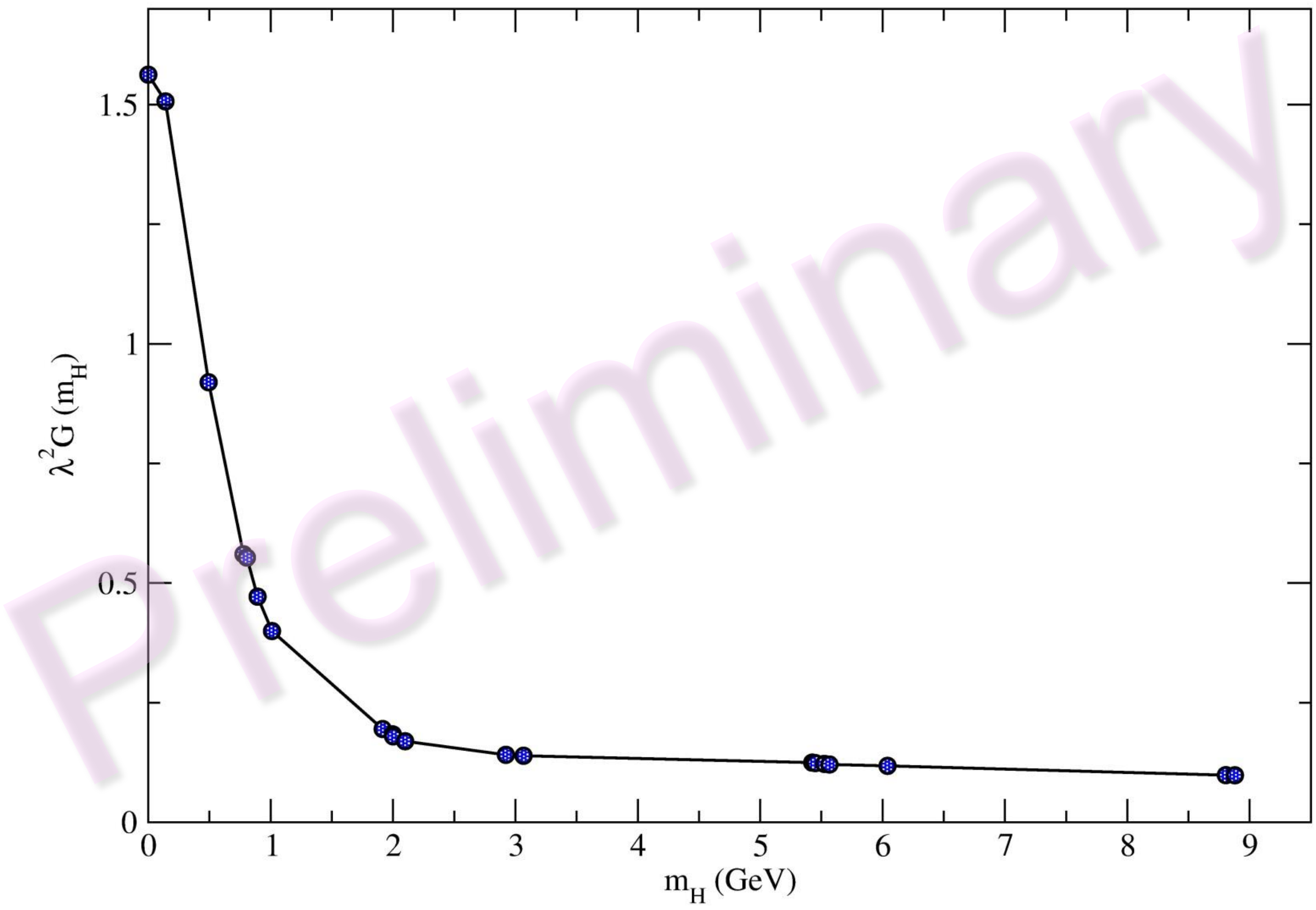
$J^{PC}=0^{-+}$	PDG (MeV)	λ^2 G	$J^{PC}=1^{--}$	PDF (MeV)	λ^2 G
π	139.57	1.508	**ρ**	775.26	0.576
K	493.68	0.919	**ω**	782.65	0.673
D	1869.62	0.224	**K***	891.66	0.476
D_s	1968.50	0.197	**Φ**	1019.45	0.377
H_c	2983.7	0.128	**D***	2010.29	0.168
B	5279.26	0.215	**D*s**	2112.3	0.158
B_s	5366.77	0.191	**J/ψ**	3096.92	0.129
B_c	6274.5	0.0906	**B***	5325.2	0.237
η_b	9398.0	0.0612	**B*s**	5415.8	0.231
			ϒ	9460.3	0.0601

$$\tilde{\Pi}_H(M^2) \sim [\mathrm{GeV}^2] \quad \Rightarrow \quad \lambda^2 G(M) \sim [\mathrm{dimensionless}]$$

G coupling: deriving exact meson masses



G coupling: smoothing



Numerical results for meson masses

Equation for Meson Mass: $$1 - G_{smooth} \cdot \tilde{\Pi}_H(M^2) = 0$$

$J^{PC} = 0^{-+}$ | $J^{PC} = 1^{--}$

	PDG (MeV)	λ^2 G (smooth)	M (MeV)		PDG (MeV)	λ^2 G (smooth)	M (MeV)
π	139.57	1.507	141	**ρ**	775.26	0.560	778
K	493.68	0.920	493	**ω**	782.65	0.554	806
D	1869.62	0.195	1915	**K***	891.66	0.472	893
$\mathbf{D_s}$	1968.50	0.184	1998	**Φ**	1019.45	0.401	1011
$\mathbf{\eta_c}$	2983.7	0.141	2922	**D***	2010.29	0.180	2001
B	5279.26	0.125	5425	**D*s**	2112.3	0.170	2099
$\mathbf{B_s}$	5366.77	0.122	5524	**J/ψ**	3096.92	0.139	3067
$\mathbf{B_c}$	6274.5	0.118	6041	**B***	5325.2	0.124	5450
$\mathbf{\eta_b}$	9398.0	0.0986	8806	**B*s**	5415.8	0.121	5566
				ϒ	9460.3	0.0984	8880

G coupling: A Comparison

• It might be interesting to compare the behavior of G with the effective QCD coupling constant obtained in the relativistic models with specific forms of analytically confined quark and gluon propagators.

$$L = -\frac{1}{4}\left(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g f^{abc} A_\mu^b A_\nu^c\right)^2 + \sum_f \left(\bar{q}_f^a \left[\gamma_\alpha \partial_\alpha - m_f + ig\,\gamma_\alpha t^C A_\alpha^C\right]^{ab} q_f^b\right)$$

$$\tilde{S}_{IR}(\hat{p}) = \left(i\hat{p} + m\right) \cdot \int_0^{1/\Lambda^2} dt \exp\left\{-t\cdot(p^2 + m^2)\right\};$$

$$D_{IR}(x) = \int_{\Lambda^2/4}^{\infty} ds\, e^{-s x^2} = \frac{e^{-x^2\Lambda^2/4}}{4\pi^2 x^2} \quad .$$

G.Ganbold,
Phys. Rev. D 79, 034034 (2009).
Phys. Part. Nucl. **43**, 79, (2012)
Phys. Part. Nucl. **45**, 10, (2014)

• In that models the four-quark nonlocal interaction is induced by one-gluon exchange between bi-quark currents. Since the quark currents are connected via the confined gluon propagator having the dimension of the inverse mass squared in the momentum space, the resulting coupling (α_s) is dimensionless.

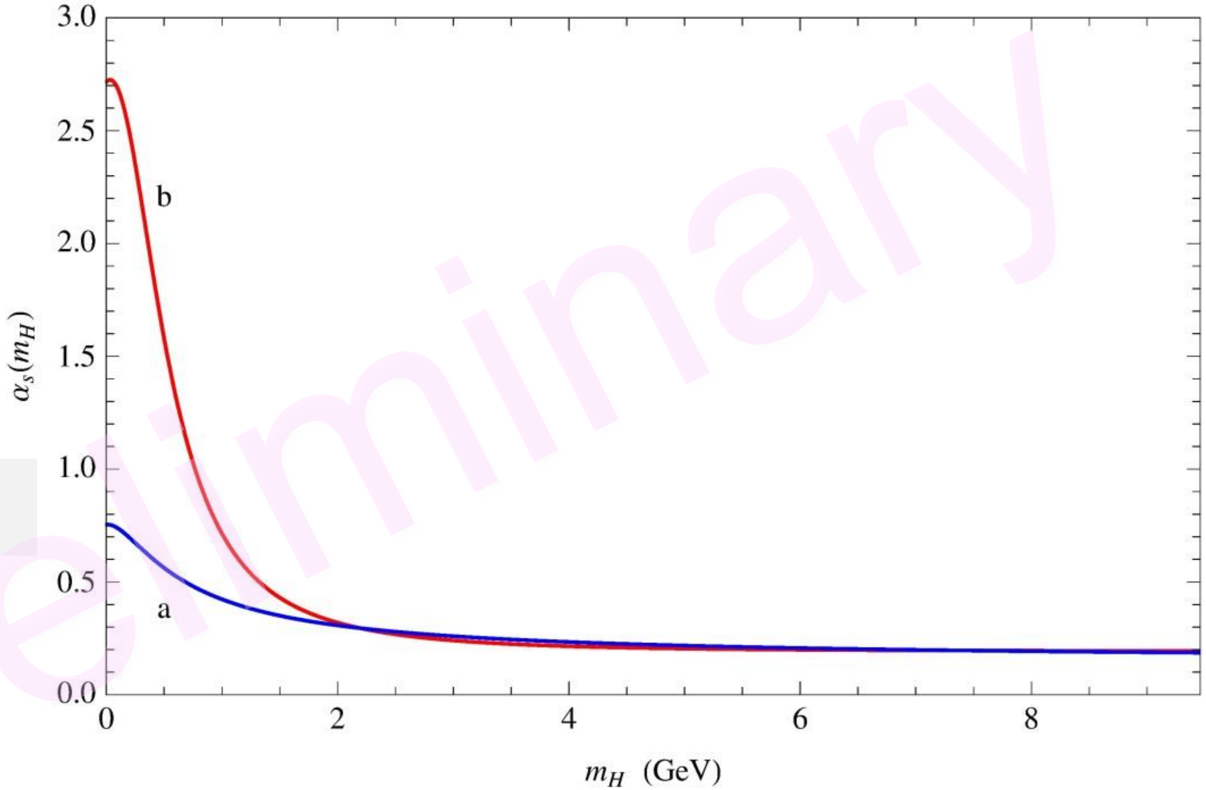
$$1 - \alpha_s \cdot \frac{8C_J}{3\pi^3} \int d^4k\, V_J(k) \cdot \Pi_J(p,k) \cdot V_J(-k) = 0; \qquad (p^2 = -M_J^2)$$

$$V_J(k) = \int dx\, \sqrt{D(x)}\, U_J(x)\, e^{ikx}$$

In order to be able to compare two curves obtained in these different approaches we re-scale our dimensionless coupling λ^2*G (red curve) by a factor of ~ 1.74 to approach α_s (blue curve).

$$\Lambda = 345 \text{ MeV}$$
$$m_{ud} = 193 \text{ MeV}$$
$$m_s = 293 \text{ MeV}$$
$$m_c = 1848 \text{ MeV}$$
$$m_b = 4693 \text{ MeV}$$

G.Ganbold,
Phys. Rev. D 81, 094008 (2010).



- Despite the different model origins and input parameter values, the behaviors of two curves are very similar each other in the region above ~ 2 GeV.
- Their values at origin are mostly determined by the confinement mechanisms realized in different ways in these models. This could explain why they have different behaviors in the low-energy region below 2 GeV.

Summary and Outlook:

- ♣ A brief sketch of an approach to the bound state problem in QFT based on the compositeness condition is represented.

- ♣ We have explicitly demonstrated that the four-fermion theory with the Fermi coupling *G* is equivalent to the Yukawa-type theory if,
  - the wave function renormalization constant in the Yukawa theory is equal to zero,
  - the Fermi coupling *G* is inversely proportional to the meson mass function calculated at physical meson mass.

- ♣ This approach has been applied to the mass spectrum of conventional mesons within CCQ model. We updated the fit of model parameters and calculated the Fermi coupling *G* as a function of physical mass.

- ♣ A *smoothness criterion* has been suggested – just by varying the meson masses in such a way to obtain the smooth behavior of the Fermi coupling *G*. The mass spectrum obtained in this manner is found to be in good agreement with the experimental data (from $\pi(140)$ up to $\Upsilon(9460)$).

- ♣ We have compared the behavior of *G* with the strong QCD coupling α_s calculated in the QCD-inspired approach.
- ♣ Despite its model origin, the approximations used, and questions about the very definition of the coupling, our approach exhibits a specific IR-finite behavior of G coupling, similar to the QCD running coupling, especially for energy > 3GeV.
- ♣ The approach is able to address simultaneously different sections of the low-energy particle physics. Consideration can be extended to other problems (exotic hadrons, glueballs, mixed and multiquark states…etc.).