Bethe-Salpeter approach with separable potential. Review and results (Twenty Years).

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1. ***Introduction***
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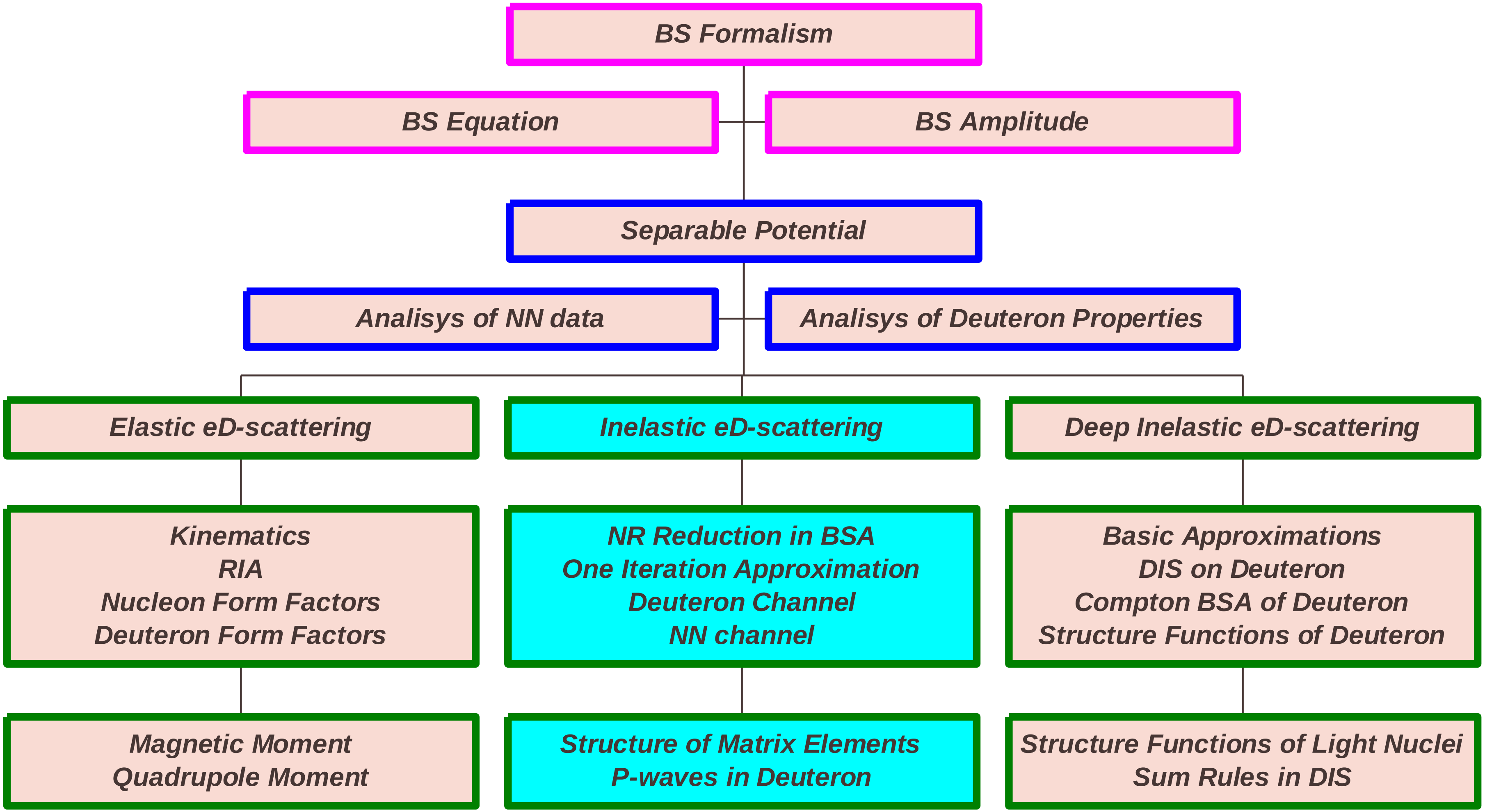
The first publication: S.G. Bondarenko, Burov V.V., S.M. Dorkin, Elastic Form factors of the Deuteron in Bethe – Salpeter Approach and Charge Form Factor of Neutron, Proceedings of the XII International Seminar on High Energy Physics Problems, JINR, Dubna, 1994, p.90.

Bethe-Salpeter Approach. Twenty Years.



Introduction

- Study of static and dynamic electromagnetic properties of light nuclei enables us to understand more deeply a nature of strong interactions and, in particular, the nucleon - nucleon interaction.
- This investigations are connected to a large amount of experimental data, and also with planned new experiments, which will allow to move in region of the large transfer momenta in elastic, inelastic, and deep-inelastic lepton - nucleus reactions.
- At such energies an assumptions of nucleus as a nucleon system is not well justified. For this reason the problems to study in intermediate energy region the nonnucleonic degrees of freedom (Δ-isobars, quarks etc.) and Mesonic Exchange Currents (MEC) are widely discussed.

Introduction

- However, in spite of the significant progress being achieved in this way, the relativistic effects (which ***a priori*** are very important at large transfer momenta) are needed to be included.
- Other actively discussed problem is the extraction of the information about the structure bound nucleons from experiments with light nuclei . Such tasks require to take into account relativistic kinematics of reaction and dynamics of ***NN*** interaction. For this reason const-ruction of covariant approach and detailed analysis of relativistic effects in electromagnetic reactions with light nuclei are very important and interesting.
- *Bethe - Salpeter approach give a possibility to take into account relativistic effects in a consistent way.*

First Stage.
Bethe-Salpeter Formalism

Bethe –Salpeter Formalism

- Let us define full two particle Green Function:

$$\mathrm{G}_{\alpha,\beta;\gamma,\delta}\left(x_1,x_2;y_1,y_2\right)=-\left\langle 0\left|T\left[\psi_\alpha\left(x_1\right)\psi_\beta\left(x_2\right)\bar{\psi}_\gamma\left(y_1\right)\bar{\psi}_\delta\left(y_2\right)\right]\right|0\right\rangle,$$

- Bethe –Salpeter Equation for G:

$$\mathrm{G}_{\alpha,\beta;\gamma,\delta}\left(x_1,x_2;y_1,y_2\right)=\mathrm{G}^{(0)}{}_{\alpha,\beta;\gamma,\delta}\left(x_1,x_2;y_1,y_2\right)+$$
$$+i\int\prod_{k=1}^{4}dw_k\,\mathrm{G}^{(0)}{}_{\alpha,\beta;\sigma,\rho}\left(x_1,x_2;w_1,w_2\right)\times$$
$$\times K_{\sigma,\rho;\lambda,\omega}\left(w_1,w_2;w_3,w_4\right)\mathrm{G}_{\lambda,\omega;\gamma,\delta}\left(w_3,w_4;y_1,y_2\right),$$

where

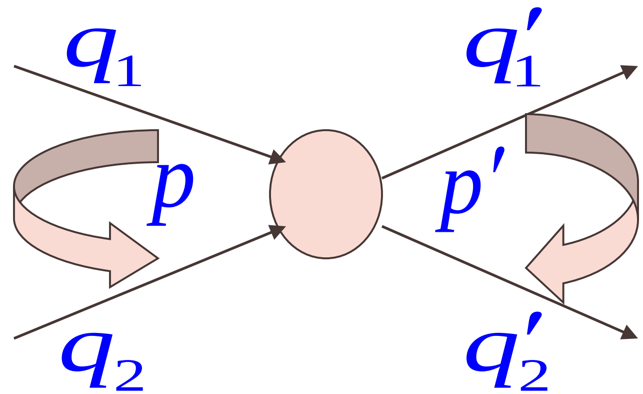
$$\mathrm{G}^{(0)}{}_{\alpha,\beta;\gamma,\delta}\left(x_1,x_2;y_1,y_2\right)=S'_{F\alpha\gamma}{}^{(1)}\left(x_1-x_2\right)S'_{F\beta\delta}{}^{(2)}\left(y_1-y_2\right),$$

- Let us make Fourier transformation of $K, G, G^{(0)}$:

$$V\left(p', p; P\right) = \int dx_1 dx_2 dy_1 dy_2 K\left(x_1, x_2; y_1, y_2\right) \times$$
$$\exp\left[iP\left(\frac{x_1 + x_2}{2} - \frac{y_1 + y_2}{2} \right) + ip'\left(x_1 - x_2\right) - ip\left(y_1 - y_2\right) \right],$$

where P is total, $p\left(p'\right)$ are relative 4-momentum

$$P = q_1 + q_2 \qquad q_1 = P/2 + p$$
$$p = \left(q_1 - q_2\right)/2 \qquad q_2 = P/2 - p$$



The expressions for G and $G^{(0)}$ are similar.

- Full Green function for two particle system is:

$$G_{\alpha\beta;\gamma\delta}(p', p; P) = S'^{(1)}_{F\alpha\gamma}\left(\frac{P}{2}+p\right) S'^{(2)}_{F\beta\delta}\left(\frac{P}{2}-p\right)\delta^{(4)}(p'-p) +$$
$$+ iS'^{(1)}_{F\alpha\varepsilon}\left(\frac{P}{2}+p'\right) S'^{(2)}_{F\beta\lambda}\left(\frac{P}{2}-p'\right)\int\frac{dk}{(2\pi)^4} V_{\varepsilon\lambda;\nu\mu}(p', k; P)\, G_{\nu\mu;\gamma\delta}(k, p; P).$$

- The full one particle Green function is:

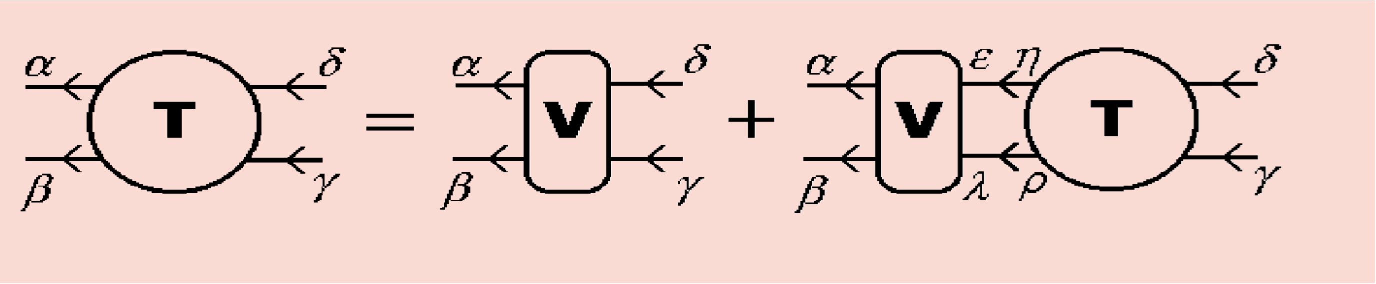
$$S'^{(1)}_{F\alpha\gamma}(p) = \frac{1}{p\cdot\gamma - m - \Sigma(p) + i0}.$$

Mass operator

We will use propagators without mass operator:

$$S'_{F\alpha\beta}(p) \Rightarrow S_{\alpha\beta}(p)$$

T-matrix



- Let us introduce *T*-matrix:

$$GV = G^{(0)}T$$

- The BSE for T-matrix is:

$$T_{\alpha\beta;\delta\gamma}\left(p',p;P\right)=V_{\alpha\beta;\delta\gamma}\left(p',p;P\right)+i\int\frac{dk}{\left(2\pi\right)^4}V_{\alpha\beta;\varepsilon\lambda}\left(p',k;P\right)\times$$
$$\times S_{\varepsilon\eta}{}^{(1)}\left(\frac{P}{2}+k\right)S_{\lambda\rho}{}^{(2)}\left(\frac{P}{2}-k\right)T_{\eta\rho;\delta\gamma}\left(k,p;P\right).$$

- Thus a bound state corresponds to a pole in a T-matrix at $P^2 = M^2$ (M is the mass of the bound state):

$$T_{\alpha\beta;\delta\gamma}\left(p', p; P\right) = \frac{\Gamma_{\alpha\beta}\left(P, p'\right)\bar{\Gamma}_{\gamma\delta}\left(P, p\right)}{P^2 - M^2} + R_{\alpha\beta;\delta\gamma}\left(p', p; P\right),$$

$R_{\alpha\beta;\delta\gamma}\left(p', p; P\right)$ is regular at $P^2 = M^2$ function,

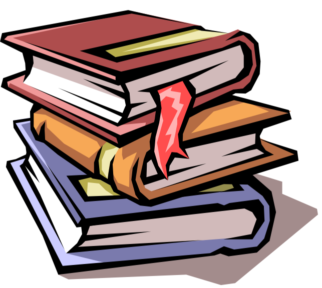
$\Gamma_{\alpha\beta}\left(P, p\right)$ is a vertex function.

The normalization condition follows from: $\left\langle D,P\left|J_\mu(0)\right|D,P\right\rangle = 2iP_\mu$

$$2P_\mu = i\int \frac{dk}{(2\pi)^4}\bar{\Gamma}(P,k)\frac{\partial}{\partial P^\mu}\left[S^{(1)}\left(\frac{P}{2}+k\right)S^{(2)}\left(\frac{P}{2}-k\right)\right]_{P^2=M^2}\Gamma(P,k).$$

BS equation for Amplitude:

$$\Phi_{\alpha\beta}(P,p') = iS_{\alpha\eta}^{(1)}\left(\frac{P}{2}+p'\right)S_{\beta\rho}^{(2)}\left(\frac{P}{2}-p'\right)\int\frac{dk}{(2\pi)^4}V_{\eta\rho;\varepsilon\lambda}(p',k;P)\Gamma_{\varepsilon\lambda}(P,k).$$



1. E.E. Salpeter and H.A.Bethe, Phys.Rev. C**84**(1951) 1232
2. S. Mandelstam, Proc.Roy.Soc. **233**A (1955) 248.
3. S.Bondarenko et.al, Prog.Part.Nucl.Phys. **48**(2002)449;
4. S.Bondarenko et.al, NP, **A832**(2010)233; NP, **A848** (2010) 75; NP, **B219-220c** (2011) 216; FBS, **49** (2011) 121; PLB, **705**(2011)264; JETP Letters, **94**(2011)800.

Second Stage.
Separable Anzats

Solution of BS Equation

- Separable Kernel of Interaction
  - BSE for T-matrix after partial expansion can be written as:

$$T_{\alpha\beta}\left(p_0',|\vec{p}'|,p_0,|\vec{p}|;s\right)=V_{\alpha\beta}\left(p_0',|\vec{p}'|,p_0,|\vec{p}|;s\right)+\frac{i}{2\pi^2}\int dq_0\vec{q}^{\,2}d|\vec{q}|\times$$
$$\sum_{\gamma\delta}V_{\alpha\gamma}\left(p_0',|\vec{p}'|,q_0,|\vec{q}|;s\right)S_{\gamma\delta}\left(q_0,|\vec{q}|;s\right)T_{\delta\beta}\left(q_0,|\vec{q}|,p_0,|\vec{p}|;s\right).$$

  - Separable *anzats*:

$$V_{\alpha\beta}\left(p_0',|\vec{p}'|,p_0,|\vec{p}|;s\right)=\sum_{ij=1}^{N}\lambda_{ij}g_i^{(\alpha)}\left(p_0',|\vec{p}'|\right)g_j^{(\beta)}\left(p_0,|\vec{p}|\right),\ \lambda_{ij}=\lambda_{ji}$$

Then for T – matrix we can write:

$$T_{\alpha\beta}\left(p_0',|\vec{p}'|,p_0,|\vec{p}|;s\right)=\sum_{ij=1}^{N}\tau_{ij}\left(s\right)g_i^{(\alpha)}\left(p_0',|\vec{p}'|\right)g_j^{(\beta)}\left(p_0,|\vec{p}|\right).$$

Substitution V, T in BSE for T–matrix we can find $\tau_{ij}\left(s\right)$:

$$\tau_{ij}^{-1}\left(s\right)=\lambda_{ij}^{-1}-H_{ij}\left(s\right),$$

where the $H_{ij}\left(s\right)$ can be written as:

$$H_{ij}\left(s\right)=\frac{i}{2\pi^2}\sum_{\alpha\beta}\int dq_0\vec{q}^{\,2}d|\vec{q}|S_{\alpha\beta}\left(q_0,|\vec{q}|;s\right)g_i^{(\alpha)}\left(q_0,|\vec{q}|\right)g_j^{(\beta)}\left(q_0,|\vec{q}|\right).$$

Then radial part of BSA has following form:

$$\phi_\alpha\left(p_0,|\vec{p}|\right)=\sum_{\beta}\sum_{\mathrm{i,j=1}}^{\mathrm{N}}S_{\alpha\beta}\left(p_0,|\vec{p}|;s\right)\lambda_{ij}g_i^{(\beta)}\left(p_0,|\vec{p}|\right)c_j\left(s\right),$$

where coefficients $c_j\left(s\right)$ satisfy the equation:

$$c_i\left(s\right)-\sum_{k,j=1}^{N}H_{ik}\left(s\right)\lambda_{kj}c_j\left(s\right)=0.$$

NN-scattering

- Let us consider *NN*-scattering in $^3S_1 - {}^3D_1$ -channel ($^{2S+1}L_J^{\rho}$ - notation). In this case nucleons are on mass shell:

$$p_0 = p_0' = 0,\ |\vec{p}| = |\vec{p}\,'| = |\vec{p}^{\,*}| = \sqrt{s/4 - m^2} = \sqrt{m\,\mathrm{E}_{lab}/2},$$

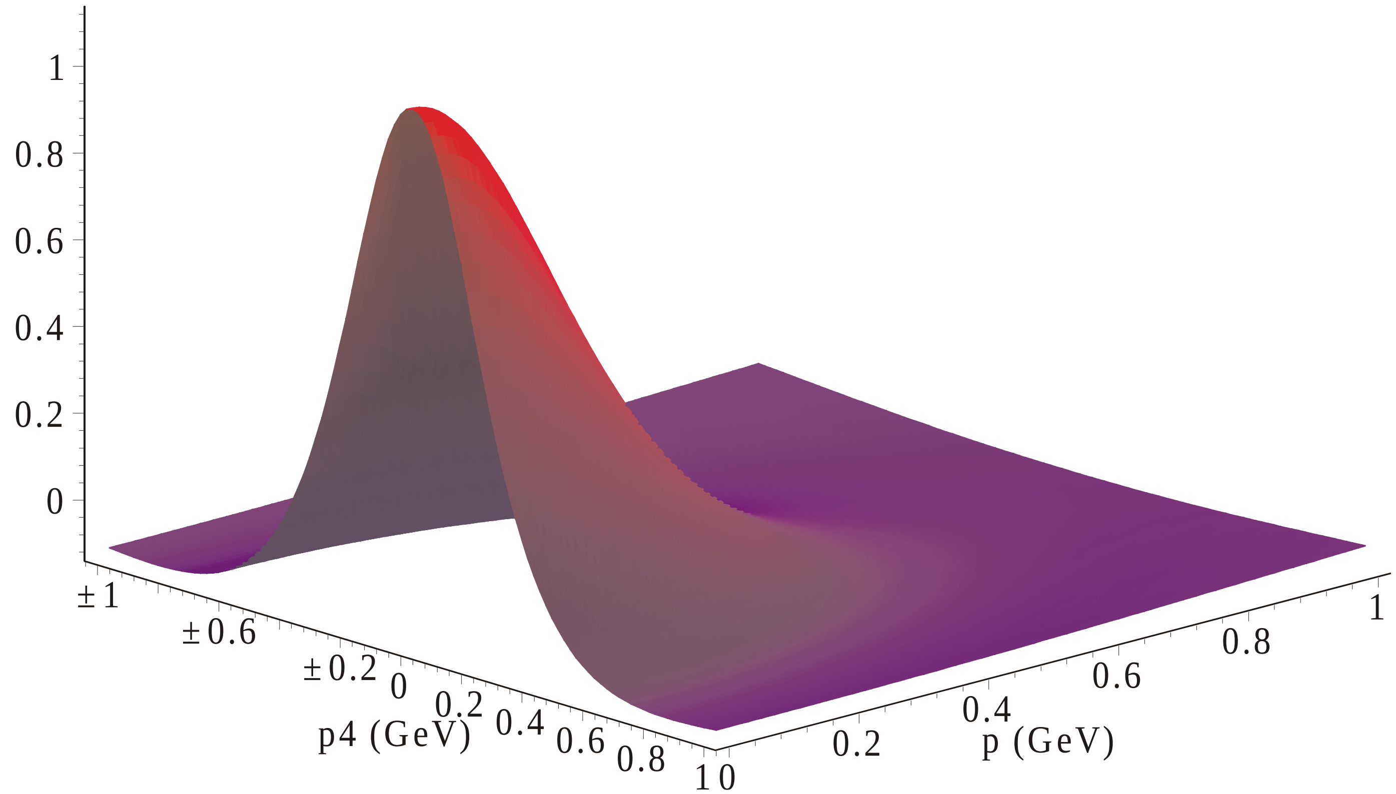
and ***T***-matrix can be parameterized as:

$$T^{(0s)} = -\frac{2i}{|\vec{p}^{\,*}|\sqrt{s}}\begin{pmatrix} \cos 2\varepsilon e^{2i\delta_S} - 1 & i\sin 2\varepsilon e^{i(\delta_S+\delta_D)} \\ i\sin 2\varepsilon e^{i(\delta_S+\delta_D)} & \cos 2\varepsilon e^{2i\delta_D} - 1 \end{pmatrix}.$$

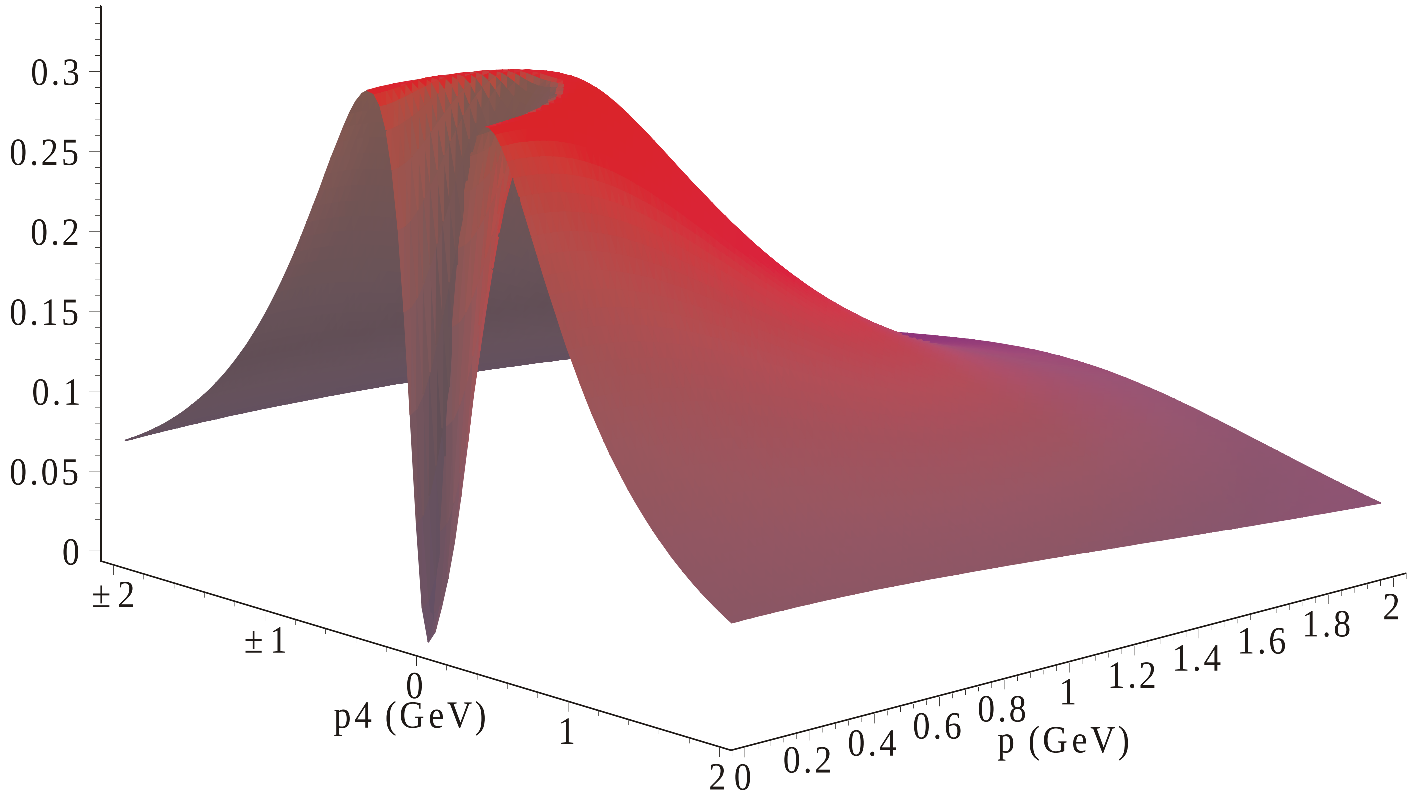
Here $\delta_S\left(\delta_D\right)$ are phase shifts of $^3S_1\left({}^3D_1\right)$ waves, ε - is mixing parameter. For low energy ***NN*** – scattering we can express phase shift through scattering length ***a***, effective radius of interaction r_0:

$$|\vec{p}^{\,*}|\cot\delta_S(s) = -\frac{1}{a} + \frac{r_0}{2}|\vec{p}^{\,*}|^2 + O\left(|\vec{p}^{\,*}|^3\right).$$

Vertex function $g_{^3S_1^+}\left(p_0 \equiv p_4, |\vec{p}| \equiv p\right)$.



Vertex function $g_{^3D_1^+}\left(p_0 \equiv p_4, |\vec{p}| \equiv p\right).$



Third Stage.
Problem of poles.

Relativistic description

Yamaguchi

$$\boldsymbol{g}(|\,\boldsymbol{p}\,|) = \frac{1}{\boldsymbol{p}^2 + \beta^2}$$

$$\boldsymbol{p}^2 \to -p^2 = -p_0^2 + \boldsymbol{p}^2 \Rightarrow \boldsymbol{g}_{\boldsymbol{p}}(p) = \frac{1}{-p^2 + \beta^2} \xrightarrow{center-of-mass} \frac{1}{-p_0^2 + \boldsymbol{p}^2 + \beta^2 + \boldsymbol{i}0}$$

Y. Avishai, T. Mizutani, Nucl. Phys. A 338 (1980) 377-412

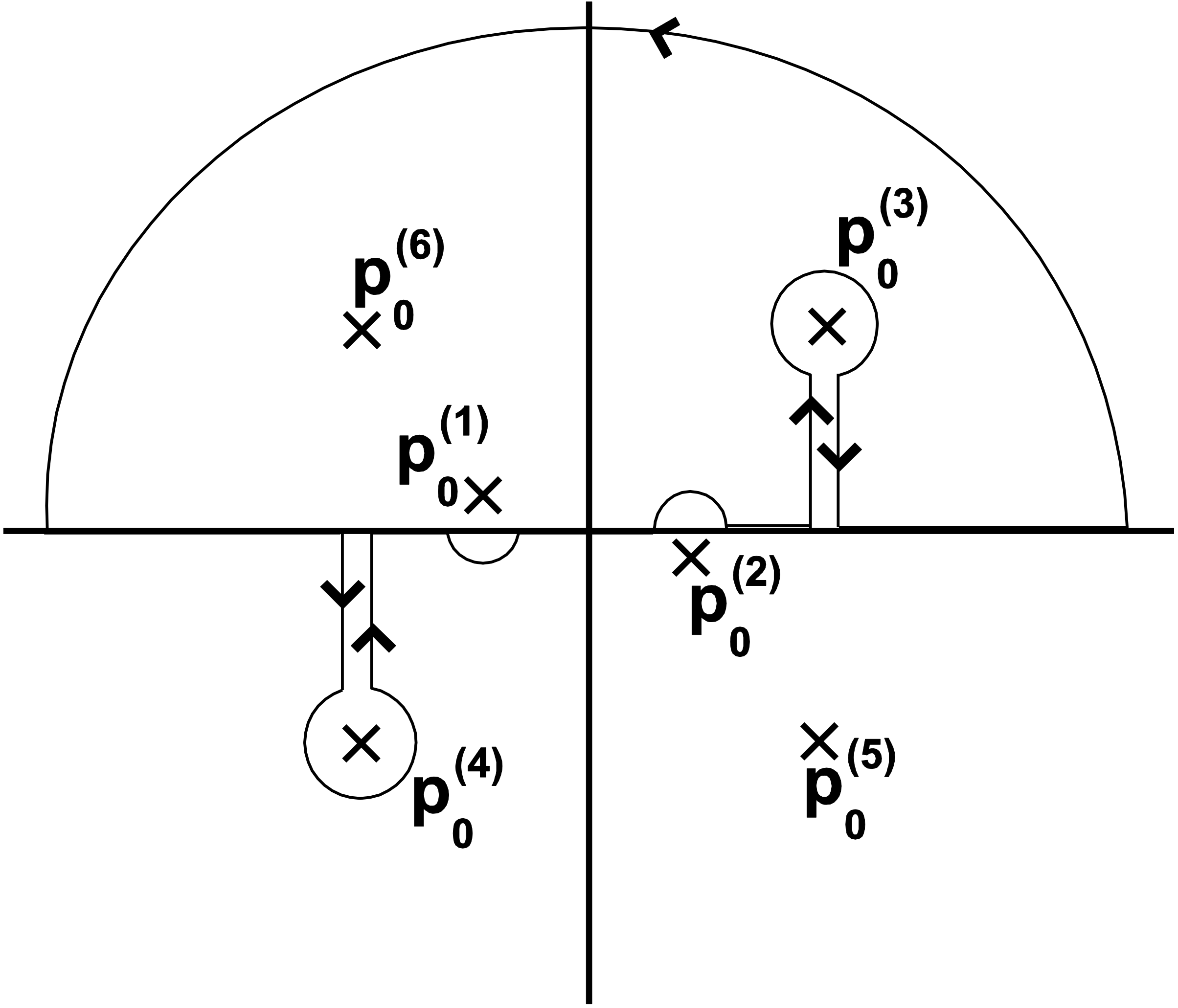
$$Q = p - \frac{P \cdot p}{s} P \Rightarrow \boldsymbol{g}_Q(p) = \frac{1}{-Q^2 + \beta^2} \xrightarrow{center-of-mass} \frac{1}{\boldsymbol{p}^2 + \beta^2}$$

$$\boldsymbol{g}_{\boldsymbol{p}} \to p_0 = \pm\sqrt{\boldsymbol{p}^2 + \beta^2} \mp \boldsymbol{i}0$$

$\boldsymbol{g}_Q \to$ No poles

K. Schwarz, J. Frohlich, H.F.K. Zingl, L. Streit, Acta Phys. Austr. 53 (1981) 191-202

$$\boldsymbol{g}_{\boldsymbol{p}}(p) = \frac{1}{(p_0^2 - \boldsymbol{p}^2 - \beta^2)^2 + \alpha^4}$$



$$p_0^{(1,2)} = \pm\sqrt{s}/2 \mp E_{\boldsymbol{p}} \pm \boldsymbol{i}\varepsilon$$

$$p_0^{(3,4)} = \pm\sqrt{\boldsymbol{p}^2 + \beta^2 + \boldsymbol{i}\alpha^2}$$

$$p_0^{(5,6)} = \pm\sqrt{\boldsymbol{p}^2 + \beta^2 - \boldsymbol{i}\alpha^2}$$

At α→0:

1) Poles do not cross the counter
2) Good limit

R.E. Cutkosky, P.V. Landshoff, D.I. Olive, J.C. Polkinghorne, Nucl. Phys. B 12 (1969) 281-300

Form factors of the separable kernel

The uncoupled channels

$^3P_0, {}^1P_1, {}^3P_1$:

$$g_1^{[P]}(p) = \frac{\sqrt{-p_0^2 + \mathbf{p}^2}}{(p_0^2 - \mathbf{p}^2 - \beta_1^2)^2 + \alpha_1^4}$$

$$g_2^{[P]}(p) = \frac{\sqrt{(-p_0^2 + \mathbf{p}^2)^3}(p_{c2} - p_0^2 + \mathbf{p}^2)}{((p_0^2 - \mathbf{p}^2 - \beta_2^2)^2 + \alpha_2^4)^2}$$

$^1S_0$:

$$g_1^{[S]}(p) = \frac{(p_{c1} - p_0^2 + \mathbf{p}^2)}{(p_0^2 - \mathbf{p}^2 - \beta_1^2)^2 + \alpha_1^4}$$

$$g_2^{[S]}(p) = \frac{(p_0^2 - \mathbf{p}^2)(p_{c2} - p_0^2 + \mathbf{p}^2)^2}{((p_0^2 - \mathbf{p}^2 - \beta_2^2)^2 + \alpha_2^4)^2}$$

$$g_3^{[S]}(p) = \frac{(p_0^2 - \mathbf{p}^2)}{(p_0^2 - \mathbf{p}^2 - \beta_3^2)^2 + \alpha_3^4}$$

Form factors of the separable kernel

The uncoupled channels

$^1D_2$:

$$g_1^{[D]}(p_0,\mathbf{p}) = \frac{-p_0^2+\mathbf{p}^2}{\left(p_0^2-\mathbf{p}^2-\beta_1^2\right)^2+\alpha_1^4}$$

$$g_2^{[D]}(p_0,\mathbf{p}) = \frac{\left(-p_0^2+\mathbf{p}^2\right)^2\left(-p_0^2+\mathbf{p}^2+pc_2\right)}{\left(\left(p_0^2-\mathbf{p}^2-\beta_{21}^2\right)^2+\alpha_{21}^4\right)\left(\left(p_0^2-\mathbf{p}^2-\beta_{22}^2\right)^2+\alpha_{22}^4\right)}$$

$$g_3^{[D]}(p_0,\mathbf{p}) = -\frac{\left(-p_0^2+\mathbf{p}^2\right)^3\left(-p_0^2+\mathbf{p}^2+pc_3\right)^2}{\left(\left(p_0^2-\mathbf{p}^2-\beta_3^2\right)^2+\alpha_3^4\right)^3}$$

$$g_4^{[D]}(p_0,\mathbf{p}) = \frac{\left(-p_0^2+\mathbf{p}^2\right)^4\left(-p_0^2+\mathbf{p}^2+pc_4\right)^3}{\left(\left(p_0^2-\mathbf{p}^2-\beta_{41}^2\right)^2+\alpha_{41}^4\right)^2\left(\left(p_0^2-\mathbf{p}^2-\beta_{42}^2\right)^2+\alpha_{42}^4\right)\left(\left(p_0^2-\mathbf{p}^2-\beta_{43}^2\right)^2+\alpha_{43}^4\right)}$$

$$g_5^{[D]}(p_0,\mathbf{p}) = \frac{\left(-p_0^2+\mathbf{p}^2\right)^9}{\left(\left(p_0^2-\mathbf{p}^2-\beta_5^2\right)^2+\alpha_5^4\right)^5}$$

Form factors of the separable kernel

The uncoupled channels

$^1F_2$:

$$g_1^{[F]}(p_0,\mathbf{p}) = \frac{\sqrt{\left(-p_0^2+\mathbf{p}^2\right)^3}\left(-p_0^2+\mathbf{p}^2+pc_1\right)}{\left(\left(p_0^2-\mathbf{p}^2-\beta_{11}^2\right)^2+\alpha_{11}^4\right)\left(\left(p_0^2-\mathbf{p}^2-\beta_{12}^2\right)^2+\alpha_{12}^4\right)}$$

$$g_2^{[F]}(p_0,\mathbf{p}) = \frac{\sqrt{\left(-p_0^2+\mathbf{p}^2\right)^5}\left(-p_0^2+\mathbf{p}^2+pc_2\right)^2}{\left(\left(p_0^2-\mathbf{p}^2-\beta_{21}^2\right)^2+\alpha_{21}^4\right)^2\left(\left(p_0^2-\mathbf{p}^2-\beta_{22}^2\right)^2+\alpha_{22}^4\right)}$$

The coupled channel $^3S_1$-$^3D_1$

$$g_1^{[S]}(p) = -\frac{(p_{c1} - p_0^2 + \boldsymbol{p}^2)}{(p_0^2 - \boldsymbol{p}^2 - \beta_1^2)^2 + \alpha_1^4}$$

$$g_3^{[S]}(p) = g_4^{[S]}(p) = g_1^{[D]}(p) = g_2^{[D]}(p) = 0$$

$$g_2^{[S]}(p) = -\frac{(p_0^2 - \boldsymbol{p}^2)(p_{c2} - p_0^2 + \boldsymbol{p}^2)^2}{((p_0^2 - \boldsymbol{p}^2 - \beta_2^2)^2 + \alpha_2^4)^2}$$

$$g_3^{[D]}(p) = \frac{(p_0^2 - \boldsymbol{p}^2)(p_{c3} - p_0^2 + \boldsymbol{p}^2)^2}{((p_0^2 - \boldsymbol{p}^2 - \beta_{31}^2)^2 + \alpha_{31}^4)((p_0^2 - \boldsymbol{p}^2 - \beta_{32}^2)^2 + \alpha_{32}^4)}$$

$$g_4^{[D]}(p) = \frac{(p_0^2 - \boldsymbol{p}^2)}{(p_0^2 - \boldsymbol{p}^2 - \beta_4^2)^2 + \alpha_4^4}$$

The vertex functions of the deuteron

$$g_{^3S_1^+}(p) = (c_1\lambda_{11} + c_2\lambda_{12} + c_3\lambda_{13} + c_4\lambda_{14})g_1^{[S]}(p) + (c_1\lambda_{11} + c_2\lambda_{22} + c_3\lambda_{23} + c_4\lambda_{24})g_2^{[S]}(p)$$

$$g_{^3D_1^+}(p) = (c_1\lambda_{13} + c_2\lambda_{23} + c_3\lambda_{33} + c_4\lambda_{34})g_3^{[D]}(p) + (c_1\lambda_{14} + c_2\lambda_{24} + c_3\lambda_{34} + c_4\lambda_{44})g_4^{[D]}(p)$$

The normalization

$$p_l = \frac{\boldsymbol{i}}{2M_d(2\pi)^4}\int dk_0 \int \boldsymbol{k}^2 d|\boldsymbol{k}| \frac{(E_k - M_d/2)[g_l(k_0,|\boldsymbol{k}|)]^2}{((M_d/2 - E_k + \boldsymbol{i}0)^2 - k_0^2)^2}$$

$^1S_0^+$:

	a_s (fm)	r_{0s} (fm)
MY3	-23.750	2.70
MYQ3	-23.754	2.78
Experiment	-23.748(10)	2.75(5)

$^3S_1^+$-$^3D_1^+$:

	p_d(%)	a_t(fm)	r_{0t}(fm)	E_d(MeV)
MY4	6	5.417	1.75	2.2246
MYQ4	6	5.417	1.75	2.2246
CD-Bonn	4.85	5.4196	1.751	2.224575
Graz II	4.82	5.42	1.78	2.225
Experiment	-	5.424(4)	1.759(5)	2.224644(46)

O. Dumbrajs et al., Nucl Phys. B 216 (1983) 277

Fourth Stage.
Inelasticity.

To describe the influence of the inelastic channels into the elastic NN scattering the inelasticity parameter is introduced.

S matrix (Arndt-Roper parametrization)

$$S = \frac{1 - K_i + iK_r}{1 + K_i - iK_r} = \eta \exp(2i\delta)$$

$$K = K_r + iK_i$$

$$K_r = \tan\delta, \quad K_i = \tan^2\rho$$

δ - the phase shift, ρ - the inelasticity parameter.

Inelasticity!

$$\eta^2 = \frac{1 + K^2 - 2K_i}{1 + K^2 + 2K_i}$$

$$K^2 = K_r^2 + K_i^2$$

$$\delta = \frac{1}{2}\{\tan^{-1}[K_r/(1 - K_i)] + \tan^{-1}[K_r/(1 + K_i)]\}$$

If there are no inelastic channels: $(\rho = 0)$, $\delta = \delta_e$, $\eta = 1$ and $S = S_e = \exp(2i\delta_e)$.

Complex separable kernel

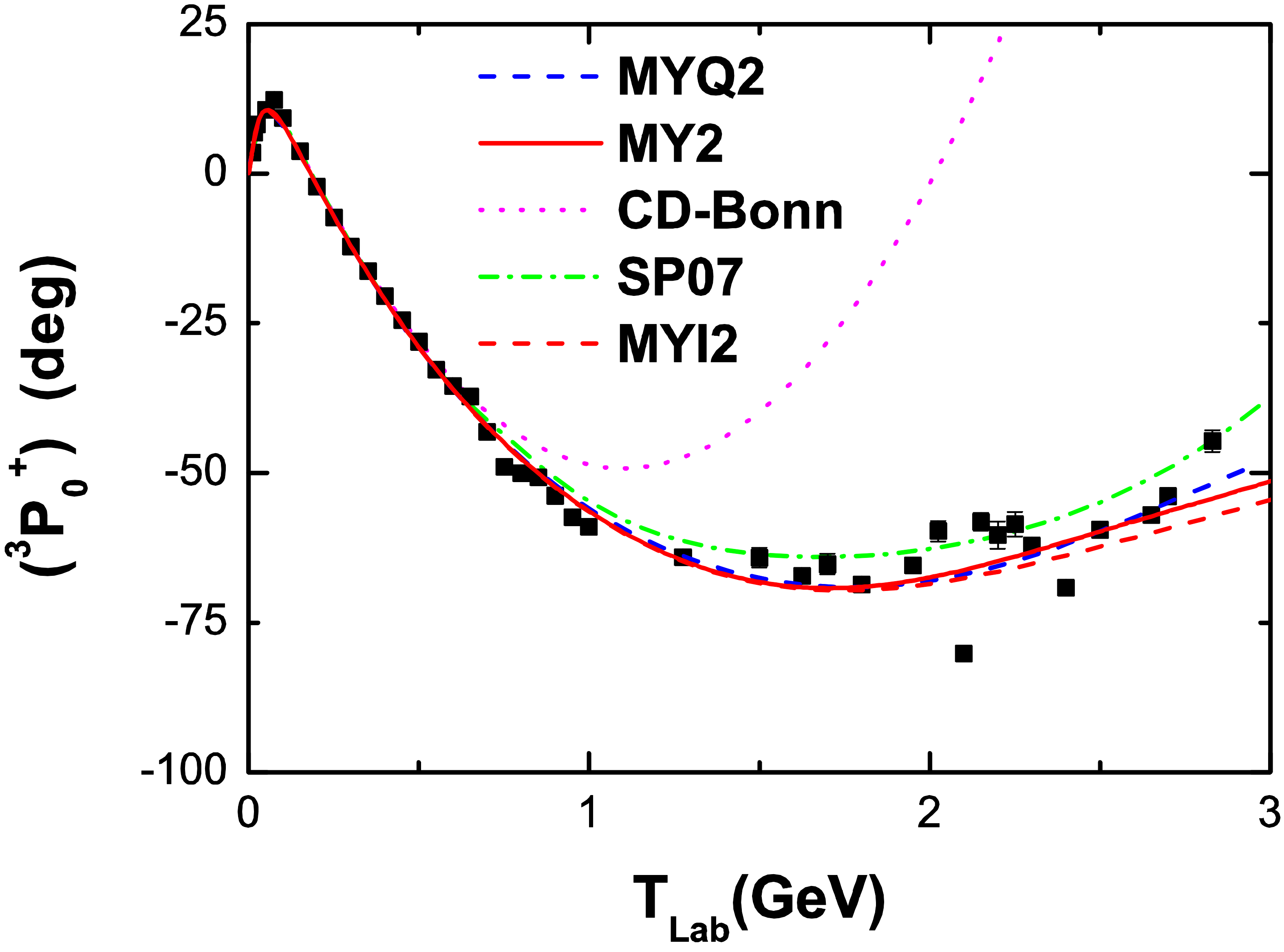
$$V_r \to V = V_r + iV_i.$$

$$V_{l'l}(p_0', |\mathbf{p}'|; p_0, |\mathbf{p}|; s) = \sum_{m,n=1}^{N} \left[\underline{\lambda_{mn}^r(s)} + i\lambda_{mn}^i(s) \right] \underline{g_i^{[l']}(p_0', |\mathbf{p}'|) g_j^{[l]}(p_0, |\mathbf{p}|)}$$

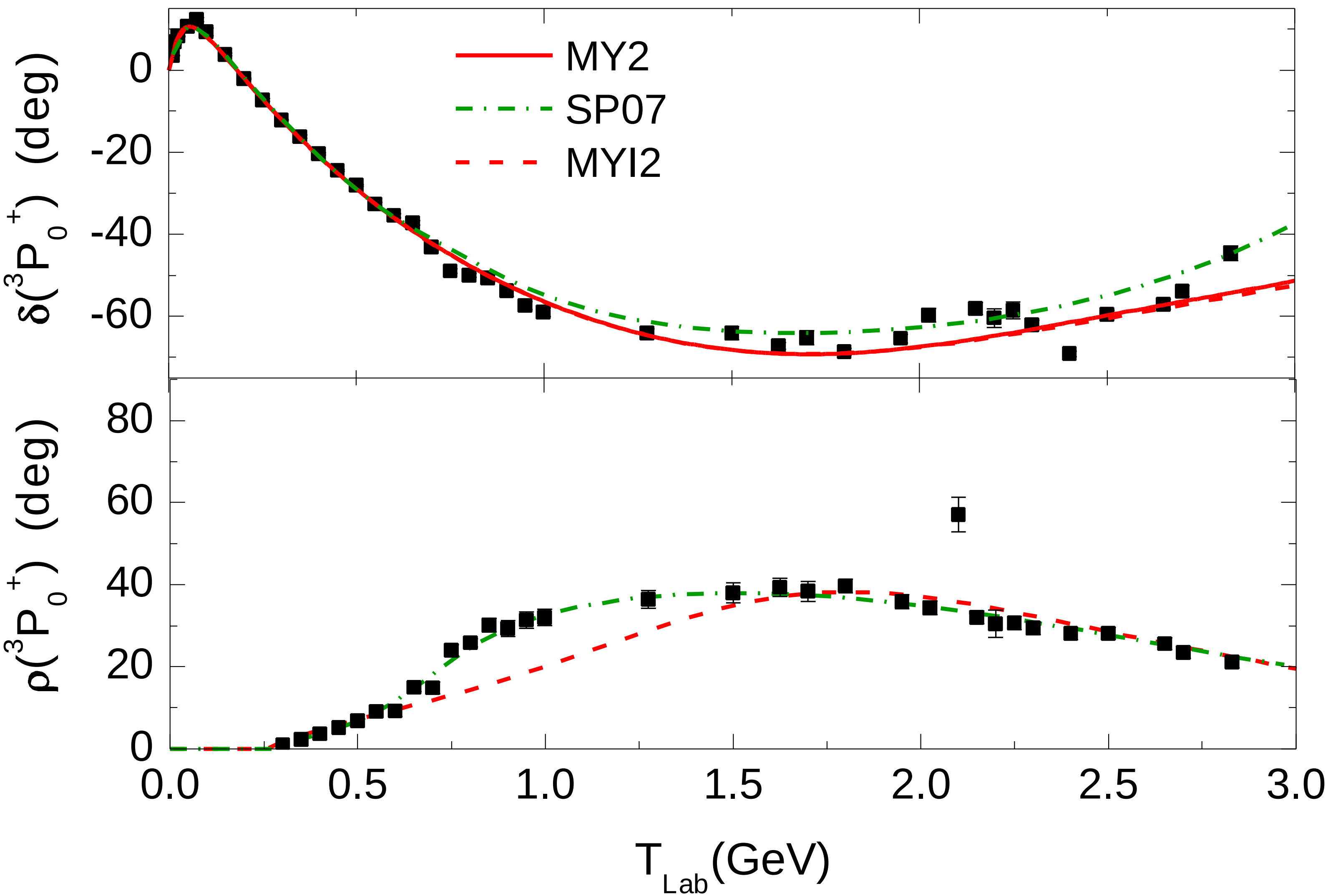
underlined part $\equiv$ MYN kernels,

$$\lambda_{mn}^i(s) = \theta(s - s_{th}) \left(1 - \frac{s_{th}}{s}\right) \bar{\lambda}_{mn}^i$$

s_{th} - the inelasticity threshold.



MYQ2
MY2
CD-Bonn
SP07
MYI2
25
0
-25
-50
-75
-100
0
1
2
3
$(^3P_0^+)$ (deg)
T_{Lab}(GeV)


MY2
SP07
MYI2
δ(³P₀⁺) (deg)
ρ(³P₀⁺) (deg)
T_Lab(GeV)
0
-20
-40
-60
80
60
40
20
0
0.0
0.5
1.0
1.5
2.0
2.5
3.0

Fifth Stage. Dibaryons

3P0

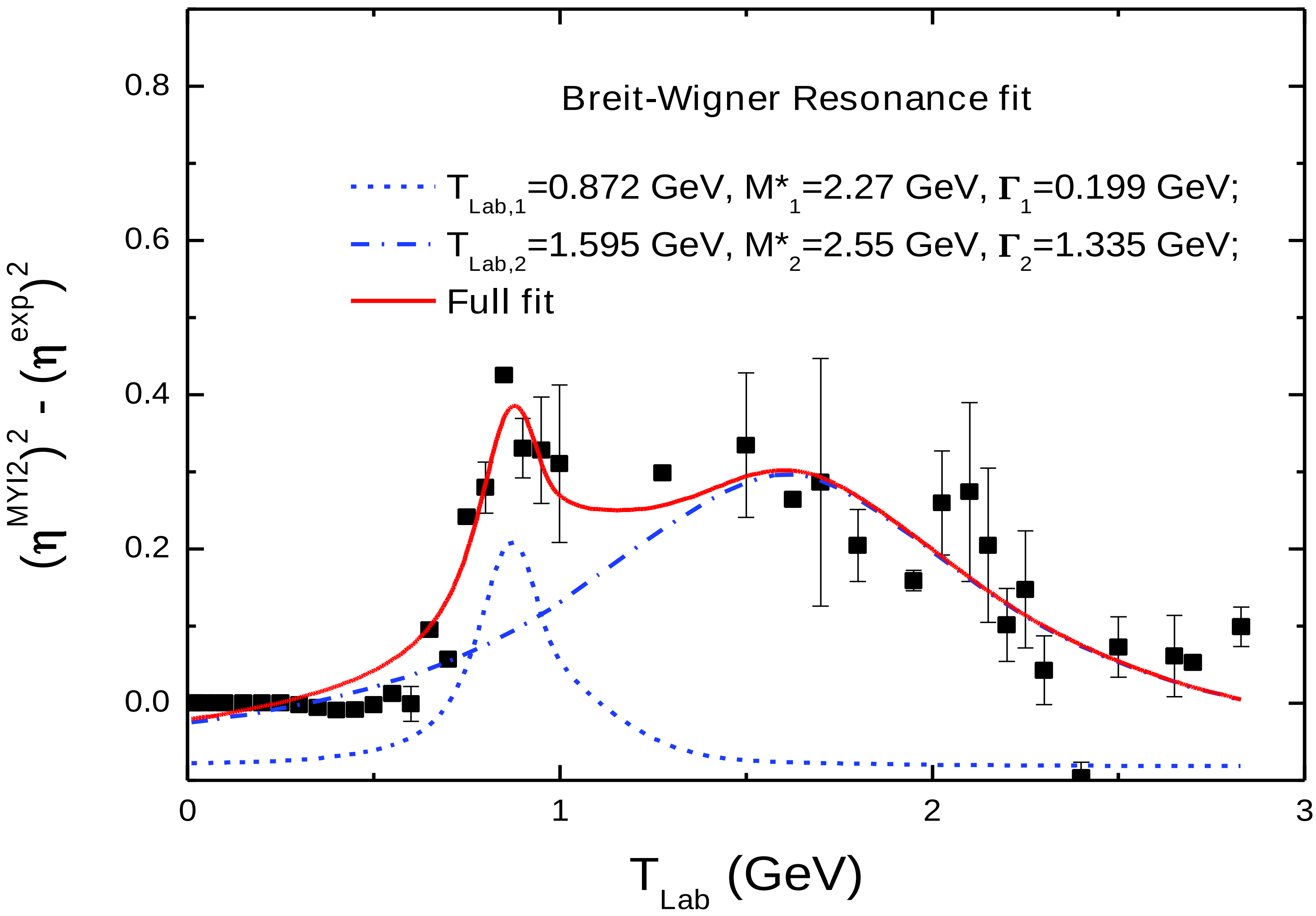
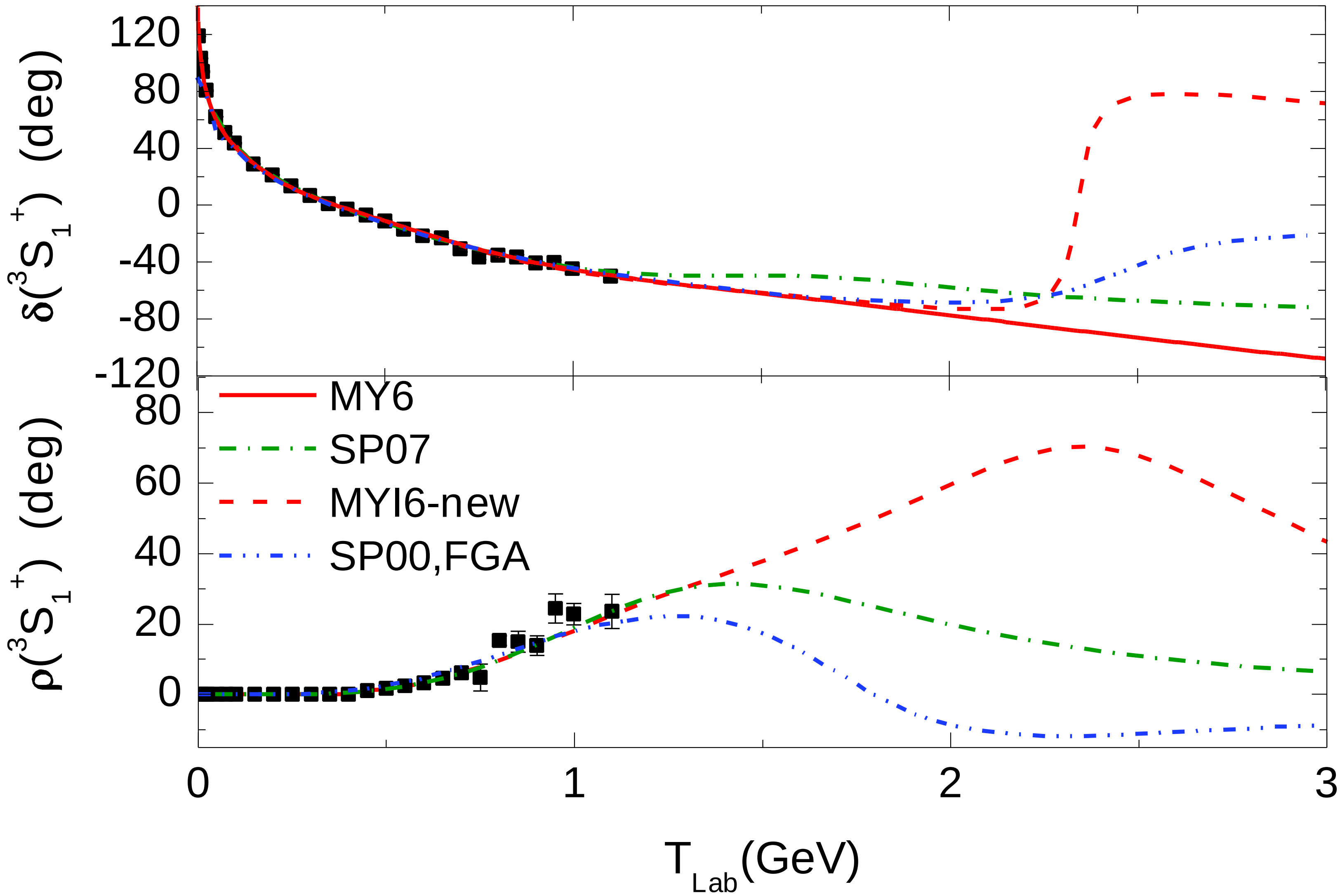

Breit-Wigner Resonance fit
T_Lab,1=0.872 GeV, M*_1=2.27 GeV, Γ_1=0.199 GeV;
T_Lab,2=1.595 GeV, M*_2=2.55 GeV, Γ_2=1.335 GeV;
Full fit
0.8
0.6
0.4
0.2
0.0
0
1
2
3
(η^MYI2)^2 - (η^exp)^2
T_Lab (GeV)

Таблица 1: Dibaryons in all considered partial-wave states

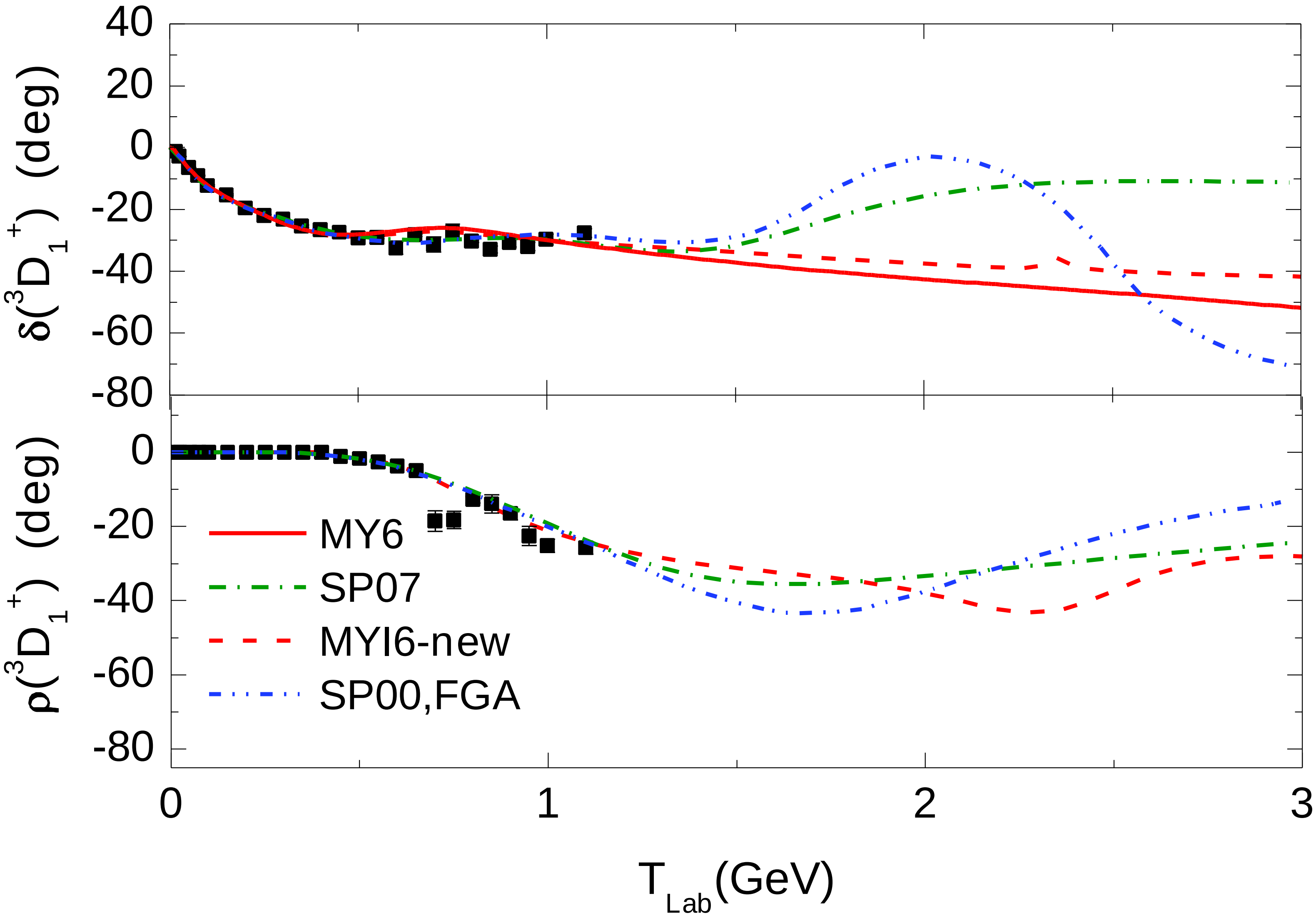
T_{Lab}(GeV)	M^*(GeV)	Γ^*(GeV)	State
0.856 (GeV)	2.26	0.200	$^3P_2$
0.872 (GeV)	2.27	0.199	$^3P_0$
0.874 (GeV)	2.27	0.206	$^3F_2$
1.161 (GeV)	2.39	0.245	$^1D_2$
1.505 (GeV)	2.52	0.639	$^3F_3$
1.555 (GeV)	2.54	0.238	$^1D_2$
1.595 (GeV)	2.55	1.335	$^3P_0$
1.760 (GeV)	2.61	2.264	$^3F_2$
2.100 (GeV)	2.73	0.118	$^3F_2$
2.700 (GeV)	2.93	0.374	$^3F_3$

Sixth Stage.

Deuteron.

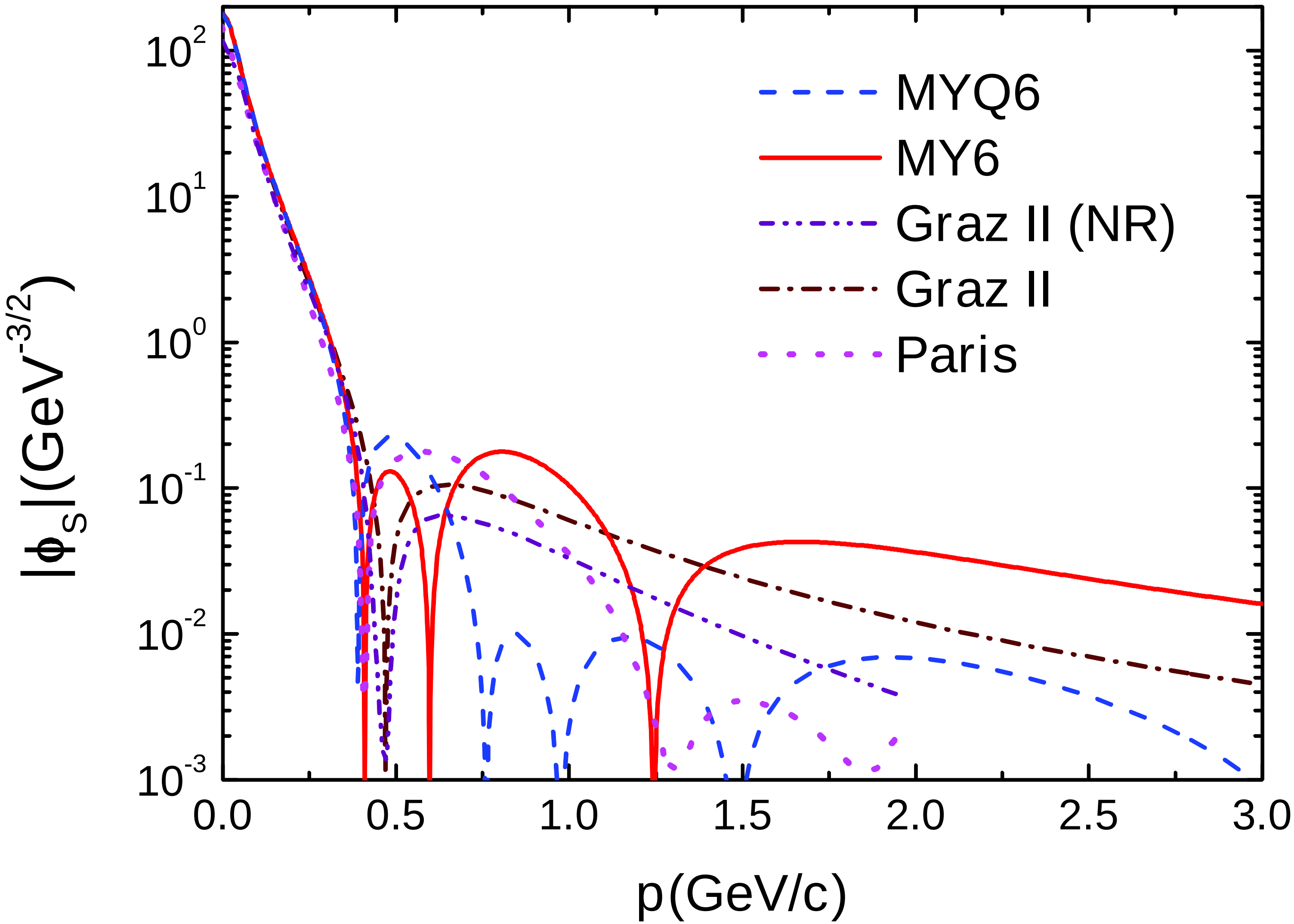


120
80
40
0
-40
-80
-120
δ(³S₁⁺) (deg)
80
60
40
20
0
ρ(³S₁⁺) (deg)
MY6
SP07
MYI6-new
SP00,FGA
0
1
2
3
T Lab (GeV)

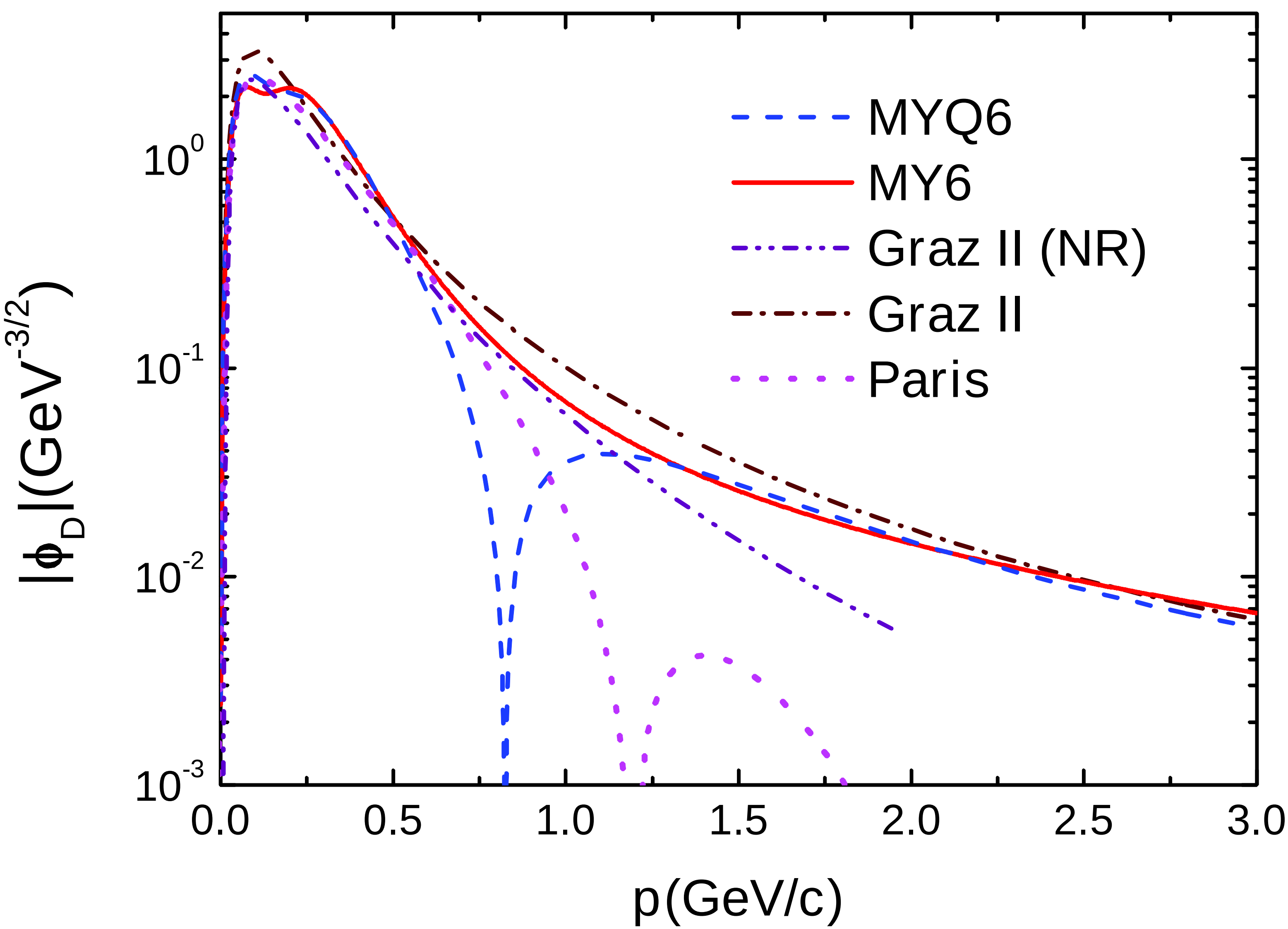


$\delta(^3D_1^+)$ (deg)
$\rho(^3D_1^+)$ (deg)
MY6
SP07
MYI6-new
SP00,FGA
T_{Lab}(GeV)

Deuteron "Wave function"



Deuteron "Wave function"

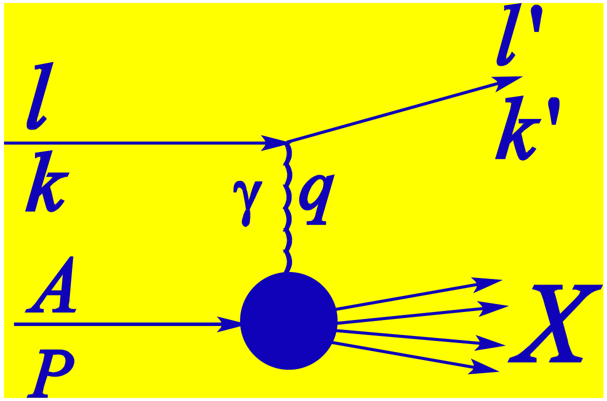


Seventh Stage.
Deep Inelastic Scattering

Evolution of Nucleon Structure in Nuclei

- Let us consider Deep Inelastic Scattering (DIS) leptons from nuclei:

$$l + A \rightarrow l' + X$$



- Cross section can be written as:

$$d\bar{\sigma} \propto \frac{\alpha^2}{q^4} L^{\mu\nu}(k,k') W_{\mu\nu}(P,p).$$

- Lepton tensor has form:

$$L^{\mu\nu}(k,k') = \frac{1}{2}\sum_{ss'} \bar{u}^{s'}(k')\gamma_\mu u^s(k)\bar{u}^s(k)\gamma_\mu u^{s'}(k').$$

- Hadron tensor we write as:

$$W_{\mu\nu}(P,p) = \frac{1}{2}\sum_n \langle P | j^+_\mu | n \rangle \langle n | j_\nu | P \rangle (2\pi)^4 \delta^4(P + q - p_n).$$

Structure functions in DIS

- Hadron tensor can be related to amplitude for forward Compton scattering *T*-matrix by means of the unitary relation:

$$W_{\mu\nu}(P,q)=\frac{1}{2\pi}\operatorname{Im}T_{\mu\nu}(P,q).$$

- Using Gauge invariance condition:

$$q_\mu W_{\mu\nu}(P,q)=0,$$

we can write ($\nu = q_0$ is the photon energy):

$$W_{\mu\nu}(P,q)=W_1(\nu,q^2)\left(-g_{\mu\nu}+\frac{q_\mu q_\nu}{q^2}\right)+\frac{W_2(\nu,q^2)}{M^2}\left(P_\mu-\frac{P\bullet q}{q^2}q_\mu\right)\left(P_\nu-\frac{P\bullet q}{q^2}q_\nu\right)$$

- In Bjorken limit

$$-q^2 = Q^2 \to \infty, \nu \to \infty,$$
$$MW_1(\nu, q^2) \to F_1(x),$$
$$\nu W_2(\nu, q^2) \to F_2(x),$$
$$x = -\frac{q^2}{2M\nu}$$

- we can write the hadron tensor in following form:

$$W_{\mu\nu}(P,q) = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) F_1(x) + \frac{1}{P \bullet q}\left(P_\mu - \frac{P \bullet q}{q^2} q_\mu\right)\left(P_\nu - \frac{P \bullet q}{q^2} q_\nu\right) F_2(x)$$

- Here $F_1(x), F_2(x)$ are scale invariant structure functions (SF).

Basic Approximations

- There are three basic groups of models for explanation of the EMC effect by taking into account:
  - Nucleon separation energy, relativistic fermi-motion, *NN*-correlations
  - Non-nucleon degrees of freedom;
  - The quark confinement radius changes.
- Basic Approximations:
  - The one boson approximation in the bound state equation;
  - Treatment of the DIS amplitude as an incoherent sum of amplitudes on individual constituents;
  - Representation of the hadron tensor of the bound nucleon in the same form as for free nucleon

Nuclear Compton Amplitude

- Nuclear Compton amplitude can be written as:

$$T_{\mu\nu}^{A}(P,q)=i\int d^4x e^{iqx}\left\langle A,P\left|T\left\{j_\mu(x)j_\nu(0)\right\}\right|A,P\right\rangle,$$

$$\left\langle A,P\left|T\left\{j_\mu(x)j_\nu(0)\right\}\right|A,P\right\rangle=\int dZdZ'\bar{\chi}_{\alpha,P}^{A}(Z)\bar{G}_{2(n+1)\mu\nu}(Z,x,Z')\bar{\chi}_{\alpha,P}^{A}(Z'),$$

where $Z=z_{1,}\ldots z_n;\ dZ=dz_{1,}\ldots dz_n$, relative time is:

$$\tau_i=\frac{1}{n}\sum_{j=1}^{n}z_{j0}-z_{i0.}$$

- Bethe-Salpeter vertex function is:

$$\bar{\chi}_{\alpha,P}^{A}(\xi)=\int dZdZ'S_{(n)}(\xi,Z)\bar{G}_{2n}(Z,Z')\bar{\chi}_{\alpha,P}^{A}(Z').$$

- The kernel of the integral BS equation is:

$$\bar{G}_{2n}(Z,Z') = S_{(n)}^{-1}(Z,Z') - G_{2n}^{-1}(Z,Z').$$

- The kernel is the *anzats* of the theory:
  - One bozon exchange kernel:

$$\bar{G}_{2n}(Z,Z') = \sum_{i,j}\sum_{m}\Gamma_m^{(1)}(z_i)\otimes\Gamma_m^{(1)}(z_j)\Delta(z_i,z_j);$$

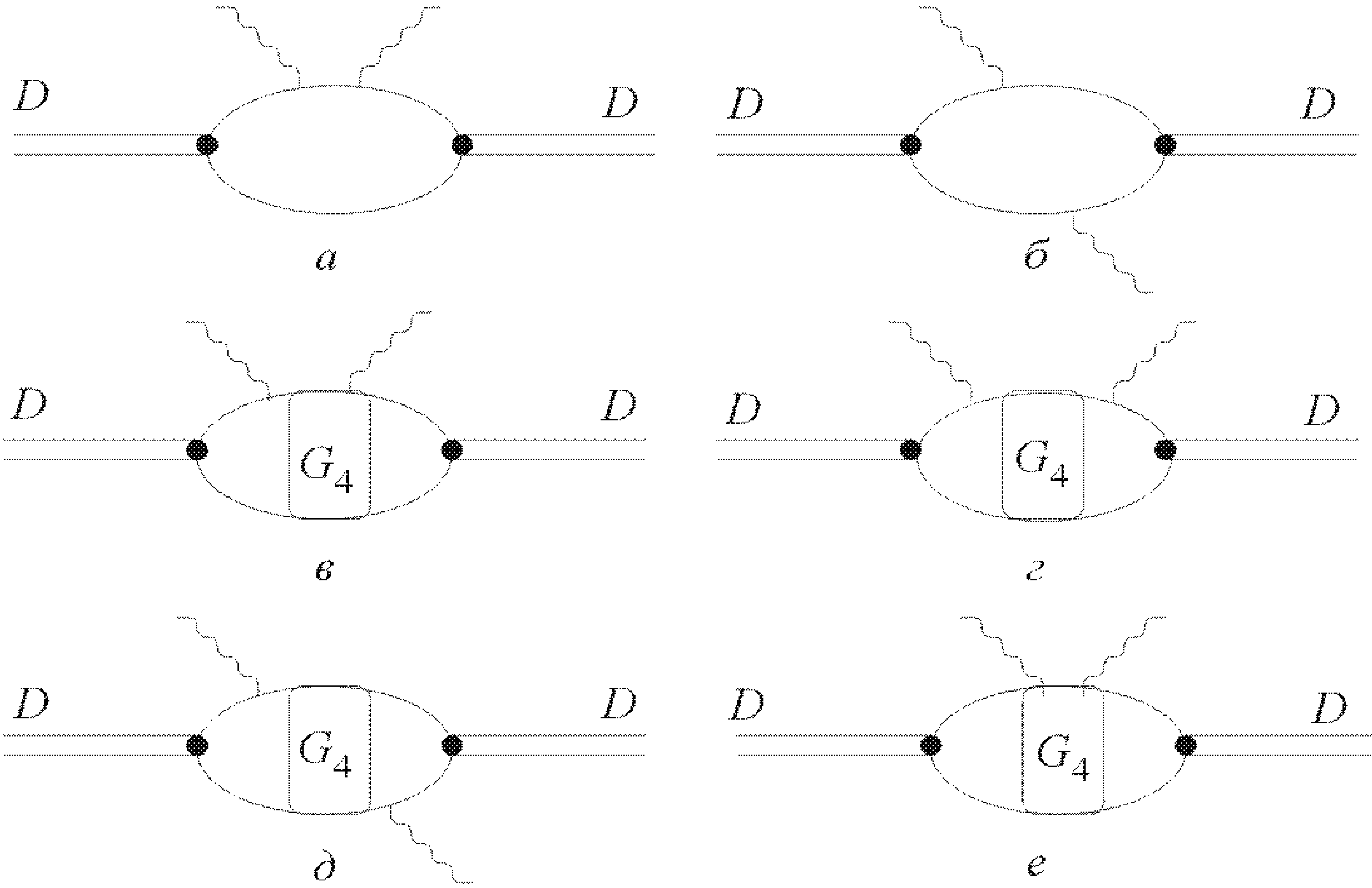
  - Separable form of the kernel:

$$\bar{G}_{2n}(Z,Z') = \sum_{i,j}\lambda_{ij}g_i(Z)g_j(Z').$$

  - BS vertex in the momentum space is:

$$S_{(n)}(P,K)\Gamma_\alpha^A(P,K) = \int dx_1^4\ldots dx_n^4 e^{-i\sum_{j=1}^{n}k_j x_j}\chi_{\alpha,P}(x_1\ldots x_n).$$

The BS Amplitude of Compton scattering for deuteron



Structure functions of deuteron

- Using Mandelstam technique and neglecting terms of order *1/Q²* and $(M_D - 2E)^2$ *we arrive to expression for structure function of deuteron:*

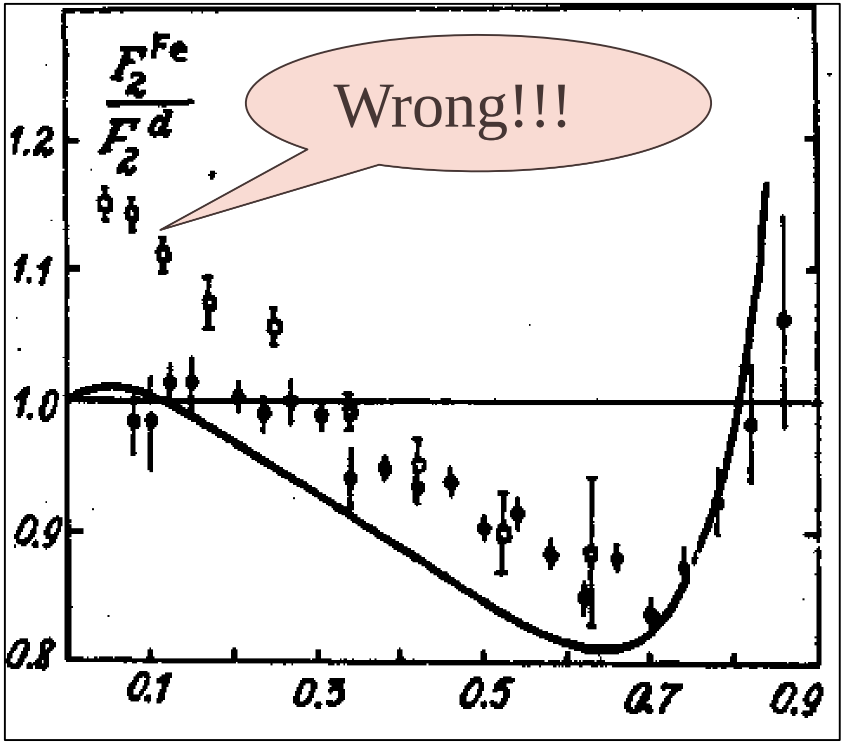
$$F_2^D(x_D) = \int \frac{d^3k}{(2\pi)^3} \frac{m^2}{4E^3 (M_D - 2E)^2} \left\{ F_2^N(x_N) \left(\frac{E - k_3}{M_D} + \frac{M_D - 2E}{2M_D} \right) \Phi^2(M_D, k) - \right.$$

$$- \frac{M_D - 2E}{M_D} x_N \frac{dF_2^N(x_N)}{dx_N} \Phi^2(M_D, k) +$$

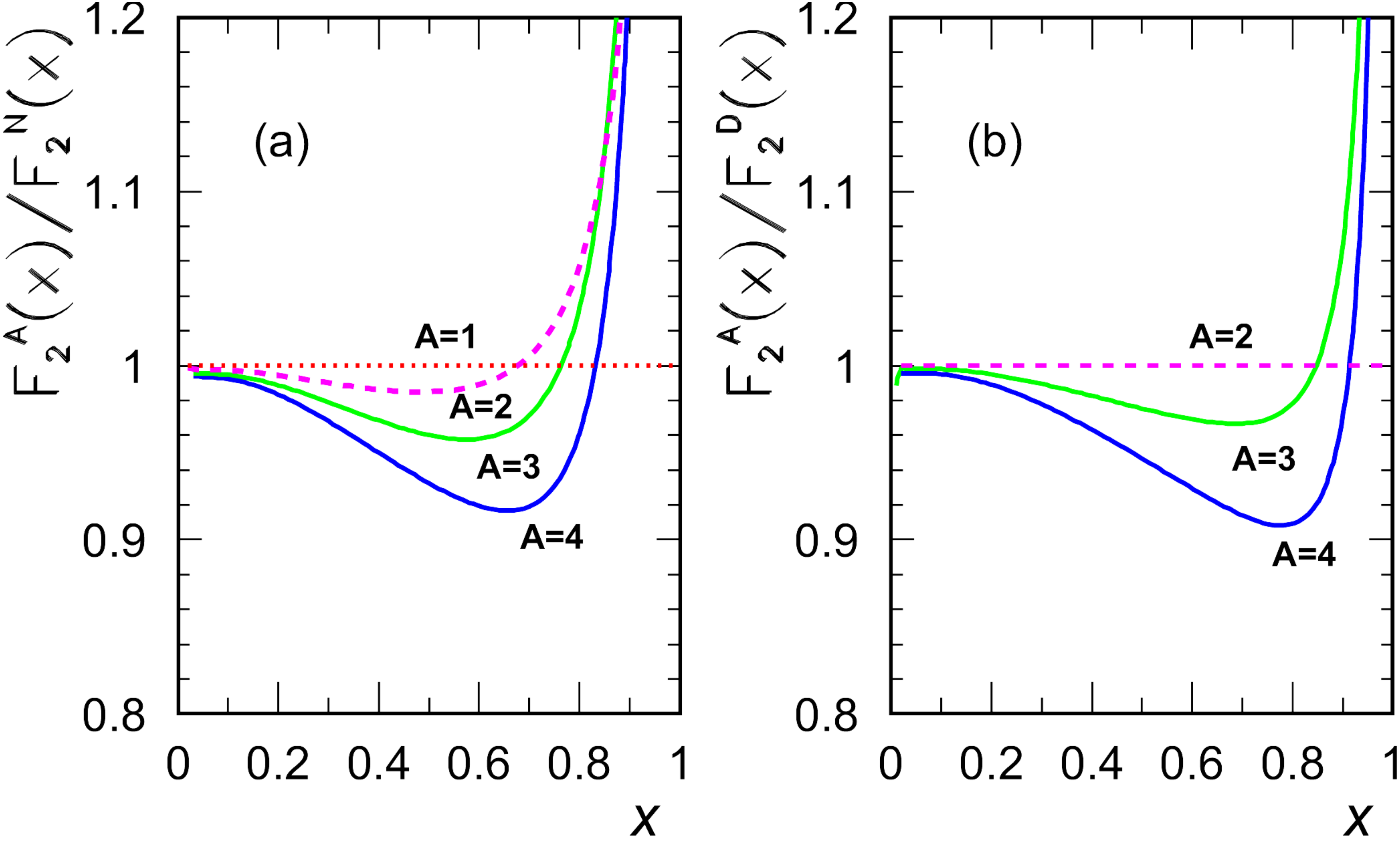
$$\left. + F_2^N(x_N) \frac{E - k_3}{M_D} (M_D - 2E) \frac{\partial}{\partial k_0} \Phi^2(M_D, k) \right\}_{k_0 = E - M_{D/2}} .$$

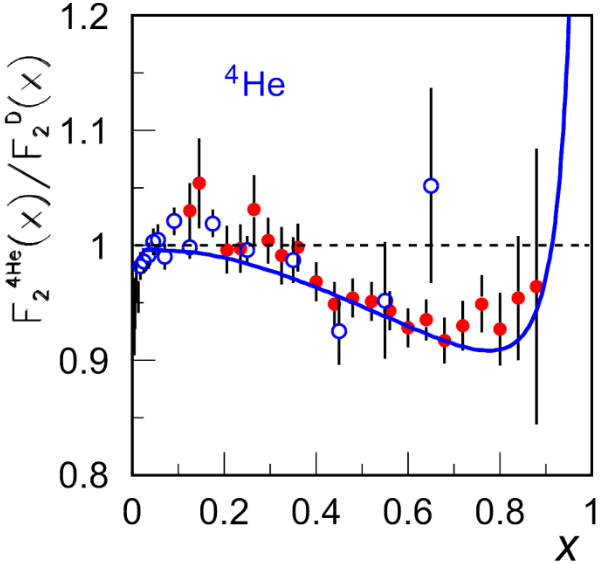
European Muon Collaboration

Data for the ratio of iron and deuterium structure functions are from the EMC [Aubert J.J. e.al. PL, 123B, 275(1983)] (□) and from the SLAC [Arnold R.G. at al., PRL, 52,727(1984)] (•) experiments. Theory [Akulinichev et al. Preprint INR P-0382(1984)]: the values V= -50Mev, P_F= 270 Mev/c have been used in numerical calculations

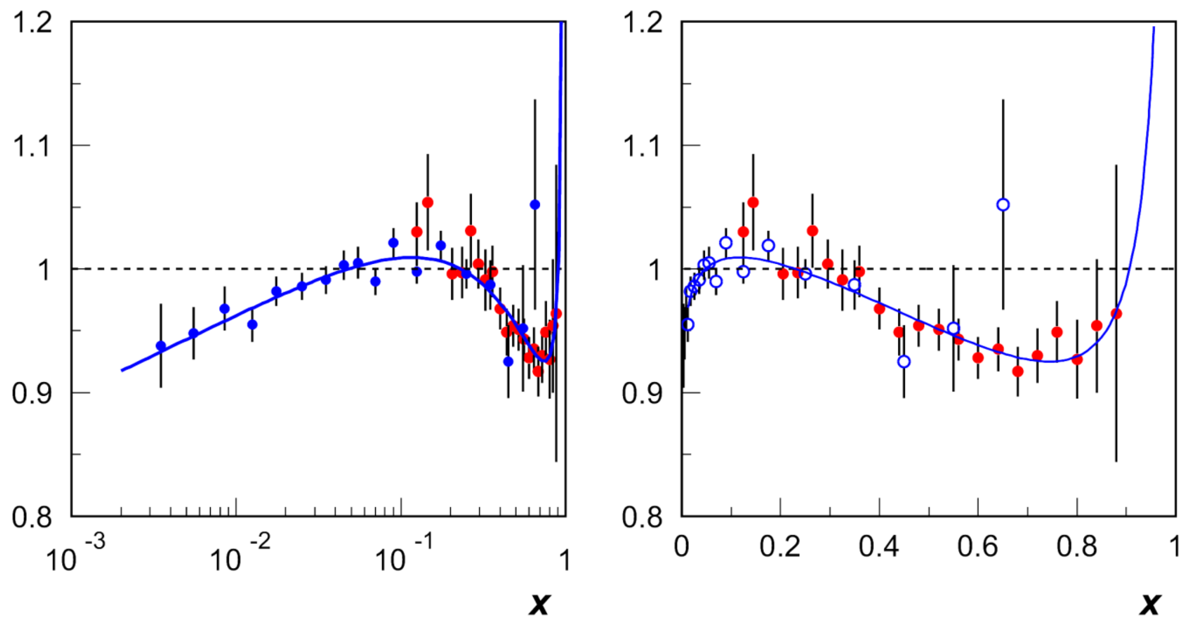


Ratio of SF's in BS approach





The ratio $F_2^{^4\mathrm{He}}(x)/F_2^{\mathrm{D}}(x)$. The data are from NMC (1995) (blue circles) and SLAC (1994) (red circles).



The ratio $F_2^{^4\mathrm{He}}(x)/F_2^{\mathrm{D}}(x)$ with corrected $F_2^{\mathrm{N}}(x)$.

Eighth Stage.
Structure Functions.

Introduction

Extraction of the *neutron* structure functions from deuteron data:

1. Theoretical models
2. Naive assumption: $2F_2^D = F_2^p + F_2^n$
3. Extrapolation of the EMC effect from heavy nuclei to $A = 2$

Most dangerous is option (3) : one can *approximately* esimate the amplitude of modification, but *never* the form of x dependence

We suggest an approach to extraction of $F_2^n(x)$ from the data collected in *deep inelastic scattering* experiments, which relies on relativistic theoretical description of $F_2^D(x)$ and well defined assumptions on the high x asymptotics for the ratio $F_2^n(x) / F_2^p(x)$.

It is based on the covariant Bethe-Salpeter formalism and allows to express the hadronic part of the *nuclear* deep inelastic amplitude W^A in terms of the off-mass-shell nucleon and antinucleon amplitudes.

Hadronic Tensor (bound states)

The space-time distribution of nucleons inside a nucleus:

$$\phi(x_1, \ldots x_n) = \langle 0|T\psi(x_1)\ldots\psi(x_n)|A\rangle, \tag{1}$$

The distribution depends on the relative time!

$$\boxed{\tau_{i,j} = x_{0j} - x_{0i}}$$

$$W^A_{\mu\nu}(P_A, q) = \int \frac{d^4p}{(2\pi)^4} \frac{W^N_{\mu\nu}(p,q) f^{N/A}(P,p)}{(p^2 - m^2)^2 \left((P-p)^2 - M^2_{A-1}\right)}. \tag{2}$$

$$W^A_{\mu\nu}(P_A, q) = \int \frac{d^3p}{(2\pi)^3} \frac{dp_0}{(2\pi)} \frac{W^N_{\mu\nu}(p,q)\, f^{N/A}(P,p)}{(p^2 - m^2)^2 \left((P-p)^2 - M^2_{A-1}\right)}. \tag{3}$$

The covariant Bethe-Salpeter formalism provides the *integral equation*

relating F_2^p, F_2^n and F_2^D.

It is solved iteratively by using input structure functions

F_2^p and F_2^D

Additionally, this allows *extrapolation* of F_2^D into a wide range of x and Q^2

Deuteron structure function

The integral equation:

$$F_2^D(x_D) = \int \frac{d^3k}{(2\pi)^3} \frac{m^2}{4E^3(M_D - 2E)^2} \left\{ F_2^N(x_N) \left(\frac{E - k_3}{M_D} + \frac{M_D - 2E}{2M_D} \right) \Phi^2(M_D, k) - \frac{M_D - 2E}{M_D} x_N \frac{dF_2^N(x_N)}{dx_N} \Phi^2(M_D, k) \right\}, \quad (4)$$

where $F_2^N(x_N) = F_2^p(x_N) + F_2^n(x_N)$.

Second term results from the Fermi motion along time axis

EMC-effect cannot be explained without changing the nucleon structure in a nucleus –

4D – radius of a bound nucleon changes

SMC fit for F_2^p and its modification

$$F_2^p(x,Q^2) = x^{\lambda_1}(1-x)^{\lambda_2} \sum_{n=1..5} C_n(1-x)^{n-1} \left(\frac{ln(\frac{Q^2}{\Lambda})}{ln(\frac{Q_0^2}{\Lambda})}\right)^{B(x)} \left(1+\frac{\sum_{n=1..4}\kappa_n x^n}{Q^2}\right) \quad (A.1)$$

where

$$B(x) = \rho_1 + \rho_2 x + \frac{\rho_3}{\rho_4 + x}.$$

The parametrization is restricted to the kinematic region $3.5 \cdot 10^{-5} < x < 0.85$.

i	λ_i	ρ_i	κ_i	C_i
1	−0.2499713175097	0.1141083888210	−1.451744104784	0.2289630236346
2	2.396344728724	−2.235597858569	8.474547402342	0.08498360257578
3	−	0.03115195484229	−34.37914208393	3.860797992943
4	−	0.02135222381130	45.88805973036	−7.414275585348
5	−	−	−	3.434223579597

Table 1: Values of the parameters for F_2^p given in Eq. (A.1).

Modification suggested by BMST ($x \to 1$) : $\lambda_2 \to \lambda_2 + (3 - \lambda_2)\, x^{15}$

Assumptions

- $\lim_{x\to 0} \frac{F_2^n(x)}{F_2^p(x)} = 1$

- $\lim_{x\to 1} F_2^p(x) = Const \cdot (1-x)^3$

  No elastic limit $\rightarrow$ $F_2^p(1) = 0.$

- $\lim_{x\to 1} \frac{F_2^n(x)}{F_2^p(x)} = C$

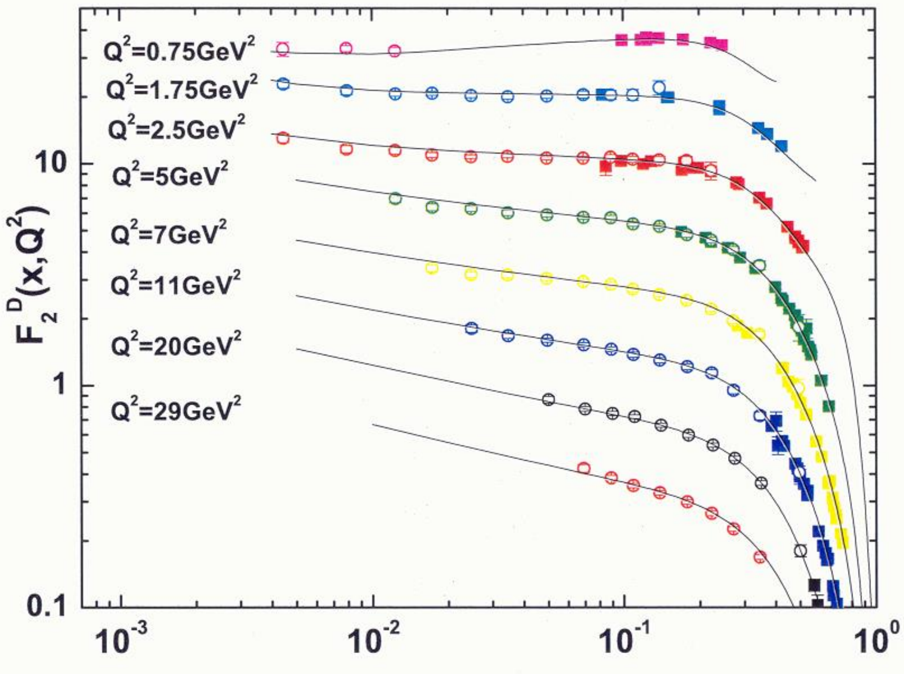
  Three different models for the u/d ratio at $x = 1$:

  1) SU(6) symmetry: $C = 2/3$,
  2) Elastic limit: $C = \left(\frac{\mu_n}{\mu_p}\right)^2 \simeq 0.47$,
  3) Scalar diquark dominance: $C = 1/4$.

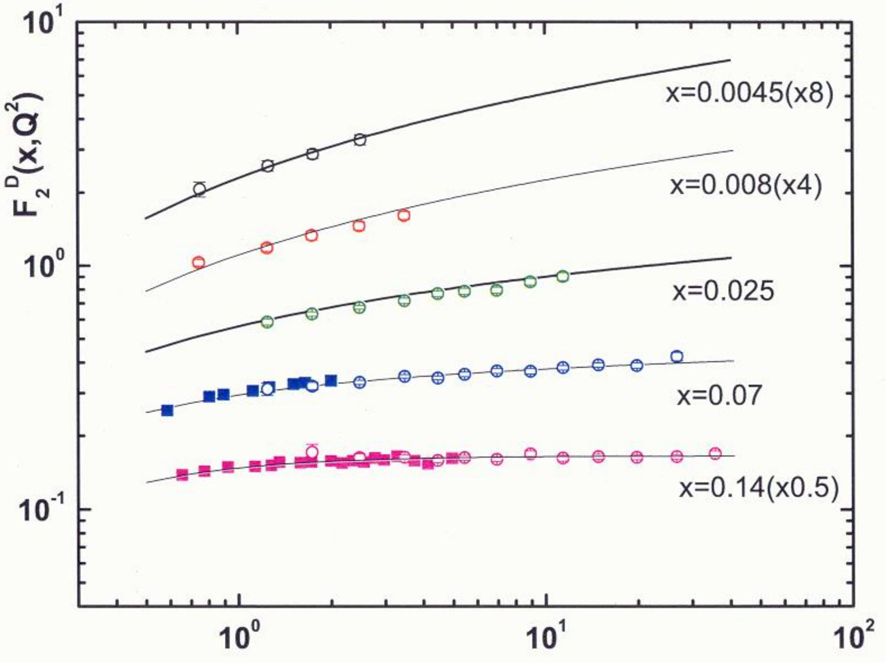
- $\lim_{x\to 1} \frac{d}{dx} \left(\frac{F_2^n(x)}{F_2^p(x)}\right) = 0$

Main assumption: $F_2^p(x)$ is known in the range $3.5\ 10^{-5} < x < 0.85$

Deuteron structure function compared with data from SLAC and NMC experiments (low *x* region)



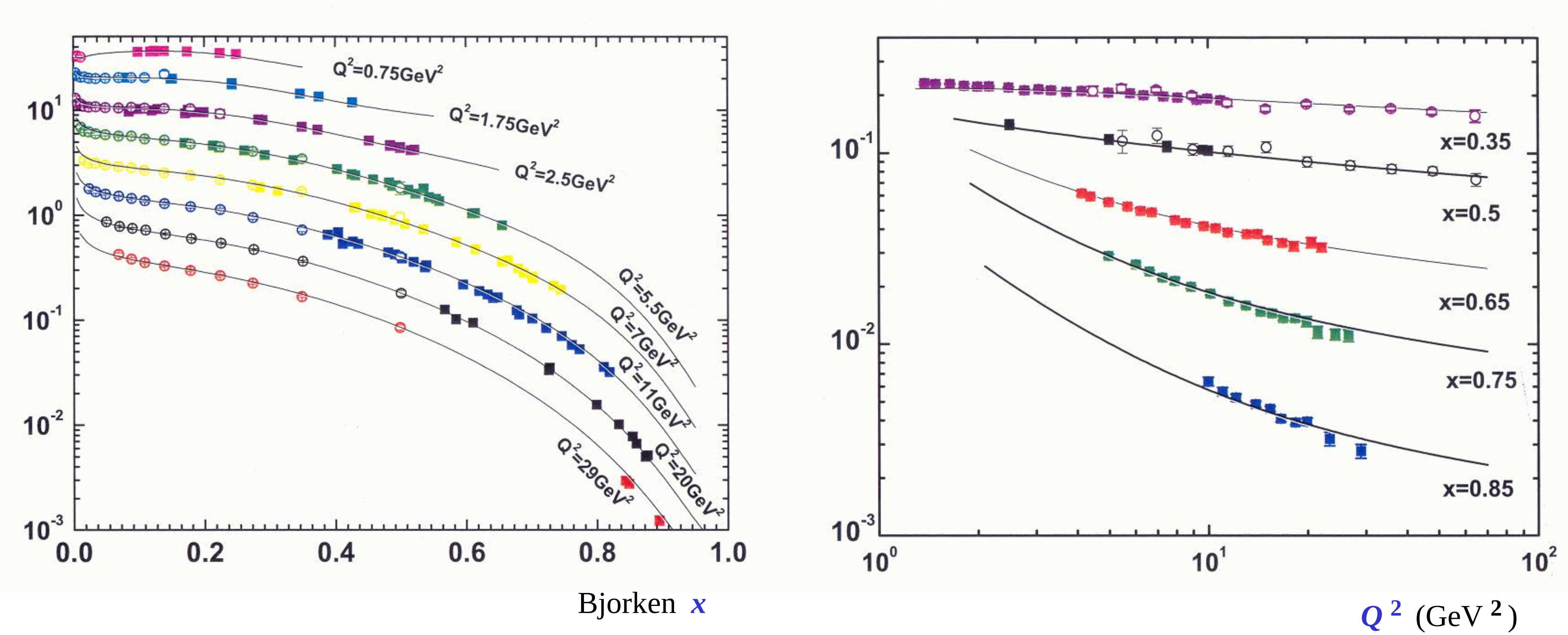
Bjorken *x*



Q^2 (GeV ^{2})

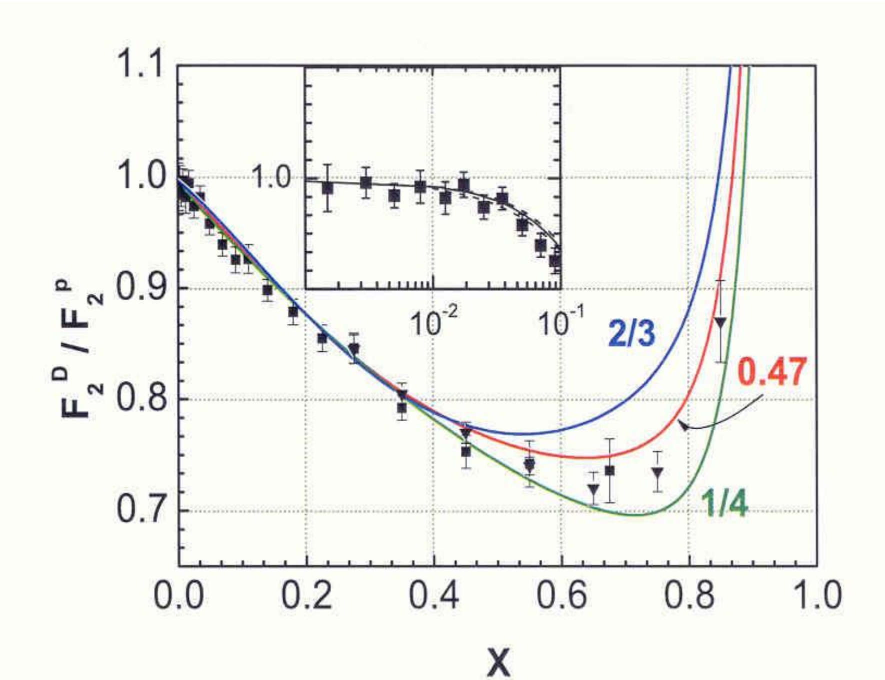
$F_2^D(x)$ approximated with Eq. (4) in the range $10^{-3} < x < 0.6$ with constraints listed in « Assumptions »

Deuteron structure function compared with data from SLAC and NMC experiments (high *x* region)

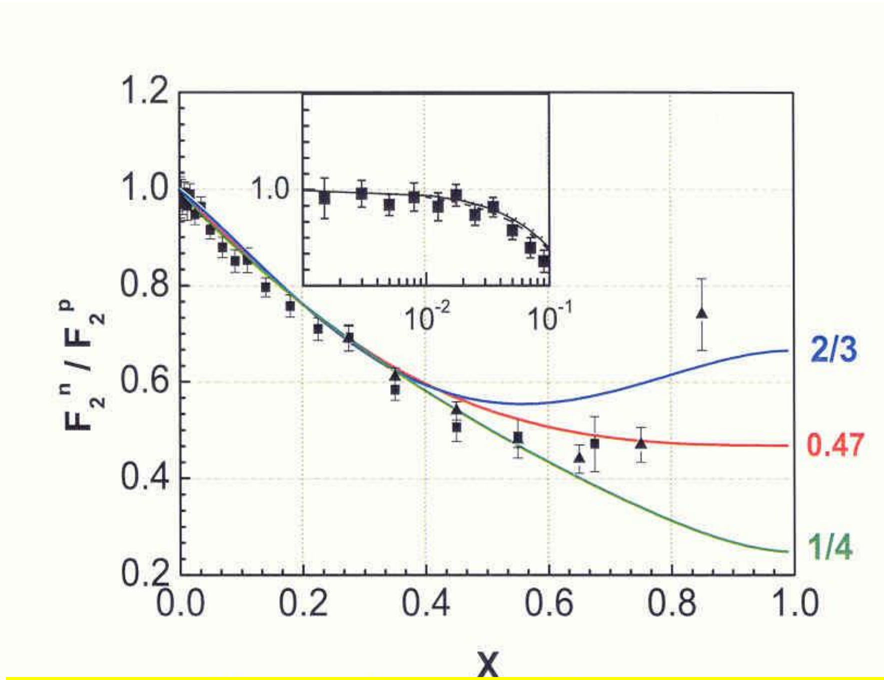


$F_2^D(x)$ **approximated with Eq. (4) in the range $10^{-3} < x < 0.6$ with constraints listed in « Assumptions »**

Results representing ratios of structure functions versus Bjorken *x*

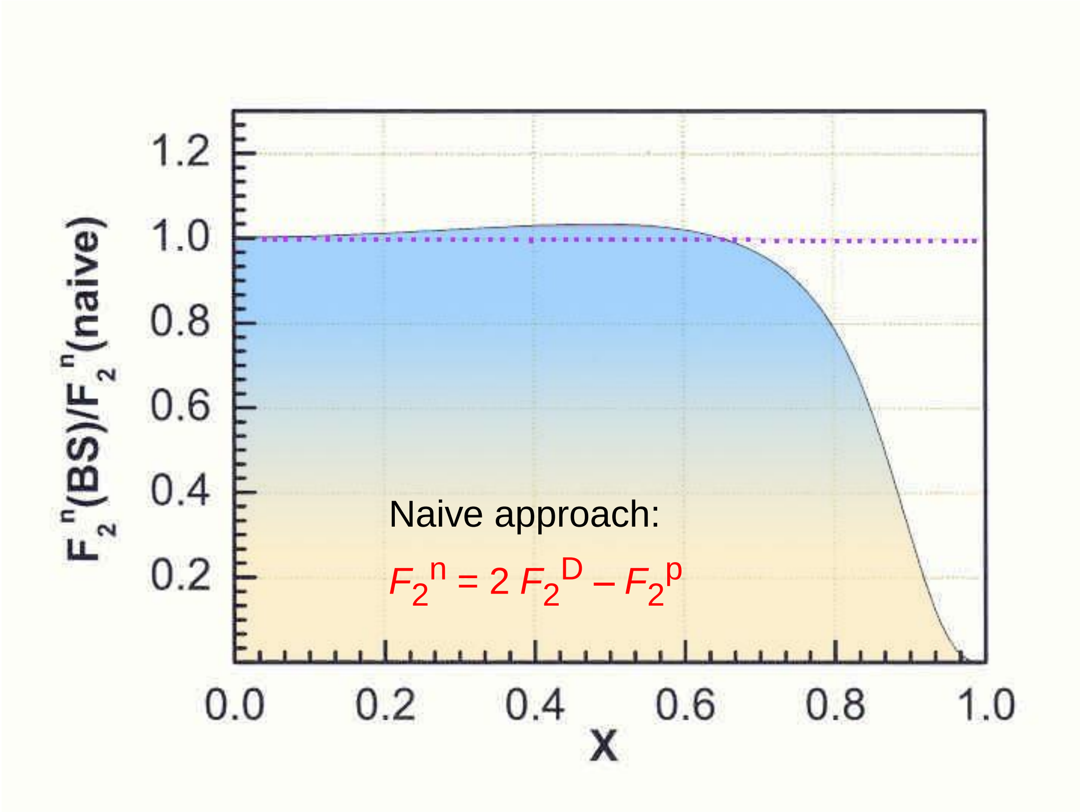


Squares and triangles — results of NMC and SLAC, respectively.



Squares and triangles — results of NMC and SLAC, respectively, obtained in naive approach.

Comparison with the naive approximation for the $F_2^n(x)$ evaluation



The ratio is virtually 1.0 below x = 0.7 due to cancellation of contributions from 3D Fermi motion and the Fermi motion along time axis

Ratio of the neutron and proton structure functions

$$R^{n/p}(x) = (1-x) + a_2 x^{\alpha_2} + \frac{\alpha_2 a_2 - 1}{1+c_1} x^{\beta_1}(1-x)(1+c_1 x^{\gamma_1}) \qquad \text{(A.2)}$$

	$a_2 = 2/3$	$a_2 = 0.47$	$a_2 = 1/4$
α_2	3.13971	2.2262	1.15416
β_1	2.2129	1.61188	0.88126
c_1	−1.01176	−1.00692	0.86217
γ_1	0.01901	0.08483	5.65744

Three values of a_2 correspond to three different assumptions on

$F_2^n(x)$ / $F_2^p(x)$ at $x = 1$

Conclusions

Theoretically justified and fully consistent procedure for extracting $F_2^n(x)$ in the kinematic range $10^{-3} < x < 1$ under three different assumptions on $F_2^n(x) / F_2^p(x)$ at $x = 1$ is proposed.

Increase in experimental accuracy in measurements of $F_2^p(x)$ and $F_2^D(x)$ in the range $0.6 < x < 0.8$ *by factor of two* will be sufficient for verification of models suggested for the evaluation of the ***d/u*** ratio at $x = 1$.

Procedure relies on a good approximation of $F_2^D(x)$ which is not sensitive to different high x limits of the neutron structure function.

This also means that $F_2^D(x)$ measured by already completed DIS experiments ($x < 0.9$) can be described without introducing nonbaryonic degrees of freedom. The interval which remains unmeasured can in principle accommodate dibaryon states or some other exotica.

Summary

- BS approach:
  - is full covariant descriptions of two body system;
  - allows to describe the properties of deuteron with separable potential!
  - can give very reasonable explanation structure functions, form factors and tensor polarization of deuteron in elastic ***eD***-scattering;
  - gives in one iteration approximation pair mesonic currents
  - gives foundations of light cone dynamics approaches;
  - gives good instrument to study polarization phenomena in elastic, inelastic, deep-inelastic lepton deuteron scattering;

Summary

- BS approach:
  - allows by the model-independently the SF of light nuclei to be calculated in terms of SF of nuclear fragments and three-dimensional momentum distribution;
  - gives the good explanation of the behavior for SF's ratios of the light nuclei to the SF of the free nucleon;
  - indicates that the modification of the nucleon structure of lightest nuclei is a manifestation of unsynchronous behavior of bound nucleon;
  - gives new understanding fundamental properties of nucleon, mainly its time deformation in relativistic bound system.