Separable kernels of the NN interaction within Bethe-Salpeter approach

Serge Bondarenko, V. Burov, E. Rogochaya
Joint Institute for Nuclear Research, Dubna, Russia

N. Hamamoto, A. Hosaka, Y. Manabe, H. Toki
Research Center for Nuclear Physics, Osaka University, Osaka, Japan

W-Y. Pauchy Hwang
National Taipei University, Taipei, Taiwan

K.Yu. Kazakov, D.V. Shulga
Far Eastern National University, Vladivostok, Russia

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The Bethe-Salpeter approach is a powerful tool to investigate few-body compounds such as the deuteron, unbound neutron-proton (np) system, three-nucleon system.
In our investigations the separable kernel of the nucleon-nucleon interaction is widely used. We consider separable kernel as a *nonlocal* covariant interaction representing complex nature of the space-time continuum.

Separable kernel for Schrodinger equation with separable potential

Yoshio Yamaguchi "Two-Nucleon Problem When the Potential Is Nonlocal but Separable. I" Phys.Rev.95, 1628 (1954)
Yoshio Yamaguchi, Yoriko Yamaguchi "Two-Nucleon Problem When the Potential Is Nonlocal but Separable. II" Phys.Rev.95, 1635 (1954)

Nonlocal: $\langle \mathbf{r}|V|\mathbf{r}'\rangle \neq \delta^{(3)}(\mathbf{r}-\mathbf{r}')$
in configuration space

$$\langle \mathbf{r}|V|\mathbf{r}'\rangle = -(\lambda/m_N)v^*(\mathbf{r})v^*(\mathbf{r}')$$

in momentum space

$$\langle \mathbf{p}|V|\mathbf{p}'\rangle = (\lambda/m_N)g^*(\mathbf{p})g^*(\mathbf{p}')$$

for S-state: $g(p) = 1/(p^2+\beta^2)$
for D-state: $g(p) = p^2/(p^2+\beta^2)^2$
for the deuteron and scattering problem.

Separable nucleon-nucleon potential was widely uses for the two- and three-nucleon calculations in nonrelativistic nuclear physics

Willibald Plessas et al. Graz, Graz-II potentials, separable representation of the popular Bonn and Paris potentials

K. Schwarz, Willibald Plessas, L. Mathelitsch "Deuteron Form-factors And E D Polarization Observables For The Paris And Graz-II Potentials" Nuovo Cim. A76 (1983) 322-329.
J. Haidenbauer, Willibald Plessas "Separable Representation Of The Paris Nucleon Nucleon Potential" Phys.Rev. C30 (1984) 1822-1839.
Johann Haidenbauer, Y. Koike, Willibald Plessas "Separable representation of the Bonn nucleon-nucleon potential" Phys.Rev. C33 (1986) 439-446.

$$g(p) = \sum_n p^{2m}/(p^2+\beta_n^2)^n,$$

m corresponds to angular momentum

Lippmann-Schwinger equation → Bethe-Salpeter equation

G. Rupp and J. A. Tjon "Relativistic contributions to the deuteron electromagnetic form factors" Phys. Rev. C41. 472 (1990)

$$\mathbf{p}^2 \to -p^2 = -p_0^2 + \mathbf{p}^2$$

$$g_p(p,P) = \frac{1}{-p^2 + \beta^2} \xrightarrow{\text{c.m.}} \frac{1}{-p_0^2 + \mathbf{p}^2 + \beta^2 + i\epsilon}$$

singularities: $p^0 = \pm\sqrt{\mathbf{p}^2 + \beta^2} \mp i\epsilon$

Graz-II covariant kernel, rank III

$$g_1^{(S)}(p_0,|\mathbf{p}|) = \frac{1-\gamma_1(p_0^2-\mathbf{p}^2)}{(p_0^2-\mathbf{p}^2-\beta_{11}^2)^2},$$
$$g_2^{(S)}(p_0,\mathbf{p}) = -\frac{(p_0^2-\mathbf{p}^2)}{(p_0^2-\mathbf{p}^2-\beta_{12}^2)^2},$$
$$g_3^{(D)}(p_0,|\mathbf{p}|) = \frac{(p_0^2-\mathbf{p}^2)(1-\gamma_2(p_0^2-\mathbf{p}^2))}{(p_0^2-\mathbf{p}^2-\beta_{21}^2)(p_0^2-\mathbf{p}^2-\beta_{22}^2)^2},$$
$$g_1^{(D)}(p_0,|\mathbf{p}|) = g_2^{(D)}(p_0,|\mathbf{p}|) = g_3^{(S)}(p_0,|\mathbf{p}|) \equiv 0. \quad (1)$$

Table : Deuteron and low-energy scattering properties

	$p_D(\%)$	ϵ_D (MeV)	Q_D (Fm^{-2})	μ_D $(e/2m)$	$\rho_{D/S}$	r_0 (Fm)	a (Fm)
Covariant Graz-II	4	2.225	0.2484	0.8279	0.02408	1.7861	5.4188
Experimental data		2.2246	0.286	0.8574	0.0263	1.759	5.424

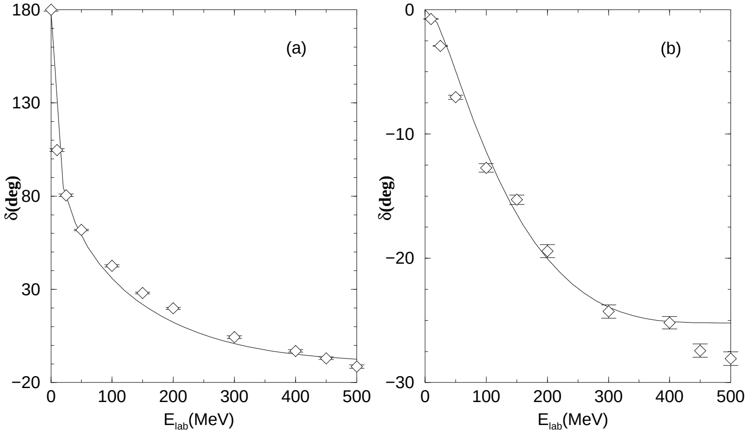


Figure : Phase shifts of the $^3S_1$ and $^3D_1$ partial states

Elastic eD scattering cross section

$$\frac{d\sigma}{d\Omega'_{\rm e}} = \left(\frac{d\sigma}{d\Omega'_{\rm e}}\right)_{\rm Mott}\left[A(q^2) + B(q^2)\tan^2\frac{\theta_{\rm e}}{2}\right],$$

$$\left(\frac{d\sigma}{d\Omega'_{\rm e}}\right)_{\rm Mott} = \frac{\alpha^2\cos^2\theta_{\rm e}/2}{4E_{\rm e}^2(1+2E_{\rm e}/M_d\sin^4\theta_{\rm e}/2)},$$

where $\theta_{\rm e}$ is the electron scattering angle, M_d is the deuteron mass, E_e is the incident electron energy.

Deuteron structure functions $A(q^2)$ and $B(q^2)$

$$A(q^2) = F_{\rm C}^2(q^2) + \frac{8}{9}\eta^2 F_{\rm Q}^2(q^2) + \frac{2}{3}\eta F_{\rm M}^2(q^2)$$

$$B(q^2) = \frac{4}{3}\eta(1+\eta)F_{\rm M}^2(q^2)$$

where $\eta = -q^2/4M_d^2 = Q^2/4M_d^2$

Relativistic impulse approximation (RIA)

Deuteron current matrix element

$$\langle D'\mathcal{M}'|J_\mu^{RIA}|D\mathcal{M}\rangle =$$

$$ie\int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}\Big\{\bar{\chi}^{1\mathcal{M}'}(P',k')\Gamma_\mu^{(\mathrm{S})}(q)\chi^{1\mathcal{M}}(P,k)(P\cdot\gamma/2-k\cdot\gamma+m)\Big\}$$

$\chi^{1\mathcal{M}}(P,k)$ - the BS amplitude of the deuteron, $P'=P+q$ and $k'=k+q/2$.

The vertex of γNN interaction

$$\Gamma_\mu^{(\mathrm{S})}(q)=\gamma_\mu F_1^{(\mathrm{S})}(q^2)-\frac{\gamma_\mu q\cdot\gamma-q\cdot\gamma\gamma_\mu}{4m}F_2^{(\mathrm{S})}(q^2)$$

is chosen to be the form factor on mass shell.

The isoscalar form factors of the nucleon

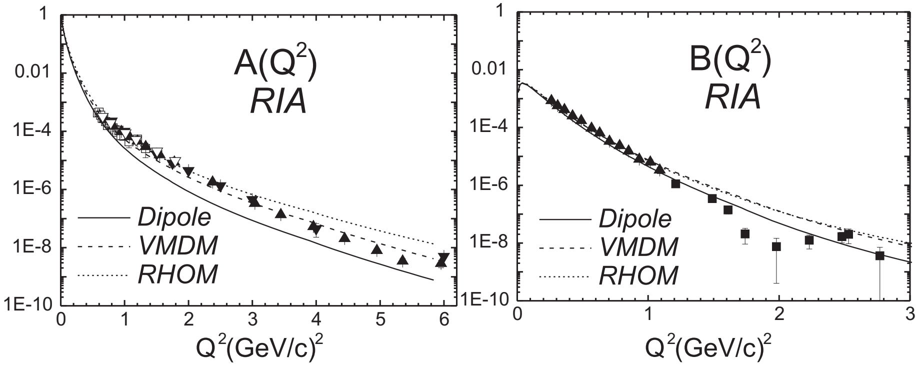
$$F_{1,2}^{(\mathrm{S})}(q^2)=(F_{1,2}^{(\mathrm{p})}(q^2)+F_{1,2}^{(\mathrm{n})}(q^2))/2$$

with normalization condition

$$F_1^{(\mathrm{S})}(0)=1/2,\quad F_2^{(\mathrm{S})}(0)=(\varkappa_p+\varkappa_n)/2$$

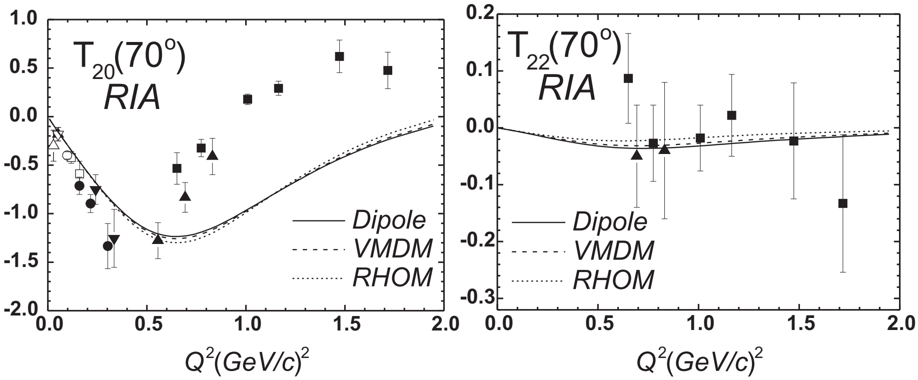
with $\varkappa_p=\mu_p-1$ and $\varkappa_n=\mu_n$ being anomalous parts of the proton μ_p and neutron μ_n magnetic moments, respectively.

Structure functions $A(q^2)$ and $B(q^2)$



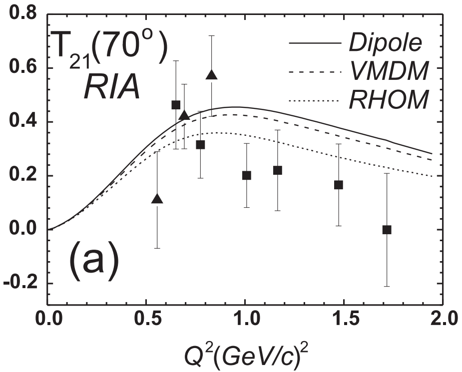
Long and short dashes represent calculations with the VMDM and RHOM nucleon form factors, respectively. The solid curve corresponds to the dipole fit.

Tensor polarization components $T_{20}(q^2)$ and $T_{22}(q^2)$ calculated at $\theta_e = 70°$.



Long and short dashes represent calculations with the VMDM and RHOM nucleon form factors, respectively. The solid curve corresponds to the dipole fit.

Tensor polarization component $T_{21}(q^2)$ calculated at $\theta_e = 70^\circ$



Long and short dashes represent calculations with the VMDM and RHOM nucleon form factors, respectively. The solid curve corresponds to the dipole fit.

Separable ansatz for kernel

$$V_{a'a}(p_0', |\mathbf{p}'|; p_0, |\mathbf{p}|; s) = \sum_{m,n=1}^{N} \lambda_{mn}^{r[a'a]}(s) g_m^{[a']}(p_0', |\mathbf{p}'|) g_n^{[a]}(p_0, |\mathbf{p}|)$$

Solution for the T matrix

$$T_{a'a}(p_0', |\mathbf{p}'|; p_0, |\mathbf{p}|; s) = \sum_{i,j=1}^{N} \tau_{ij}(s)\, g_i^{[a']}(p_0', |\mathbf{p}'|)\, g_j^{[a]}(p_0, |\mathbf{p}|)$$

where

$$\left[\tau_{ij}(s)\right]^{-1} = \left[\lambda_{mn}^{r[a'a]}(s)\right]^{-1} + h_{ij}(s),$$

$$h_{ij}(s) = -\frac{i}{4\pi^3} \sum_a \int dk_0 \int \mathbf{k}^2 d|\mathbf{k}| \frac{g_i^{[a]}(k_0, |\mathbf{k}|) g_j^{[a]}(k_0, |\mathbf{k}|)}{(\sqrt{s}/2 - E_{\mathbf{k}} + i\epsilon)^2 - k_0^2},$$

$g_j^{[a]}$ - the model functions, $\lambda_{ij}^{[a'a]}(s)$ - a matrix of model parameters.

Analysis

$V(p,p') \to T_{off-mass-shell}(p,p') \to T_{on-mass-shell}(\bar{p},\bar{p}) \to (\delta, a_0, E_d ...)$

δ - the phase shifts, a_0, r_0 - the low-energy parameters (the scattering length, the effective range), E_d - the deuteron binding energy.

Procedure $(J = 0, 1)$

calculate the kernel parameters – $\lambda_{ij}(s)$-matrix and parameter of the g-functions – to minimize the function χ^2:

$\chi^2 =$

$\sum_{i=1}^{n} (\delta^{\exp}(s_i) - \delta(s_i))^2/(\Delta\delta^{\exp}(s_i))^2$ – for all partial-wave states

$+(a_0^{\exp} - a_0)^2/(\Delta a_0^{\exp})^2$ – for the ${}^1S_0^+$ and ${}^3S_1^+$ partial-wave states

$+(E_d^{\exp} - E_d)^2/(\Delta E_d^{\exp})^2$ – for the ${}^3S_1^+$-${}^3D_1^+$ partial-wave states

$\{+...\}$

Covariant generalization of the *Yamaguchi*-functions

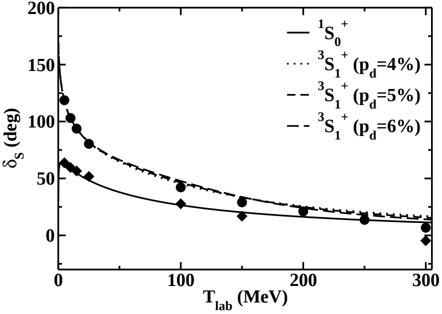
functions for $g^{[L]}(p_0,p)$:

$$g^{[S]}(p_0,p) = \frac{1}{p_0^2 - p^2 - \beta_0^2 + i0}, \quad (2)$$

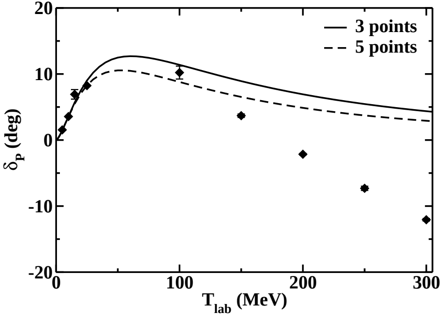
$$g^{[P]}(p_0,p) = \frac{\sqrt{|-p_0^2 + p^2|}}{(p_0^2 - p^2 - \beta_1^2 + i0)^2}, \quad (3)$$

$$g^{[D]}(p_0,p) = \frac{C(p_0^2 - p^2)}{(p_0^2 - p^2 - \beta_2^2 + i0)^2}. \quad (4)$$

Results for the phase-shifts

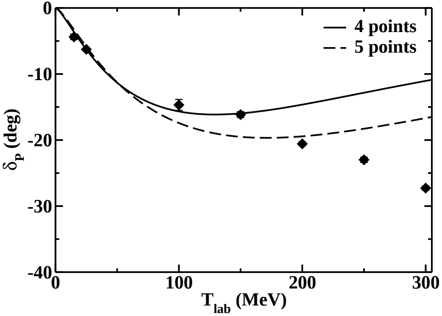


$^1S_0^+$ and $^3S_1^+$ channels phase shifts

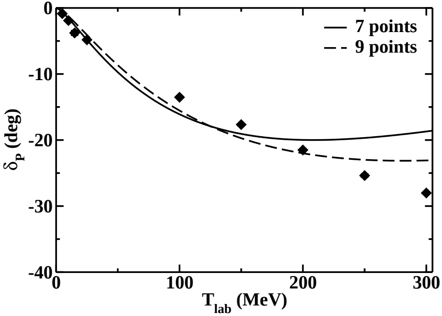


$^3P_0^+$ channel phase shifts

Results for the phase-shifts



$^1P_1^+$ channel phase shifts



$^3P_1^+$ channel phase shifts

Cross-section in CMS

$$\frac{d\sigma}{d\Omega_{\hat{\mathbf{p}}}} = \frac{\alpha}{16\pi s}\frac{|\hat{\mathbf{p}}|}{\omega}\overline{|\mathcal{M}_{fi}|^2}$$

with $\alpha = e^2/4\pi$ - the fine structure constant

$$\begin{aligned}
\mathcal{M}_{fi,IA}^{PWA} &= -\varepsilon(\lambda)\,\mathrm{Sp}\Big\{\bar{\chi}^{(0)}_{Sm_S}(p_f,P_f)\,\left[\Gamma^{(1)}(q)+(-1)^{S+1}\Gamma^{(2)}(q)\right]\Lambda(\mathcal{L})\\
&\quad\times S\left(\mathcal{L}^{-1}(\tfrac{1}{2}(K_i-q)+p_f)\right)\Gamma_{m_d}\left(\mathcal{L}^{-1}(p_f-\tfrac{1}{2}q);K_{i(0)}\right)\Lambda(\mathcal{L})^{-1}\Big\},\\
\mathcal{M}_{fi,IA}^{FSI} &= -\varepsilon(\lambda)\int d^4p\ \mathrm{Sp}\Big\{\bar{\chi}^{(t)}_{Sm_S}(p;p_f,P_f)\,\left[\Gamma^{(1)}(q)+(-1)^{S+1}\Gamma^{(2)}(q)\right]\Lambda(\mathcal{L})\\
&\quad\times S\left(\mathcal{L}^{-1}(\tfrac{1}{2}(K_i-q)+p)\right)\Gamma_{m_d}\left(\mathcal{L}^{-1}(p-\tfrac{1}{2}q);K_{i(0)}\right)\Lambda(\mathcal{L})^{-1}\Big\}
\end{aligned}$$

$$\bar{\chi}^{(t)}_{Sm_s}(p;p_f,P_f) = \frac{i}{4\pi^3} S_{++}(p_0,|\mathbf{p}|;s) \tag{5}$$

$$\times\Big[\quad \delta_{S0}\,\delta_{m_S0}\,\frac{1}{\sqrt{4\pi}}\,\bar{\mathcal{Y}}_{^1S_0^+}(\mathbf{p})\,t^*_{^1S_0^+,{}^1S_0^+}(0,|\hat{\mathbf{p}}|;p_0,|\mathbf{p}|;s)$$

$$+ \quad \delta_{S0}\,\delta_{m_S0}\,\sum_M Y_{1M}(\theta_{\hat{\mathbf{p}}},\varphi_{\hat{\mathbf{p}}})\,\bar{\mathcal{Y}}_{^1P_1^+M}(\mathbf{p})\,t^*_{^1P_1^+,{}^1P_1^+}(0,|\hat{\mathbf{p}}|;p_0,|\mathbf{p}|;s)$$

$$+ \quad \delta_{S1}\,(-)^{1-m_S}\,\frac{1}{\sqrt{3}}\,Y_{1-m_S}(\theta_{\hat{\mathbf{p}}},\varphi_{\hat{\mathbf{p}}})\,\bar{\mathcal{Y}}_{^3P_0^+}(\mathbf{p})\,t^*_{^3P_0^+,{}^3P_0^+}(0,|\hat{\mathbf{p}}|;p_0,|\mathbf{p}|;s)$$

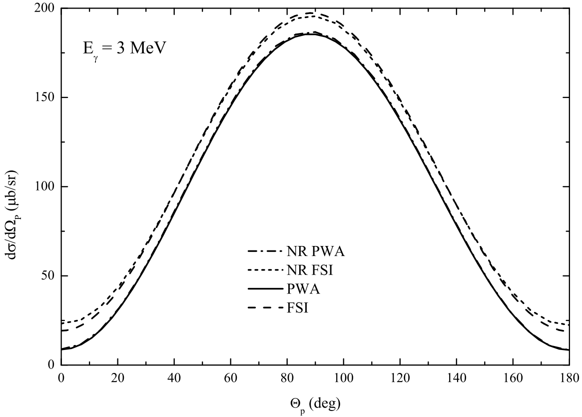
$$+ \quad \delta_{S1}\,\sum_{Mm} C^{1M}_{1m1m_S}\,Y_{1m}(\theta_{\hat{\mathbf{p}}},\varphi_{\hat{\mathbf{p}}})\,\bar{\mathcal{Y}}_{^3P_1^+M}(\mathbf{p})\,t^*_{^3P_1^+,{}^3P_1^+}(0,|\hat{\mathbf{p}}|;p_0,|\mathbf{p}|;s)$$

$$+ \quad \delta_{S1}\,\frac{1}{\sqrt{4\pi}}\,\bar{\mathcal{Y}}_{^3S_1^+m_S}(\mathbf{p})\,t^*_{^3S_1^+,{}^3S_1^+}(0,|\hat{\mathbf{p}}|;p_0,|\mathbf{p}|;s)$$

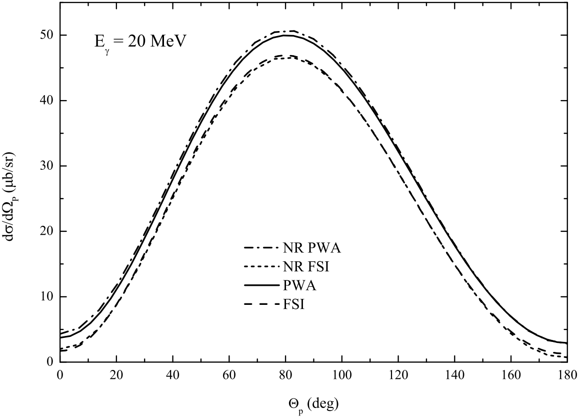
$$+ \quad \delta_{S1}\,\sum_M C^{1M}_{2M-m_S1m_S}\,Y_{2M-m_S}(\theta_{\hat{\mathbf{p}}},\varphi_{\hat{\mathbf{p}}})\,\bar{\mathcal{Y}}_{^3S_1^+M}(\mathbf{p})\,t^*_{^3S_1^+,{}^3D_1^+}(0,|\hat{\mathbf{p}}|;p_0,|\mathbf{p}|;s)$$

$$+ \quad \delta_{S1}\,\frac{1}{\sqrt{4\pi}}\,\bar{\mathcal{Y}}_{^3D_1^+m_S}(\mathbf{p})\,t^*_{^3D_1^+,{}^3S_1^+}(0,\hat{\mathbf{p}};p_0,|\mathbf{p}|;s)$$

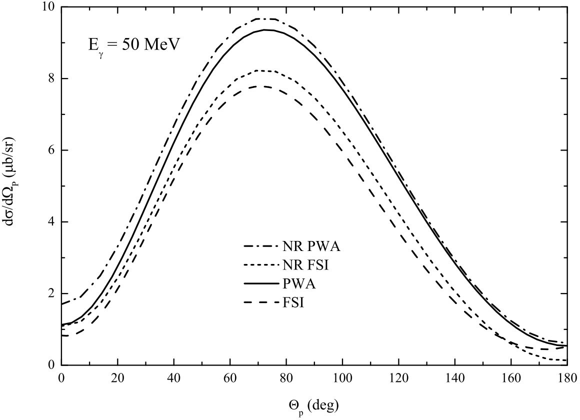
$$+ \quad \delta_{S1}\,\sum_M C^{1M}_{2M-m_S1m_S}\,Y_{2M-m_S}(\theta_{\hat{\mathbf{p}}},\varphi_{\hat{\mathbf{p}}})\,\bar{\mathcal{Y}}_{^3D_1^+M}(\mathbf{p})\,t^*_{^3D_1^+,{}^3D_1^+}(0,|\hat{\mathbf{p}}|;p_0,|\boldsymbol{p}|;s)\Big]$$



CMS differential cross section at the laboratory photon energies $E_\gamma = 3$ MeV



CMS differential cross section at the laboratory photon energies $E_\gamma = 20$ MeV



CMS differential cross section at the laboratory photon energies $E_\gamma = 50$ MeV

Motivation for MY(I)N kernel

Consider integral

$$\int dk_0 \int \mathbf{k}^2 d|\mathbf{k}| \frac{[g(k_0, |\mathbf{k}|)]^2}{(\sqrt{s}/2 - E_\mathbf{k} + i\epsilon)^2 - k_0^2}$$

Simple poles of propagators

$$k_0^{(1,2)} = \pm\sqrt{s}/2 \mp E_\mathbf{k} \pm i\epsilon$$

Taking into account pole and Yamaguchi-like type of $g(k_0, |\mathbf{k}|) = 1/(k_0^2 - \mathbf{k}^2 - \beta^2)$

$$(2\pi i) \int \mathbf{k}^2 d|\mathbf{k}| \frac{1}{(s/4 - \sqrt{s}E_\mathbf{k} + m^2 - \beta^2)^2} \frac{1}{\sqrt{s} - 2E_\mathbf{k} + i\epsilon}$$

Motivation for MY(I)N kernel

Consider numerator $f = s/4 - \sqrt{s}E_{\mathbf{k}} + m^2 - \beta^2$

- If $4(m-\beta)^2 < s < 4(m+\beta)^2$ then always $f < 0$ and function $1/f^n$ is integrable for any integer n and any $E_{\mathbf{k}}$.
- For bound state $s = M_d^2 = (2m - \epsilon_D)^2$. Since for $\beta_{min} = 0.2$ GeV always $\beta_{min} > \epsilon_D/2$ then function $1/f^n$ is integrable for any integer n and any $E_{\mathbf{k}}$.
- If $4(m-\beta)^2 > s > 4(m+\beta)^2$ then f can be positive and negative and $1/f^n$ is non-integrable for even n and any $E_{\mathbf{k}}$.

Critical value for $s^c = 4(m+\beta)^2$ corresponds to laboratory kinetic energy of np pair $T^c_{lab} = 4\beta + 2\beta^2/m \simeq 4\beta$. If $\beta_{min} = 0.2$ GeV then $T^{min}_{lab} = 0.8$ GeV.

The solution is to change $g(k_0, |\mathbf{k}|)$

$$g_{\rm Y}(k_0, |\mathbf{k}|) = 1/(k_0^2 - \mathbf{k}^2 - \beta^2) \to g_{\rm MY}(k_0, |\mathbf{k}|) = 1/((k_0^2 - \mathbf{k}^2 - \beta^2)^2 + \alpha^4)$$

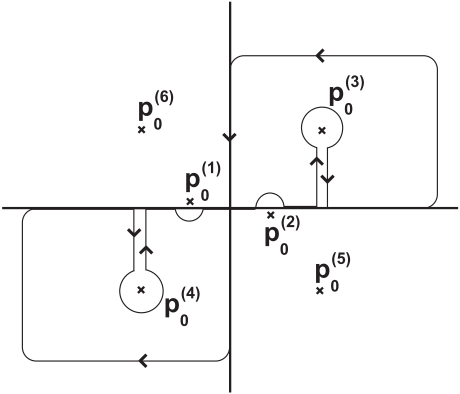
here Y stands for Yamaguchi-like and MY - for Modified Yamaguchi

Modified Yamaguchi functions

$$g_i^{[a]}(p_0,|\boldsymbol{p}|) = \frac{(p_{ci} - p_0^2 + \mathbf{p}^2)^{n_i}(p_0^2 - \mathbf{p}^2)^{m_i}}{((p_0^2 - \mathbf{p}^2 - \beta_{1i}^2)^2 + \alpha_{1i}^4)^{k_i}((p_0^2 - \mathbf{p}^2 - \beta_{1i}^2)^2 + \alpha_{1i}^4)^{l_i}}$$

All parameters - n_i, m_i, k_i, l_i (integer), $p_{ci}, \beta_{1i}, \beta_{2i}, \alpha_{1i}, \alpha_{2i}$ (real) - depend on channel $[a]$.

Contour for integration over p_0: the Wick rotation



Separable ansatz for complex kernel

$$V_{a'a}(p_0', |\mathbf{p}'|; p_0, |\mathbf{p}|; s) = \sum_{m,n=1}^{N} \left[\underline{\lambda_{mn}^{r[a'a]}(s)} + i\lambda_{mn}^{r[a'a]}(s) \right] \underline{g_m^{[a']}(p_0', |\mathbf{p}'|) g_n^{[a]}(p_0, |\mathbf{p}|)}$$

underlined part $\equiv$ MYN kernels, the sum $\equiv$ MYIN kernels

$$\lambda_{mn}^{i}(s) = \theta(s - s_{th}) \left(1 - \frac{s_{th}}{s}\right) \bar{\lambda}_{mn}^{i}$$

s_{th} - the inelasticity threshold. Here MY stands for Modified Yamaguchi.

Solution for the T matrix

$$T_{a'a}(p_0', |\mathbf{p}'|; p_0, |\mathbf{p}|; s) = \sum_{i,j=1}^{N} \tau_{ij}(s)\, g_i^{[a']}(p_0', |\mathbf{p}'|)\, g_j^{[a]}(p_0, |\mathbf{p}|)$$

where

$$\left[\tau_{ij}(s)\right]^{-1} = \left[\lambda_{mn}^{r[a'a]}(s) + i\lambda_{mn}^{r[a'a]}(s)\right]^{-1} + h_{ij}(s),$$

$$h_{ij}(s) = -\frac{i}{4\pi^3} \sum_a \int dk_0 \int \mathbf{k}^2 d|\mathbf{k}| \frac{g_i^{[a]}(k_0, |\mathbf{k}|) g_j^{[a]}(k_0, |\mathbf{k}|)}{(\sqrt{s}/2 - E_{\mathbf{k}} + i\epsilon)^2 - k_0^2},$$

$g_j^{[a]}$ - the model functions, $\lambda_{ij}^{[a'a]}(s)$ - a matrix of model parameters

Analysis

$V(p,p') \to T_{off-mass-shell}(p,p') \to T_{on-mass-shell}(\bar{p},\bar{p}) \to (\delta, \underline{\rho}, a_0, E_d ...)$

δ - the phase shifts, a_0, r_0 - the low-energy parameters (the scattering length, the effective range), E_d - the deuteron binding energy.

Procedure $(J = 0, 1)$

calculate the kernel parameters – $\lambda_{ij}(s)$-matrix and parameter of the g-functions – to minimize the function χ^2:

$\chi^2 =$

$\sum_{i=1}^{n} (\delta^{\exp}(s_i) - \delta(s_i))^2 / (\Delta\delta^{\exp}(s_i))^2$ – for all partial-wave states

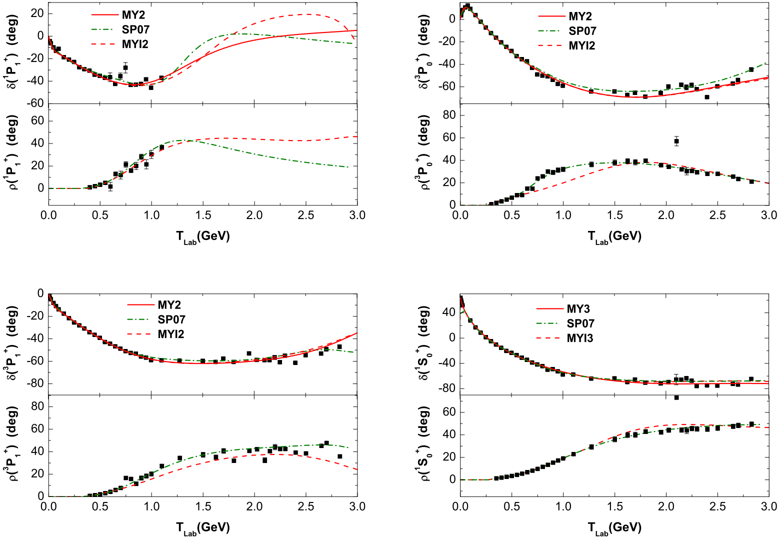
$\sum_{i=1}^{n} (\rho^{\exp}(s_i) - \rho(s_i))^2 / (\Delta\rho^{\exp}(s_i))^2$ – for all partial-wave states

$+(a_0^{\exp} - a_0)^2 / (\Delta a_0^{\exp})^2$ – for the ${}^1S_0^+$ and ${}^3S_1^+$ partial-wave states

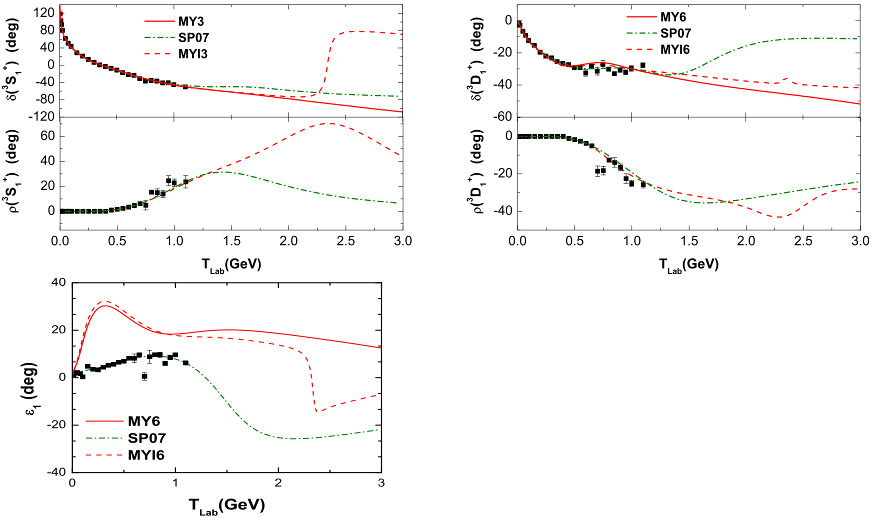
$+(E_d^{\exp} - E_d)^2 / (\Delta E_d^{\exp})^2$ – for the ${}^3S_1^+$-${}^3D_1^+$ partial-wave states

$\{+ ...\}$

Phase shifts for $^1P_1^+$, $^3P_0^+$, $^3P_1^+$ and $^1S_0^+$ partial-wave states



Phase shifts and inelasticity parameter for triplet $^3S_1$-$^3D_1$ state



Phase shifts and inelasticity parameter for triplet $^3S_1$-$^3D_1$ state

Table : Deuteron and low-energy scattering properties

	a_{0t} (fm)	r_{0t} (fm)	p_D (%)	E_d (MeV)	$\rho_{D/S}$	μ_d $(e/2m)$
MY6	5.42	1.800	4.92	2.2246	0.0255	0.8500
Graz-II	5.42	1.779	5	2.2254	0.0269	0.8512
Paris	5.43	1.770	5.77	2.2249	0.0261	0.8469
CD-Bonn	5.4196	1.751	4.85	2.2246	0.0256	0.8522
Exp.	5.424(4)	1.759(5)	4-7	2.224644(46)	0.0256(4)	0.8574

Unpolarized exclusive cross-section

The 5-fold differential cross section in one-photon approximation (in laboratory system - LS)

$$\frac{d^3\sigma}{dE'_e d\Omega'_e d\Omega_p} = \frac{\sigma_{\text{Mott}}}{8M_d(2\pi)^3} \frac{\boldsymbol{p}_p^2\sqrt{s}}{\sqrt{1+\eta}|\boldsymbol{p}_p| - E_p\sqrt{\eta}\cos\theta_p}$$

$$\times \left[l^0_{00}W_{00} + l^0_{++}(W_{++}+W_{--}) + 2l^0_{+-}\cos 2\phi\ \mathrm{Re}W_{+-} - 2l^0_{+-}\sin 2\phi\ \mathrm{Im}W_{+-} \right.$$
$$\left. -2l^0_{0+}\cos\phi\ \mathrm{Re}(W_{0+}-W_{0-}) - 2l^0_{0+}\sin\phi\ \mathrm{Im}(W_{0+}+W_{0-})\right]$$

$\sigma_{\text{Mott}} = (\alpha\cos\frac{\theta}{2}/2E_e\sin^2\frac{\theta}{2})^2$ - Mott cross section
$\alpha = e^2/(4\pi)$ - fine structure constant
M_d - mass of the deuteron, m - mass of the nucleon
$q = p_e - p'_e = (\omega, \boldsymbol{q})$ - momentum transfer
$p_e = (E_e, \boldsymbol{l})$ and $p'_e = (E'_e, \boldsymbol{l}')$ - initial and final electron momenta
Ω'_e - outgoing electron solid angle
$\boldsymbol{p}_p$ - momentum of outgoing proton
$\Omega_p = (\theta_p, \phi)$ - outgoing proton solid angle
$\eta = \boldsymbol{q}^2/s$ - Lorentz boost factor

Matrix element

Plane-wave approximation

$$< np : SM_S|j_\mu|d : 1M >^{(0)} = i \sum_{n=1,2} \{\Lambda(\mathcal{L}^{-1})\bar{\chi}_{SM_S}(p^{*\,\mathsf{CM}}; P^{\mathsf{CM}})\Lambda(\mathcal{L})\Gamma_\mu^{(n)}(q)$$

$$\times S^{(n)}(\frac{K_{(0)}}{2} - (-1)^n p^* - \frac{q}{2})\Gamma^M(p^* + (-1)^n \frac{q}{2}; K_{(0)})\}$$

Final-state interaction

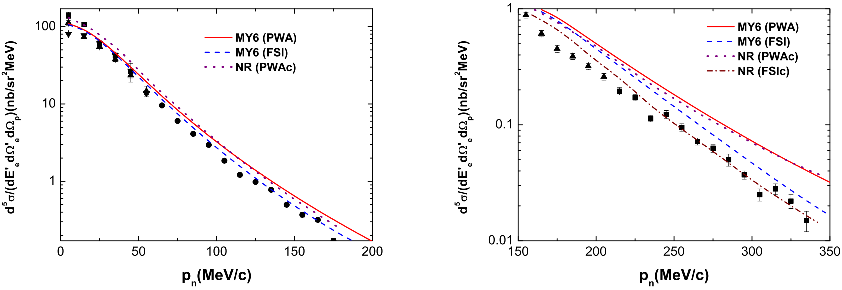
$$< np : SM_S|j_\mu|d : 1M >^{(t)} = \frac{i}{4\pi^3} \sum_{n=1,2} \sum_{LmJM_JL'lm'} C^{JM_J}_{LmJM_J} Y_{Lm}(\hat{\boldsymbol{p}}^*)$$

$$\int\limits_{-\infty}^{\infty} dp_0^{\mathsf{CM}} \int\limits_0^{\infty} (\boldsymbol{p}^{\mathsf{CM}})^2 d|\boldsymbol{p}^{\mathsf{CM}}| \int\limits_{-1}^{1} d\cos\theta_{\boldsymbol{p}}^{\mathsf{CM}} \int\limits_0^{2\pi} d\phi$$

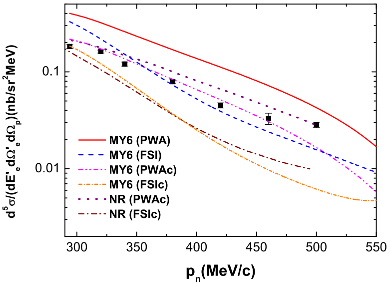
$$\mathrm{Sp}\left\{\Lambda(\mathcal{L}^{-1})\bar{\mathcal{Y}}_{JL'SM_J}(\boldsymbol{p}^{\mathsf{CM}})\Lambda(\mathcal{L})\Gamma_\mu^{(n)}(q)S^{(n)}\left(\frac{K_{(0)}}{2} - (-1)^n p - \frac{q}{2}\right)\right.$$

$$\left.\mathcal{Y}_{JlSm'}\left(\boldsymbol{p} + (-1)^n \frac{\boldsymbol{q}}{2}\right)\right\}$$

$$\frac{T^*_{L'L}(p_0^{\mathsf{CM}}, |\boldsymbol{p}^{\mathsf{CM}}|; 0, |\boldsymbol{p}^*|; s)}{(\sqrt{s}/2 - E_{\boldsymbol{p}} + i\epsilon)^2 - p_0^2} g_l\left(p_0 + (-1)^n \frac{\omega}{2}, \left|\boldsymbol{p} + (-1)^n \frac{\boldsymbol{q}}{2}\right|\right)$$



Cross section depending on recoil neutron momentum $|\boldsymbol{p}_n|$ calculated under kinematic conditions set I, II of the Sacley experiment (S$_I$,S$_{II}$). MY6 (PWA) (red solid line) - relativistic calculation in the plane-wave approximation with the MY6 potential; MY6 (FSI) (blue dashed line) - relativistic calculation including FSI effects; NR (PWAc) (violet dotted line) - nonrelativistic calculation (Shebeko *et al.*)



The same as in previous figures but under kinematic conditions set III of the Scale experiment (S_{III}). Two additional results are presented for comparison: MY6 (PWAc) (pink dashed-dotted-dotted line) - relativistic PWA calculation; MY6 (FSIc) (orange dashed-dotted line) - relativistic calculation with FSI effects; both obtained under current conservation condition $\omega J_0 = q_z J_z$

Conclusion

- The multirank separable complex kernels of the neutron-proton interaction for the partial-states with the total angular momentum $J = 0, 1$ were constructed using the analyses of the NN-scattering observables
- The obtained kernels for the states with the total angular momentum J=0,1 were used to calculate final-state interaction effects for the photo- and electrodisintegration of the deuteron