On gluon propagator and Gribov noise in Landau gauge gluodynamics

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Abstract First we give concise summary of our long-term project aimed at obtaining continuum-limit extrapolated data for the Landau-gauge gluon and ghost propagators of $SU(2)$ lattice gauge theory. In our simulations we keep the physical volume fixed at $(9.6fm)^4$ and vary the lattice coupling parameter such that lattice spacing from 0.17 down to 0.09 fermi are reached. Next I will propose a way of a Gribov noise suppression and show the results for the gluon propagator which have been obtained by this technique in the SU(2) Landau gauge quenched QCD.

Introduction

- Nonperturbative studies of Landau gauge gluon and ghost propagators

$$D^{ab}_{\mu\nu} = \delta^{ab}\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\frac{Z(q^2)}{q^2}, \quad G^{ab} = \delta^{ab}\frac{J(q^2)}{q^2}$$

with continuum Dyson-Schwinger (DS) or Funct. Renorm. Group (FRG) Eqs. and within the lattice approach should be compared.

- DS and FRG Eqs. have 2 different solutions:
1) conformal, or scaling
[von Smekal, Hauck, Alkofer '98; Zwanziger '02; Lerche, von Smekal '02]

$$J(q^2) \propto (q^2)^{\alpha_{gh}} \quad \text{and} \quad Z(q^2) \propto (q^2)^{\alpha_{gl}}$$

with $$\alpha_{gl} + 2\alpha_{gh} = 0,$$

$D(q^2) = \frac{Z}{q^2} \to 0, \quad J(q^2) \to \infty$ for $q^2 \to 0$, and
2) decoupling, or regular
[Boucaud et al. '05 - '07; Aguilar et al. '04 - '08]
$D(q^2) \to D_0 = const, \quad J(q^2) \to J_0 = const$

- Note that just decoupling solution is found by lattice supercomputer simulations. Our present aim is to get closer to continuum limit for all q^2.

Gauge fixing: SA+OR

In order to fix the Landau gauge we apply a gauge transformation $g(x)$ to link variables $U_{x,\mu} \in SU(3)$ or $SU(2))$ such that the gauge functional is maximized

$$F_U[g] = \sum_{x,\mu} \frac{1}{N_c} \mathfrak{Re}\,\mathrm{Tr}\ {}^gU_{x,\mu}\,.$$

$\Rightarrow$ For $A_\mu(x+\hat{\mu}/2) := (1/2ig_0)\left(U_{x,\mu} - U^\dagger_{x,\mu}\right)_{\mathrm{traceless}}$
this is equivalent to $\Delta_\mu A_\mu = 0\,,$
$\Rightarrow$ but not unique: Gribov copies,
$\Rightarrow$ search for global maxima -
fundamental modular region (FMR), for this search we use Simulated Annealing (SA), finalized by Overrelaxation (OR).

- SA is a "stochastic optimization method" – here with the statistical weight $W[g] \propto \exp\{F_U[g]/T\}$ –
  allowing quasi-equilibrium tunneling through functional barriers, in the course of a "temperature" T decrease.

- In principle - with infinitely slow cooling down - it allows to reach global extrema.

Finite-volume effects

Compute gluon propagator for $\beta = 2.3$ and various L: $L = 40, 56, 80, 112$

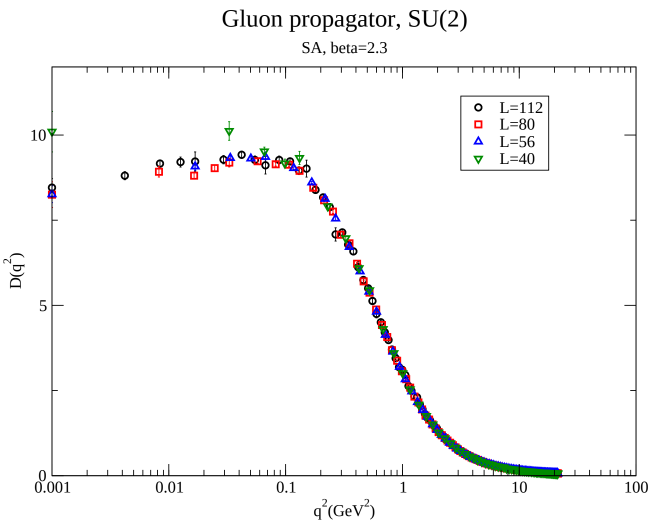

Figure 1: The gluon propagator for $\beta = 2.3$ and various lattice sizes . The data points drawn at $q^2 = 0.001$ represent the zero-momentum gluon propagator $D(0)$.

Fig.1 shows that finite volume effects (FVEs) for $\beta = 2.3$ are negligible if lattice size $L \geq 56$. For $L = 56$ and $\beta = 2.3$ physical lattice size $L_{FV} = L * a(\beta) = 9.6 fm$

One can assume that for all $\beta \in [2.3, 2.6]$ FVEs are small if $L * a(\beta) \geq 9.6 fm$, but this needs checking !

Interpolation of D data

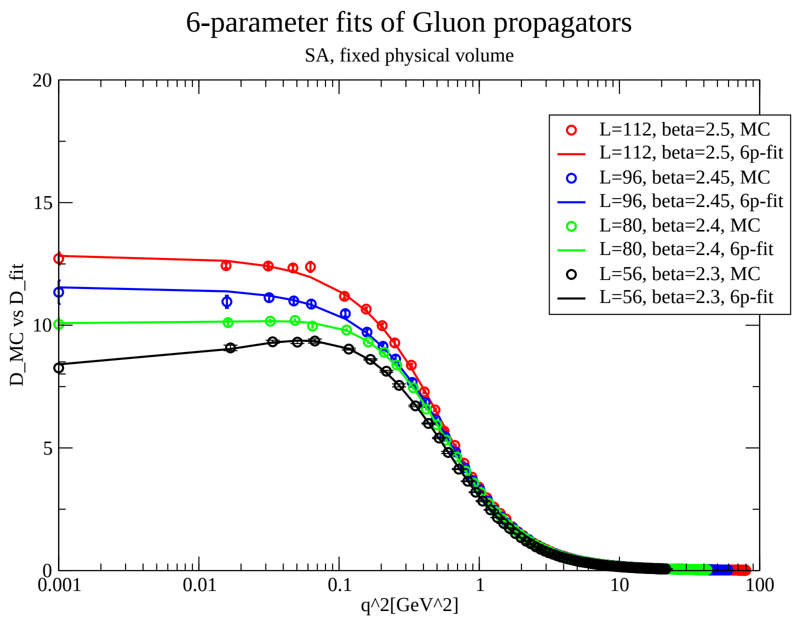

Figure 2: Fitting of bare gluon propagators for various β at fixed physical volume

We use the 6 parameters fit (Cucchieri et al'11):

$$D(q^2) = C\frac{q^4 + Aq^2 + B}{q^6 + Dq^4 + Fq^2 + G}$$

Interpolation of J data

For fitting of $J(q^2)$ the 5-parameter formula

$$J(q^2) = \frac{Cq^{2Z} + Bq^T}{q^4 + Dq^2}$$

proved to be pretty effective.

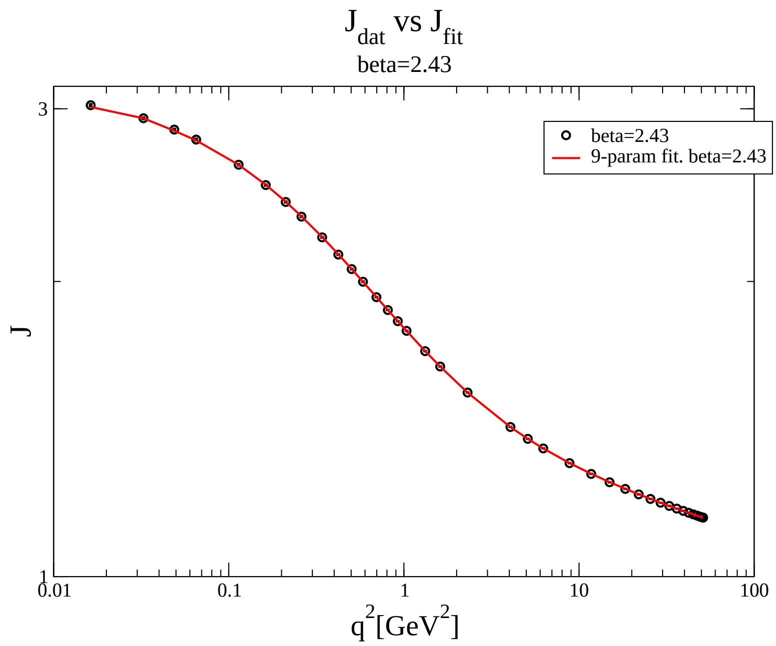

Figure 3: Fitting of bare ghost dressing function for $\beta = 2.43$

$\Rightarrow$ Note universality of fitting formulas for all β and q^2; Presently we are looking for even better fit for $D(q^2)$.

$\Rightarrow$ Fitted D and J curves are used further, in particular for renormalization and interpolating between data points.

Renormalizing gluon and ghost

- Gluon propagators and ghost dressing functions have been renormalized requiring that values of dressing functions are equal to each other at some μ_1, for which lattice artefacts (both UV and IR ones) are small, i.e. $\mu_1 = 1.0$ GeV.

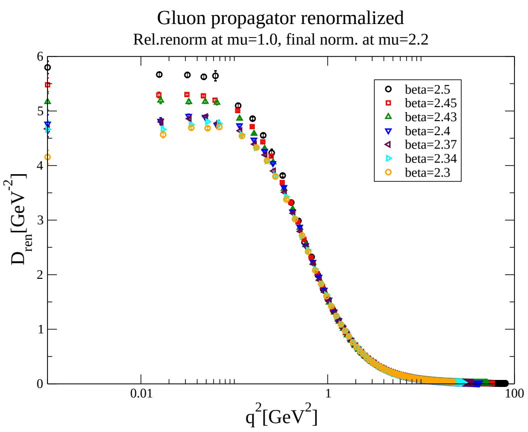

Figure 4: Renormalized fitted gluon propagators for various β and fixed physical volume, $\mu_1 = 1.0$ GeV

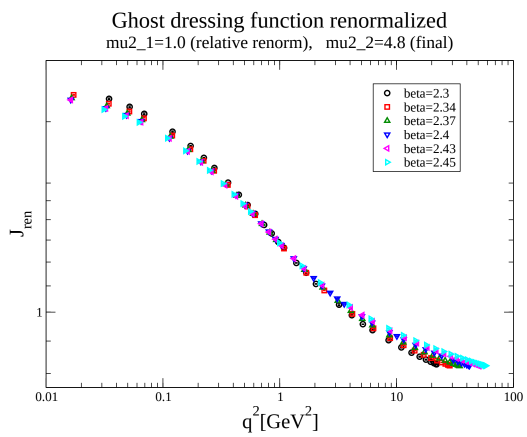

Figure 5: Renormalized ghost dressing functions for various β and fixed physical volume, $\mu_1 = 1.0$ GeV

Running coupling

Now compute running coupling $\alpha_s(q^2) = \frac{g_0^2}{4\pi} J^2(q^2)\, Z(q^2)$.

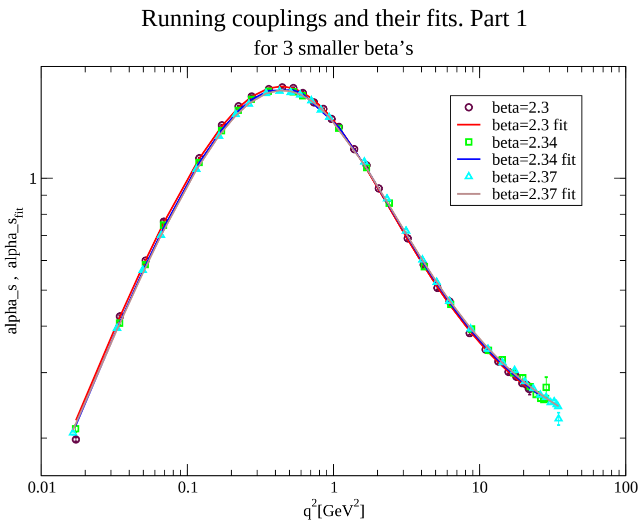

Figure 6: Running coupling $\alpha_s(q^2)$ computed for various β at fixed physical volume

$\Rightarrow$ No IR fixed point seen for $SU(2)$ $\alpha_s(q^2)$: decoupling solutions! $\Rightarrow$ Continuum limit of $\alpha_s(q^2)$ for intermediate region, $q^2 \approx 1.0$ GeV seems to be reached already at $L = 56, \beta = 2.3$.

IR and UV artefacts: D

- We compute ratios of bare gluon dressing functions $Z(q^2)$ for various β's: $Z(q^2, \beta)/Z(q^2, \beta = 2.5)$

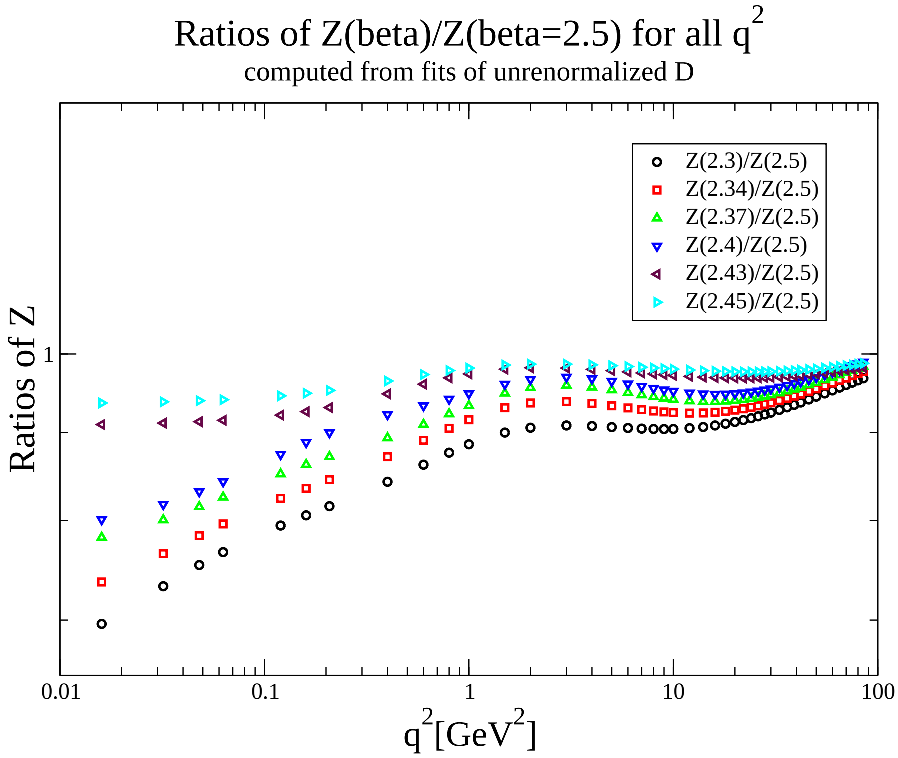

Figure 7: Ratios of bare $Z(\beta)/Z(\beta = 2.5)$ as a sign of distortions due to discretization artefacts in IR and UV momenta regions

$\Rightarrow$ UV hypercubic distortions can hopefully be accounted for by the Lattice Perturbation Theory (with lattice spacing a as a small parameter): see talk by A.Sternbeck and J.Simeth at LATTICE-2013

Lattice artefacts: J

- Analogously compute ratios of bare ghost dressing functions: $J(q^2, \beta)/J(q^2, \beta = 2.45)$ for various β's:

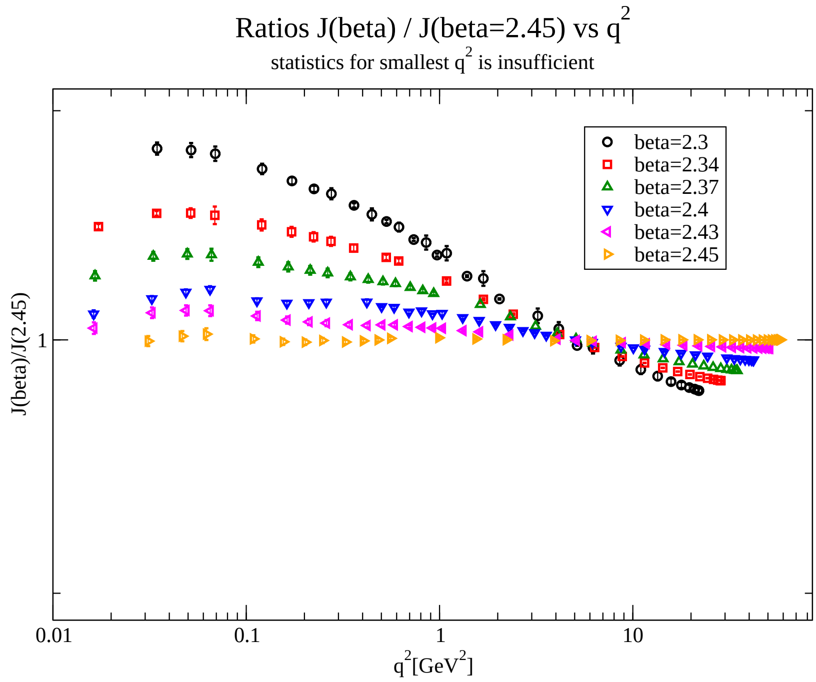

Figure 8: Ratios $J(q^2, \beta)/J(q^2, \beta = 2.45)$ as a sign of distortions in IR and UV momenta regions.

$\Rightarrow$ IR lattice artefacts can also be due to hypercubic discretization effects in small q^2 region and hopefully will be accounted for by the LPT in $1/a$.

Lattice artefacts: α_s

- Finally compute ratios of running coupling: $\alpha_s(q^2, \beta)/\alpha_s(q^2, \beta = 2.45)$ for various β's:

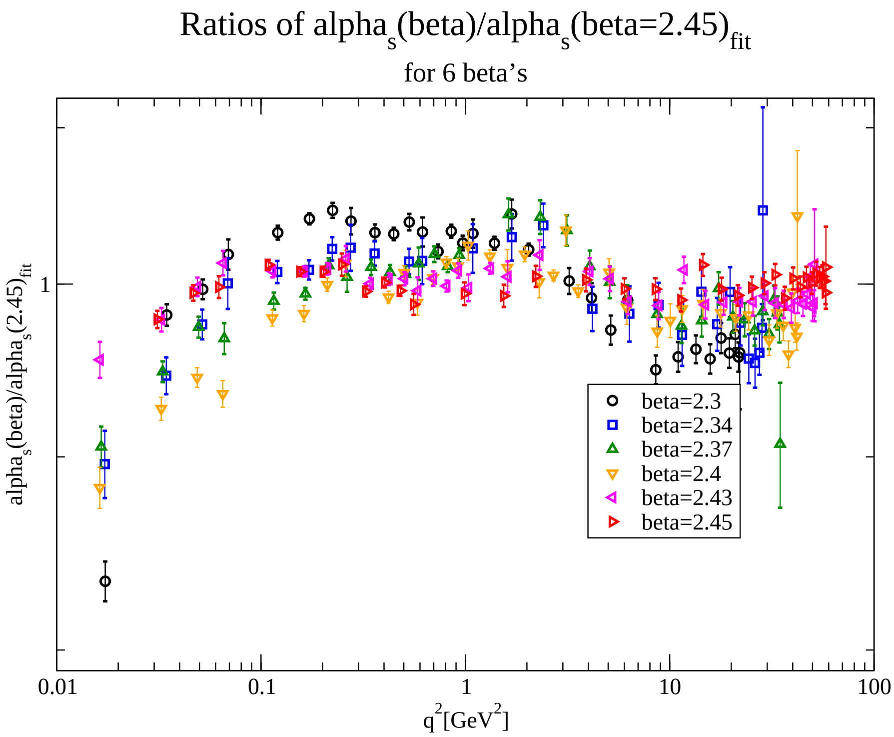

Figure 9: Ratios of $\alpha_s(q^2, \beta)/\alpha_s(q^2, \beta = 2.45)$ as a sign of distortions due to hypercubic artefacts in IR and UV momenta regions

$\Rightarrow$ One can assume that IR lattice artefacts can also be due to hypercubic effects in small q^2 region and hopefully will be accounted for by the LPT (with $1/a$ as a small parameter).

Gribov noise for gluon (1)

Study gluon propagator for $\beta = 2.4$ and $L = 80$. Consider fixed MC configuration, say, e.g., 5th, and generate various Gribov copies, again with "long SA + OR" Landau gauge-fixing. Here "various" means different random starting points of the gauge fixing procedure. Apriori one can think that in this approach Gribov ambiguity and its influence on gluon observables will be small/moderate, because "long SA + OR" approach produces Gribov copies with F_G close to the global maximum for the MC configuration chosen.

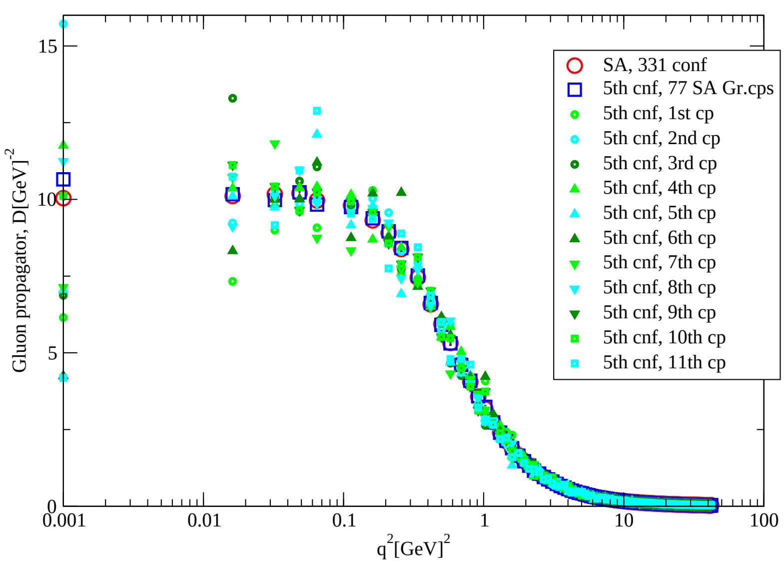

Figure 10: The gluon observable $D(q^2)$ for $\beta = 2.4, L = 80$, the 5th MC configuration and various Gribov copies compared with $D(q^2)$ averaged over 77 SA Gribov copies

Gribov noise for gluon (2)

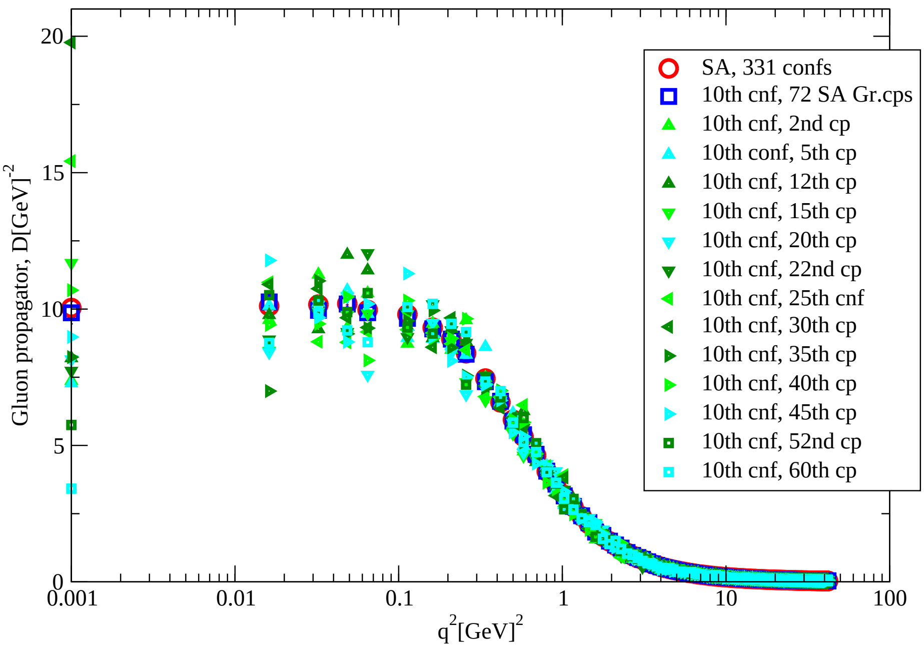

Figure 11: The gluon observable $D(q^2)$ for $\beta = 2.4, L = 80$, the 10th MC configuration and various Gribov copies compared with $D(q^2)$ averaged over 72 SA Gribov copies

$\Rightarrow$ One can see that
(i)Gribov noise is suprisingly high
(ii) $D(q^2)$ averaged over copies (both for the 5th and 10th configurations) clearly shows decoupling behaviour (plateau for small q^2)

Gribov noise for gluon (3)

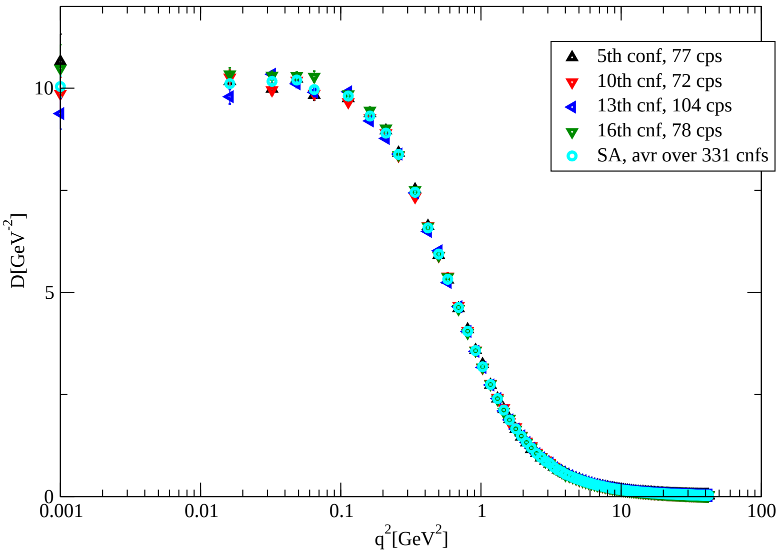

Figure 12: The gluon observable $D(q^2)$ for $\beta = 2.4, L = 80$, averaged over Gribov SA copies for fixed MC configurations versus gluon propagator $D(q^2)$ obtained by averaging over 331 MC configurations

$\Rightarrow$ One can see that averaging over various Gribov copies generated from one Yang-Mills MC configuration typically leads to the $D(q^2)$ dependence pretty close to the standard $D(q^2)$ propagator obtained by commonly used averaging over MC configurations

$\Rightarrow$ This way of suppressing Gribov noise may be used for obtaining gluon propagator when only few MC configurations are available.

Conclusions and Questions

- Finite-volume effects for lattice gluon propagator at $\beta = 2.3$ are small when lattice extension $L \geq 56$ which corresponds to linear size $La(\beta) \geq 9.6$ fm .

- Systematic lattice simulations at various β and fixed physical volume $L^4 a(\beta)^4$ allow to study evolution of gluon and ghost propagators under decrease of lattice spacing $a(\beta)$

- Our studies confirmed so-called "decoupling" behaviour of gluon and ghost propagators for decreasing spacing a

- Discretization (hypercubic) lattice artefacts found for gluon and ghost propagators in IR (for few smallest momenta) turned out to be surprisingly large

- Averaging over many "long SA+OR" Gribov copies obtained for the fixed MC configuration hopefully allows to find gluon propagator $D(q^2)$ when only few MC configurations are available

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