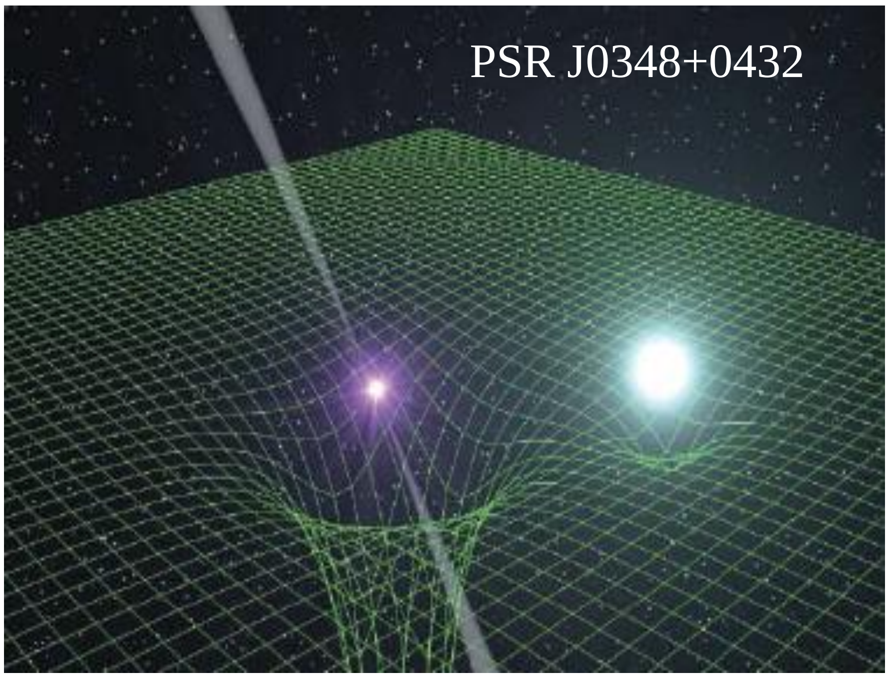
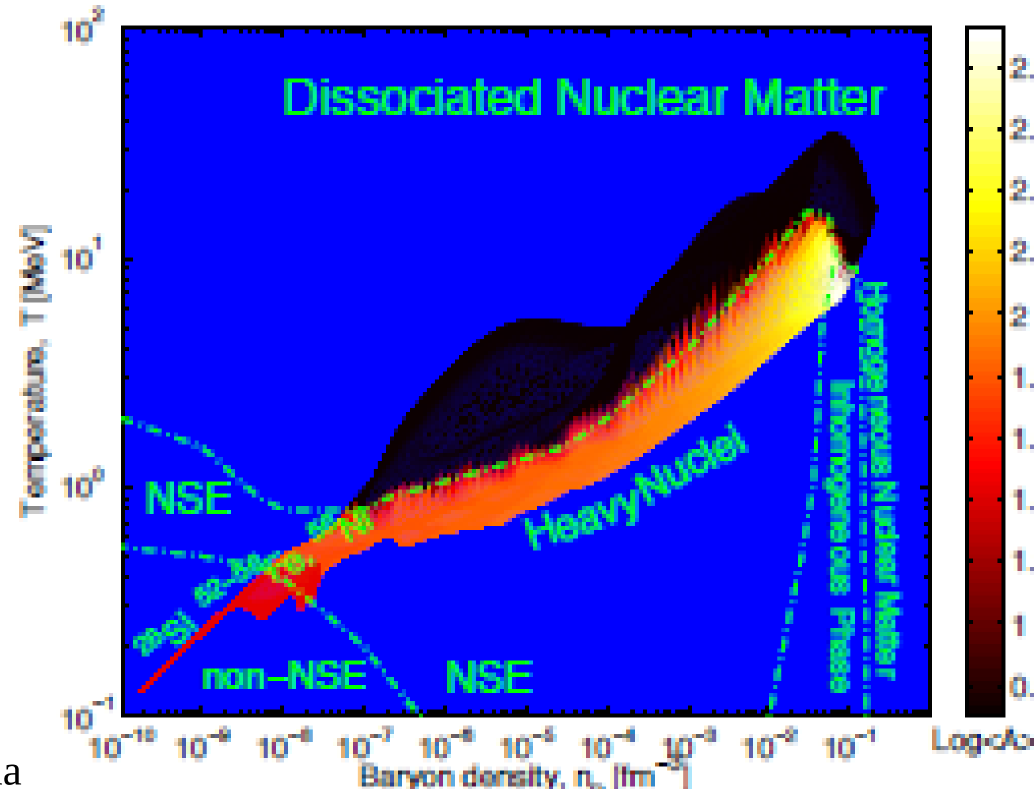
Cluster virial expansion for quark/nuclear matter

David Blaschke (University of Wroclaw, Poland & JINR Dubna, Russia)



Antoniadis et al., Science 340 (2013) 448



T. Fischer, Talk at "Dense Matter 2010", Dubna

Baldin Seminar 'QFT Problems & QCD', Dubna, September 18, 2014

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Cluster virial expansion for quark/nuclear matter

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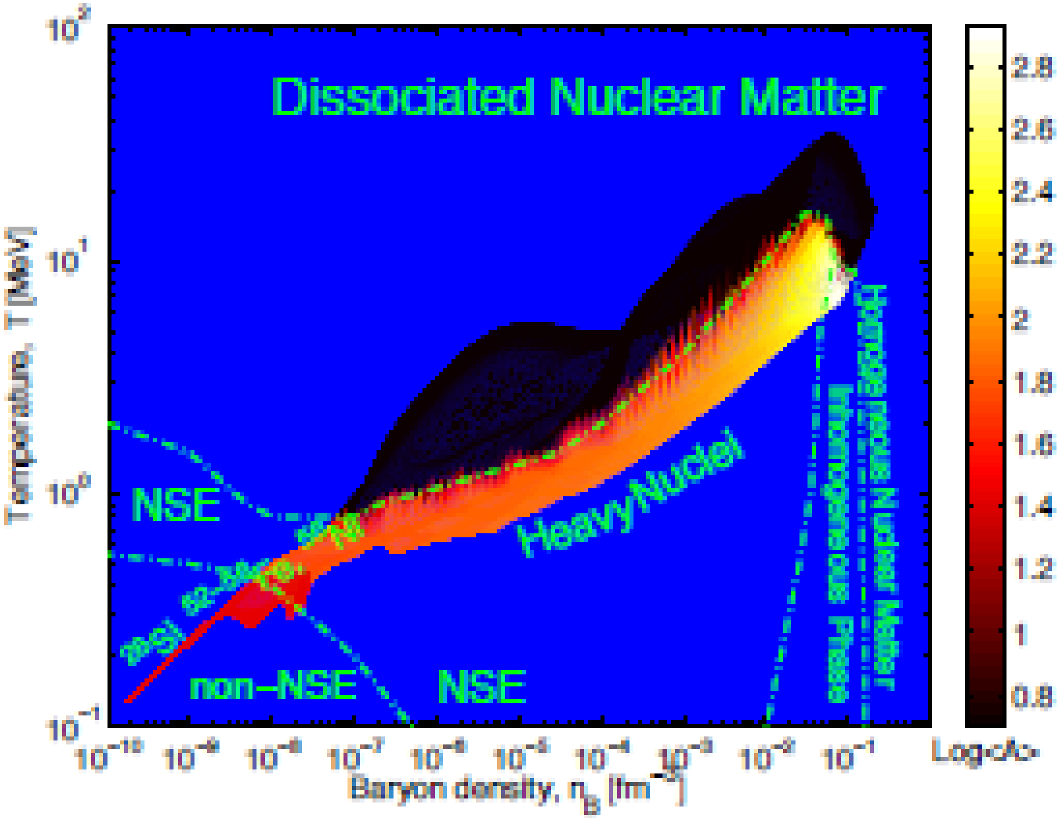
1. Introduction:
- cluster expansion & virial corr.
- nuclear matter vs. quark matter

2. Clusters in nuclear matter:
- NSE = nuclear statistical equil.
- Mott transition, virial corr.
- Φ – derivable formulation ?

3. Clusters in quark matter:
- HRG = hadron resonance gas
- Mott transition, PNJL, virial c.

4. Outlook:
- unified quark-nuclear matter
- supernova explosion modeling?



Introduction: Cluster expansion and virial corrections

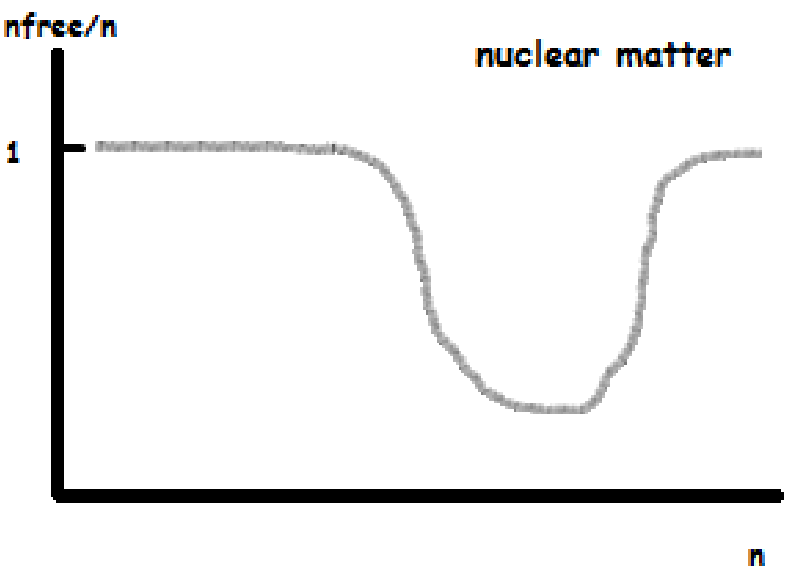
Nuclear Matter:

Low density:

$n = n_{free} + \sum A\, n_A + n_{corr}$

High density:

$n = n_{free} +$ ~~$\sum A\, n_A$~~ $+ n_{corr}$



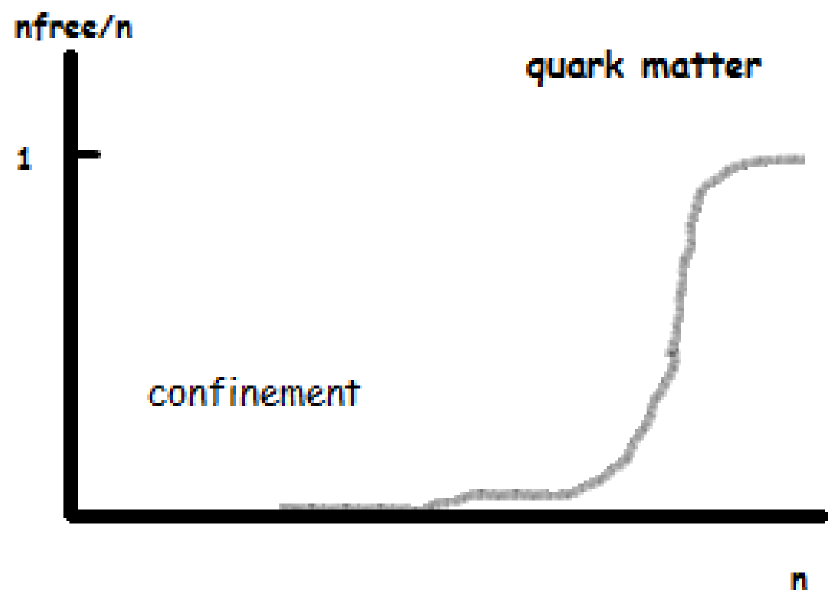
Quark matter:

Low density:

$n =$ ~~n_{free}~~ $+ \sum n_{M,B} + n_{corr}$

High density:

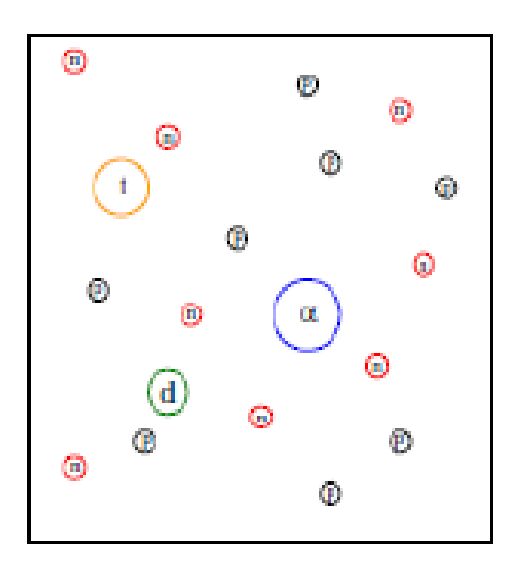
$n = n_{free} +$ ~~$\sum n_{M,B}$~~ $+ n_{corr}$



Clusters in nuclear matter: different concepts

Composition of Nuclear Matter

- depends strongly on density, temperature and neutron-proton asymmetry
- affects thermodynamical properties



Theoretical Models: different points of view

- **chemical picture**
  mixture of different nuclear species and nucleons
  in chemical equilibrium
  - properties of constituents independent of medium
  - interaction between particles ?
  - dissolution of nuclei at high densities ?

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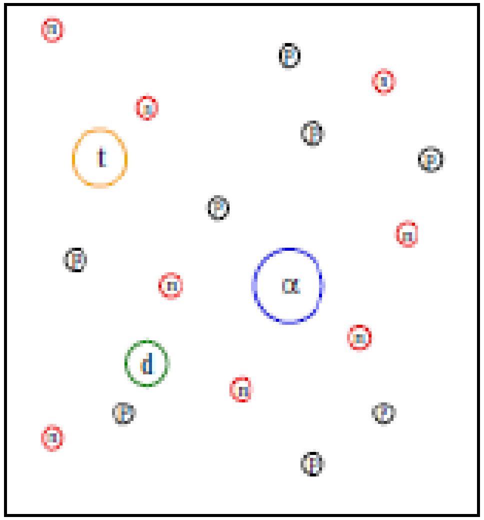

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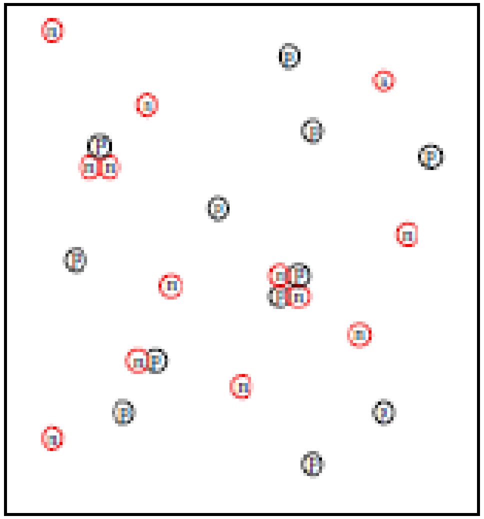


- **physical picture**
  interaction between nucleons ⇒ correlations
  ⇒ formation of bound states/resonances
  - treatment of two-, three-, . . . many-body correlations ?
  - choice of interaction ?

Strategy: successive improvement

Improving the description step by step:

- most simple approach, ideal mixture of independent particles, no interaction
  $\Rightarrow$ **Nuclear Statistical Equilibrium**
- low-density limit, with interactions/correlations
  $\Rightarrow$ **Virial Equation of State**
- consider medium effects with increasing density
  $\Rightarrow$ **Generalized Beth-Uhlenbeck Approach**
- connecting to densities around nuclear saturation
  $\Rightarrow$ **Generalized Relativistic Density Functional**

system of units such that $\hbar = c = k_B = 1$

Thermodynamical relationships

Grand Canonical Ensemble

- particles i with chemical potentials μ_i at Temperature T in Volume V

  $\Rightarrow$ natural variables: μ_i, T, V
- thermodynamical potential: grand canonical potential $\Omega = \Omega(\mu_i, T, V) = -pV$

  $\Rightarrow$ contains all information of the system
- equations of state:

$$n_i(\mu_i, T, V) = \frac{N_i}{V} = -\frac{1}{V}\frac{\partial\Omega}{\partial\mu_i}\bigg|_{T,V}$$ particle number densities

$$S(\mu_i, T, V) = -\frac{\partial\Omega}{\partial T}\bigg|_{\mu_i,V}$$ entropy

$$p(\mu_i, T, V) = -\frac{\partial\Omega}{\partial V}\bigg|_{\mu_i,V} = -\frac{\Omega}{V}$$ pressure

- connection to microphysics: grand canonical partition function

$$\mathcal{Z}(\mu_i, T, V) = \mathrm{trace}\exp\left[-\beta\left(\hat{H} - \sum_i \mu_i \hat{N}_i\right)\right] \quad \Rightarrow \quad \Omega = -T\ln\mathcal{Z}$$

with Hamilton operator $\hat{H}$, particle number operators $\hat{N}_i$ and $\beta = 1/T$

Nuclear statistical equilibrium (NSE)

most simple approach

- ideal mixture of nucleons (p, n) and nuclei X in chemical equilibrium

  $$Zp + Nn \Leftrightarrow {}^{A}_{Z}X_N \quad \Rightarrow \quad Z\tilde{\mu}_p + N\tilde{\mu}_n = \tilde{\mu}_X \quad \text{or} \quad Z\mu_p + N\mu_n = \mu_X - B_X$$

  with relativistic chemical potentials $\tilde{\mu}_i = \mu_i + m_i$ and
  binding energy B_X of nucleus X with mass m_X

- independent particles without mutual interaction

  $\Rightarrow$ factorization $\mathcal{Z} = \prod_i \mathcal{Z}_i$

- Maxwell-Boltzmann statistics

  $$\mathcal{Z}_i = \sum_{N_i=0}^{\infty} \frac{1}{N_i!} Q_i^{N_i} z_i^{N_i} = \exp\left(Q_i z_i\right)$$ with fugacities $$z_i = \exp\left(\frac{\mu_i}{T}\right)$$

  and single-particle canonical partition functions $$Q_i = \text{trace} \exp\left(-\beta \hat{H}_i\right)$$

  with single-particle Hamilton operator $\hat{H}_i$

Nuclear statistical equilibrium (NSE)

- nonrelativistic kinematics: $\hat{H}_i = \hat{p}_i^2/(2m_i) \quad \Rightarrow \quad E_i = p_i^2/(2m_i)$

$$\Rightarrow \quad Q_i = \frac{g_i}{(2\pi)^3} \int d^3r_i \int d^3p_i \, \exp\left(-\frac{E_i}{T}\right) = g_i \frac{V}{\lambda_i^3}$$

with degeneracy factor $g_i = (2J_i + 1)$ ($= 2$ for nucleons)
and thermal wavelength $\lambda_i = \sqrt{\frac{2\pi}{m_i T}}$

$$\Rightarrow \quad \mathcal{Z} = \prod_i \exp\left(g_i \frac{V}{\lambda_i^3} z_i\right) \quad \text{and} \quad \Omega = -T \ln \mathcal{Z} = -TV \sum_i \frac{g_i}{\lambda_i^3} \exp\left(\frac{\mu_i}{T}\right)$$

- including excited states (x) of nuclei

$$g_i \to g_i(T) = (2J_i^{gs} + 1) + \sum_x (2J_x + 1) \exp\left(-\frac{E_x}{T}\right)$$

$$\to (2J_i^{gs} + 1) + \int dE \, \varrho_i(E) \exp\left(-\frac{E}{T}\right)$$ with level density $\varrho_i(E)$

Nuclear statistical equilibrium (NSE)

- equations of state

$$n_i(\mu_i, T, V) = -\frac{1}{V}\frac{\partial \Omega}{\partial \mu_i}\bigg|_{T,V} = \frac{g_i}{\lambda_i^3}\exp\left(\frac{\mu_i}{T}\right)$$ particle number densities

$\Rightarrow \Omega = -TV\sum_i n_i$

$$S(\mu_i, T, V) = -\frac{\partial \Omega}{\partial T}\bigg|_{\mu_i,V} = -\frac{5\Omega}{2T} - \frac{V}{T}\sum_i \mu_i n_i$$ entropy

$$p(\mu_i, T, V) = -\frac{\partial \Omega}{\partial V}\bigg|_{\mu_i,V} = -\frac{\Omega}{V}$$ pressure

$\Rightarrow pV = NT$ with $N = \sum_i N_i$

$$E = TS - pV + \sum_i \mu_i N_i = \frac{3}{2}NT$$ internal energy

$\Rightarrow$ mixture of ideal gases

Nuclear statistical equilibrium (NSE)

example

- ideal mixture of neutrons, protons and deuterons ($d = {}^2$H)

$$p + n \Leftrightarrow d \quad \Rightarrow \quad \mu_p + \mu_n = \mu_d - B_d$$

with deuteron binding energy $B_d = 2.225$ MeV

- particle number densities

$$n_{n,p} = \frac{2}{\lambda_{n,p}^3} \exp\left(\frac{\mu_{n,p}}{T}\right) \qquad n_d = \frac{3}{\lambda_d^3} \exp\left(\frac{\mu_d}{T}\right)$$

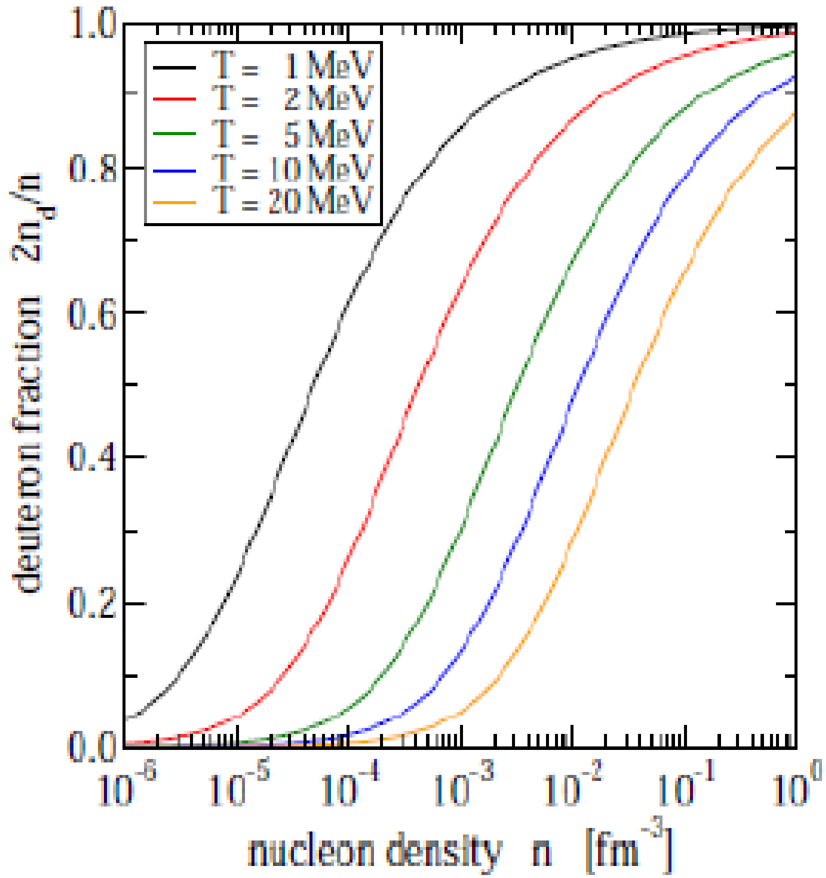
$\Rightarrow$ law of mass action

$$\frac{n_d}{n_n n_p} = \frac{3}{4}\left(\frac{2\pi m_d}{m_n m_p T}\right)^{\frac{3}{2}} \exp\left(\frac{B_d}{T}\right)$$

- symmetric nuclear matter ($n_n = n_p$)

total nucleon density $n = n_n + n_p + 2n_d$

deuteron fraction $X_d = \frac{2n_d}{n}$



Virial Equation of State (VEoS)

consider two-, (three-, . . . many-) body correlations !

- expansion of grand canonical partition function in powers of fugacities $z_i = \exp\left(\frac{\mu_i}{T}\right)$

$$\mathcal{Z}(\mu_i, T, V) = 1 + \sum_i Q_i z_i + \frac{1}{2}\sum_{ij} Q_{ij} z_i z_j + \frac{1}{6}\sum_{ijk} Q_{ijk} z_i z_j z_k + \ldots$$

  with one-, two-, three-, . . . many-body canonical partition functions

$$Q_i = \text{trace}_1 \exp\left(-\beta \hat{H}_i\right) \qquad Q_{ij} = \text{trace}_2 \exp\left(-\beta \hat{H}_{ij}\right)$$

$$Q_{ijk} = \text{trace}_3 \exp\left(-\beta \hat{H}_{ijk}\right)$$ and Hamiltonians $\hat{H}_i$, $\hat{H}_{ij}$, $\hat{H}_{ijk}$, . . .

- expansion valid only for $z_i \ll 1$

- for independent particles without interaction:
  $\hat{H}_{ij} = \hat{H}_i + \hat{H}_j$, $\hat{H}_{ijk} = \hat{H}_i + \hat{H}_j + \hat{H}_k$, . . .
  $\Rightarrow$ factorization $Q_{ij} = Q_i Q_j \qquad Q_{ijk} = Q_i Q_j Q_k \qquad \ldots$

Virial Equation of State (VEoS)

- use $\ln(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \ldots$

$$\ln \mathcal{Z} = \sum_i Q_i z_i + \frac{1}{2}\sum_{ij} \left(Q_{ij} - Q_i Q_j\right) z_i z_j + \ldots$$

- introduce (dimensionless) cluster (virial) coefficients

$$b_i = g_i \qquad b_{ij} = \lambda_i^{3/2}\lambda_j^{3/2}\left(Q_{ij} - Q_i Q_j\right)/(2V)$$

$$b_{ijk} = \lambda_i \lambda_j \lambda_k \left(Q_{ijk} - Q_i Q_{jk} - Q_j Q_{ik} - Q_k Q_{ij} + 2 Q_i Q_j Q_k\right)/(6V)$$

$\Rightarrow$ grand canonical potential

$$\Omega = -T \ln \mathcal{Z} = -TV \left(\sum_i b_i \frac{z_i}{\lambda_i^3} + \sum_{ij} b_{ij} \frac{z_i z_j}{\lambda_i^{3/2}\lambda_j^{3/2}} + \sum_{ijk} b_{ijk} \frac{z_i z_j z_k}{\lambda_i \lambda_j \lambda_k} + \ldots \right)$$

- $b_{ij}(T)$, $b_{ijk}(T)$ encode effects of two- and three-body correlations

no correlations $\Rightarrow$ independent particles $\Rightarrow$ $b_{ij} = 0,\ b_{ijk} = 0,\ \ldots$

Virial Equation of State (VEoS)

- particle number densities

$$n_i = -\frac{1}{V}\frac{\partial \Omega}{\partial \mu_i}\bigg|_{T,V} = b_i \frac{z_i}{\lambda_i^3} + 2\sum_j b_{ij}\frac{z_i z_j}{\lambda_i^{3/2}\lambda_j^{3/2}} + 3\sum_{jk} b_{ijk}\frac{z_i z_j z_k}{\lambda_i \lambda_j \lambda_k} + \ldots$$

contributions from free particles and correlated particles

- virial equation of state

$$\frac{pV}{NT} = -\frac{\Omega}{\sum_i n_i V T}$$

$$= 1 + \sum_{ij} a_{ij}(\lambda_i^3 n_i)^{1/2}(\lambda_j^3 n_j)^{1/2} + \sum_{ijk} a_{ijk}(\lambda_i^3 n_i)^{2/3}(\lambda_j^3 n_j)^{2/3}(\lambda_k^3 n_k)^{2/3} + \ldots$$

with virial coefficients a_{ij}, total particle number $N = \sum_i N_i$ by eliminating fugacities

- final task: determine cluster (virial) coefficients !
  - b_{ij} simple !
  - $b_{ijk}, \ldots$ difficult

Virial Equation of State (VEoS)

determination of second cluster (virial) coefficient

- interaction between two-particles independent of c.m. momentum
  $\Rightarrow$ transformation to c.m. and relative coordinates

$$\vec{R}_{ij} = \frac{1}{M_{ij}} \left(m_i \vec{r}_i + m_j \vec{r}_j \right) \qquad \vec{r}_{ij} = \vec{r}_i - \vec{r}_j$$

$$\vec{P}_{ij} = \vec{p}_i + \vec{p}_j \qquad \vec{p}_{ij} = \mu_{ij} \left(\frac{\vec{p}_i}{m_i} - \frac{\vec{p}_j}{m_j} \right)$$

  with total mass $M_{ij} = m_i + m_j$ and reduced mass $\mu_{ij} = m_i m_j / M_{ij}$

- second cluster (virial) coefficient in classical mechanics

$$b_{ij} = \lambda_i^{3/2} \lambda_j^{3/2} \left(Q_{ij} - Q_i Q_j \right) / (2V)$$

$$= \frac{1}{2} \frac{g_{ij}}{\lambda_i^{3/2} \lambda_j^{3/2}} \int d^3 r_{ij} \, \left\{ \exp \left[-\beta V_{ij}(r_{ij}) \right] - 1 \right\}$$

  with two-body Hamiltonian $H_{ij} = \frac{p_i^2}{2m_i} + \frac{p_j^2}{2m_j} + V_{ij}(r_{ij}) = \frac{P_{ij}^2}{2M_{ij}} + \frac{p_{ij}^2}{2\mu_{ij}} + V_{ij}(r_{ij})$

Virial Equation of State (VEoS)

- second cluster (virial) coefficient in quantum mechanics

  (G. E. Beth and E. Uhlenbeck Physica 3 (1936) 729, Physica 4 (1937) 915)

$$b_{ij}(T) = \frac{1+\delta_{ij}}{2}\left(\frac{m_i + m_j}{\sqrt{m_i m_j}}\right)^{3/2} \int dE\, D_{ij}(E) \exp\left(-\frac{E}{T}\right)$$

  with two-body density of states

$$D_{ij}(E) = \sum_k g_k^{(ij)} \delta(E - E_k^{(ij)}) + \sum_l \frac{g_l^{(ij)}}{\pi} \frac{d\delta_l^{(ij)}}{dE}$$

  with contributions of bound states at energies $E_k^{(ij)} < 0$

  and scattering states with phase shifts $\delta_l^{(ij)}(E)$

- experimental bound state energies/phase shifts available

  $\Rightarrow$ low-density behaviour of EoS established model-independently

  (see e.g. C. J. Horowitz, A. Schwenk, Nucl. Phys. A 776 (2006) 55)

- limitation: $n_i \lambda_i^3 \ll 1$ $\Rightarrow$ (very) low densities

Generalized Beth-Uhlenbeck Approach

extension to higher densities $\Rightarrow$ consider effects of the medium

(M. Schmidt, G. Röpke, H. Schulz, Ann. Phys. (N.Y.) 202 (1990) 57)

- use quantum statistical (QS) approach with thermodynamic Green's functions
- derive equation of state of interacting many-body system from single-particle Green's function
- Green's function of noninteracting single-particle state $j \equiv (\vec{p}_j, \sigma_j, \tau_j)$

$$G_0(j,z) = [z - E(j)]^{-1} \quad \text{with} \quad E(j) = p_j^2/(2m_j)$$

- Green's function $G(j,z)$ of interacting single-particle state j

$$G(j,z) = G_0(j,z) + G_0(j,z)\Sigma(j,z)G(j,z) \quad \text{(Dyson equation)}$$

  with self-energy $\Sigma(j,z)$ (contains information on the interaction)
- consider spectral function

$$A(j,E) = i\left[G(j,E+i0) - G(j,E-i0)\right]$$

  - probability distribution in $E, \vec{p}$
  - replaces dispersion relation $E_j = E_j(\vec{p}_j)$ of particle j in non-interacting system

Generalized Beth-Uhlenbeck Approach

- total particle density ⇒ equation of state

$$n(\mu,T,V)=\sum_j\int\frac{dE}{2\pi}f_+(E)A(j,E)$$

with Fermi distribution function $f_+(E)=\{\exp\left[\beta\left(E-\mu\right)\right]+1\}^{-1}$

- formal solution of Dyson equation

$$G(j,z)=\left[z-E(j)-\Sigma(j,z)\right]^{-1}$$ with self-energy $\Sigma=\Sigma_R+i\Sigma_I$

$$\Rightarrow A(j,E)=\frac{2\Sigma_I(j,E-i0)}{\left[E-E(j)-\Sigma_R(j,E)\right]^2+\left[\Sigma_I(j,E-i0)\right]^2}$$

- expansion of $A(j,E)$ for small $\Sigma_I=\varepsilon$ with $\frac{\varepsilon}{x^2+\varepsilon^2}=\pi\delta(x)-\varepsilon\frac{d}{dx}\mathcal{P}\frac{1}{x}+\ldots$

$$\Rightarrow A(j,E)=2\pi\delta\left[E-E(j)-\Sigma_R(j,E)\right]$$ quasiparticle contribution

$$-2\Sigma_I(j,E-i0)\,\frac{d}{dx}\mathcal{P}\frac{1}{x}\Big|_{x=E-E(j)-\Sigma_R(j,E)}+\ldots$$ correlation contribution

⇒ define quasparticle energy $e(j)$: solution of $e(j)=E(j)+\Sigma_R(j,e(j))$

Generalized Beth-Uhlenbeck Approach

⇒ total particle density

$$n(\mu, T, V) = \sum_j \int \frac{dE}{2\pi} f_+(E) A(j, E) = n_{\rm free} + 2 n_{\rm corr}$$ with

- $n_{\rm free} = \sum_j f_+[e(j)]$ density of free quasiparticles

  with medium dependent self-energies

- $n_{\rm corr}$ correlation contribution from two-body states
  - ∘ properties of two-body states depend on c.m. momentum $\vec{P}$
  - ∘ no bound two-body states for $P < P_{\rm Mott}$ Mott momentum (Pauli principle !)
  - ∘ binding energies $B_k = B_k(P, \mu, T)$ and two-body scattering phase shifts $\delta_l = \delta_l(P, \mu, T)$ depend on $\vec{P}$ and medium properties (μ, T)
  - ∘ define continuum edge $E_{\rm cont} = E_{\rm cont}(P, \mu, T) \equiv$ energy of scattering state with zero relative momentum

Generalized Beth-Uhlenbeck Approach

- correlation density

$$n_{\rm corr} = \sum_k g_k^{(2)} \sum_{\vec{P}, P > P_{\rm Mott}} f_-(E_{\rm cont} - B_k)$$ contribution of bound states

$$+ \sum_l g_l^{(2)} \sum_{\vec{P}} \int \frac{dE}{\pi} 2\sin^2\delta_l \frac{d\delta_l}{dE} f_-(E_{\rm cont} + E)$$ continuum contribution

with Bose-Einstein distribution function $f_-(E) = \{\exp[\beta(E - \mu)] - 1\}^{-1}$

- comparison with standard second cluster (virial) coefficient
  - explicit summation over c.m. momentum $\vec{P}$
  - Bose-Einstein statistics
  - additional $2\sin^2\delta_l$ factor
  - medium-dependent binding energies B_k and phase shifts δ_l

Generalized Beth-Uhlenbeck Approach

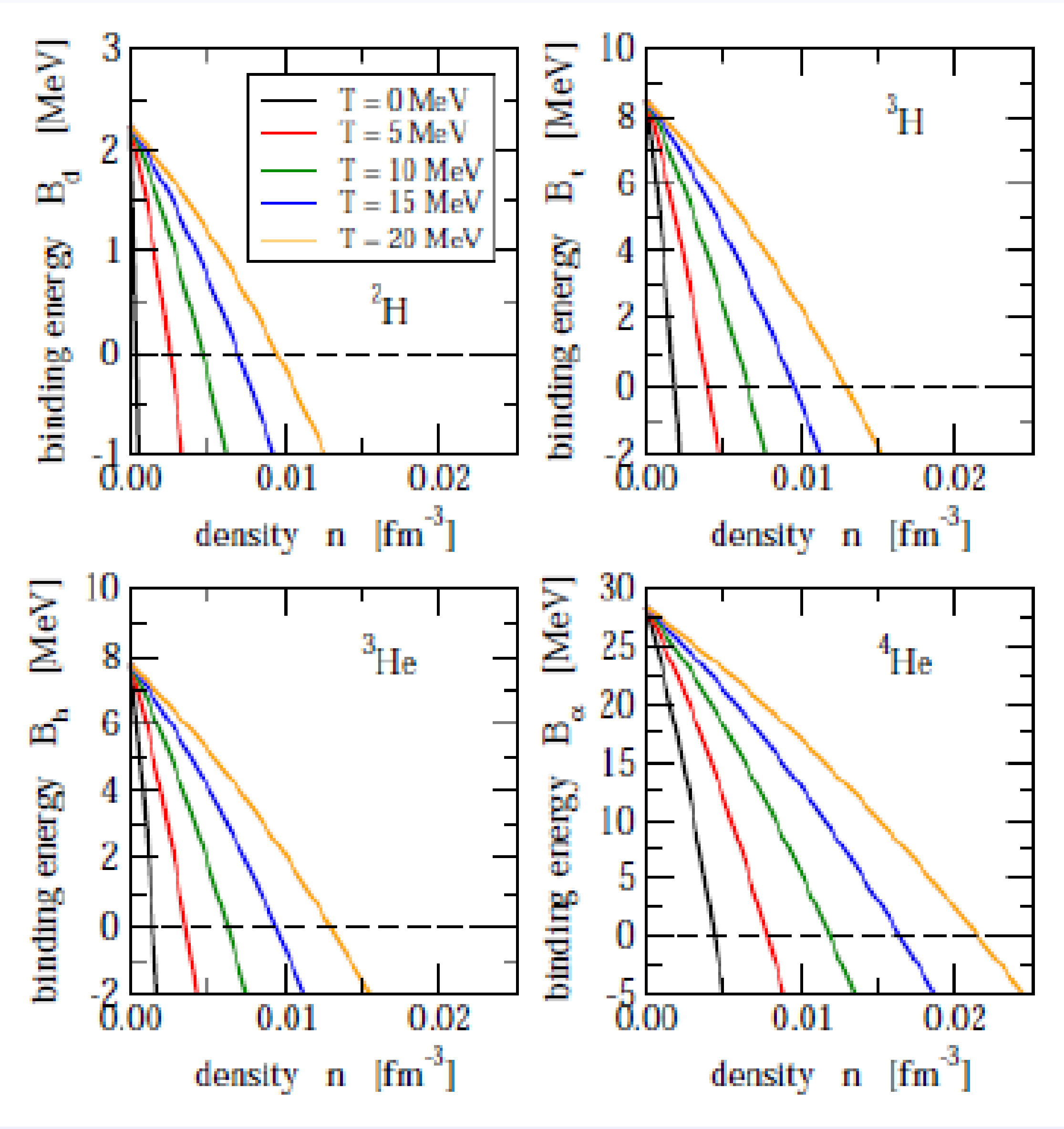
important features of QS approach

- medium-dependent self-energies
  - for free quasiparticles
  - for states of correlated particles
  - calculation e.g. in Skyrme-Hartree-Fock (SHF) or relativistic mean-field (RMF) models
- medium-dependent shift of binding energies
  - main effect: Pauli principle ⇒ blocking of states by medium !
  - calculation perturbatively/variationally with (separable) realistic nucleon-nucleon potentials
- scattering phase shifts from in-medium T-matrix
  - part of continuum strength moved to self-energies
- dissolution of clusters at high densities ⇒ Mott effect
- extensions possible to include three- and four-body correlations

Generalized Beth-Uhlenbeck Approach

shift of binding energies

- example: symmetric nuclear matter, nuclei at rest in medium
- in vacuum: experimental binding energies
- nuclei become unbound $(B_i < 0)$ with increasing density of medium
- parametrization of results used in generalized relativistic density functional



Generalized Relativistic Density Functional

nuclear matter around saturation density

- successful methods: phenomenological mean-field approaches
  - effective interactions, parameters fitted to properties of finite nuclei
  - nonrelativistic Hartree-Fock calculations
    with, e.g., Skyrme/Gogny interaction
  - relativistic mean-field models
    with nonlinear meson self-interactions
    or density dependent meson-nucleon couplings

- problem: no correlations !

  idea: include two-, three-, four-body correlations as new degrees of freedom (clusters) with medium-dependent properties

Generalized Relativistic Density Functional

generalization of relativistic mean-field (RMF) models

- extended relativistic density functional
  with nucleons (ψ_p, ψ_n), deuterons (φ_d^μ), tritons (ψ_t), helions (ψ_h), α-particles (φ_α), mesons ($\sigma, \omega_\mu, \vec{\rho}_\mu$), electrons ($\psi_e$) and photons ($A_\mu$) as degrees of freedom
  - only minimal (linear) meson-nucleon couplings
  - density-dependent meson-nucleon couplings Γ_i
    - functional form as suggested by Dirac-Brueckner calculations of nuclear matter
    - more flexible approach than models with non-linear meson self-interactions
  - parameters: nucleon/meson masses, coupling strengths/density dependence
    - fitted to properties of finite nuclei
  - cluster binding energies/effective resonance energies
    - density dependence replaced by dependence on vector meson fields

$\Rightarrow$ nucleon/cluster/meson/photon field equations, solved selfconsistently

Generalized Relativistic Density Functional

- grand canonical thermodynamical potential

$$\Omega = -pV = \int d^3r\, \omega_g(T, \mu_i, \sigma, \omega_0, \rho_0, A, \vec{\nabla}\sigma, \vec{\nabla}\omega_0, \vec{\nabla}\rho_0, \vec{\nabla}A)$$

  with density functional ω_g depending on
  temperature T, chemical potentials μ_i, meson and photon fields $\sigma, \delta, \omega_0, \rho_0, A$

  - fields equations with additional rearrangement contributions
  - consistent derivation of thermodynamical quantities

- light clusters (^{2}H, ^{3}H, ^{3}He, ^{4}He), two-nucleon scattering correlations:
  - explicitly included
- heavy clusters: (not considered here)
  - Thomas-Fermi approximation in spherical Wigner-Seitz cells

- low-density limit, finite temperature:
  only nucleons and light clusters ⇒ reproduction of standard virial EoS

(details: S. Typel et al., Phys. Rev. C 81 (2010) 015803)

Generalized Relativistic Density Functional

Relation and Differences of Models

quantum statistical approach		generalized relativistic density functional
empirical nucleon-nucleon potential ⇓ medium dependence of cluster binding energies	⇒	parametrization of binding energy shifts
parametrization of nucleon self-energy and effective mass in nonrelativistic approximation	⇐	phenomenological meson-nucleon interaction ⇓ scalar/vector nucleon self-energies
no effect of cluster formation on nucleon mean fields		medium-dependent change of cluster properties induces change of mean fields

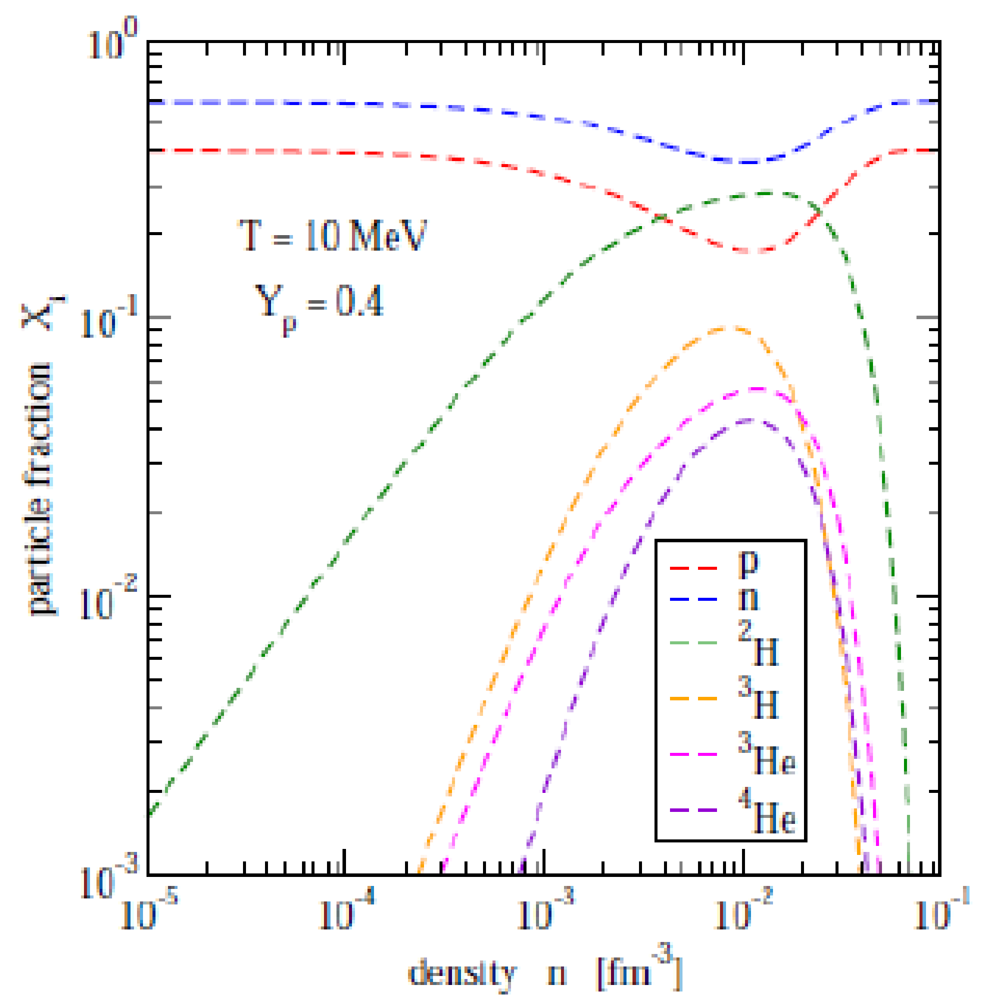
Formation and Dissociation of Clusters

- particle fractions

$$X_i = A_i \frac{n_i}{n_b} \qquad n_b = \sum_i A_i n_i$$

- low densities:
  two-body correlation most important
- high densities:
  dissolution of clusters
  $\Rightarrow$ Mott effect

generalized relativistic density functional



(without heavy clusters)

Formation and Dissociation of Clusters

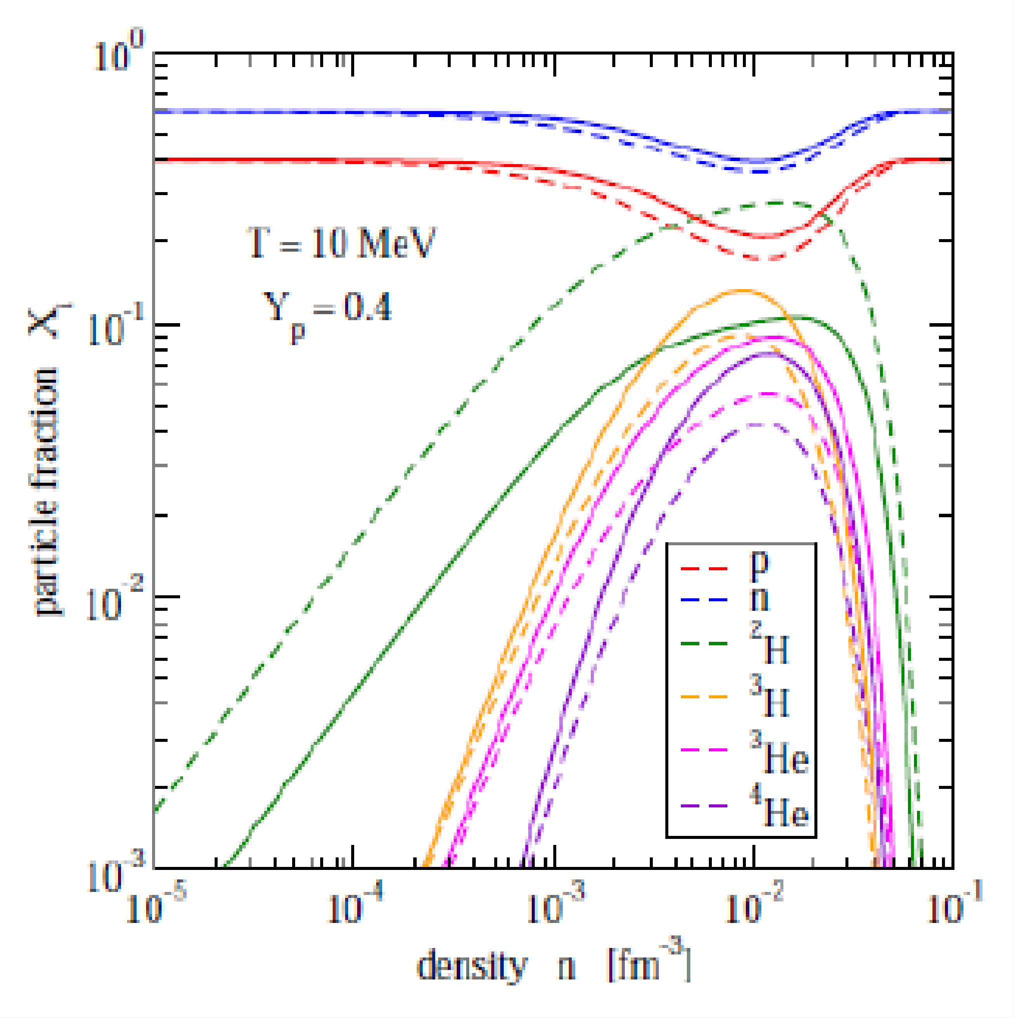
- particle fractions

$$X_i = A_i \frac{n_i}{n_b} \qquad n_b = \sum_i A_i n_i$$

- low densities:
  two-body correlation most important
- high densities:
  dissolution of clusters
  $\Rightarrow$ Mott effect
- effect of NN continuum correlations
  - dashed lines: without continuum
  - solid lines: with continuum

  $\Rightarrow$ reduction of deuteron fraction, redistribution of other particles
- correct limits in gRDF model

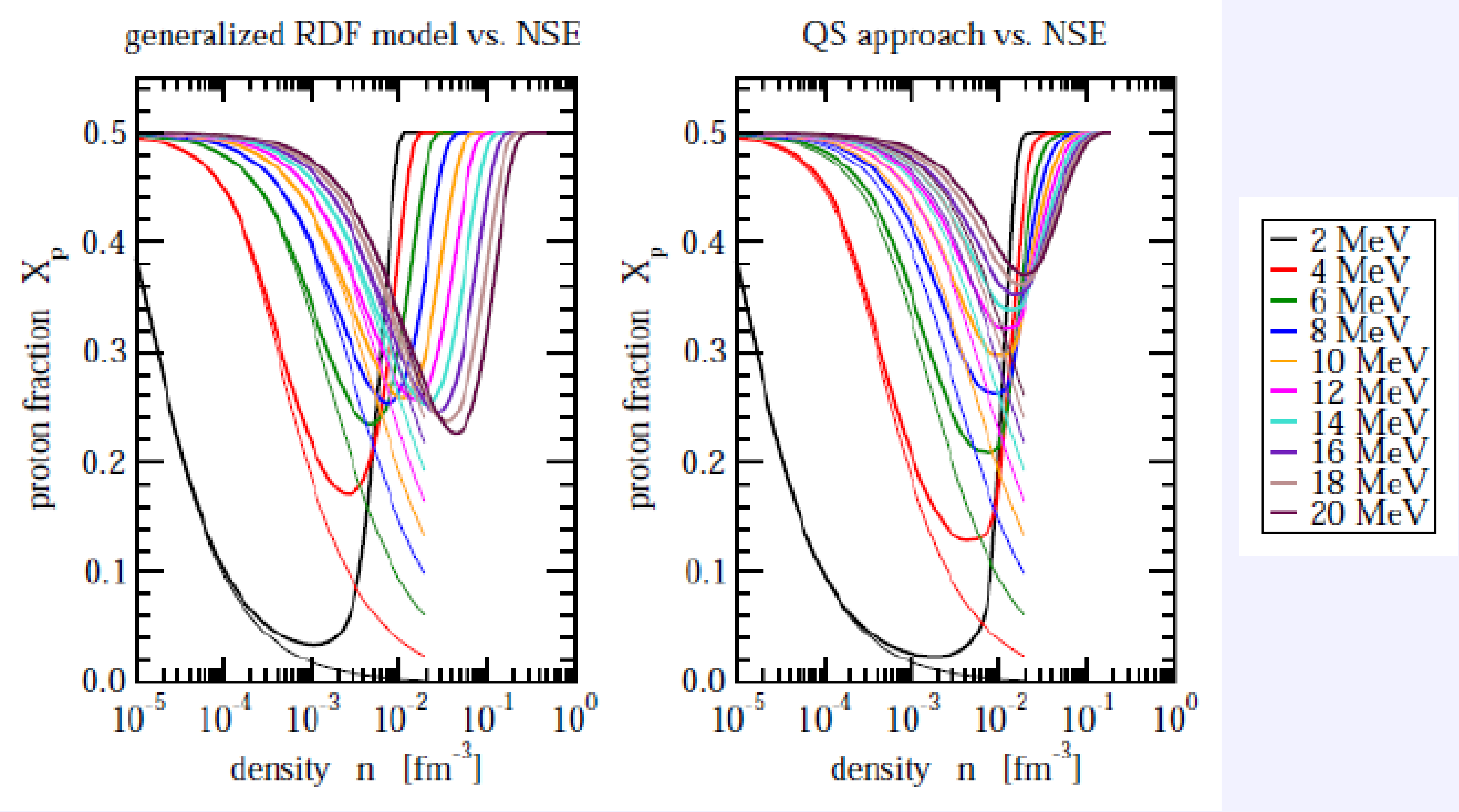
generalized relativistic density functional



(without heavy clusters)

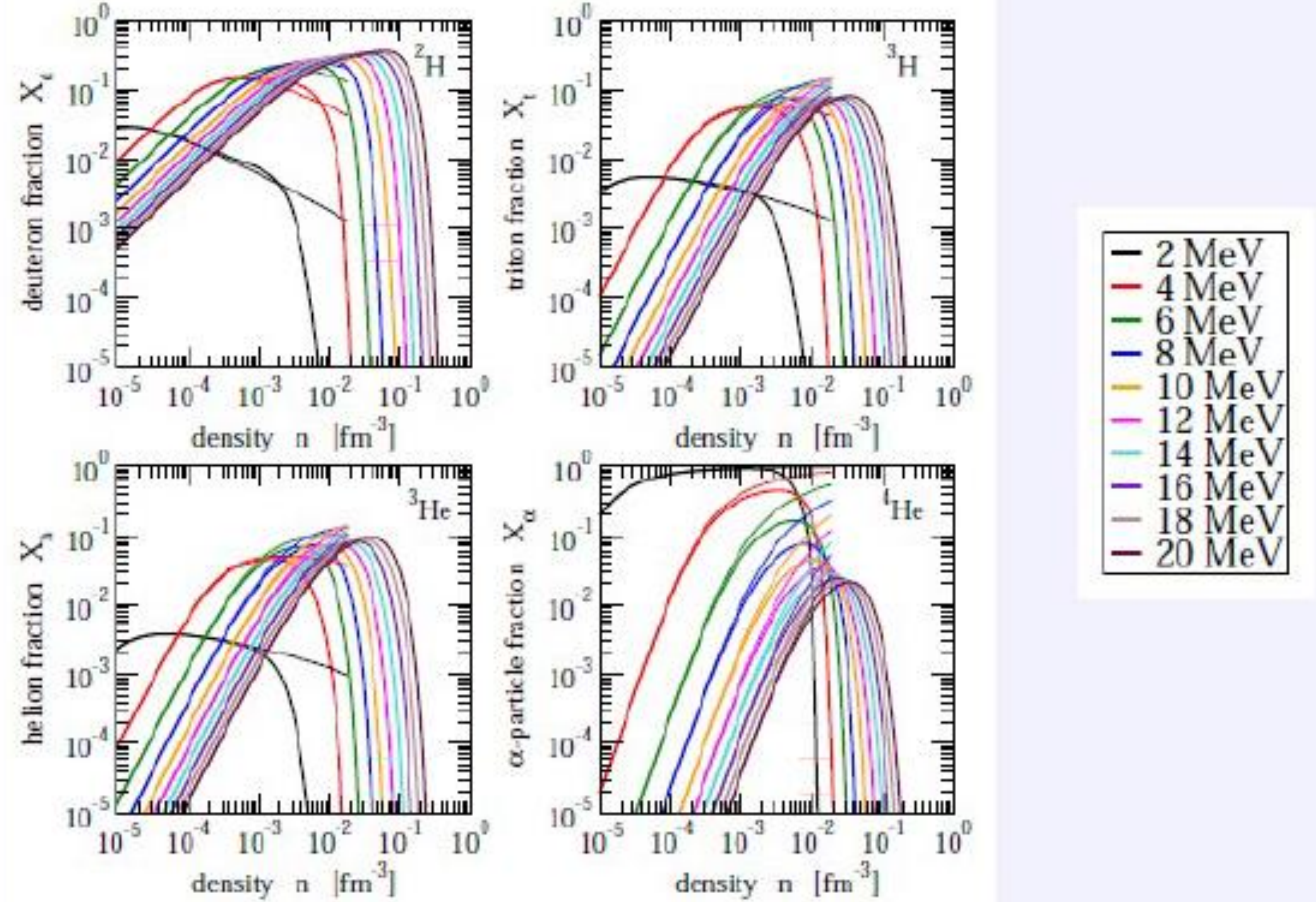
Formation and Dissociation of Clusters

fraction of free protons in symmetric nuclear matter, thin lines: NSE



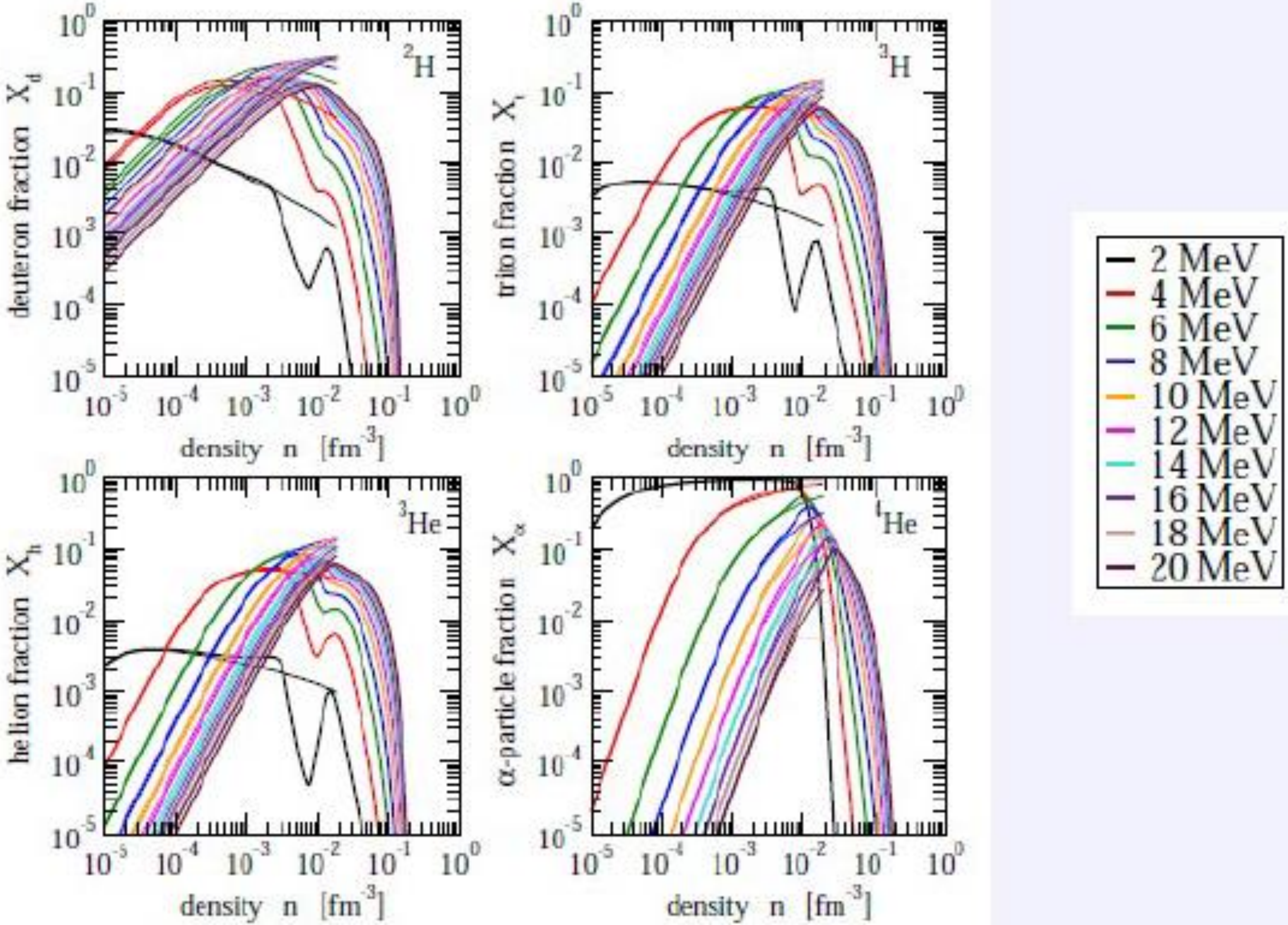
Formation and Dissociation of Clusters

fraction of clusters in symmetric nuclear matter, generalized RDF vs. NSE



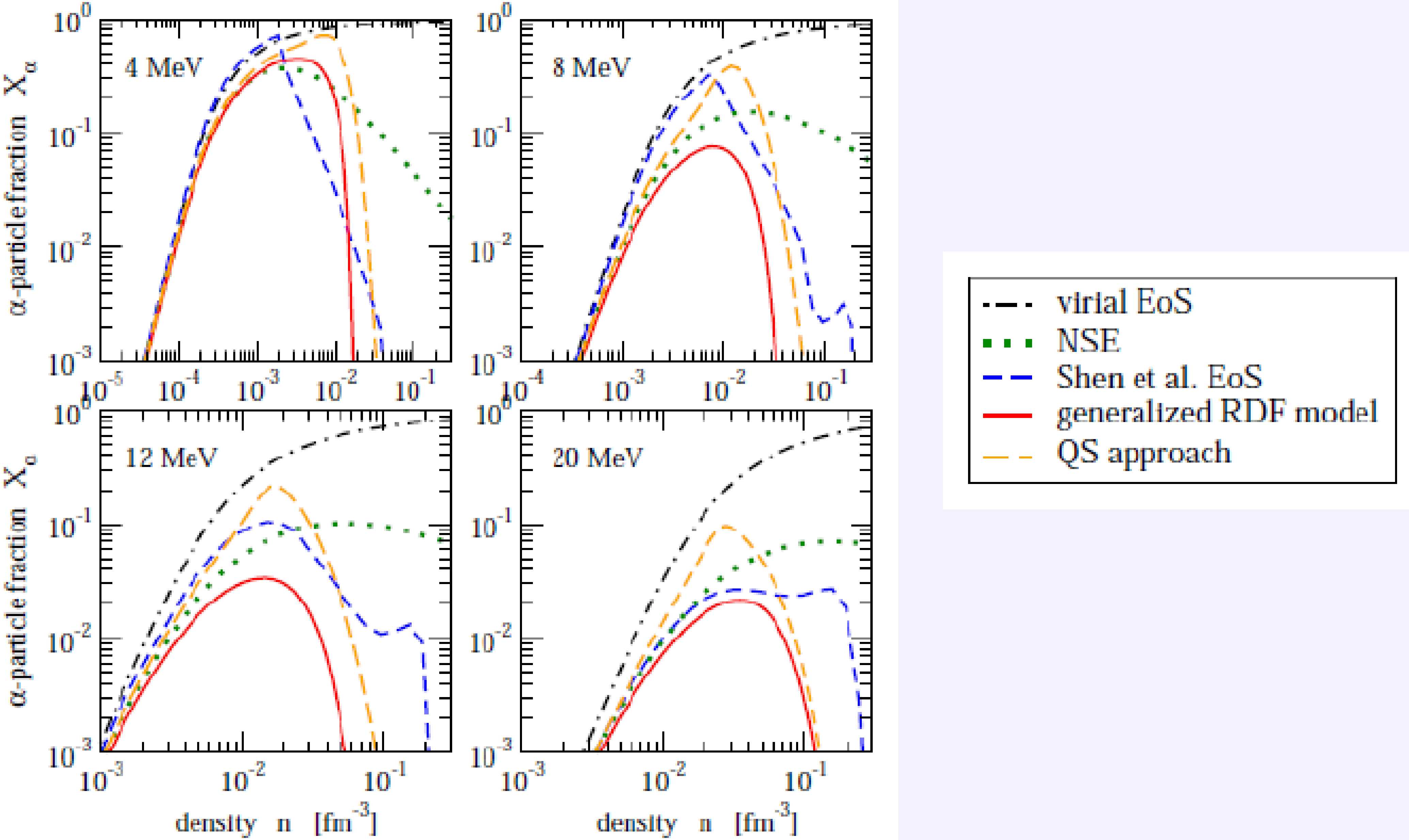
Formation and Dissociation of Clusters

fraction of clusters in symmetric nuclear matter, QS approach vs. NSE



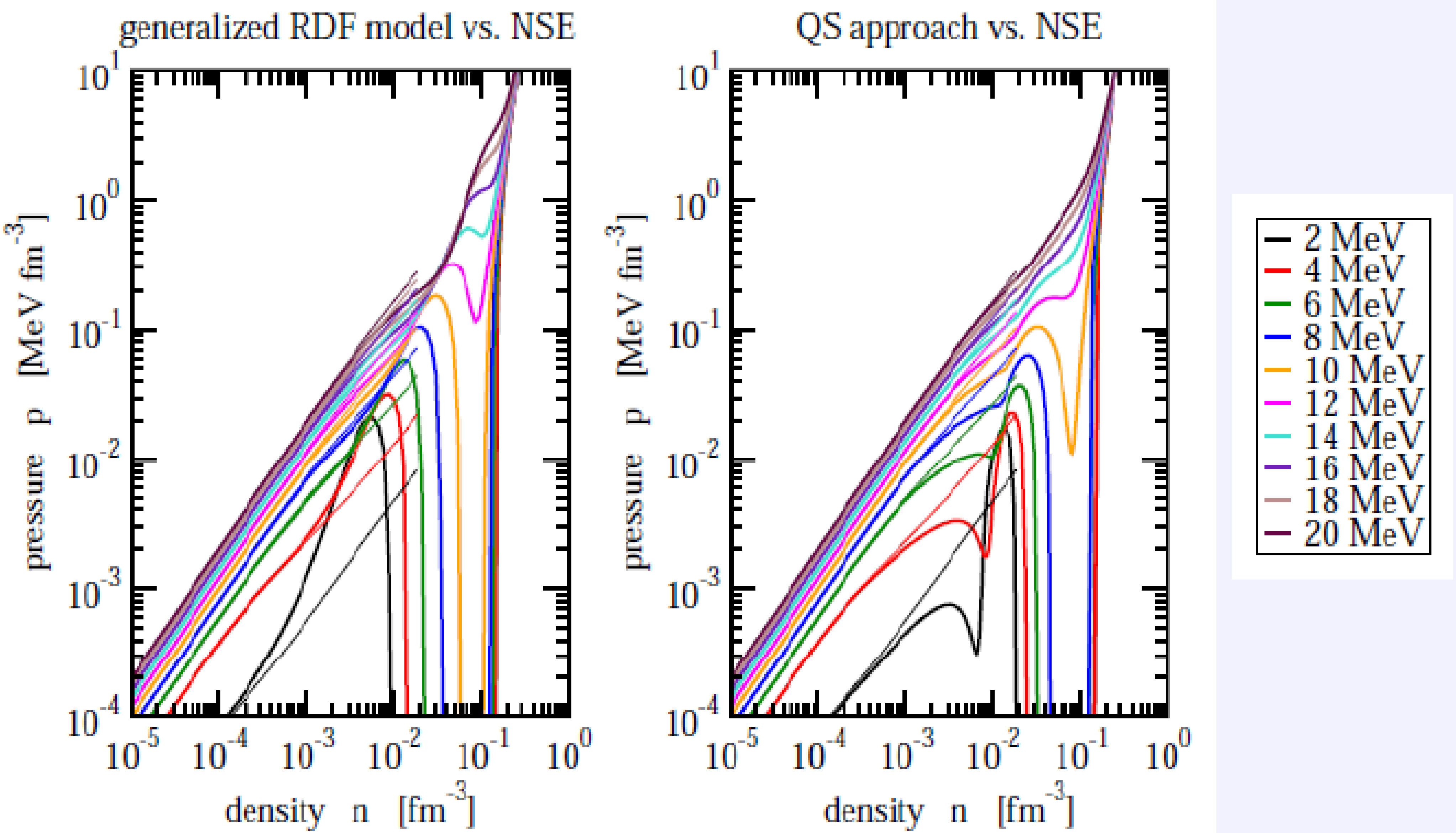
Formation and Dissociation of Clusters

alpha-particle fraction in symmetric nuclear matter



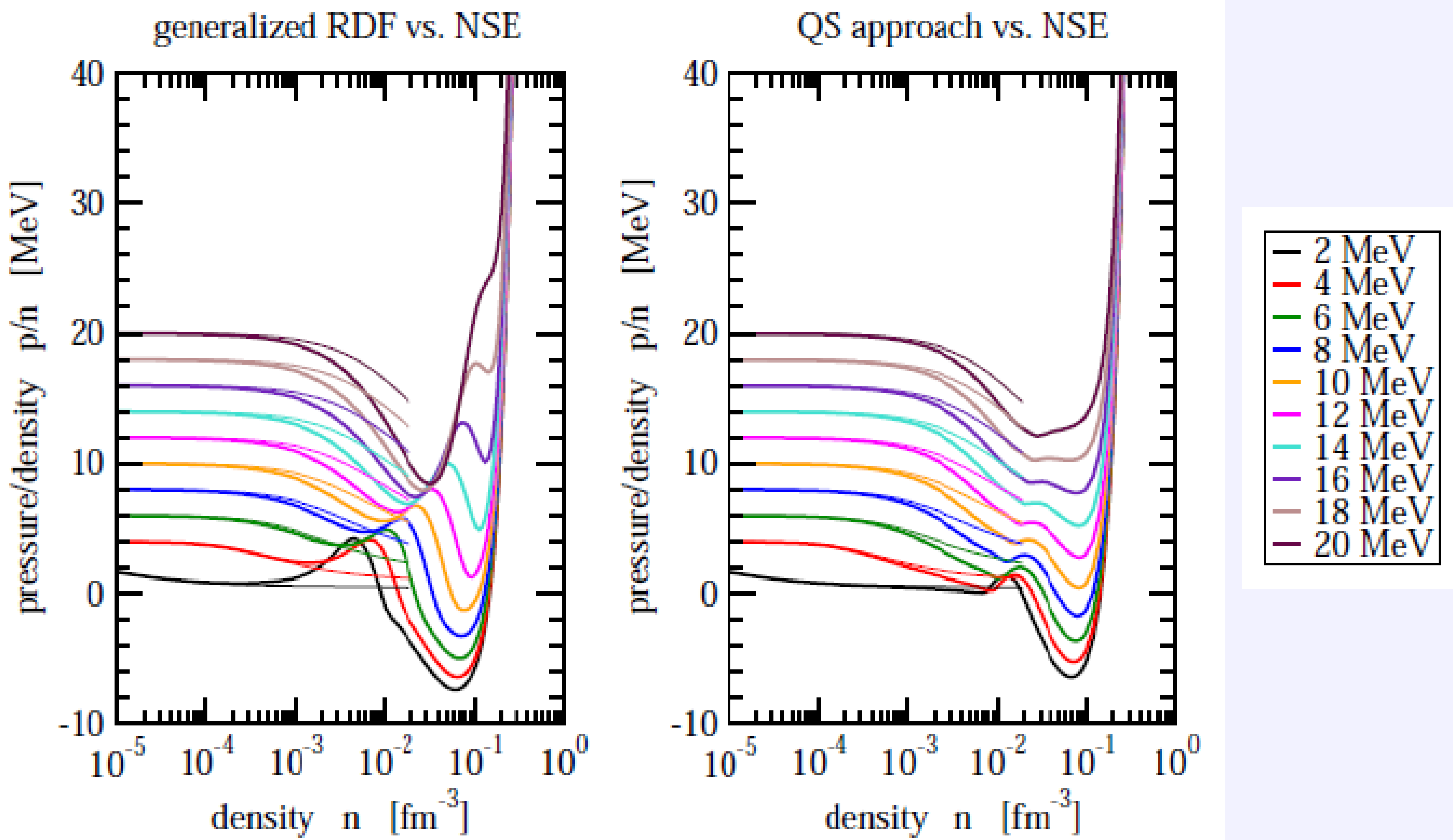
Thermodynamical Properties

pressure $p = -\Omega/V$ in symmetric nuclear matter, thin lines: NSE



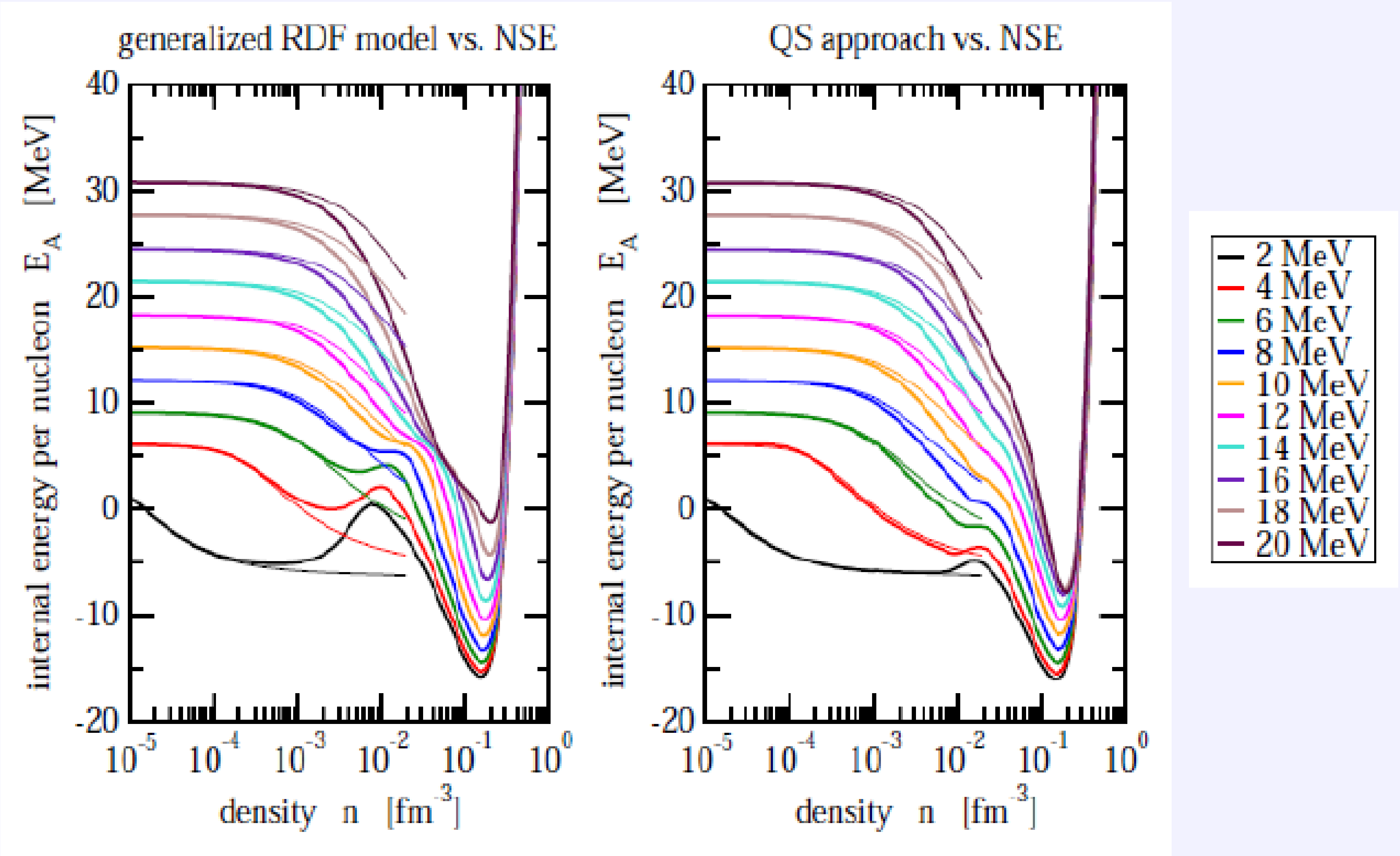
Thermodynamical Properties

p/n in symmetric nuclear matter, $\lim_{n\to 0}(p/n) = T$ (ideal gas)



Thermodynamical Properties

internal energy per nucleon E_A in symmetric nuclear matter, $\lim_{n\to 0} E_A = \frac{3}{2}T + \ldots$

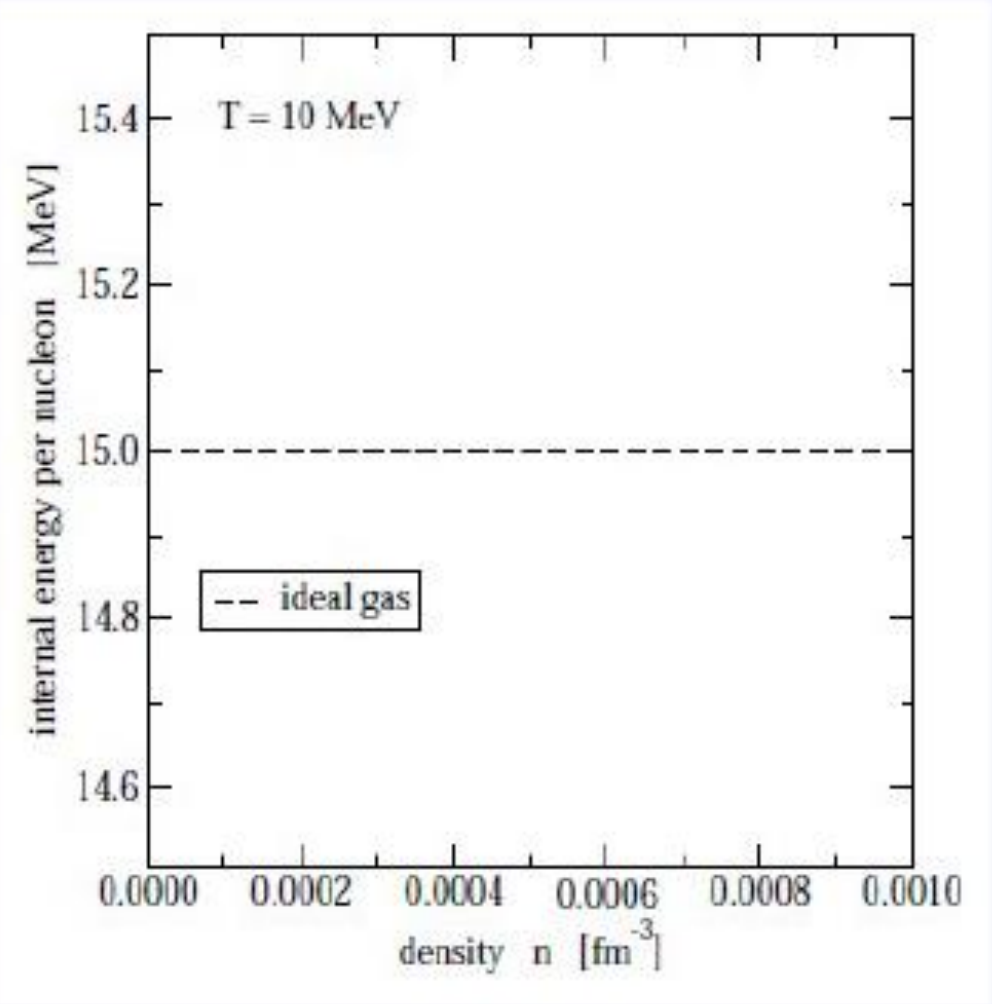


Thermodynamical Properties

neutron matter

comparison: different models and effects

- nonrelativistic ideal gas

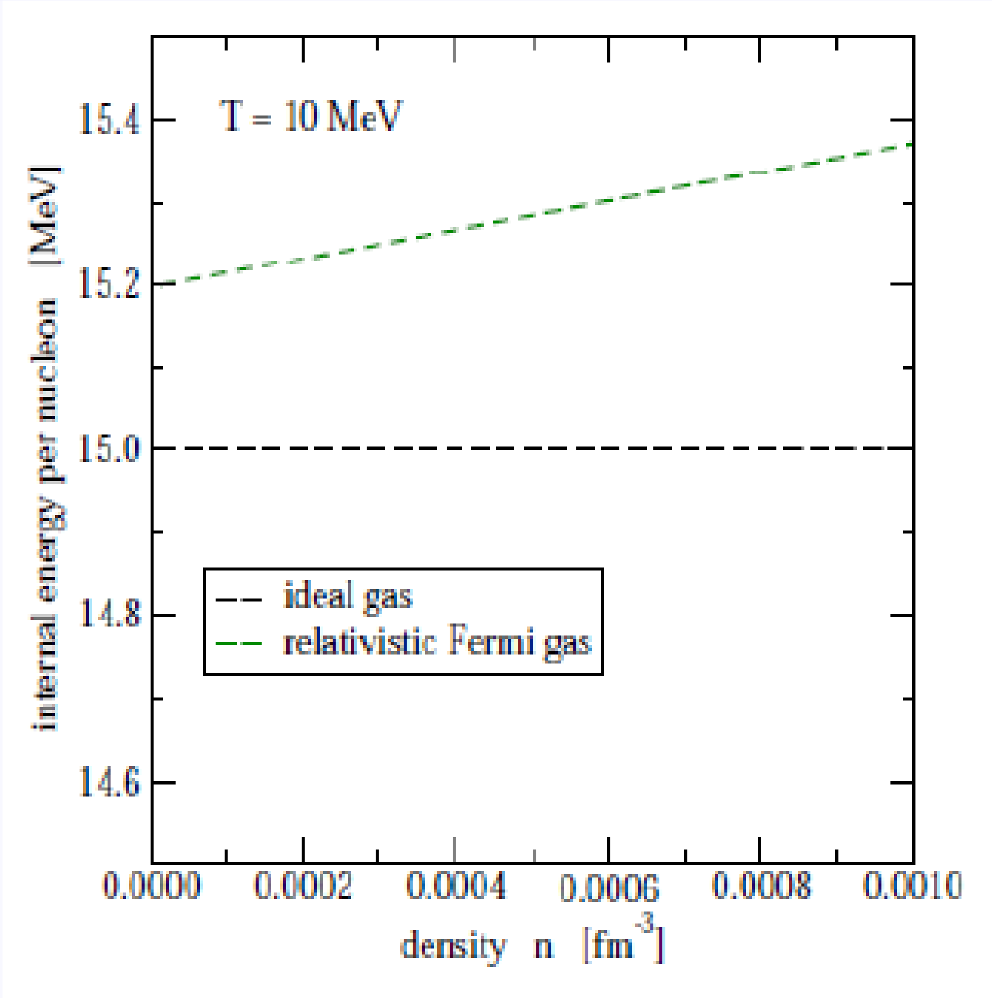


Thermodynamical Properties

neutron matter

comparison: different models and effects

- nonrelativistic ideal gas
  $\Downarrow$ rel. kinematics + statistics
- relativistic Fermi gas



Thermodynamical Properties

neutron matter

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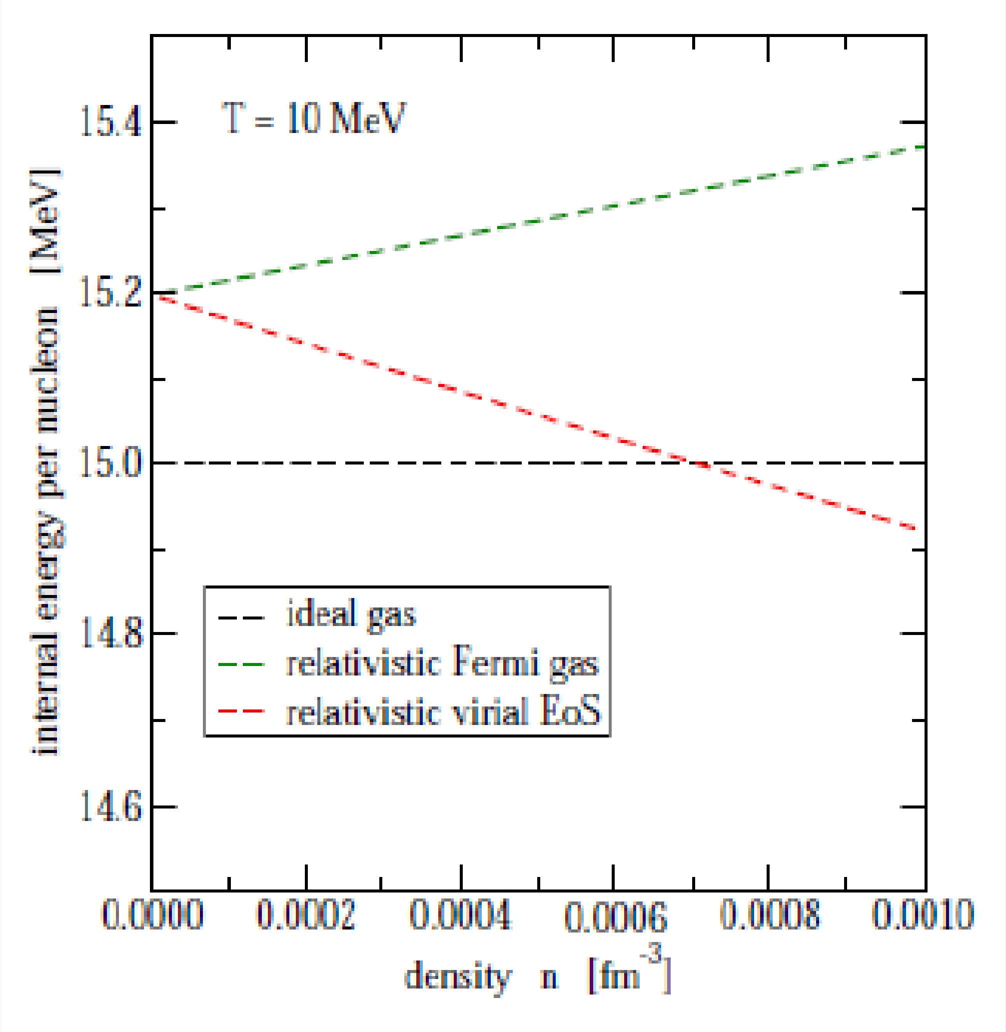
  $\Downarrow$ two-body correlations
- virial EoS with relativistic correction



Thermodynamical Properties

neutron matter

comparison: different models and effects

- nonrelativistic ideal gas
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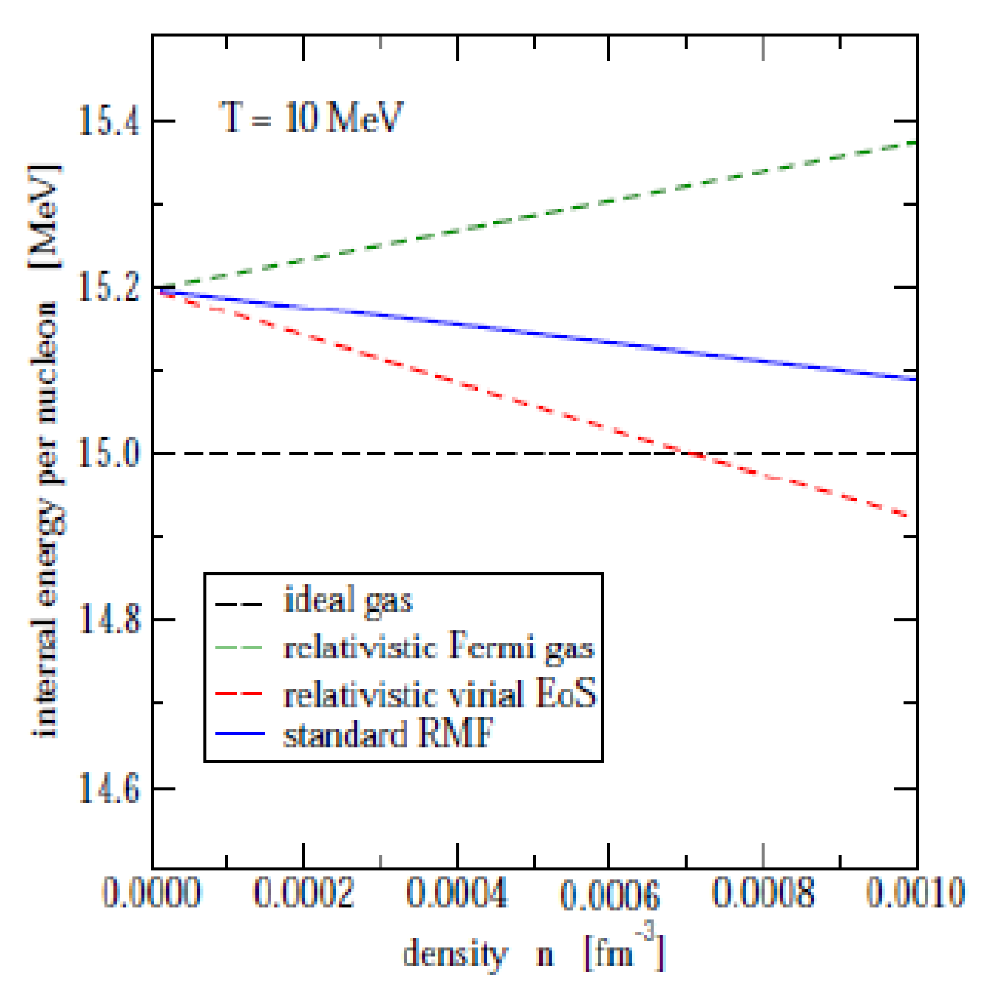
- virial EoS with relativistic correction
  (not included in standard virial EoS)

  $\Downarrow$ mean-field effects
- standard RMF model with
  density dependent couplings



Thermodynamical Properties

neutron matter

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- nonrelativistic ideal gas
  ⇓ rel. kinematics + statistics
- relativistic Fermi gas

  ⇓ two-body correlations
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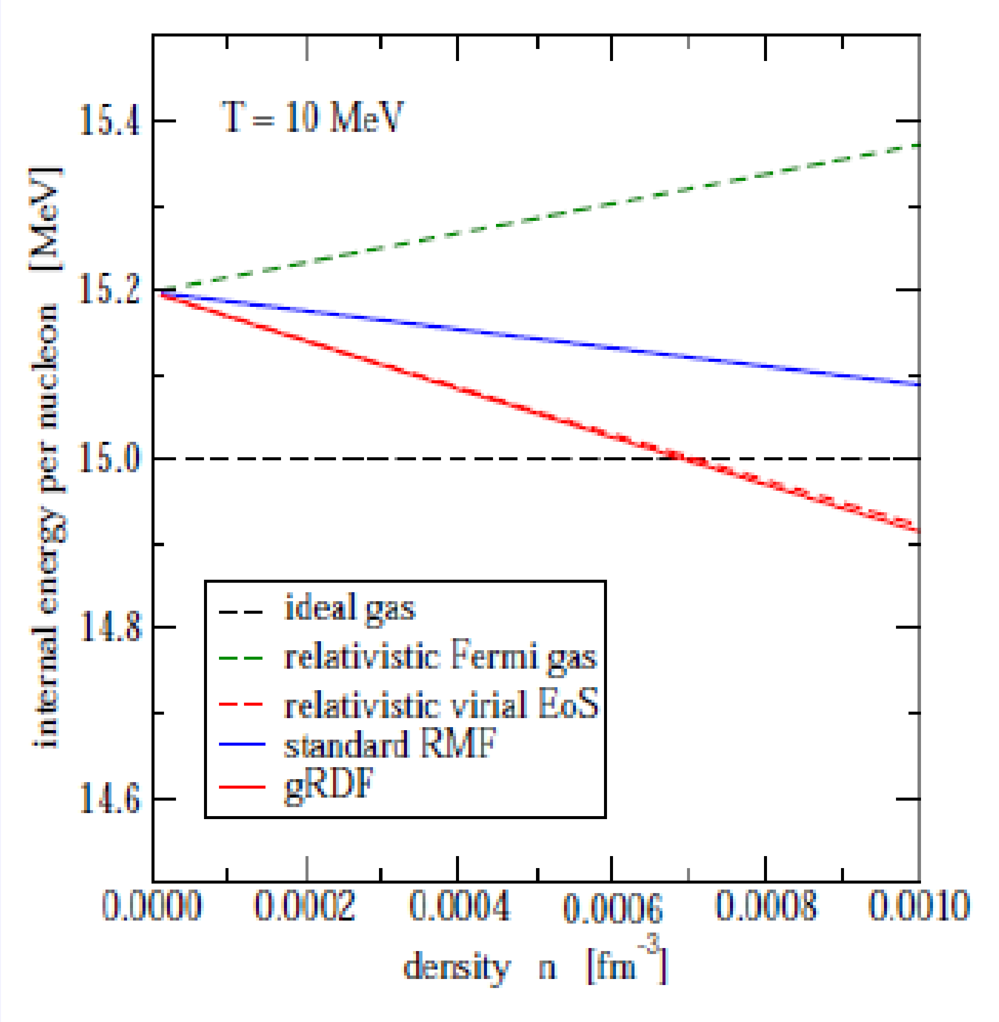
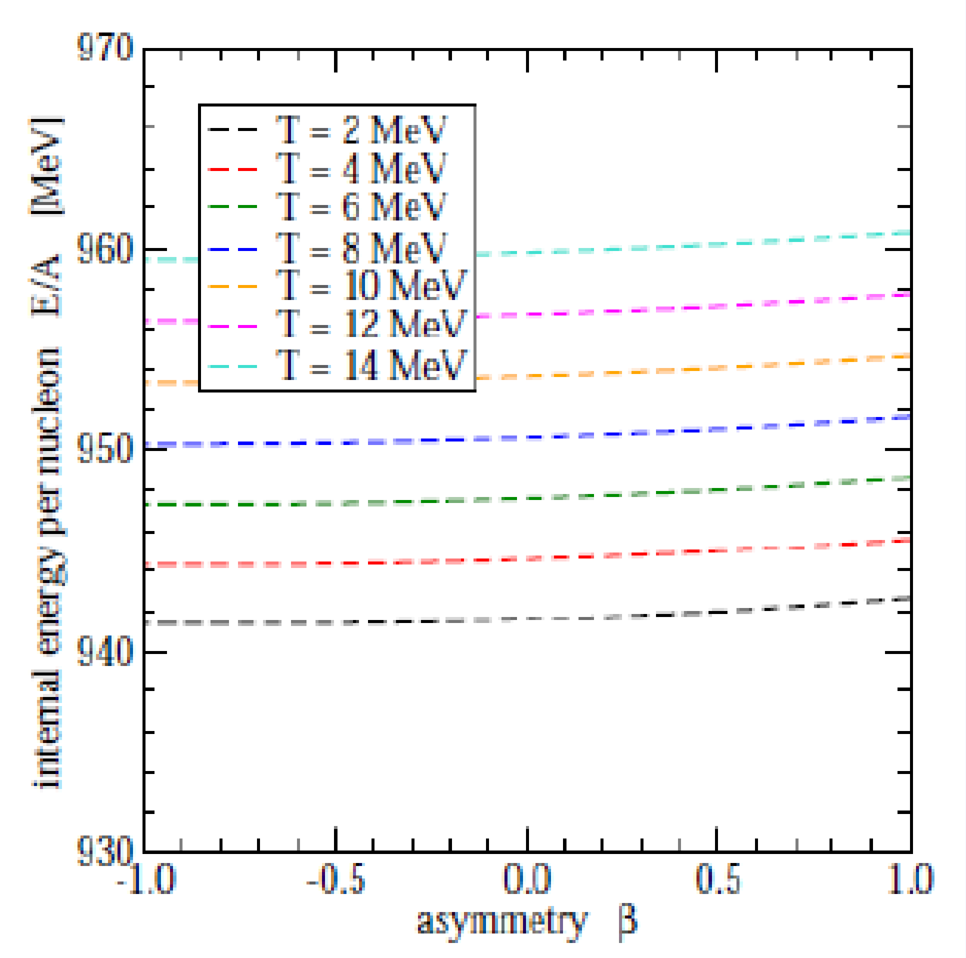
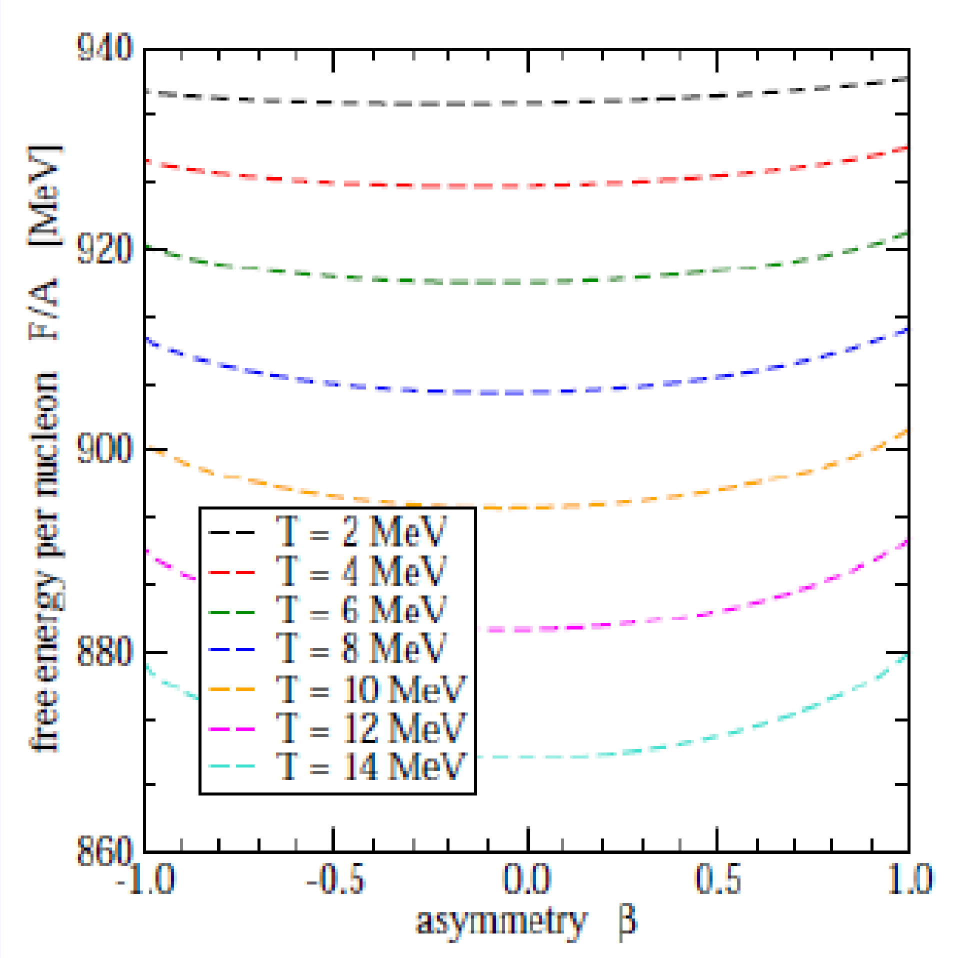
  ⇓ mean-field effects
- standard RMF model with
  density dependent couplings
  ⇓ two-body correlations
- generalized relativistic density
  functional (gRDF) with contributions
  from nn scattering



Thermodynamical Properties

- internal energy E and free energy $F = E - TS = -pV + \sum_i \mu_i N_i$
- dependence on neutron-proton asymmetry $\beta = \left(n_n^{\mathrm{tot}} - n_p^{\mathrm{tot}}\right) / \left(n_n^{\mathrm{tot}} + n_p^{\mathrm{tot}}\right)$
- constant density $n = 10^{-3}$ fm^{-3}, without (dashed lines) light clusters





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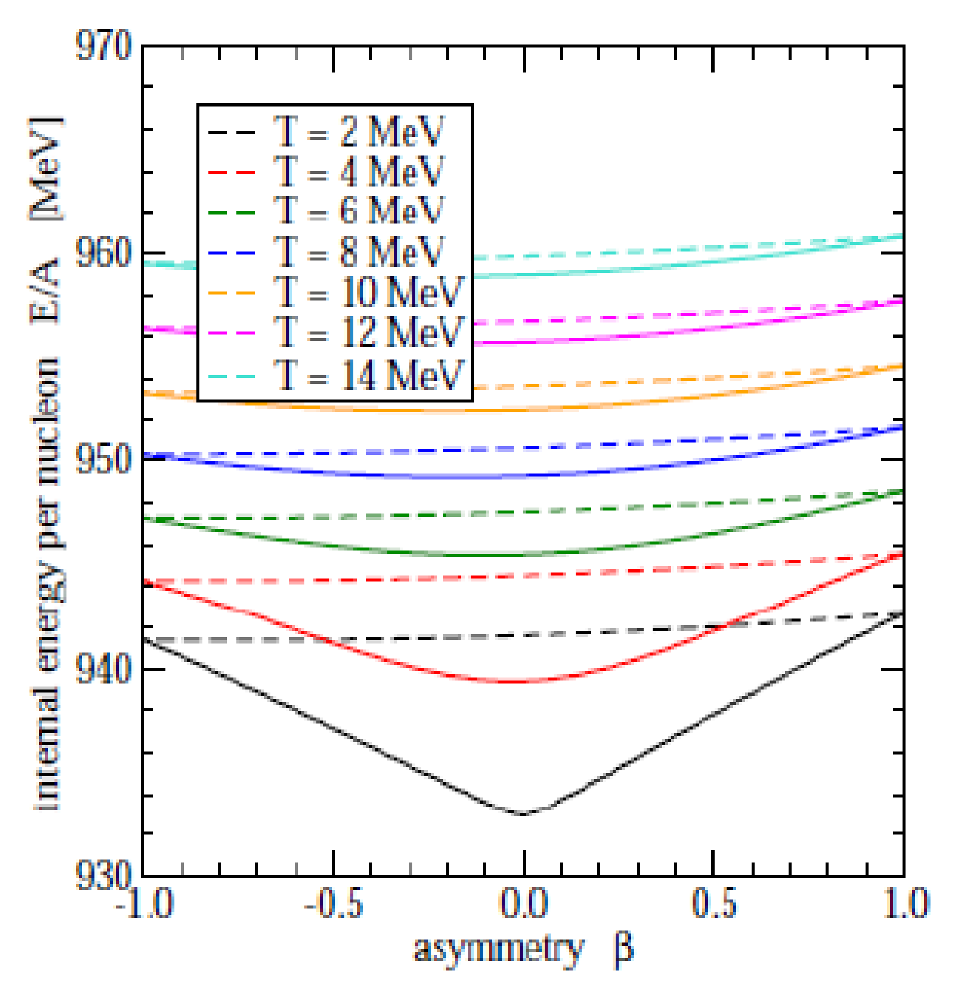
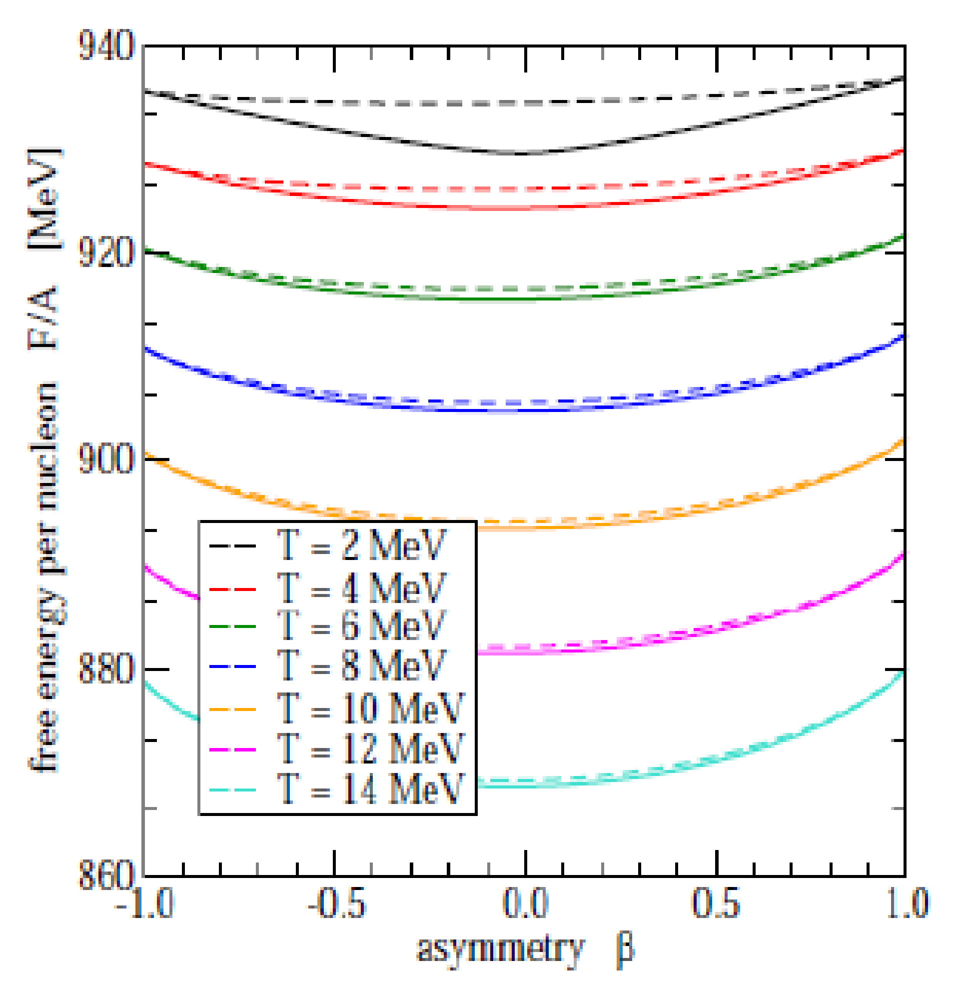




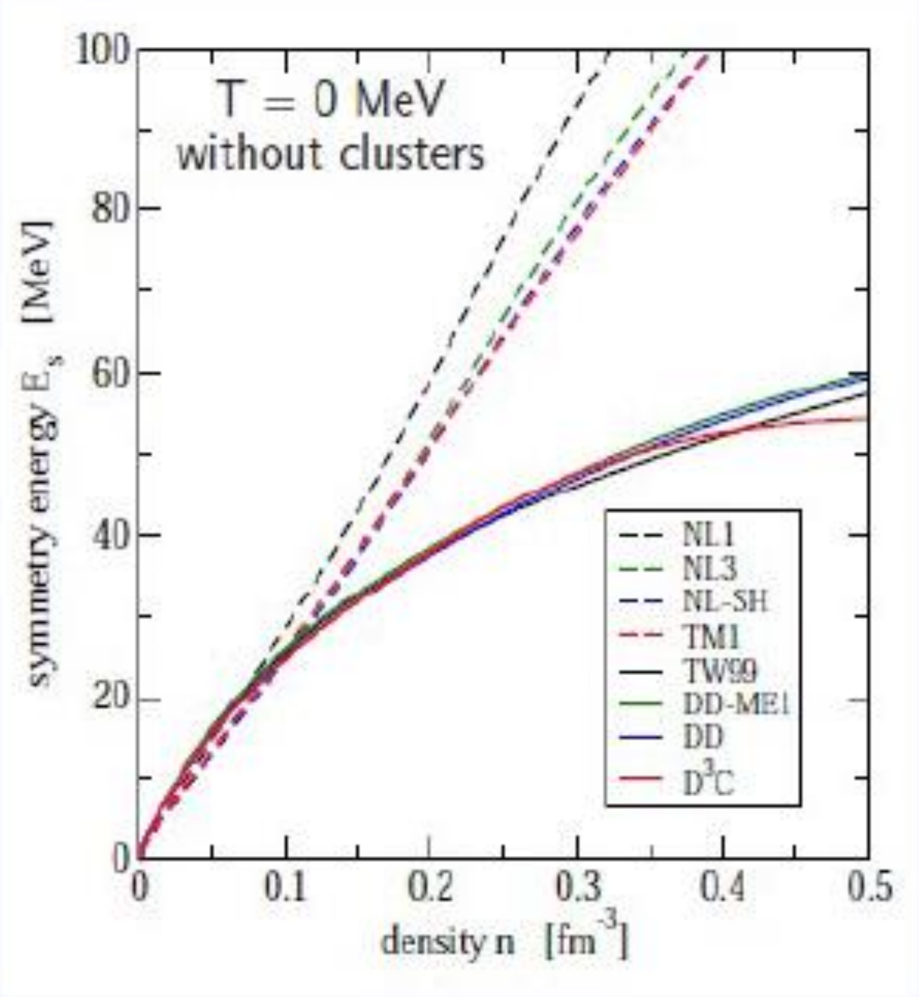
Symmetry Energy

- general definition for zero temperature:

$$E_s(n) = \frac{1}{2}\frac{\partial^2}{\partial\beta^2}\frac{E}{A}(n,\beta)\Big|_{\beta=0} \quad \beta = \frac{n_n - n_p}{n_n + n_p}$$

$\Rightarrow$ nuclear matter parameters

$$J = E_s(n_{\mathrm{sat}}) \quad L = 3n\frac{d}{dn}E_s\Big|_{n=n_{\mathrm{sat}}}$$



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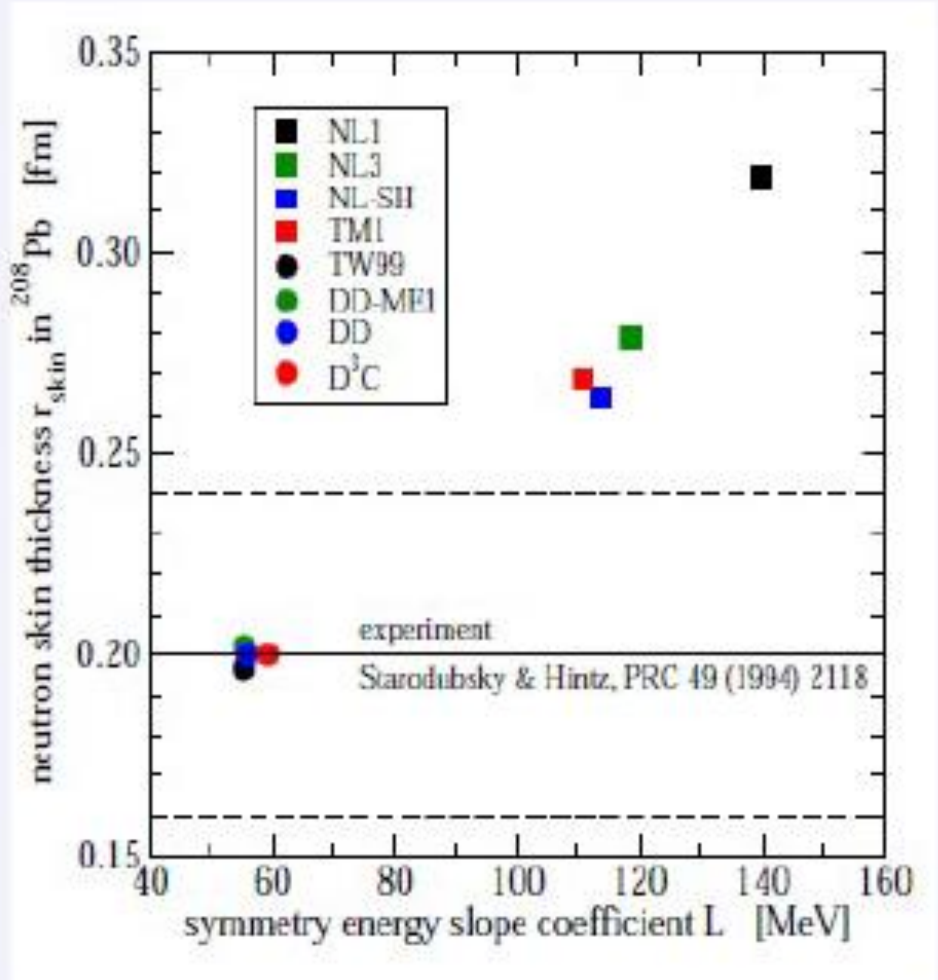
- correlation: neutron skin thickness
  $\Leftrightarrow$ slope of neutron matter EoS ($\Leftrightarrow L$)

B. A. Brown, Phys. Rev. Lett. 85 (2000) 5296,

S. Typel, B. A. Brown, Phys. Rev. C 64 (2001) 027302



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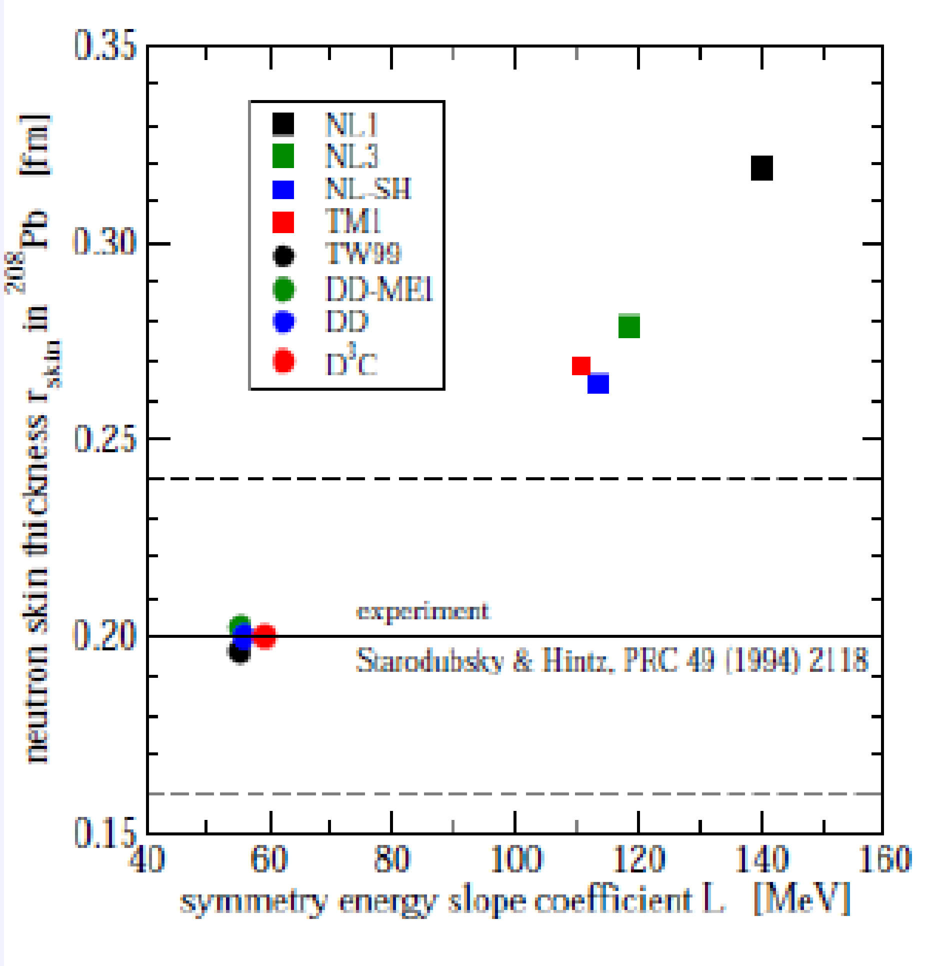


- with clusters and at finite temperatures:
  - use finite differences

$$E_s(n) = \frac{1}{2}\left[\frac{E}{A}(n,1) - 2\frac{E}{A}(n,0) + \frac{E}{A}(n,-1)\right]$$

  - distinguish free symmetry energy F_s and internal symmetry energy E_s

Symmetry Energy

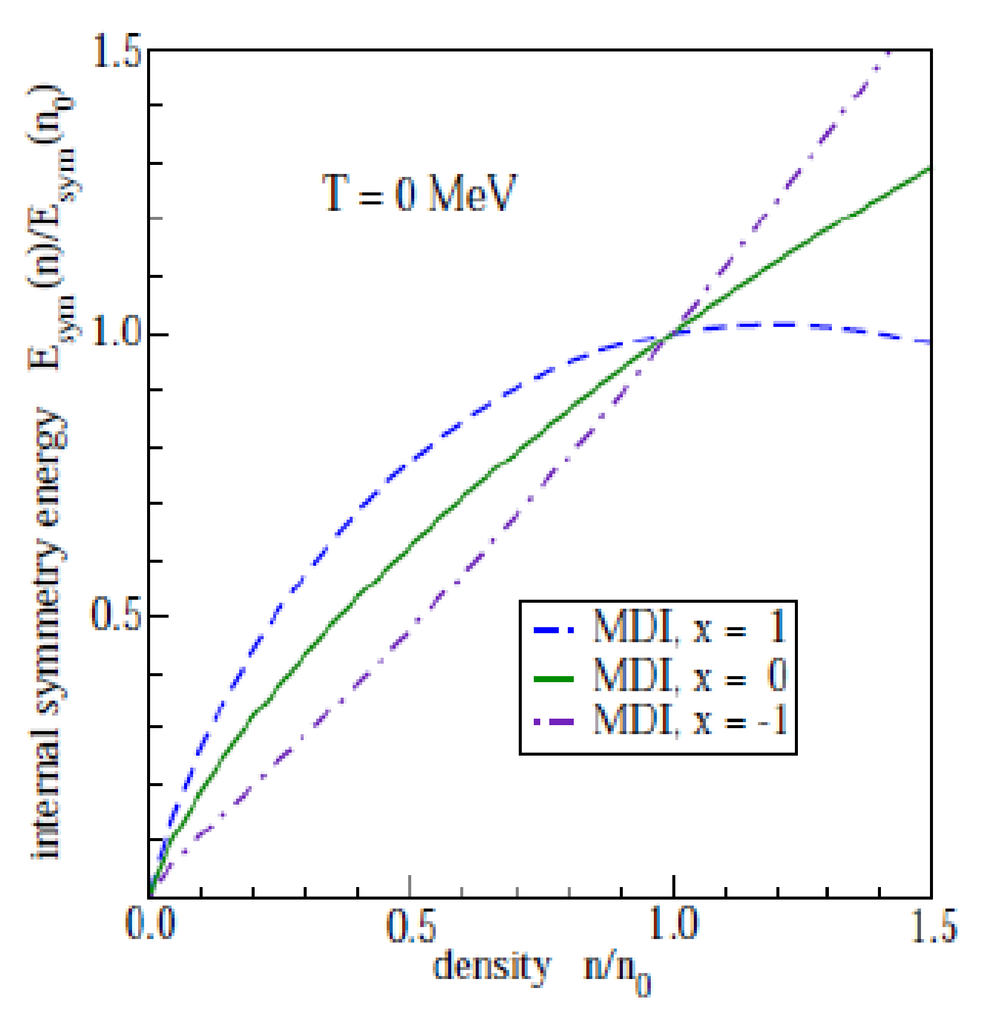
temperature $T = 0$ MeV

- mean-field models without clusters

  e.g. model with momentum-dependent interaction (MDI), parameter x controls density dependence of E_{sym}

  (B. A. Li et al., Phys. Rep. 464 (2008) 113)

  $\Rightarrow$ low-density behaviour not correct



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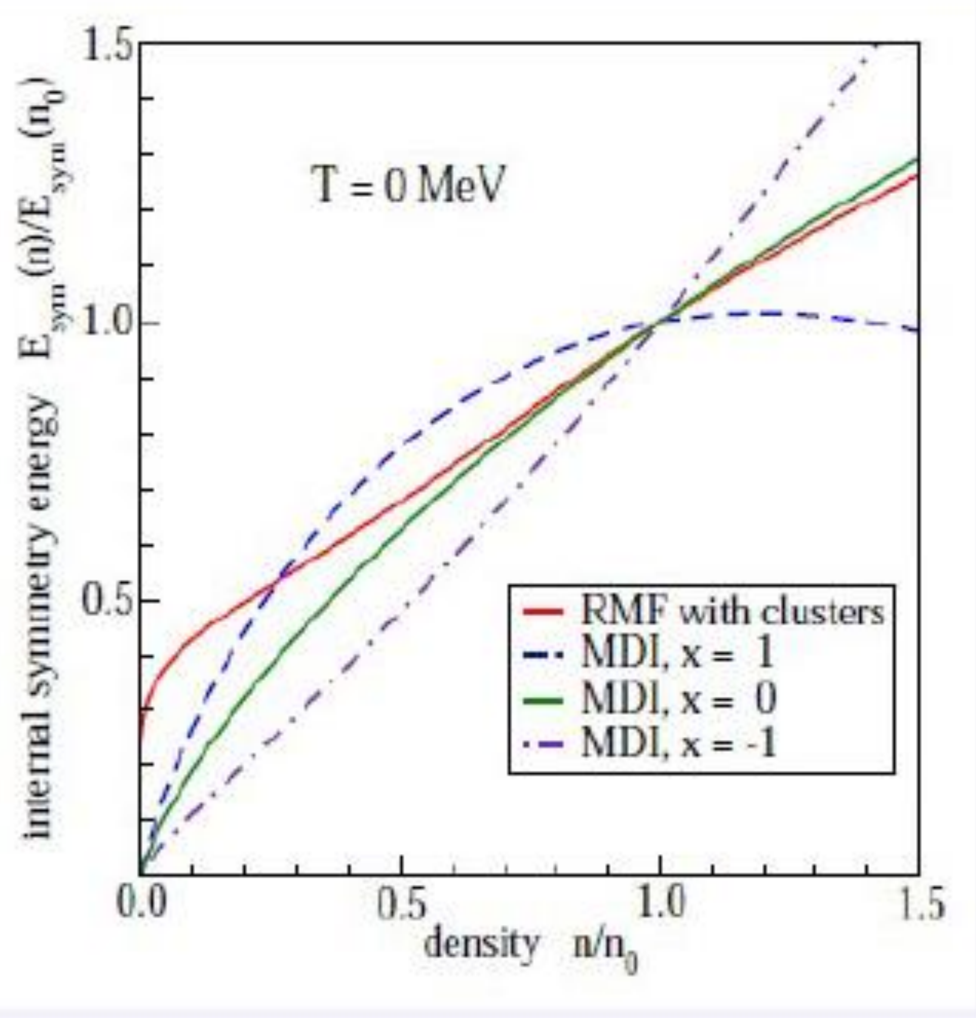
  $\Rightarrow$ low-density behaviour not correct

- gRDF with clusters

  $\Rightarrow$ increase of E_{sym} at low densities due to formation of clusters

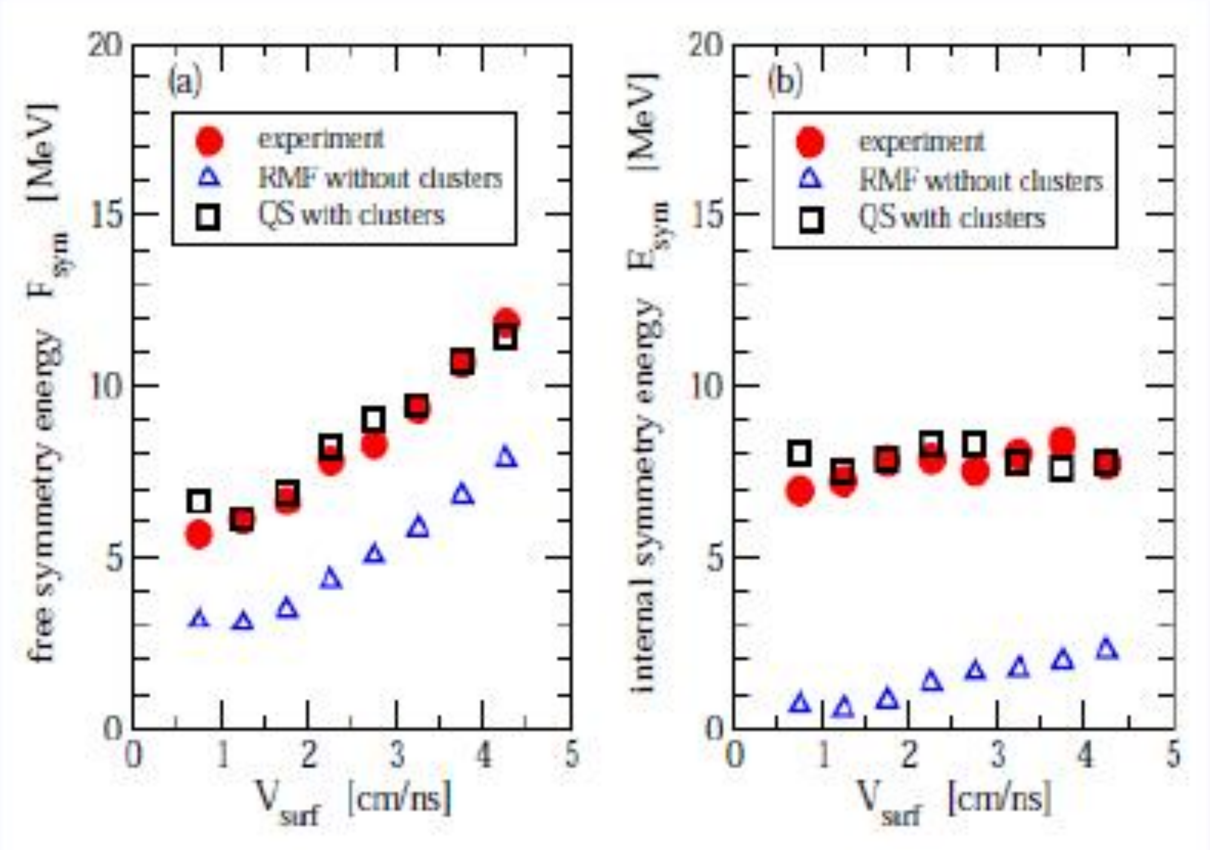
  $\Rightarrow$ finite symmetry energy in the limit $n \to 0$



Symmetry Energy

finite temperature

- experimental determination of symmetry energy
  - heavy-ion collisions of ^{64}Zn on ^{92}Mo and ^{197}Au at 35 A MeV
    temperature, density, free symmetry energy derived as functions of parameter v_{surf} (measures time when particles leave the source)
    (S. Kowalski et al., Phys. Rev. C 75 (2007) 014601)



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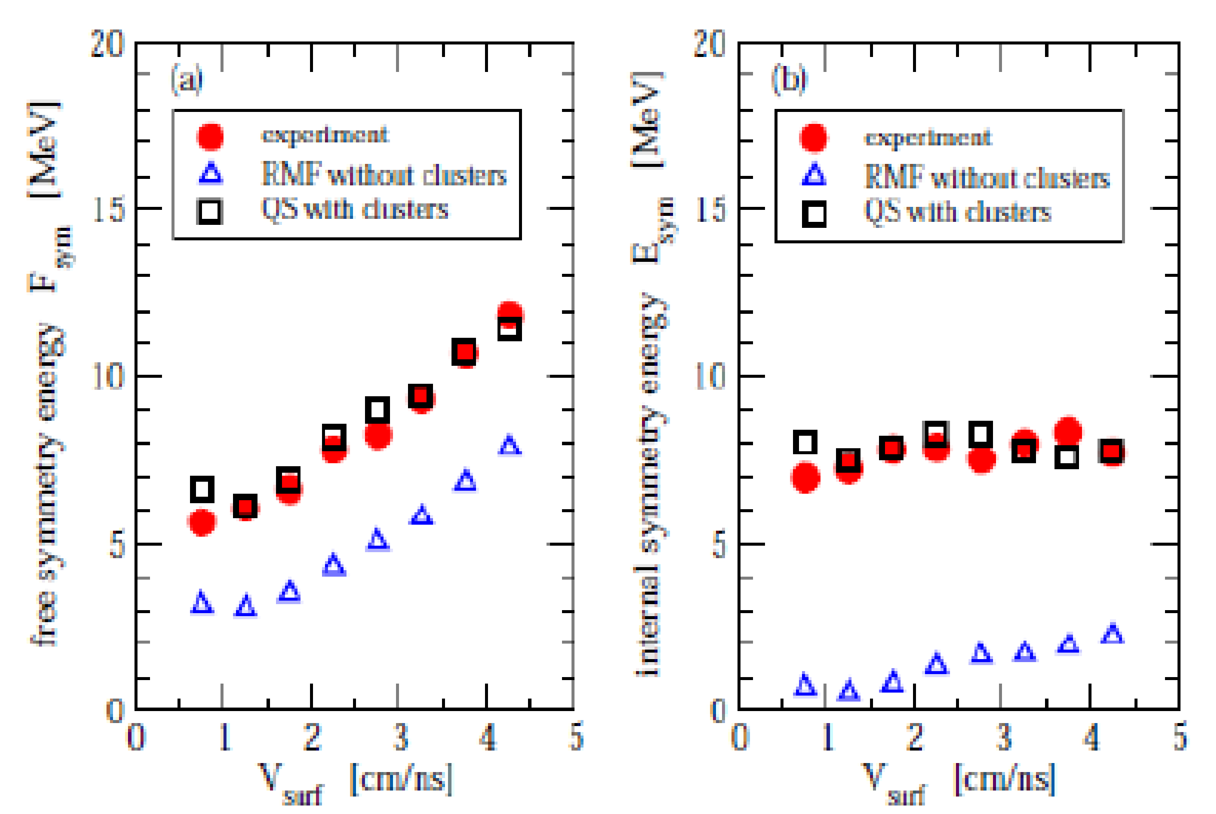
- symmetry energies in RMF calculation without clusters are too small

- very good agreement with QS calculation with light clusters

  (J. B. Natowitz et al., Phys. Rev. Lett. 104 (2010) 202501)



Summary … so far …

- **correlations in nuclear matter**
  - change of composition
    ⇒ e.g. formation and dissolution of clusters
  - modification of thermodynamical properties
    ⇒ e.g. change of symmetry energy
- **various theoretical approaches**
  - nuclear statistical equilibrium
  - virial equation of state
  - generalized Beth-Uhlenbeck approach
  - generalized relativistic density functional (gRDF)
- **important features**
  - quasiparticles with medium dependent properties
  - bound state and continuum contributions to correlations
  - dissolution of clusters ⇒ Mott effect
  - correct limits at low and high densities

Quantum Statistics & Cluster Virial Expansion

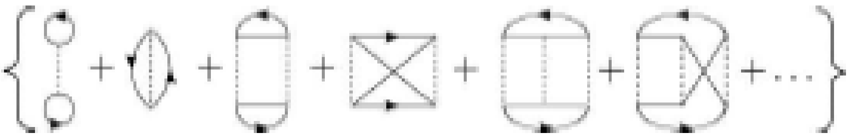
- Grand canonical thermodynamic potential

$$J = -P\Omega = -T \ln \mathrm{Tr}\, e^{-(H-\mu N)/T},$$

- Perturbation expansion

$$P = \frac{1}{\Omega}\mathrm{Tr}\ln[-G_1^{(0)}] - \frac{1}{2\Omega}\int_0^1 \frac{d\lambda}{\lambda}\mathrm{Tr}\Sigma_\lambda G_\lambda,$$

$$P = P_0 - \frac{1}{2\Omega}\int_0^1 \frac{d\lambda}{\lambda}\left\{ \ldots \right\}$$



- Single-particle propagator: $G_1^{-1}(1,z) = [G_1^{(0)}(1,z)]^{-1} - \Sigma_1(1,z)$, $G_1^{(0)}(1,z) = [z - E_1(p)_1]^{-1}$
- Alternative: nucleon density (sum rule)

$$n_{\tau_1}(T,\mu_p,\mu_n) = \frac{2}{\Omega}\sum_1 \int_{-\infty}^{\infty}\frac{d\omega}{2\pi} f_1(\omega) S_1(1,\omega),$$

$$S_1(1,\omega) = \frac{2\mathrm{Im}\,\Sigma_1(1,\omega - i0)}{(\omega - E(1) - \mathrm{Re}\,\Sigma_1(1,\omega))^2 + (\mathrm{Im}\,\Sigma_1(1,\omega - i0))^2},$$

Röpke, Bastian, DB, Klaähn, Typel, Wolter, NPA 897 (2013) 70

Quantum Statistics & Cluster Virial Expansion

Quantum Statistics & Cluster Virial Expansion

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Quantum Statistics & Cluster Virial Expansion

Φ–derivable formulation of cluster virial expansions

- $\Phi-$ derivable approach to the grand canonical thermodynamic potential [Baym, Phys. Rev. 127 (1962) 139]

$$J = -\mathrm{Tr}\,\{\ln(-G_1)\} - \mathrm{Tr}\{\Sigma_1 G_1\} + \mathrm{Tr}\,\{\ln(-G_2)\} + \mathrm{Tr}\{\Sigma_2 G_2\} + \Phi[G_1, G_2]$$

with full propagators:
$G_1^{-1}(1,z) = z - E_1(p_1) - \Sigma_1(1,z)$; $G_2^{-1}(12,1'2',z) = z - E_1(p_1) - E_2(p_2) - \Sigma_2(12,1'2',z)$
and selfenergies

$$\Sigma_1(1,1') = \frac{\delta\Phi}{\delta G_1(1,1')}\,;\Sigma_2(12,1'2',z) = \frac{\delta\Phi}{\delta G_2^{-1}(12,1'2',z)}.$$

Because of stationarity equivalent to

$$n = -\frac{1}{\Omega}\frac{\partial J}{\partial\mu} = \frac{1}{\Omega}\sum_1\int_{-\infty}^{\infty}\frac{d\omega}{\pi}f_1(\omega)S_1(1,\omega)\,,$$

(baryon number conservation)

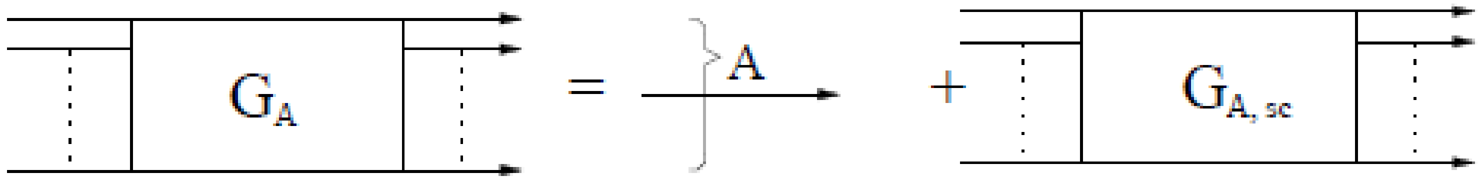
- Generalization to A-nucleon clusters in nuclear matter

$$\Omega \;=\; \sum_A(-1)^A\left[\mathrm{Tr}\ln\left(-G_A^{-1}\right) + \mathrm{Tr}\left(\Sigma_A\,G_A\right)\right] + \Phi\,,$$

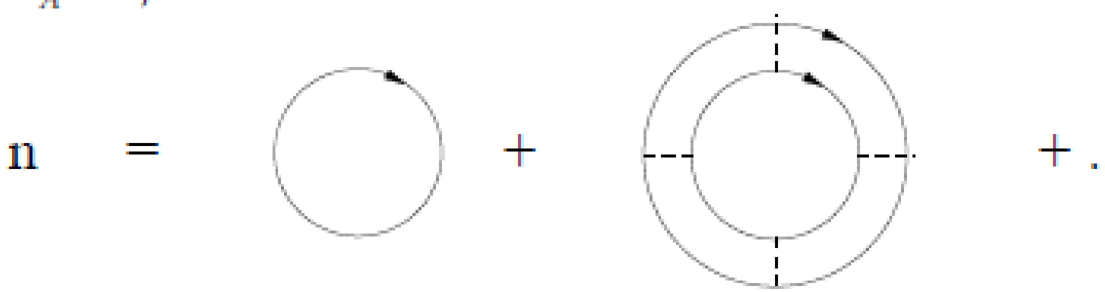
$$G_A^{-1} \;=\; G_A^{(0)^{-1}} - \Sigma_A\,,\;\; \Sigma_A(1\ldots A,1'\ldots A',z_A) = \frac{\delta\Phi}{\delta G_A(1\ldots A,1'\ldots A',z_A)}\,.$$

Φ–derivable formulation of cluster virial expansions

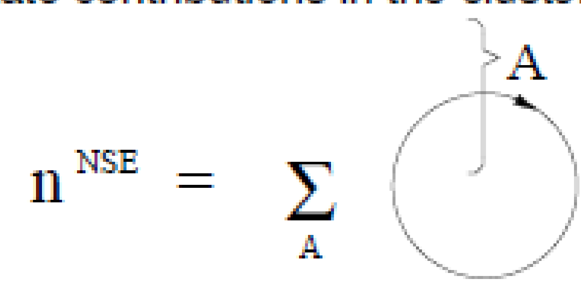
The full $A-$nucleon (cluster) propagator may be decomposed into a bound and scattering contribution, given diagrammatically



Neglecting explicit correlations between the clusters, i.e., setting $\Phi = 0$, results in a vanishing cluster selfenergy, $\Sigma_A = 0$, so that

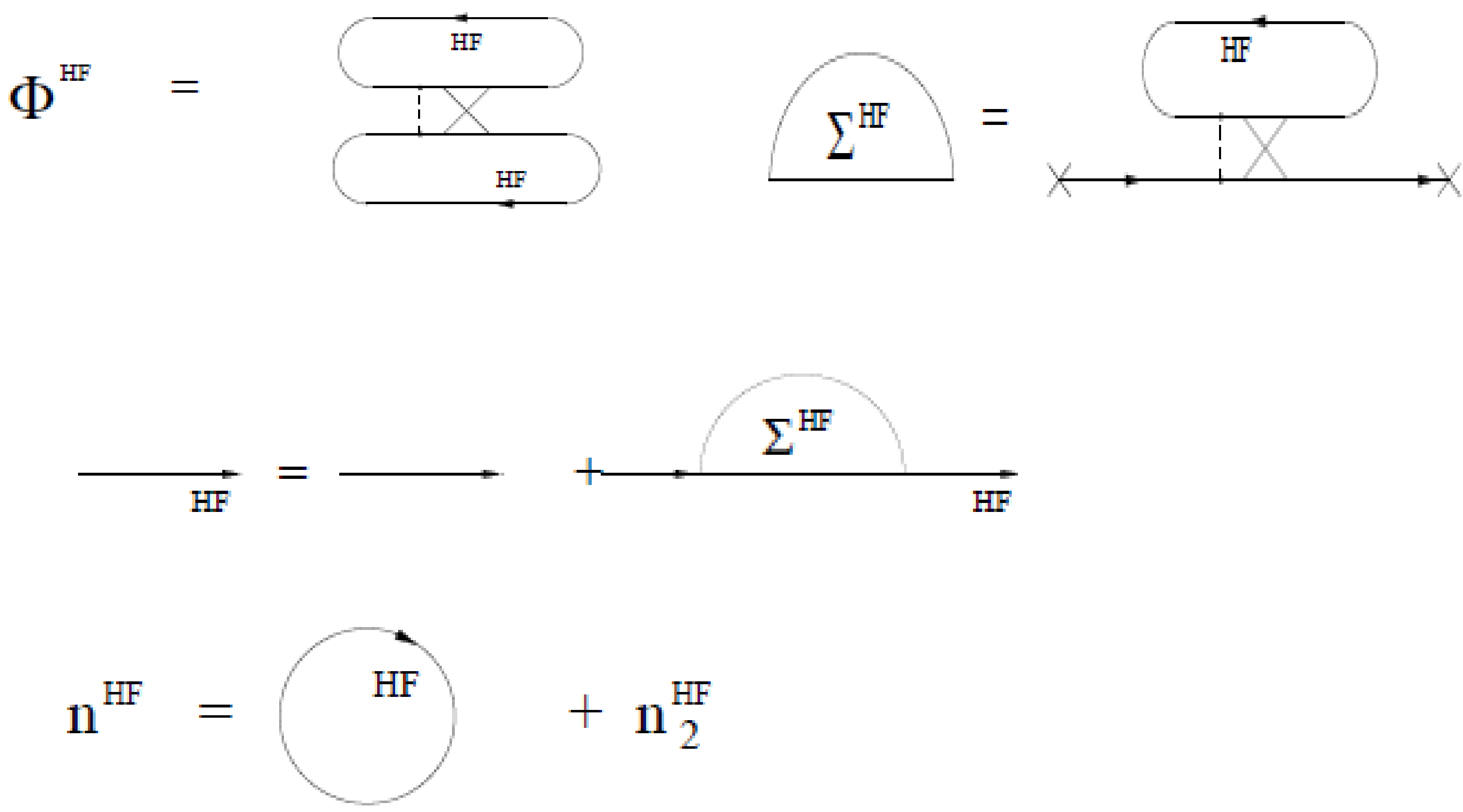


Restricting to only the bound state contributions in the cluster Greens function,



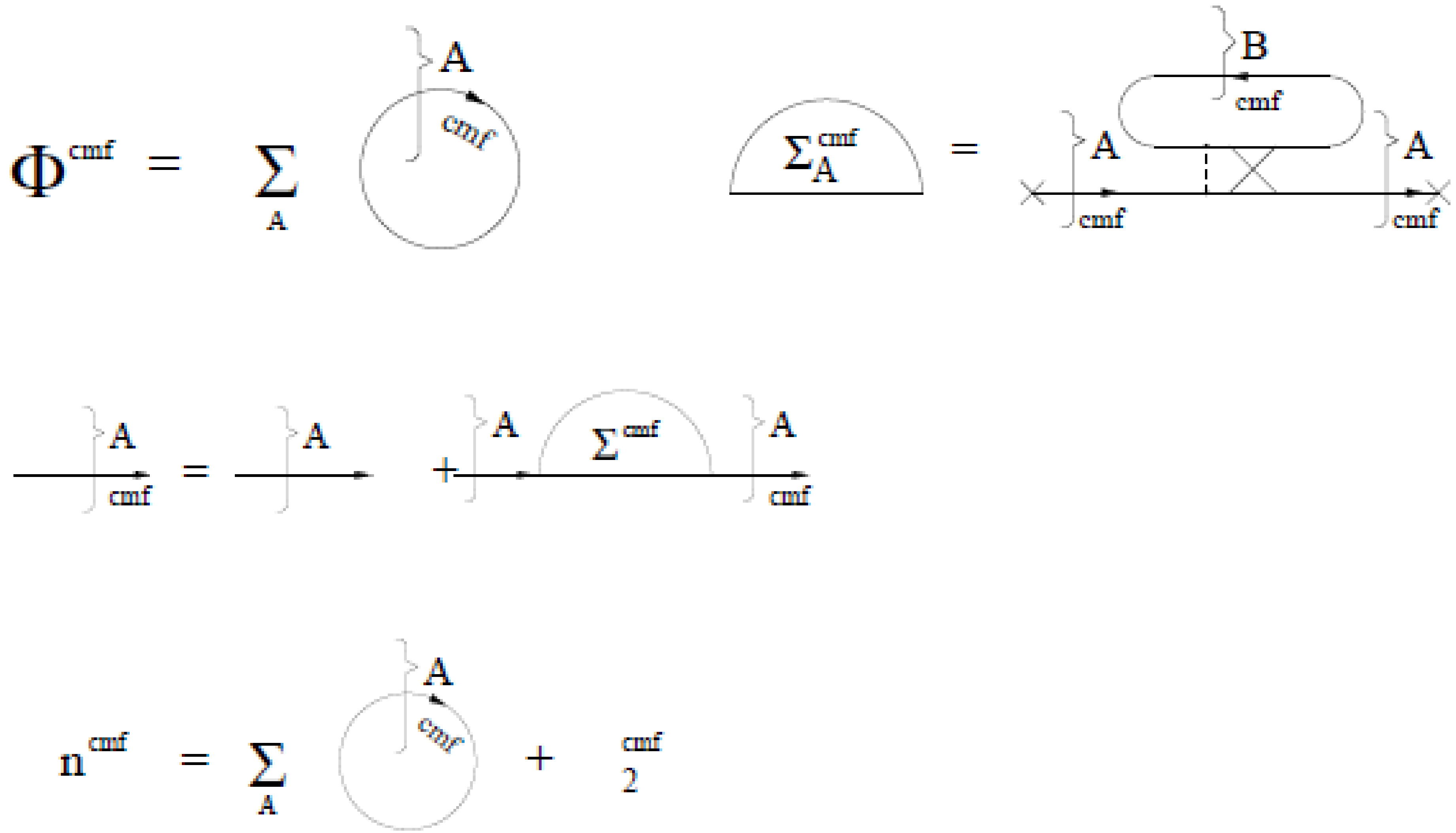
Φ–derivable formulation of cluster virial expansions

Consistent approximation schemes in the Φ– derivable form: Hartree-Fock



Φ–derivable formulation of cluster virial expansions

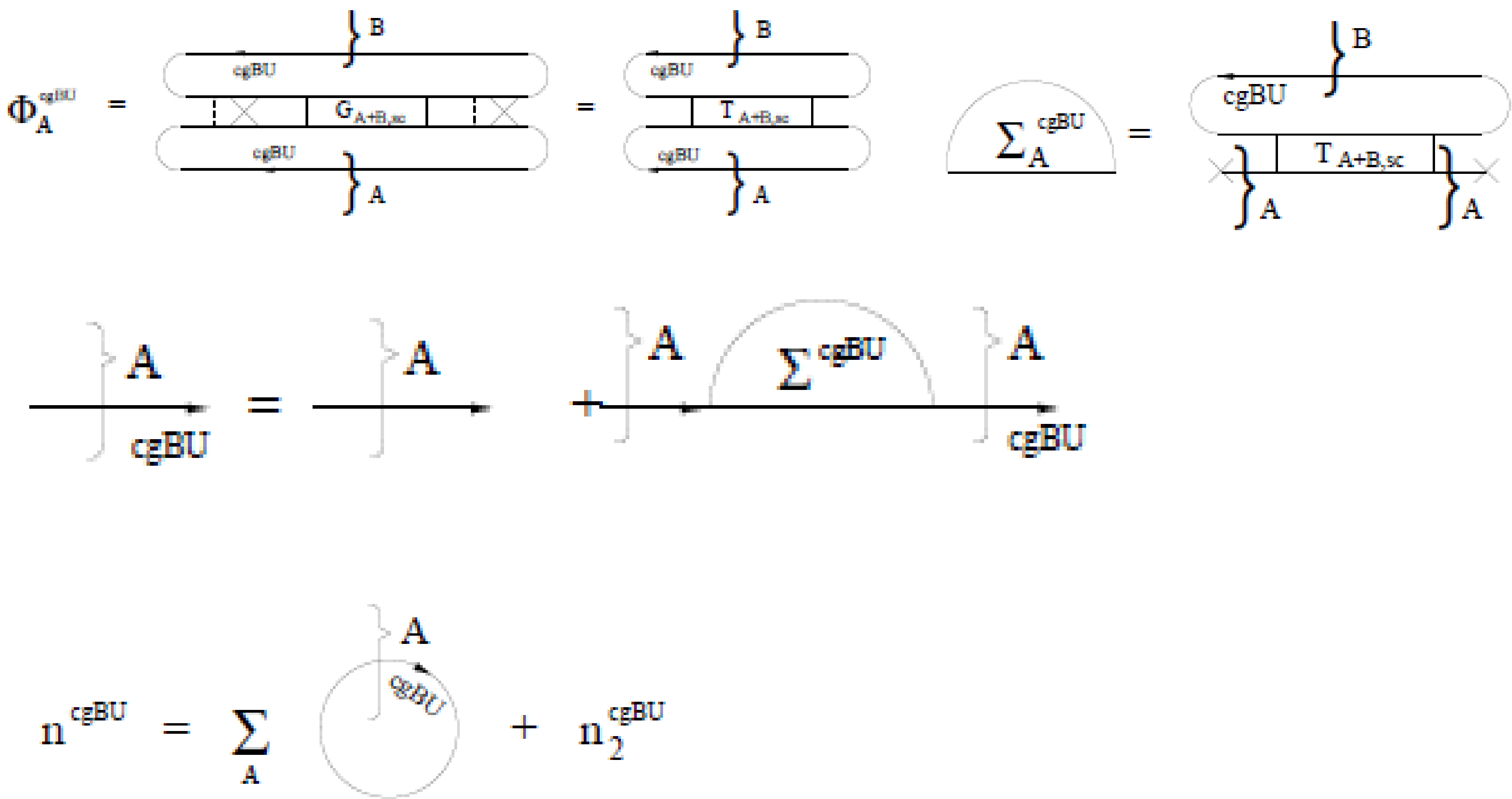
Consistent approximation schemes in the Φ– derivable form: Cluster Meanfield



Φ–derivable formulation of cluster virial expansions

Consistent approximation schemes in the Φ– derivable form:

Cluster Generalized Beth-Uhlenbeck



Φ–derivable formulation of cluster virial expansions

More to come …

Mott Dissociation of Nucleons as Quark Clusters

One-particle Green function (a = quark, diquark) approximation:

$$G_a(1,z_a) = [z_a - E_a(1) - \Sigma_a(1,z_a)]^{-1}, \quad E_a(1) = E_a + p_1^2/(2m_a)$$

Spectral function

$$A_a(1,E) = i[G_a(1,E+i0) - G_a(1,E-i0)] = \frac{2\Im\Sigma_a(1,E-i0)}{[E - E_a(1) - \Re\Sigma_a(1,E)]^2 + [\Im\Sigma_a(1,E-i0)]^2}$$

mean occupation number $n_a(1)$ and distribution function $f_a(E)$

$$n_a(1) = \eta_a \int \frac{dE}{2\pi} f_a(E) A_a(1,E), \quad f_a(E) = \eta_a[\exp[(E-\mu_a)/T] + \eta_a]^{-1}$$

Thermodynamics: via density to pressure as thermodynamical potential

$$n(T,\mu_1,\ldots\mu_c) = \frac{1}{\Omega}\sum_{a=1}^{c}\sum_{1} n_a(1) ,$$

$$p(T,\mu_1,\ldots\mu_c) = \sum_a \int_{-\infty}^{\mu_a} d\mu' n_a(T,\mu_1\ldots\mu'\ldots\mu_c)$$

$\Longrightarrow$ Generalized Beth-Uhlenbeck EoS for (n.r.) Quark Matter.

D.B., H. Grigorian, G. Röpke, in preparation

Mott Dissociation of Nucleons as Quark Clusters

Hartree-Fock: $\Im\Sigma(1,z)$ vanishes, spectral function δ – shaped at quasiparticle energies:

$$e_a(1) = E_a(1) + v_a(1), \quad v_a(1) = \Re\Sigma_a(1, e_a(1))$$

Consider contributions up to first order in $\Im\Sigma(1,z)$

$$A_a(1,E) = A_a^{free}(1,E) + A_a^{corr}(1,E)\,, \quad A_a^{free}(1,E) = 2\pi\delta(E - e_a(1)),$$
$$A_a^{corr}(1,E) = 2\int dE' \Im\Sigma_a(1, E' - i0)[\delta(E - e_a(1)) - \delta(E - E')]\frac{d}{dE'}\frac{\mathcal{P}}{E' - e_a(1)}$$

Bound states: selfenergy in terms of the T–matrix

$$\Sigma_a(1, z_\nu^a) = T\sum_{2,b}\sum_{z_\nu^b} T_{ab}(12,12, z_\nu^a + z_\nu^b)G_b(2, z_\nu^b)$$
$$= \Sigma^{HF}(1) + \sum_{2,b}\int dE\, \Im T_{ab}(12,12,E + i0)\int \frac{dE'}{2\pi}A_b(2,E')\frac{[f_b(E') - f_{ab}(E)]}{E - E' - z_a}$$

Two-particle distribution function $f_{ab}(E)$ describes fermionic quark-diquark states, $\eta_{ab} = -\eta_a\eta_b$,

$$f_{ab}(E) = \eta_{ab}[\exp(E - \mu_a - \mu_b)/T + \eta_{ab}]^{-1}$$

Mott Dissociation of Nucleons as Quark Clusters

T-matrix from solution of the Bethe–Salpeter equation:

$$T_{ab}(12,1'2',E) = V(12,1'2') + \sum_{343'4'} V(12,34) G_{ab}(34,3'4',E) T_{ab}(3'4',1'2',E)$$

$V(12,1',2')$ - interaction potential model, intermediate propagation given by Green's function

$$\begin{aligned} G_{ab}(12,1'2',Z_{ab}) &= \sum_{z_a} G_a(1,z_a) G_b(2,Z_{ab}-z_a)\delta_{11'}\delta_{22'} \\ &= \delta_{11'}\delta_{22'} \int \frac{d\omega}{2\pi}\frac{d\omega'}{2\pi}\frac{1-f_a(\omega)-f_b(\omega')}{\omega+\omega'-Z} A_a(1,\omega) A_b(2,\omega') \\ &= \frac{1-n_a(1)-n_b(2)}{e_a(1)+e_b(2)-Z_{ab}}\delta_{11'}\delta_{22'} + \Delta G_{ab}(12,1'2',Z_{ab}) \end{aligned}$$

$$n_a(1) = n_a^{free}(1) + \sum_{2,b} n_{ab}(12)\,, \quad n_{ab}(12) = \eta_{ab}\int \frac{dE}{2\pi} f_{ab}(E) D_{ab}(12,E)$$

Two-particle spectral density (Generalized Beth-Uhlenbeck)

$$D_{ab}(12,E) = 2\pi\delta(E-E_{\alpha,P}) + \sin^2\delta_{ab}(P,E)\frac{d}{dE}\delta_{ab}(P,E)\,.$$

Mott Dissociation of Nucleons as Quark Clusters

Schrödinger equation equivalent to Bethe–Salpeter equation:

$$(E_{\alpha,P} - e^0_{ab}(p,P))\Psi_{\alpha,P}(p) = \sum_{p'} V_{ab}(p,p')\Psi_{\alpha,P}(p')$$

Bilinear expansion to the T–matrix

$$T_{ab}(12,1'2',E) = \sum_\alpha T_{ab,\alpha}(p,p',P,E), = \sum_\alpha \frac{\Psi_{\alpha,P}(p)\Psi^*_{\alpha,P}(p')}{E_{\alpha,P} - E}(e^0_{ab}(p,P) - E)(e^0_{ab}(p',P) - E_{\alpha,P})$$

Separable confining-type model potential

$$V(p,p') = (2\pi)^3 C\delta^{(3)}(\vec{p} - \vec{p}') - V_0 W(p,\beta)W(p',\beta)$$

Binding energy E_B and two-particle quasi-energies $e^0_{ab}(p,P)$ (continuum edge)

$$E_B = -\frac{p_B^2}{2m} = E_{\alpha,P} - M - \frac{P^2}{2M} - C\,, \quad e^0_{ab}(p,P) = M + \frac{p^2}{2m} + \frac{P^2}{2M} + C$$

Mott Dissociation of Nucleons as Quark Clusters

Wave function and loop integral $J(E,P)=\sum V_{ab}(p,p)G_{ab}(p,P,E)$

$$\Psi_0(p)=\mathcal{N}\frac{V_0W(p,\beta)}{p^2+p_B^2}\,,\quad J(p_B,P;V_0,\beta)=\frac{mV_0}{\pi^2}\int_0^\infty\frac{W(p,\beta)^2}{p^2+p_B^2}p^2dp=1$$

Separable Yamaguchi potential for $W(p,\beta)=\beta^2/(p^2+\beta^2)$

Mass $M_N=M+C(V_0,\beta)-|E_B(V_0,\beta)|=939$ MeV and radius of the nucleon $<r^2>=0.7\ \mathrm{fm}^2$

$$4\pi(1+x)^2=mV_0\beta\,,\quad\frac{(1+x^3)(1+x)^3-16x^3}{8x^2(1-x^2)^2}=\beta^2R^2/4\,,\quad x=p_B/\beta$$

V_0[GeV^{-2}]	β[fm^{-1}]	C[GeV]	E_B[GeV]	$n_{\mathrm{Mott}}[n_0]$;SNM	$n_{\mathrm{Mott}}[n_0]$;PNM
150	3.51997	0.0867326	0.147833	3.8496	1.92465
200	3.03763	0.115714	0.176814	4.1836	2.09181
250	2.73902	0.143152	0.204252	4.46062	2.23005
450	2.16666	0.24039	0.30149	5.19448	2.59723
500	2.08783	0.262325	0.323425	5.31906	2.65924
650	1.91431	0.324144	0.385244	5.61316	2.80667

Mott Dissociation of Nucleons as Quark Clusters

In-medium T-matrix describes quark-diquark subsructure effects

$$T_{ab}(p,p',P,E) = \frac{V_{ab}(p,p')}{1-J(E,P)}, \quad J(E,P) = \sum_{a,b,p} V_{ab}(p,p) G_{ab}(p,P,E).$$

Two-particle energies (angle-averaged) in the medium

$$e_{ab}(p,P) = e^0_{ab}(p,P) + u_{ab}(p,P)\ , \ \ u_{ab}(p,P) = < v_a(1) + v_b(2) >_{pP}$$

Generalized (angle-averaged) Pauli blocking operator

$$Q(p,P) = 1 - < n_a^{(B)} >_{pP} - < n_b^{(B)} >_{pP}$$

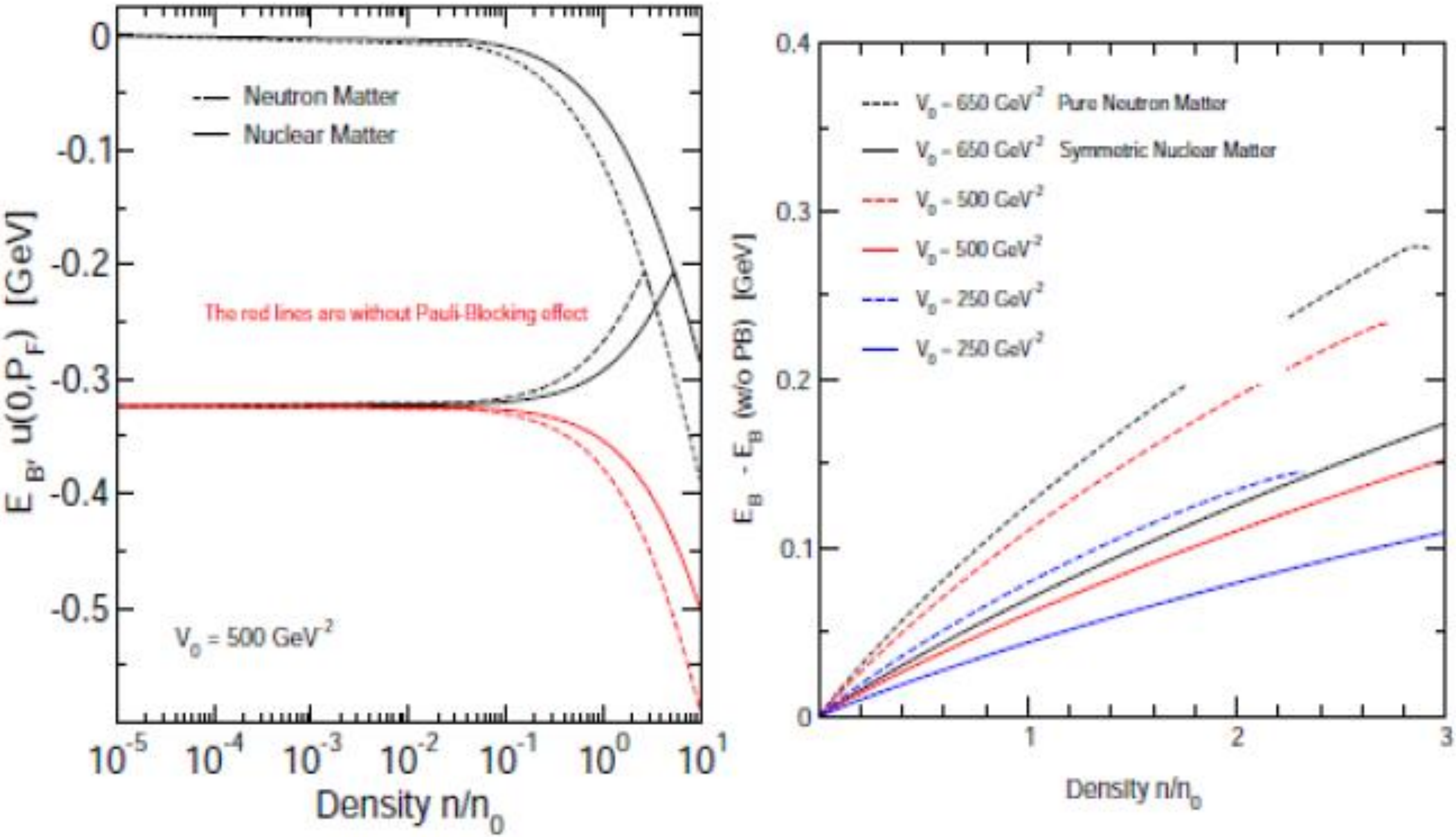
$$J_R(E,P) = \mathcal{P} \int_0^\infty \frac{p^2 dp}{2\pi^2} V_0 W^2(p) \frac{Q(p,P)}{e_{ab}(p,P) - E}; \ \ e_{ab}(p,P) > E$$

Pair distribution function

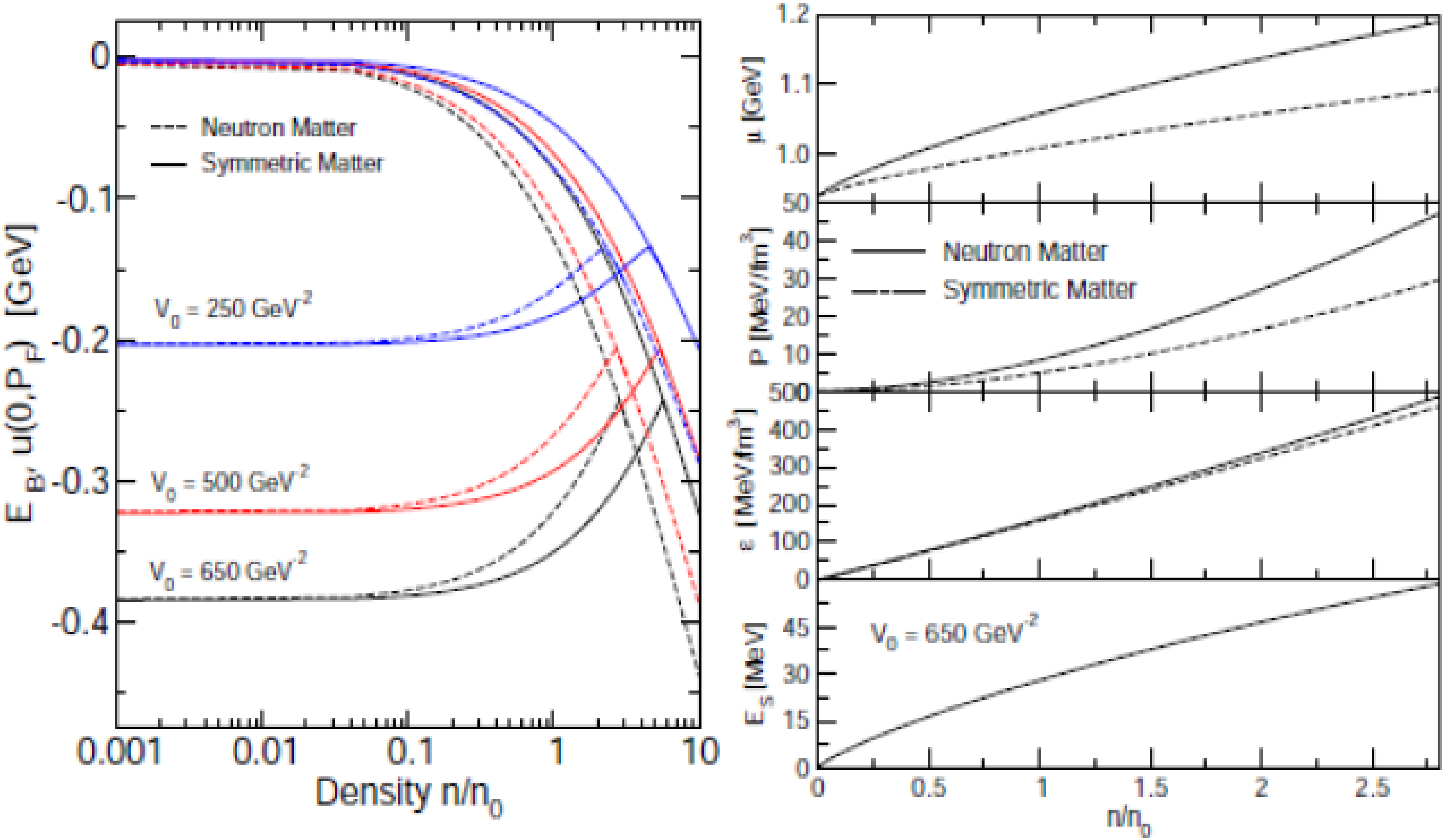
$$n_{ab}^{(B)}(p,P) = \eta_{ab} f_{ab}\left(E_B + E_0(P)\right) |\Psi_P(p)|^2\ , \ \ \Psi_P(p) = \frac{V_0 \mathcal{N} W(p) \sqrt{Q(p,P)}}{e_{ab}(p,P) - E_B - E_0(P)}$$

Nucleon Mott dissociation follows when bound state energy equals continnum.

Mott Dissociation of Nucleons as Quark Clusters



Mott Dissociation of Nucleons as Quark Clusters



D.B., H. Grigorian, G. Röpke, in preparation

Baryon dissociation inevitable – in neutron star matter at 2.5 ... 3.0 n_0

Mott Dissociation of Hadrons in Quark Matter

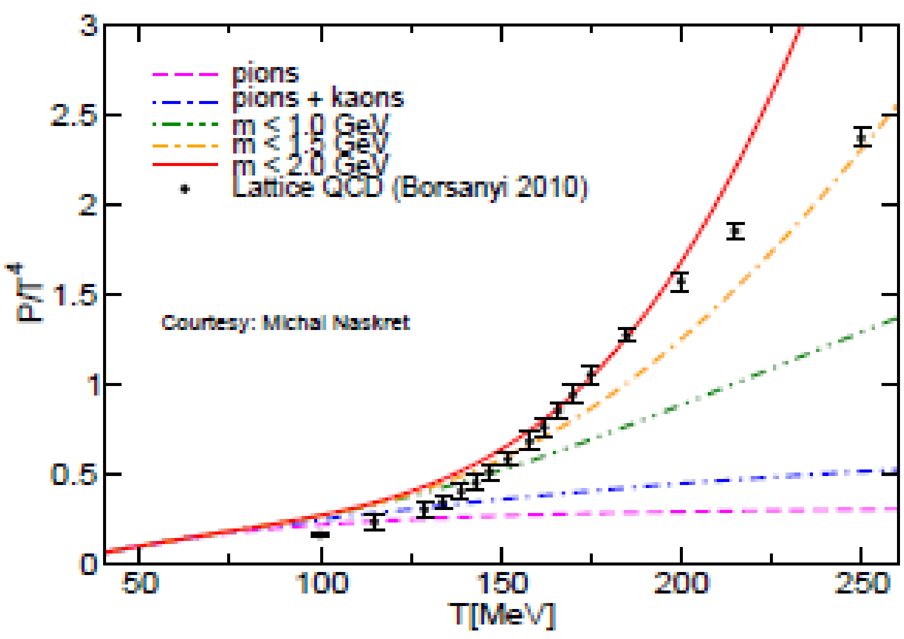
- Partition function as a Path Integral (imaginary time $\tau = i\,t$, $0 \le \tau \le \beta = 1/T$) $\Rightarrow$ PS I

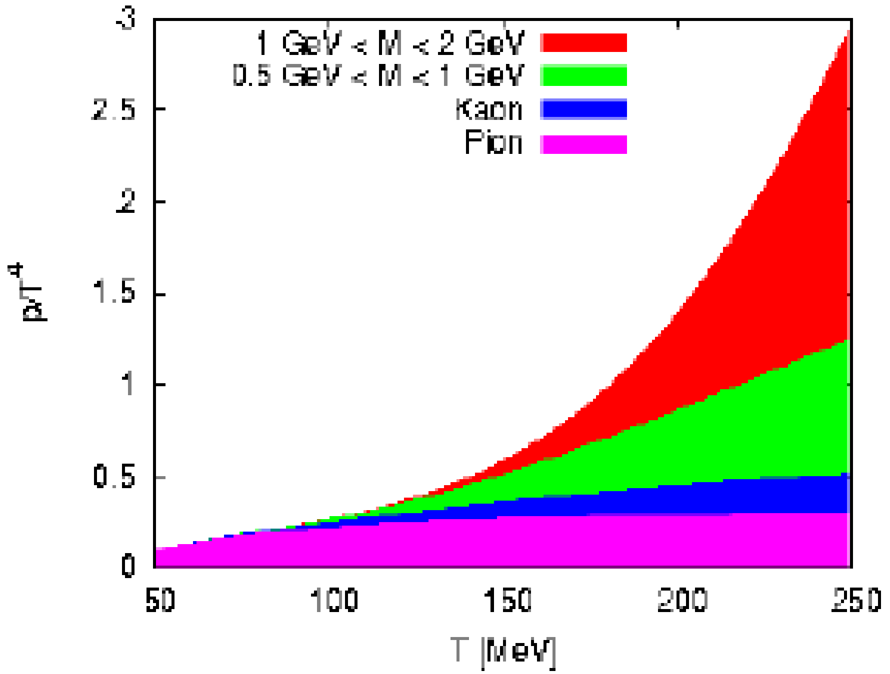
$$Z[T,V,\mu] = \int \mathcal{D}\bar\psi \mathcal{D}\psi \mathcal{D}A \exp\left\{-\int_0^\beta d\tau \int_V d^3x\, \mathcal{L}_{QCD}(\psi,\bar\psi,A)\right\}$$

- QCD Lagrangian, non-Abelian gluon field strength: $F^a_{\mu\nu}(A) = \partial_\mu A^a\nu - \partial_\nu A^a_\mu + g\ f^{abc}[A^b_\mu, A^c_\nu]$

$$\mathcal{L}_{QCD}(\psi,\bar\psi,A) = \bar\psi[i\gamma^\mu(\partial_\mu - igA_\mu) - m - \gamma^0\mu]\psi - \frac{1}{4}F^a_{\mu\nu}(A)F^{a,\mu\nu}(A)$$

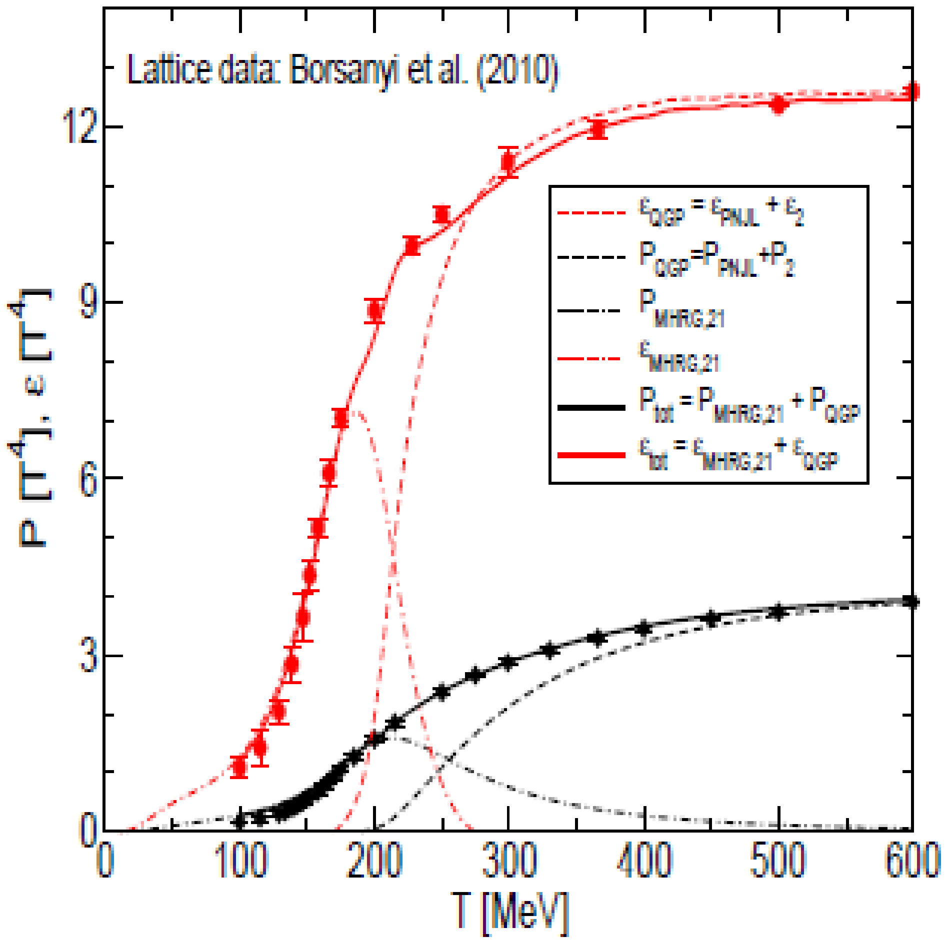
- Numerical evaluation: Lattice gauge theory simulations (hotQCD, Wuppertal-Budapest)





Mott Dissociation of Hadrons in Quark Matter

$$P_{\rm tot}(T,\{\mu_j\}) = P_{\rm PNJL}(T,\{\mu_i\}) + \sum_{r=M,B} \delta_r g_r \int ds\, A_r(s,m_r;T) \int \frac{d^3p}{(2\pi)^3} T \ln\left\{1+\delta_r \exp\left(\frac{\sqrt{p^2+s}-\mu_r}{T}\right)\right\}$$



Spectral function for hadronic resonances:

$$A_r(s,m;T) = N_s \frac{m\Gamma_r(T)}{(s-m^2)^2 + m^2\Gamma_r^2(T)}$$

Ansatz motivated by chemical freeze-out model:

$$\Gamma_r(T) = \tau_r^{-1}(T) = \sum_h \lambda < r_r^2 >_T < r_h^2 >_T n_h(T)$$

Apparent phase transition at $T_c \sim 165$ MeV

Hadron resonances present up to $T_{\rm max} \sim 250$ MeV

Blaschke & Bugaev, Fizika B13, 491 (2004)

Prog. Part. Nucl. Phys. 53, 197 (2004)

Turko, Blaschke, Prorok & Berdermann,

APPS 5, 485 (2012); J. Phys. Conf. Ser. 455, 012056 (2013)

Hadronic states above T_c ! See also: Ratti, Bellwied et al., arXiv:1109.6243 [hep-ph]

Mott Dissociation of Mesons in Quark Matter

- Partition function as a Path Integral (imaginary time $\tau = i\, t$)

$$Z[T,V,\mu] = \int \mathcal{D}\bar{q}\mathcal{D}q \exp\left\{ - \int^{\beta} d\tau \int_V d^3x [\bar{q}(i\gamma^\mu \partial_\mu - m_0 - \gamma^0 \mu) q + \sum_{M=\pi,\sigma} G_M (\bar{q}\Gamma_M q)^2] \right\}$$

- Couplings: $G_\pi = G_\sigma = G_S$ (chiral symmetry)
- Vertices: $\Gamma_\sigma = \mathbf{1}_D \otimes \mathbf{1}_f \otimes \mathbf{1}_c$; $\Gamma_\pi = i\gamma_5 \otimes \vec{\tau} \otimes \mathbf{1}_c$
- Bosonization (Hubbard-Stratonovich Transformation)

$$\exp\left[G_S(\bar{q}\Gamma_\sigma q)^2\right] = \text{const.} \int \mathcal{D}\sigma \exp\left[\frac{\sigma^2}{4G_S} + \bar{q}\Gamma_\sigma q \sigma\right]$$

- Integrate out quark fields $\longrightarrow$ bosonized partition function

$$Z[T,V,\mu] = \int \mathcal{D}\sigma \mathcal{D}\pi \exp\left\{ -\frac{\sigma^2 + \pi^2}{4G_S} + \frac{1}{2}\mathrm{Tr}\, \ln S^{-1}[\sigma,\pi] \right\}$$

- Systematic evaluation: Mean fields + Fluctuations
  - Mean-field approximation: order parameters for phase transitions (gap equations)
  - Lowest order fluctuations: hadronic correlations (bound & scattering states)

Mott Dissociation of Mesons in Quark Matter

- Separate the mean-field part of the quark determinant

$$\mathrm{Tr}\,\ln S^{-1}[\sigma,\pi] = \mathrm{Tr}\,\ln S_{\mathrm{MF}}^{-1}[m] + \mathrm{Tr}\,\ln[1 + (\sigma + i\gamma_5\vec{\tau}\vec{\pi})S_{\mathrm{MF}}[m]]$$

- Mean-field quark propagator

$$S_{\mathrm{MF}}(\vec{p}, i\omega_n; m) = \frac{\gamma_0(i\omega_n + \mu) - \vec{\gamma}\cdot\vec{p} + m}{(i\omega_n + \mu)^2 - E_p^2}$$

- Expand the logarithm: $\ln(1+x) = -\sum_{n=1}^{\infty}(-1)^n x^n/n = x - x^2/2 + \ldots$
- Thermodynamic potential in Gaussian approximation

$$\Omega(T,\mu) \;=\; -T\ln Z(T,\mu) = \Omega_{\mathrm{MF}}(T,\mu) + \sum_M \Omega_{\mathrm{M}}^{(2)}(T,\mu) + \mathcal{O}[\phi_M^3]$$

$$\Omega_{\mathrm{M}}^{(2)}(T,\mu) \;=\; \frac{N_M}{2}\int\frac{d^2p}{(2\pi)^3}\frac{1}{\beta}\sum_n \mathrm{e}^{i\nu_n\eta}\ln S_M^{-1}(\vec{p}, i\nu_n)\;, \quad N_\sigma = 1,\; N_\pi = 3$$

- Meson propagator $S_M(\vec{p}, i\nu_n) = 1/\left[1/(2G_S) - \Pi_M(\vec{p}, i\nu_n)\right]$
- Mesonic polarization loop

$$\Pi_M(\vec{p}, i\nu_n) = -\frac{1}{\beta}\sum_{n'}\mathrm{e}^{i\nu_{n'}\eta}\int\frac{d^2k}{(2\pi)^3}\mathrm{Tr}\left[\Gamma_M S_{\mathrm{MF}}(-\vec{k}, -i\omega_{n'})\Gamma_M S_{\mathrm{MF}}(\vec{k}+\vec{p}, i\omega_{n'} + i\nu_n)\right]$$

Mott Dissociation of Mesons in Quark Matter

- Polar representation of the analytically continued quark propagator

$$S_{\mathrm{M}} = |S_{\mathrm{M}}| \mathrm{e}^{i\delta_{\mathrm{M}}} = S_R + iS_I ~,$$

- Phase shift $\delta_{\mathrm{M}}(\omega, \mathbf{q}) = -\mathrm{Im} \ln S_{\mathrm{M}}^{-1}(\omega - \mu_M + i\eta, \mathbf{q})$
- Thermodynamic potential for mesonic modes

$$\begin{aligned} \Omega_{\mathrm{M}}(T,\mu) &= \mathrm{Tr} \ln S_{\mathrm{M}}^{-1}(iz_n, \mathbf{q}) = d_{\mathrm{M}} T \sum_n \int \frac{\mathrm{d}^3 q}{(2\pi)^3} \ln S_{\mathrm{M}}^{-1}(iz_n, \mathbf{q}) ~, \\ &= -d_{\mathrm{M}} T \sum_n \int \frac{\mathrm{d}^3 q}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega}{2\pi} \frac{1}{iz_n - \omega} \mathrm{Im} \ln S_{\mathrm{M}}^{-1}(\omega + i\eta, \mathbf{q}) \end{aligned}$$

- Perform Matsubara summation $\Omega_{\mathrm{M}}(T,\mu) = d_{\mathrm{M}} \int \frac{\mathrm{d}^3 q}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega}{2\pi} n_M^-(\omega) \delta_M(\omega, \mathbf{q})$
- Using symmetries of Bose function $n_M^-(-\omega) = -[1 + n_M^+(\omega)]$ and polarization loop

$$\Omega_{\mathrm{M}}(T,\mu) = d_{\mathrm{M}} \int \frac{\mathrm{d}^3 q}{(2\pi)^3} \int_0^{\infty} \frac{\mathrm{d}\omega}{2\pi} [1 + n_M^-(\omega) + n_M^+(\omega)] \delta_{\mathrm{M}}(\omega, \mathbf{q})$$

- Partial integration gives field theoretic Beth-Uhlenbeck formula

$$\Omega_{\mathrm{M}} = -d_{\mathrm{M}} \int \frac{\mathrm{d}^3 q}{(2\pi)^3} \int_0^{\infty} \frac{\mathrm{d}\omega}{2\pi} \left[\omega + T \ln\left(1 - \mathrm{e}^{-(\omega - \mu_{\mathrm{M}})/T}\right) + T \ln\left(1 - \mathrm{e}^{-(\omega + \mu_{\mathrm{M}})/T}\right) \right] \frac{d\delta_{\mathrm{M}}(\omega, \mathbf{q})}{d\omega}$$

Mott Dissociation of Mesons in Quark Matter

- When polarization loop integral can be expressed in the form

$$\Pi_{\rm M}(z,\mathbf{q}) = \Pi_{M,0} + \Pi_{M,2}(z,\mathbf{q})$$

- Factorization of two-particle propagator possible with $R_M(z,\mathbf{q}) = \frac{1-G_{\rm M}\Pi_{M,0}}{G_{\rm M}\Pi_{M,2}(z,\mathbf{q})}$

$$S_{\rm M}(z,\mathbf{q}) = \frac{1}{G_{\rm M}^{-1} - \Pi_{M,0} - \Pi_{M,2}(z,\mathbf{q})} = \frac{1}{\Pi_{M,2}(z,\mathbf{q})}\frac{1}{R_M(z,\mathbf{q}) - 1}$$

- This entails $\ln S_{\rm M}(z,\mathbf{q})^{-1} = \ln \Pi_{M,2}(z,\mathbf{q}) + \ln[R_M(z,\mathbf{q}) - 1]$
  and thus a separation of the phase shift in two contributions

$$\delta_{\rm M}(\omega,\mathbf{q}) = \delta_{X,c}(\omega,\mathbf{q}) + \delta_{X,R}(\omega,\mathbf{q})$$

- They correspond to continuum (state independent) and resonant phases

$$\begin{aligned}
\delta_{M,c}(\omega,\mathbf{q}) &= -\arctan\left(\frac{{\rm Im}\Pi_{M,2}(\omega-\mu_M+i\eta,\mathbf{q})}{{\rm Re}\Pi_{M,2}(\omega-\mu_M+i\eta,\mathbf{q})}\right) \\
\delta_{M,R}(\omega,\mathbf{q}) &= \arctan\left(\frac{{\rm Im}R_M(\omega-\mu_M+i\eta,\mathbf{q})}{1-{\rm Re}R_M(\omega-\mu_M+i\eta,\mathbf{q})}\right)
\end{aligned}$$

Mott Dissociation of Mesons in Quark Matter

- Suppose $\delta_{X,R}(\omega, \mathbf{q})$ corresponds to a resonance at $\omega = \omega_M = \sqrt{\mathbf{q}^2 + M_{\rm M}^2}$, then the propagator shall have the representation with a complex pole at $z = z_M = \omega_M + i\Gamma_M/2$, where $\Gamma_{\rm M}$ is the width of the resonance.
- The position of the pole is found from the condition $\mathrm{Re}R_M(z_M, \mathbf{q}) = 1$, where $\delta_{M,R}(\omega \to \omega_M) \to \pi/2$ since $\tan \delta_{M,R}(\omega \to \omega_M) \to \infty$
- Expanding $R_M(z, \mathbf{q})$ at the complex pole z_M for small width, one obtains

$$1 - \mathrm{Re}R_M(z_M, \mathbf{q}) = -(\omega^2 - \omega_M^2)\frac{dR_M(z, \mathbf{q})}{d\omega^2}\Big|_{z=z_M} \quad , \quad \mathrm{Im}R_M(z_M, \mathbf{q}) = \omega_M \Gamma_M \frac{dR_M(z, \mathbf{q})}{d\omega^2}\Big|_{z=z_M} \quad (1)$$

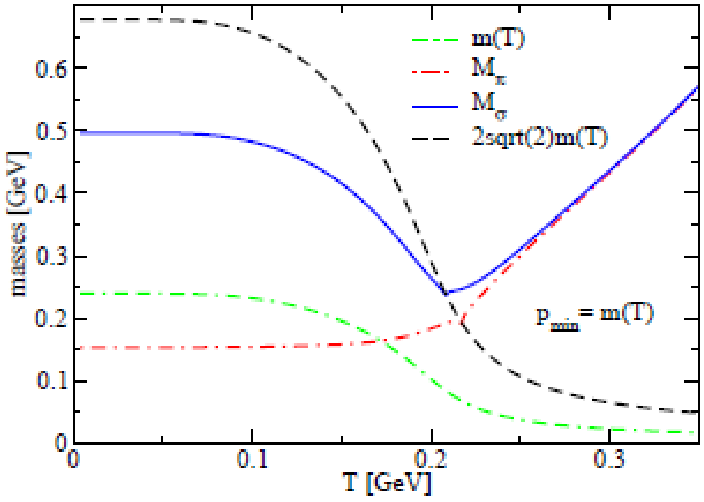
- The resonant shift becomes $\delta_{M,R}(\omega, \mathbf{q}) = -\arctan\left(\frac{\omega_{\rm M}\Gamma_{\rm M}}{\omega^2 - \omega_M^2}\right)$
  corresponding to a Breit-Wigner form of the spectral density in the Beth-Uhlenbeck EoS

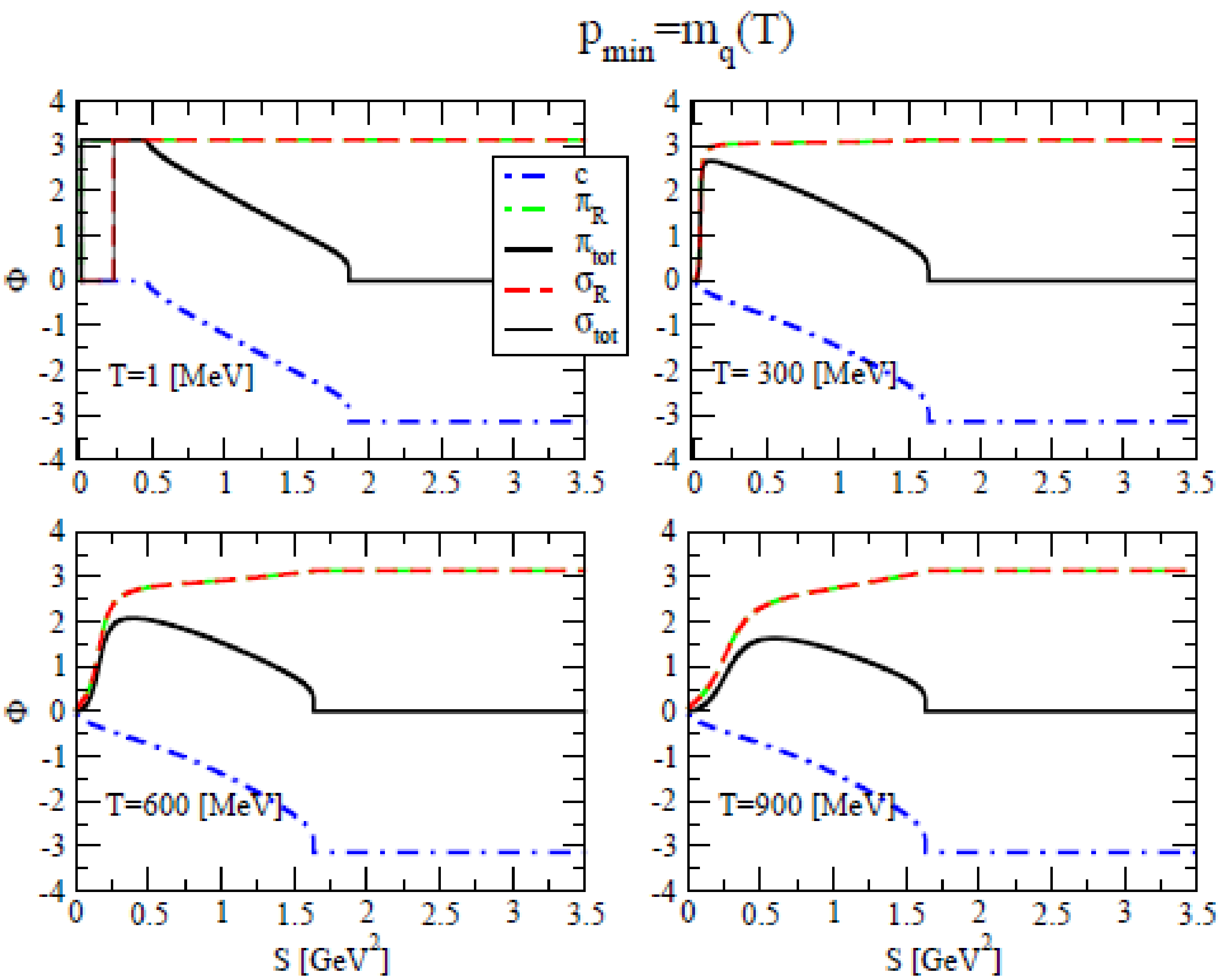
$$\frac{\mathrm{d}\delta_{M,R}}{\mathrm{d}\omega} = \frac{2\omega\omega_{\rm M}\Gamma_{\rm M}}{(\omega^2 - \omega_M^2)^2 + \omega_{\rm M}^2\Gamma_{\rm M}^2} \; .$$

- This takes the form of a bound state spectral density for $\Gamma_M \to 0$

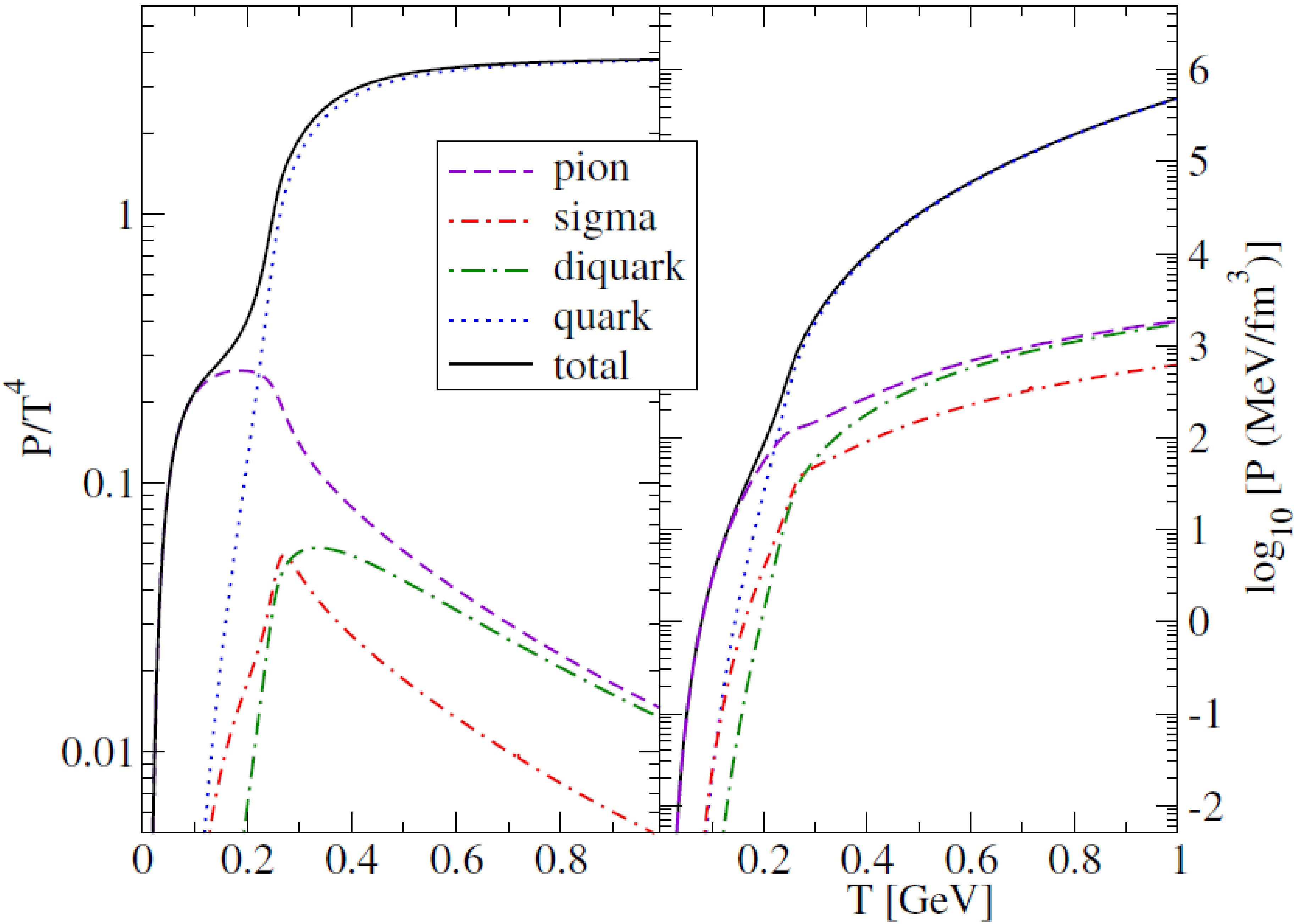
$$\lim_{\Gamma_{\rm M}\to 0} \delta'_{M,R}(\omega) = \pi\left[\delta(\omega - \omega_M) + \delta(\omega + \omega_M)\right]$$

Mott Dissociation of Mesons in Quark Matter





Mott Dissociation of Mesons and Diquarks in Quark Matter



Summary

- Generalized Beth-Uhlenbeck approach as microphysical basis to account for hadron dissociation (Mott effect) at extreme temperatures and densities
- Benchmark: pion and sigma Mott effect within NJL model, revised within nonlocal PNJL model
- Nonlocal PNJL model calibrated with lattice quark propagator data, EoS at finite T and μ, Phase diagram with critical point
- Application of GBU to interprete chemical freeze-out as Mott-Anderson localization
- Effective GBU model description: Mott-Hagedorn resonance gas + PNJL model describes Lattice QCD thermodynamics

Outlook

- RMF (Walecka) model as limit of the PNJL model: chiral transition effects in nuclear EoS
- Prospects for HIC (CBM & NICA) and Supernovae:
  color superconducting (quarkyonic) phases accessible!

Solving the Puzzles of Compact Star Interiors

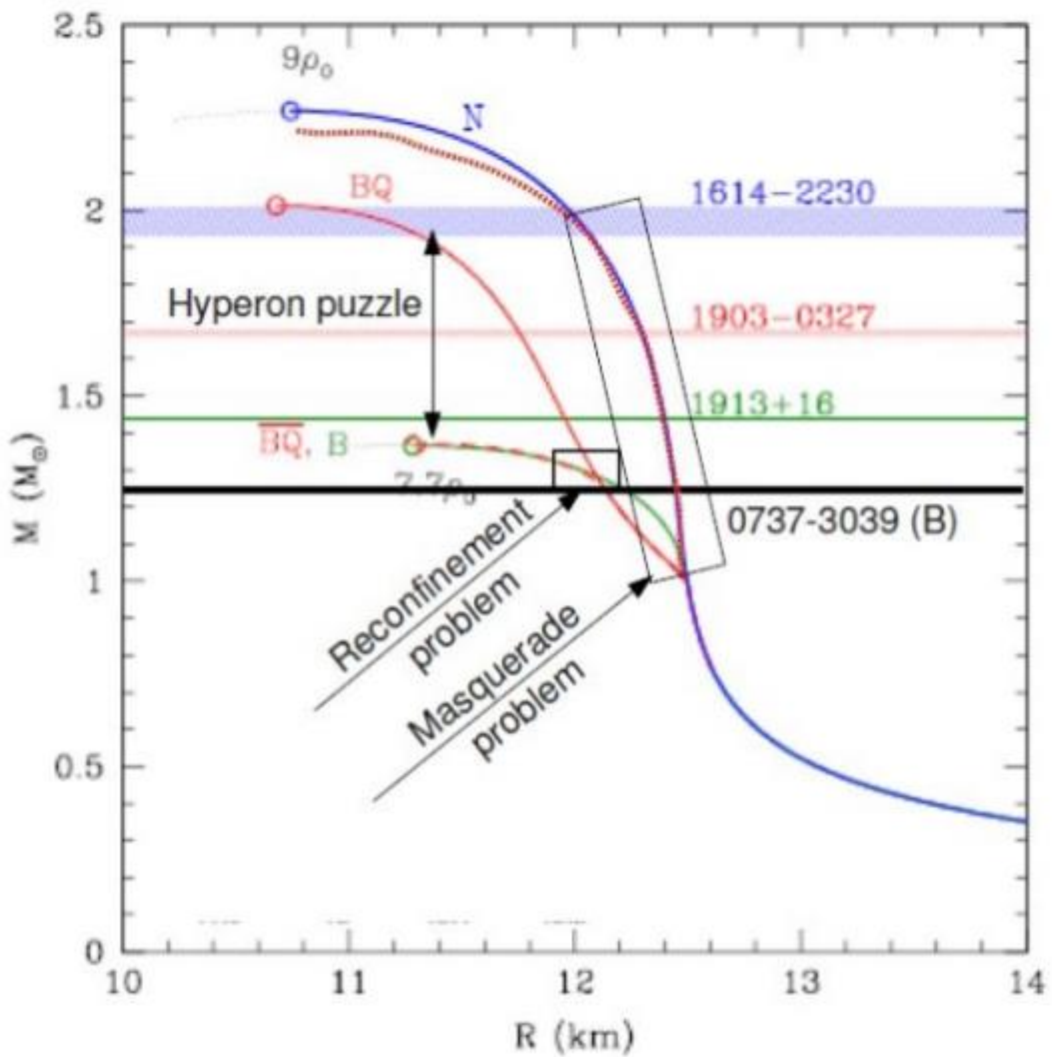
David Blaschke (University of Wroclaw, Poland & JINR Dubna, Russia)

1. The Puzzles:
- Hyperon puzzle
- Reconfinement
- Masquerade

2. The Solution:
Baryon finite size (compositeness)
→ Excluded volume Appr. (EVA)

3. The Mechanism:
Quark Pauli Blocking

4. Outlook:
- High-Mass Twins (next talk)
- Supernova explosion mechanism



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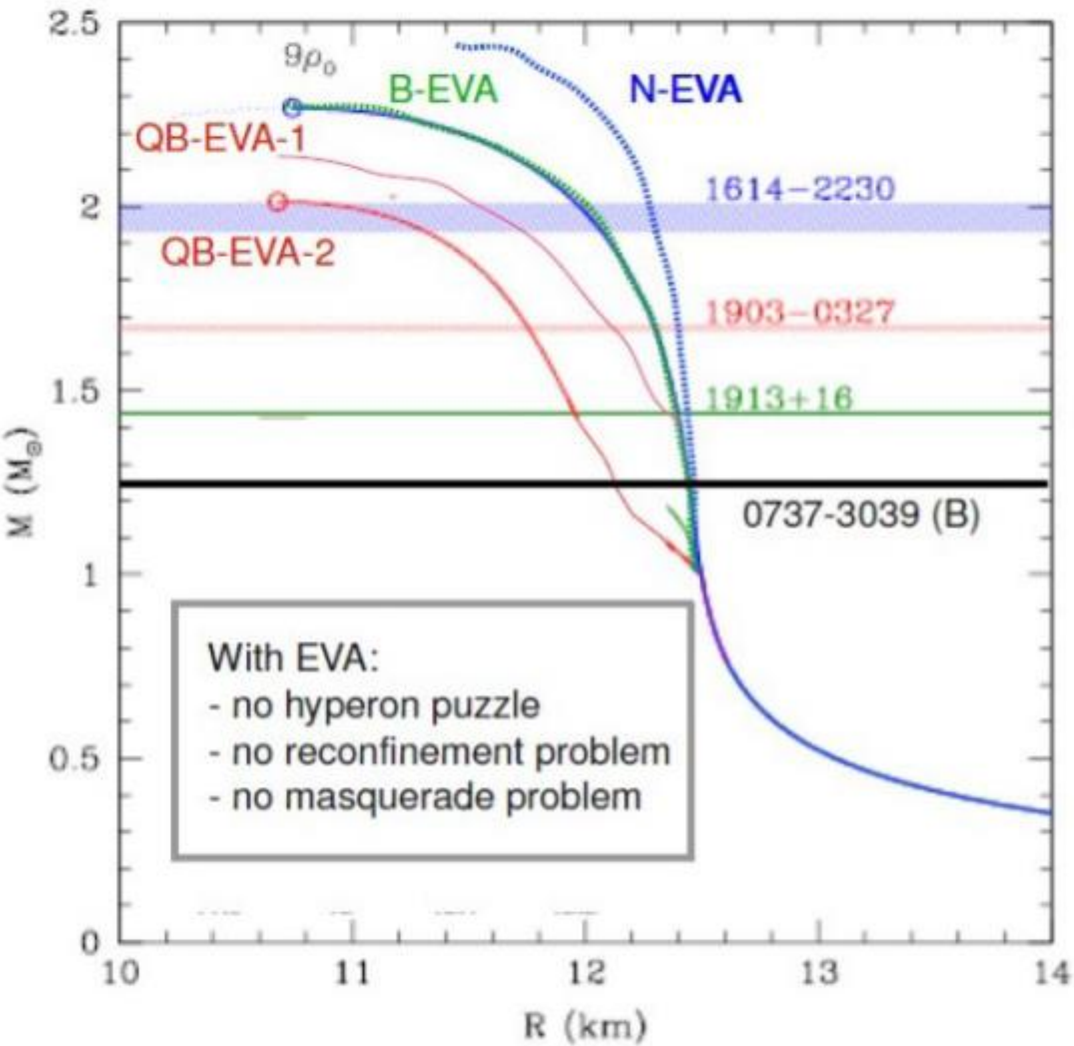

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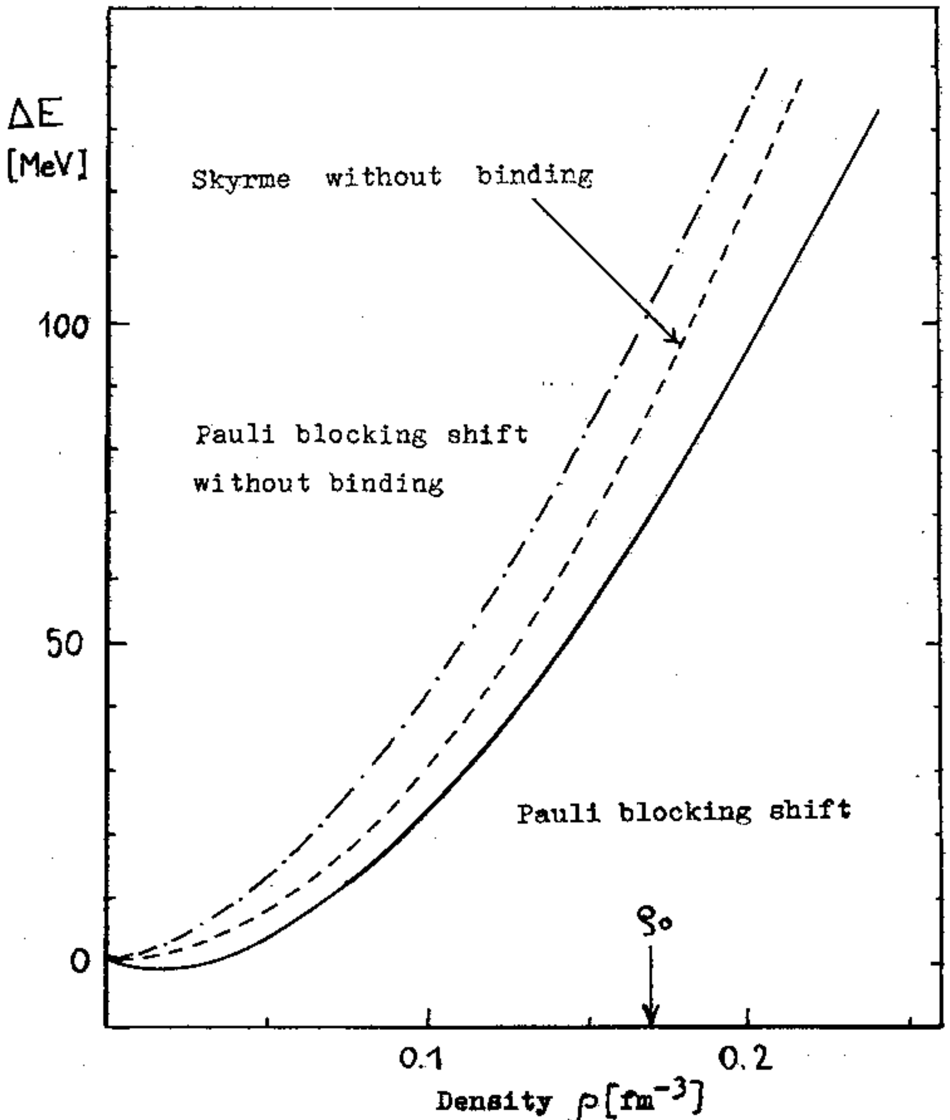

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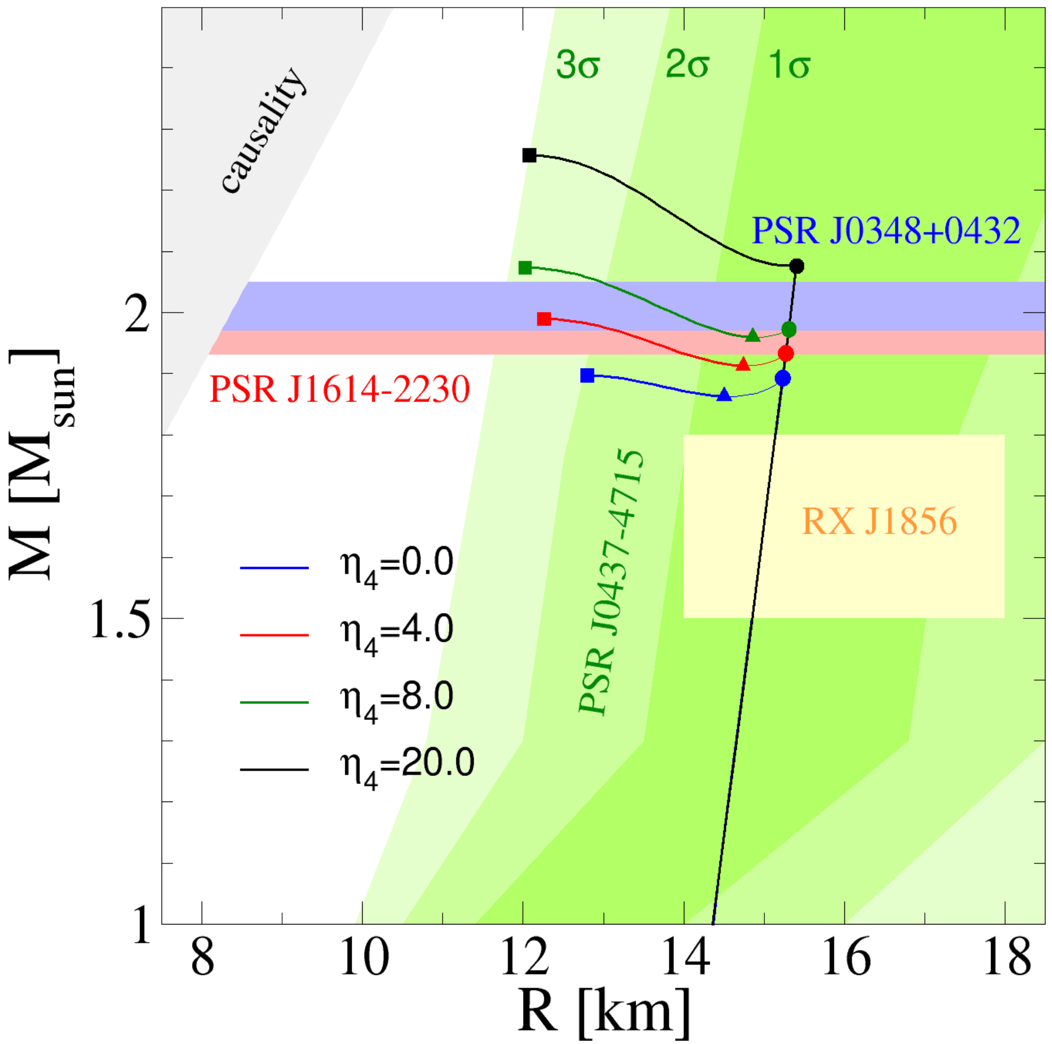
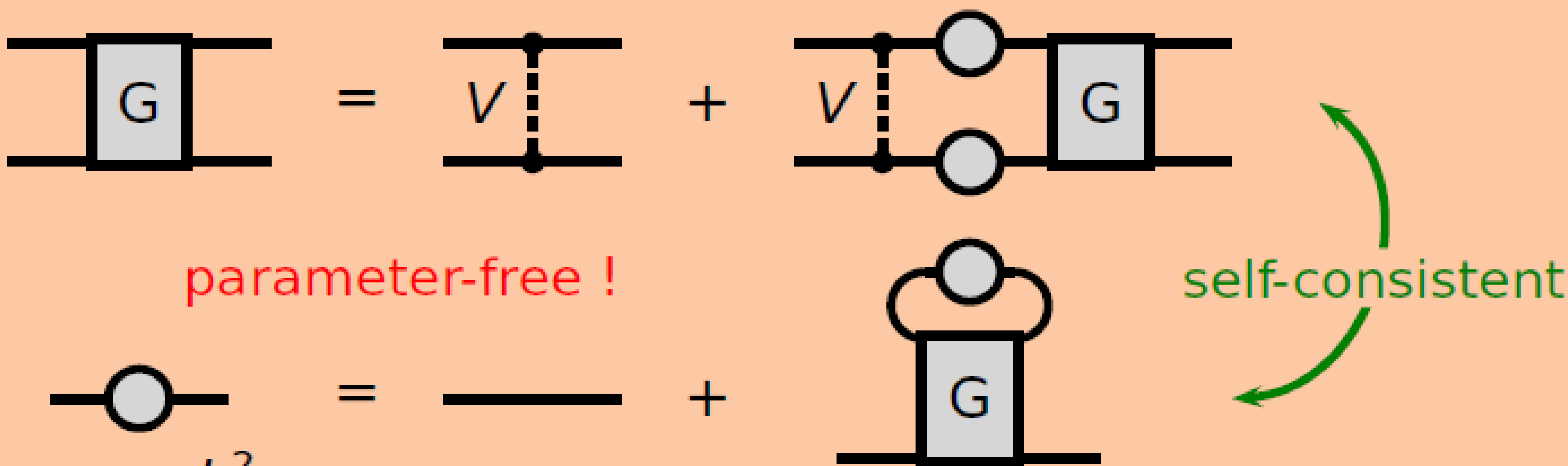


H.-J. Schulze:
Talk at HyperoNS 2012,
CAMK Warsaw

1.1. The Hyperon Puzzle

Brueckner Theory of Nuclear Matter:

- Effective in-medium interaction G from potential V:



$$e_k = \frac{k^2}{2m} + U(k)$$

Compute: binding energy $\epsilon(\rho_n, \rho_p, \rho_\Lambda, \rho_\Sigma)$,
s.p. properties, cross sections, ...

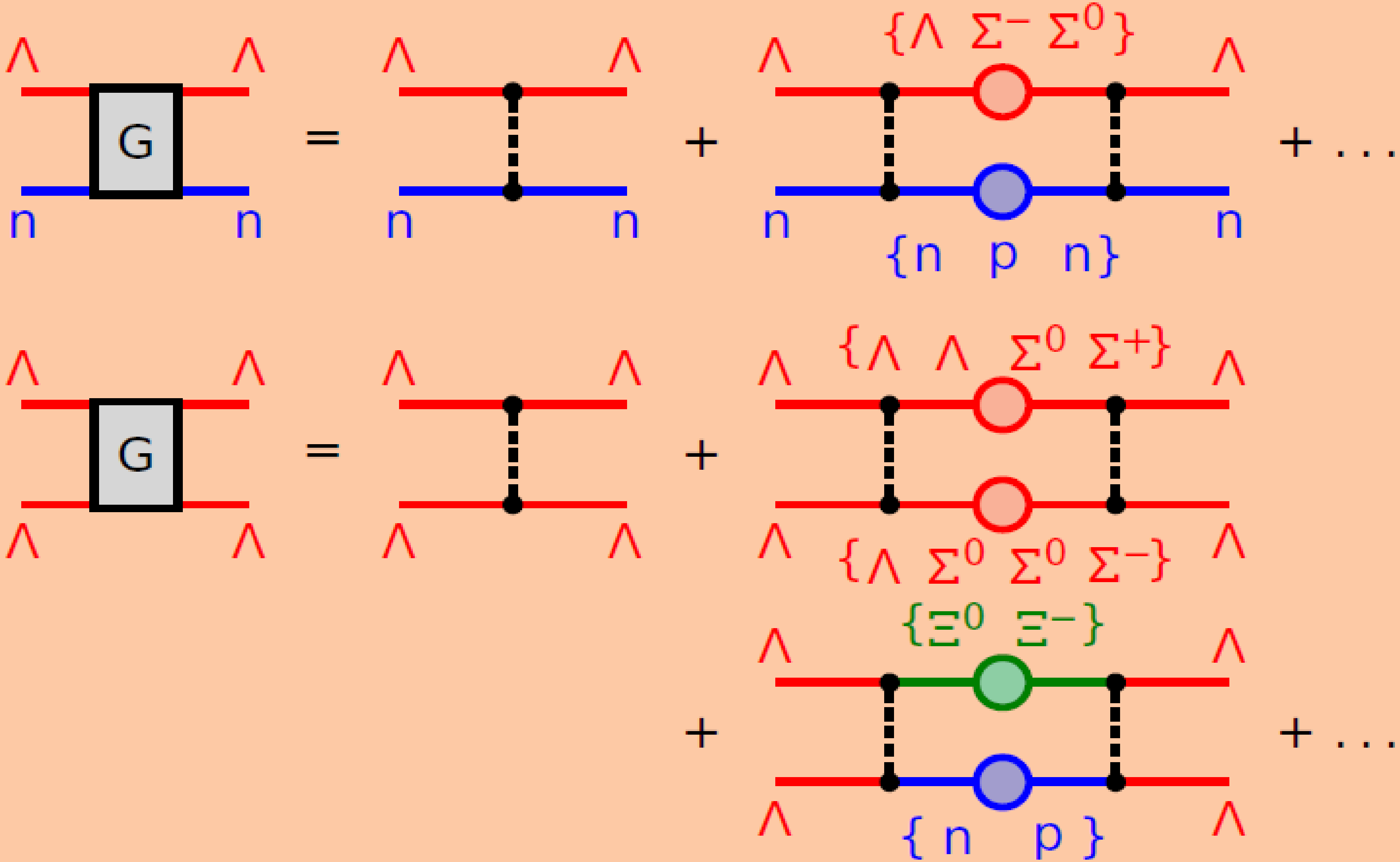
K.A. Brueckner and J.L. Gammel; PR 109, 1023 (1958) for nuclear matter

1.1. The Hyperon Puzzle

H.-J. Schulze:
Talk at HyperoNS 2012,
CAMK Warsaw

Include Hyperons:

- Technical difficulty: coupled channels:



1.1. The Hyperon Puzzle

H.-J. Schulze:
Talk at HyperoNS 2012,
CAMK Warsaw

«Recipe» for Neutron Star Structure Calculation:

Brueckner results: $\boldsymbol{\epsilon}(\boldsymbol{\rho}, \mathbf{x_e}, \mathbf{x_p}, \mathbf{x_\Lambda}, \mathbf{x_\Sigma}, \ldots)$; $x_i = \frac{\rho_i}{\rho}$

Chemical potentials: $\mu_i = \frac{\partial \epsilon}{\partial \rho_i}$

Beta-equilibrium: $\mu_i = b_i \mu_n - q_i \mu_e$

Charge neutrality: $\sum_i x_i q_i = 0$

$\mu_e = \mu_\mu = \mu_n - \mu_p$
$\mu_{\Sigma^-} = 2\mu_n - \mu_p$
$\mu_{\Sigma^0} = \mu_\Lambda = \mu_n$
$\mu_{\Sigma^+} = \mu_p$

Composition: $x_i(\rho)$

Equation of state: $p(\rho) = \rho^2 \frac{d(\epsilon/\rho)}{d\rho}(\rho, x_i(\rho))$

TOV equations:

$$\frac{dp}{dr} = -\frac{Gm}{r^2} \frac{(\epsilon + p)(1 + 4\pi r^3 p/m)}{1 - 2Gm/r}$$

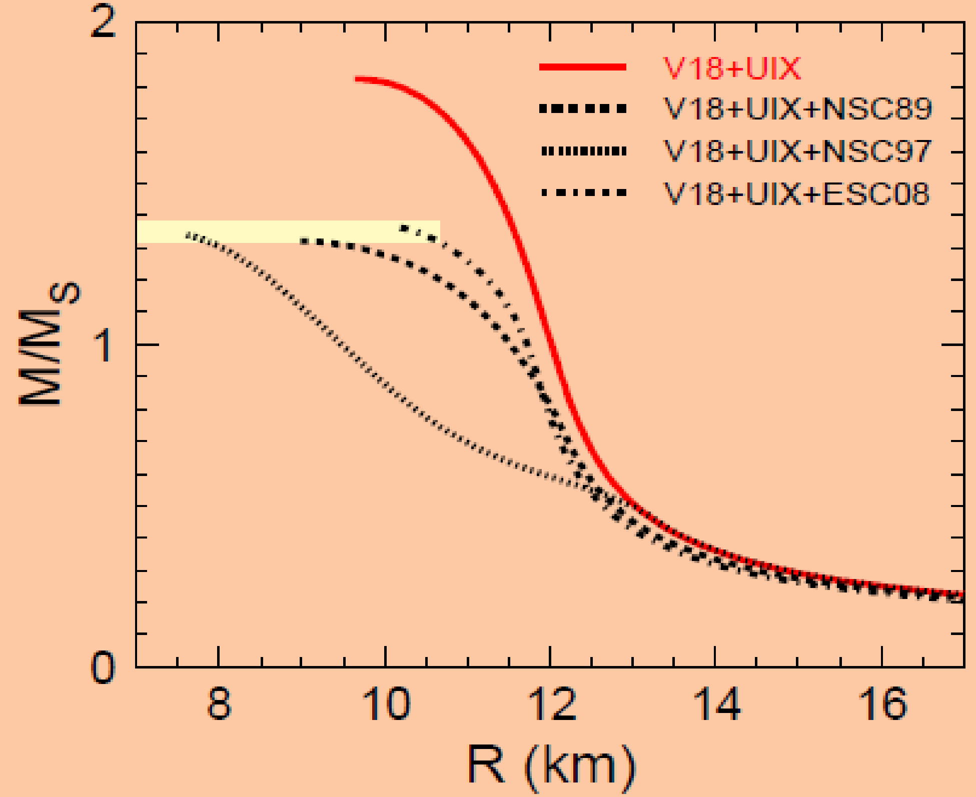
$$\frac{dm}{dr} = 4\pi r^2 \epsilon$$

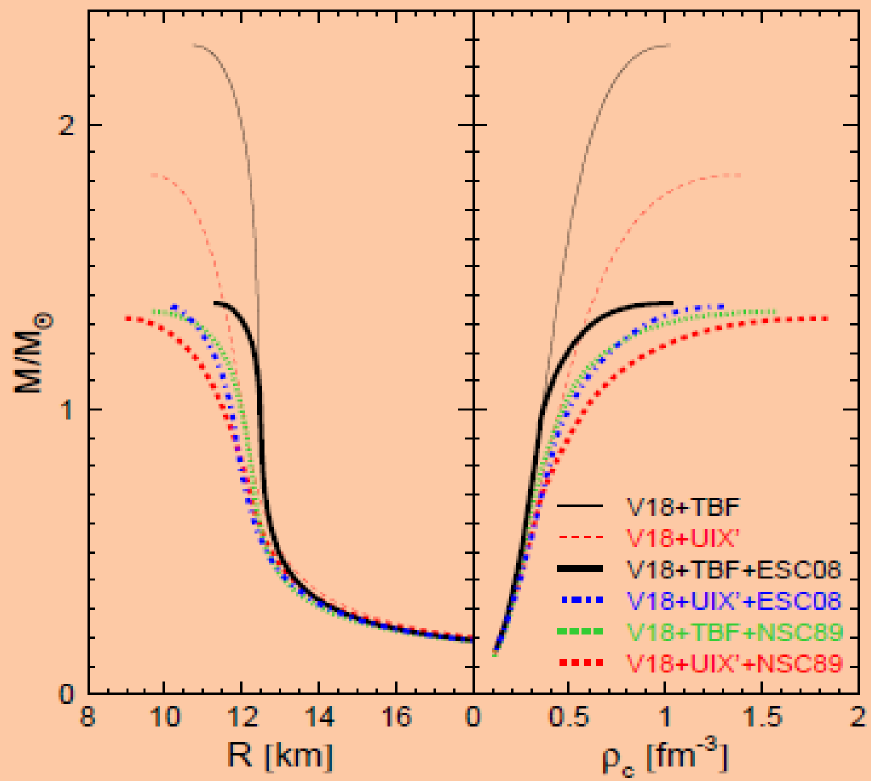
Structure of the star: $\rho(r)$, $\boldsymbol{M}(\boldsymbol{R})$ etc.

1.1. The Hyperon Puzzle

H.-J. Schulze:
Talk at HyperoNS 2012,
CAMK Warsaw

- Using different NY,YY potentials:

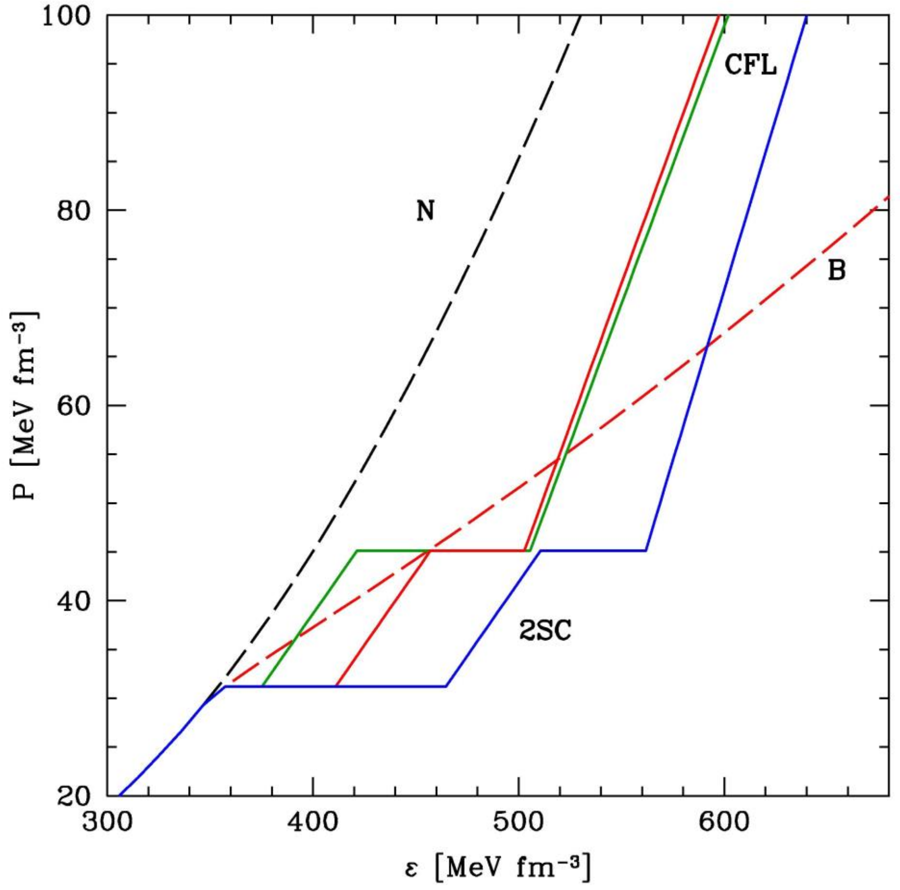


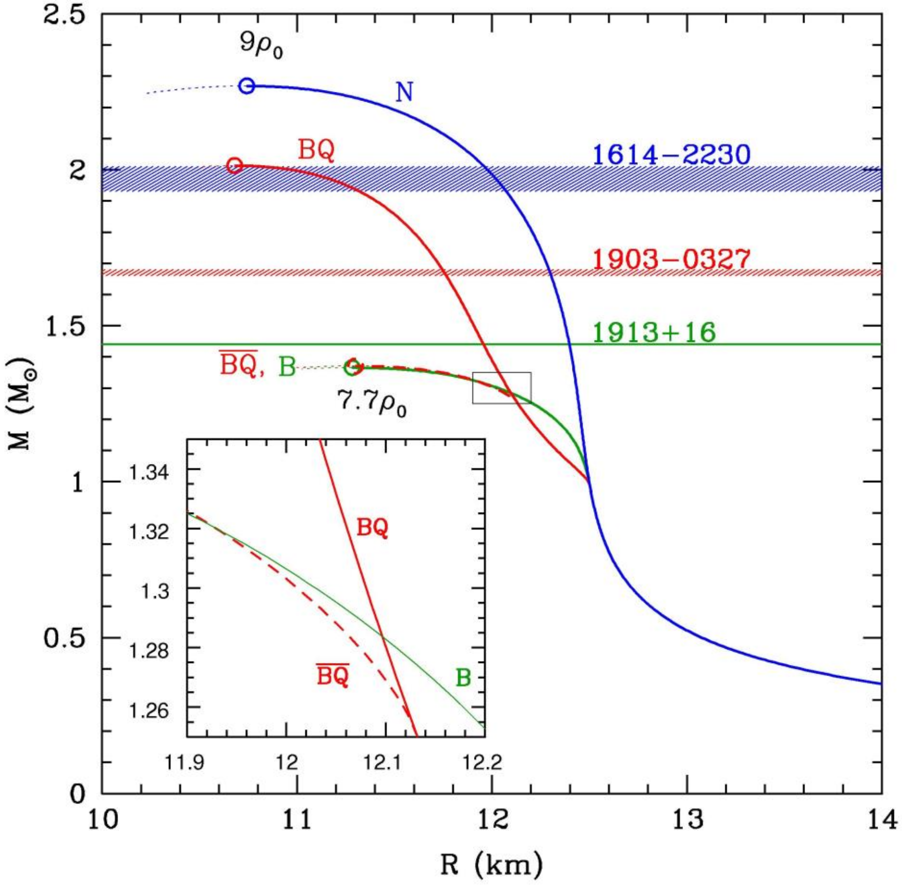


Maximum mass independent of potentials !
Maximum mass too low ($< 1.4 M_\odot$) !
Proof for "quark" matter inside neutron stars ?!

1.2. Reconfinement Problem

P. Haensel:
Talk at HyperoNS 2012,
CAMK Warsaw



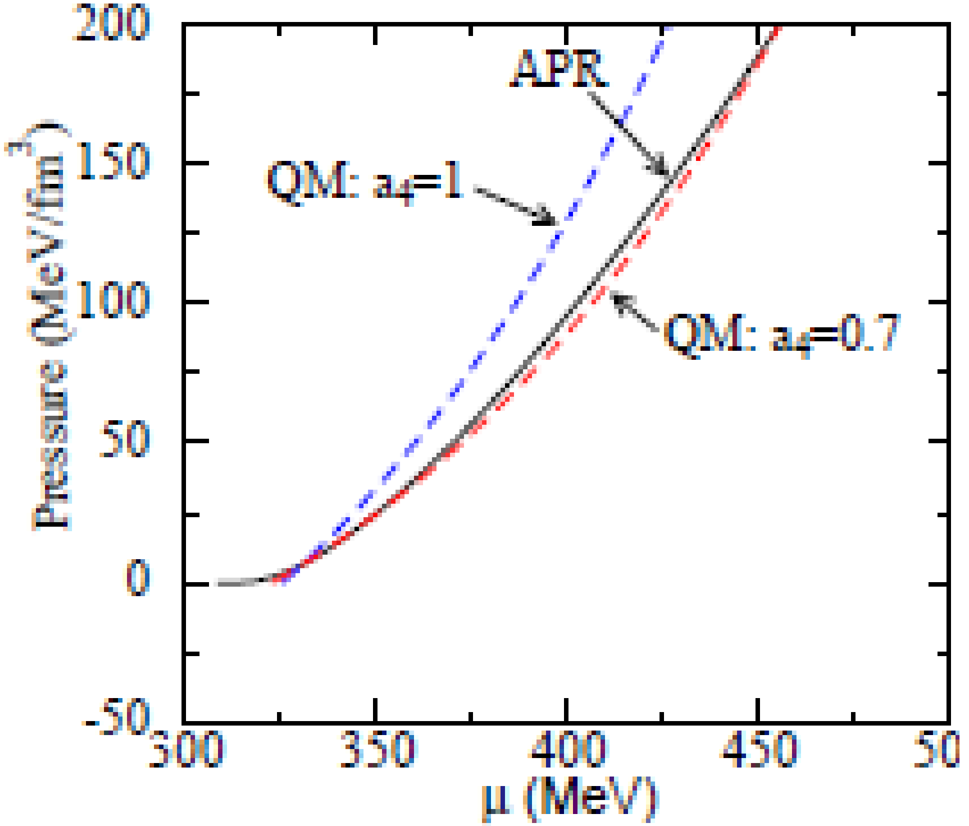


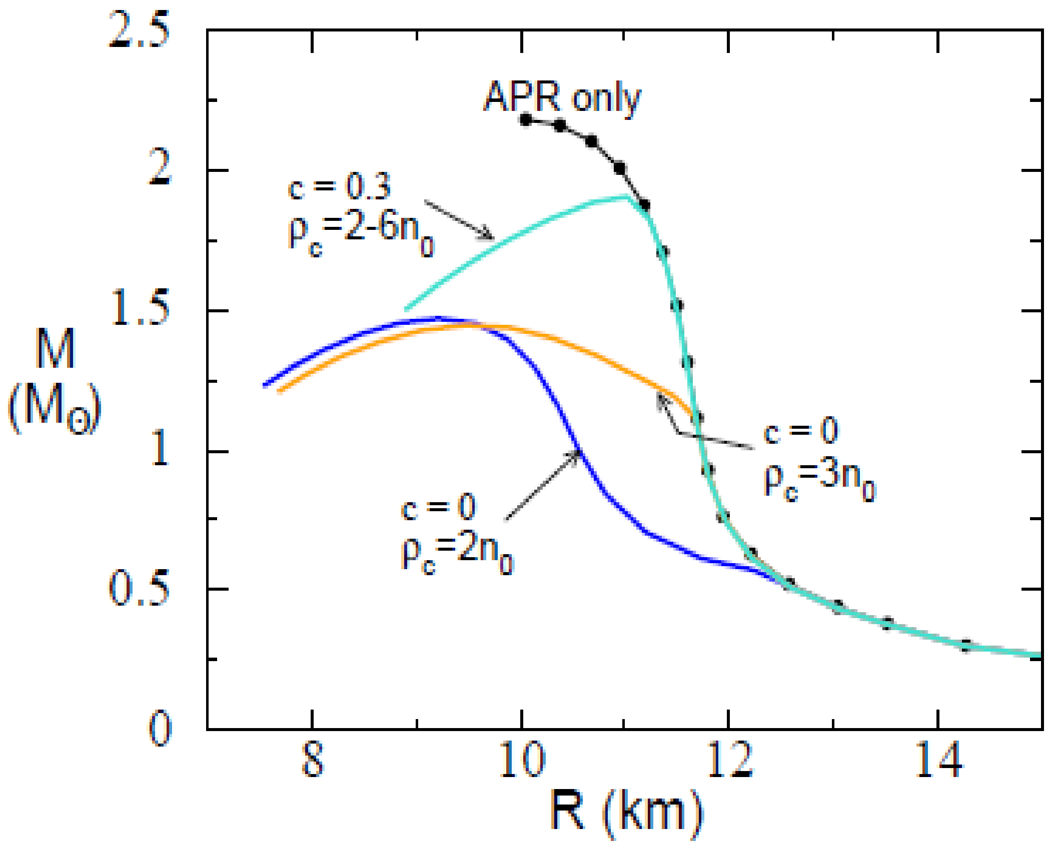
- Stability of stiff Q-core: re-confinement prohibited (see also *Lastowiecki et al. (2012)*) - indicates breakdown of the "point-particle" model of baryons

- $M_{\rm max}^{\rm (obs)} \simeq 2.2 \div 2.4\ {\rm M}_\odot$ would require $v_{\rm s}^{\rm (Q)} > 0.8 \div 0.9c$

1.3. Masquerade Problem

M. Alford et al.: "Hybrid stars that masquerade as neutron stars", ApJ 629 (2005) 969





$$\Omega_{QM} = -\frac{3}{4\pi^2} a_4 \mu^4 + \frac{3}{4\pi^2} a_2 \mu^2 + B_{eff} , \quad a_4 \equiv 1 - c$$

Quark and neutron star matter EoS are practically indistinguishable for many classes of models. Then the hybrid star branch remains indistinguishable from the neutron star branch!

2.1. Baryon finite size: Excluded volume approx. (EVA)

$$p_{\rm ex}(\mu,T) = p(\tilde{\mu},T)\ , \ \ \tilde{\mu} = \mu - v_0(\mu,T)p_{\rm ex}(\mu,T)$$

$$n_{\rm ex}(\mu,T) = \frac{\partial p_{\rm ex}}{\partial \mu} = \frac{\partial \tilde{\mu}}{\partial \mu}\frac{\partial p(\tilde{\mu},T)}{\partial \tilde{\mu}} = \left[1 - v_0 n_{\rm ex}(\mu,T) - \frac{\partial v_0}{\partial \mu}p_{\rm ex}(\mu,T)\right] n(\tilde{\mu},T)$$

Thermodynamic consistency:

$$\epsilon_{\rm ex}(\mu,T) = -p_{\rm ex}(\mu,T) + \mu n_{\rm ex}(\mu,T) + T s_{\rm ex}(\mu,T)$$

Parametrization of excluded volume with nonlinear dependence on the chemical potential:

$$v_0(\mu,T) = (4\pi/3)r^3(\mu)\ \ ,\ \ r^3(\mu) = r_0 + r_1(\mu/\mu_c)^2 + r_2(\mu/\mu_c)^4$$

S. Benic, D.B., D. Alvarez, T. Fischer: “Hybrid EoS supporting high-mass twin stars”, In preparation (2014)

2.2. Higher order quark interactions in NJL quark matter

$$\mathcal{L} = \bar{q}(i\partial\!\!\!/ - m)q + \mu_q \bar{q}\gamma^0 q + \mathcal{L}_4 + \mathcal{L}_8\ ,\quad \mathcal{L}_4 = \frac{g_{20}}{\Lambda^2}[(\bar{q}q)^2 + (\bar{q}i\gamma_5\tau q)^2] - \frac{g_{02}}{\Lambda^2}(\bar{q}\gamma_\mu q)^2\ ,$$

$$\mathcal{L}_8 = \frac{g_{40}}{\Lambda^8}[(\bar{q}q)^2 + (\bar{q}i\gamma_5\tau q)^2]^2 - \frac{g_{04}}{\Lambda^8}(\bar{q}\gamma_\mu q)^4 - \frac{g_{22}}{\Lambda^8}(\bar{q}\gamma_\mu q)^2[(\bar{q}q)^2 + (\bar{q}i\gamma_5\tau q)^2]$$

Meanfield approximation: $\mathcal{L}_{\mathrm{MF}} = \bar{q}(i\partial\!\!\!/ - M)q + \tilde{\mu}_q \bar{q}\gamma^0 q - U\ ,$

$$M = m + 2\frac{g_{20}}{\Lambda^2}\langle\bar{q}q\rangle + 4\frac{g_{40}}{\Lambda^8}\langle\bar{q}q\rangle^3 - 2\frac{g_{22}}{\Lambda^8}\langle\bar{q}q\rangle\langle q^\dagger q\rangle^2\ ,$$

$$\tilde{\mu}_q = \mu_q - 2\frac{g_{02}}{\Lambda^2}\langle q^\dagger q\rangle - 4\frac{g_{04}}{\Lambda^8}\langle q^\dagger q\rangle^3 - 2\frac{g_{22}}{\Lambda^8}\langle\bar{q}q\rangle^2\langle q^\dagger q\rangle\ ,$$

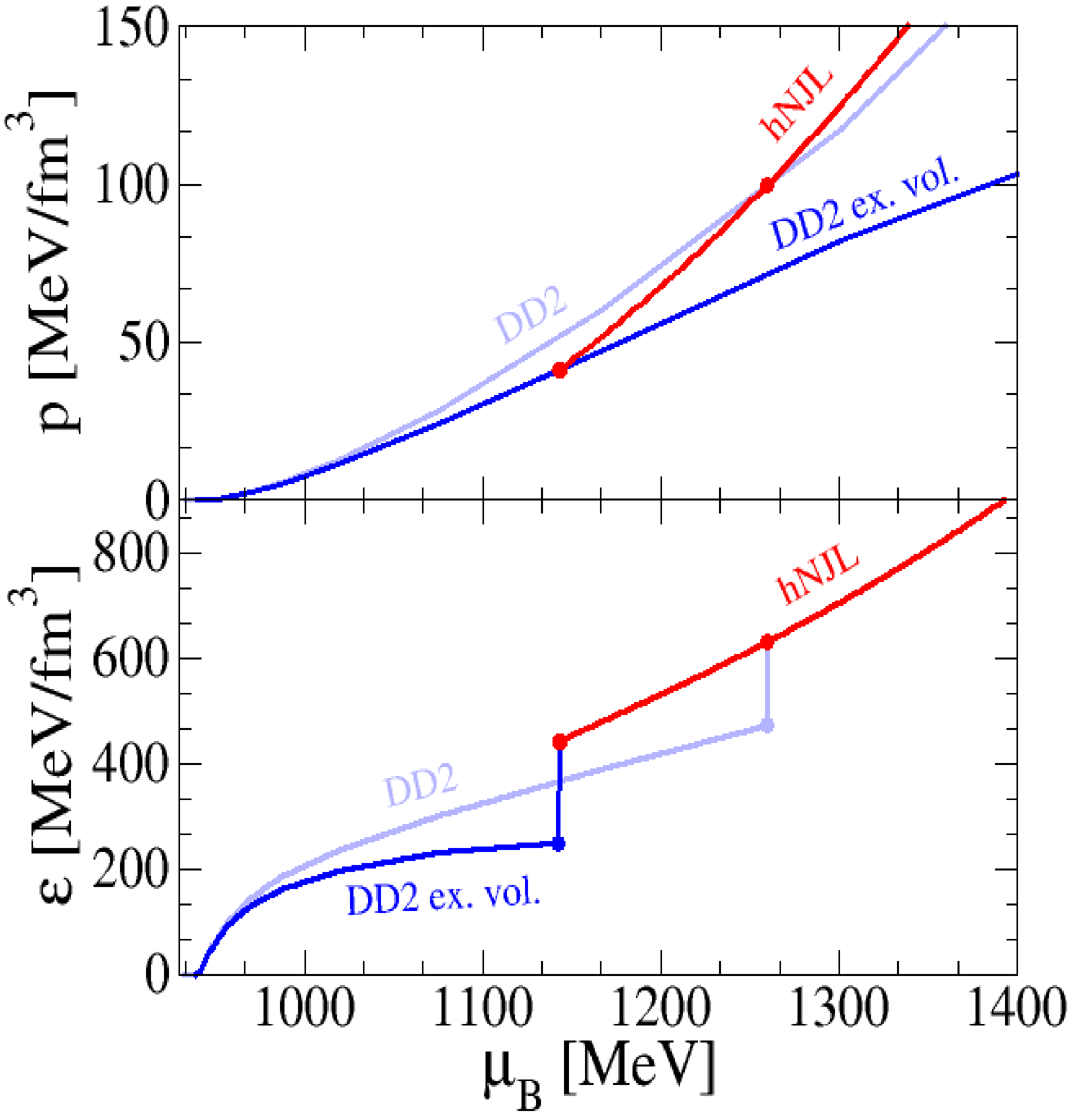
$$U = \frac{g_{20}}{\Lambda^2}\langle\bar{q}q\rangle^2 + 3\frac{g_{40}}{\Lambda^8}\langle\bar{q}q\rangle^4 - 3\frac{g_{22}}{\Lambda^8}\langle\bar{q}q\rangle^2\langle q^\dagger q\rangle^2 - \frac{g_{02}}{\Lambda^2}\langle q^\dagger q\rangle^2 - 3\frac{g_{04}}{\Lambda^8}\langle q^\dagger q\rangle^4\ .$$

Thermodynamic Potential:

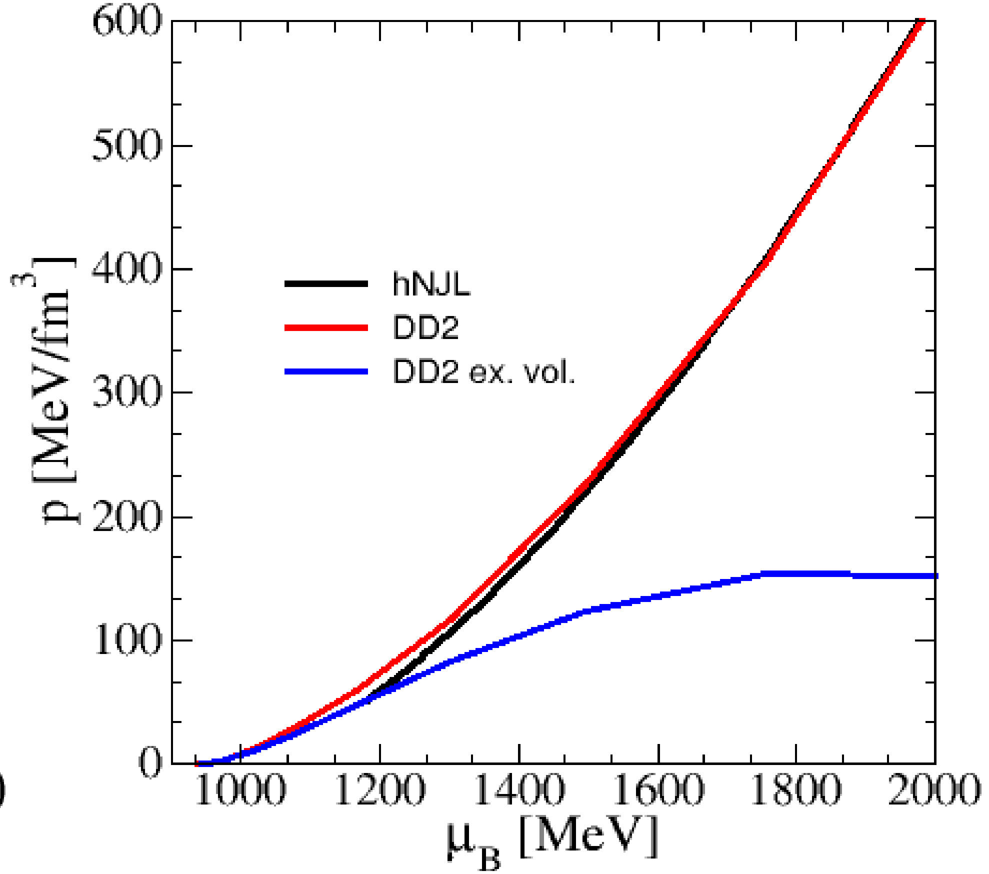
$$\Omega = U - 2N_f N_c \int \frac{d^3p}{(2\pi)^3}\left\{E + T\log[1 + e^{-\beta(E-\tilde{\mu}_q)}] + T\log[1 + e^{-\beta(E+\tilde{\mu}_q)}]\right\} + \Omega_0$$

S. Benic, D.B., D. Alvarez, T. Fischer: "Hybrid EoS supporting high-mass twin stars", in preparation (2014)

2.3. Hybrid EoS - Results



Baryon size effect (EVA):
- prevents masquerade!
- strong 1st order PT

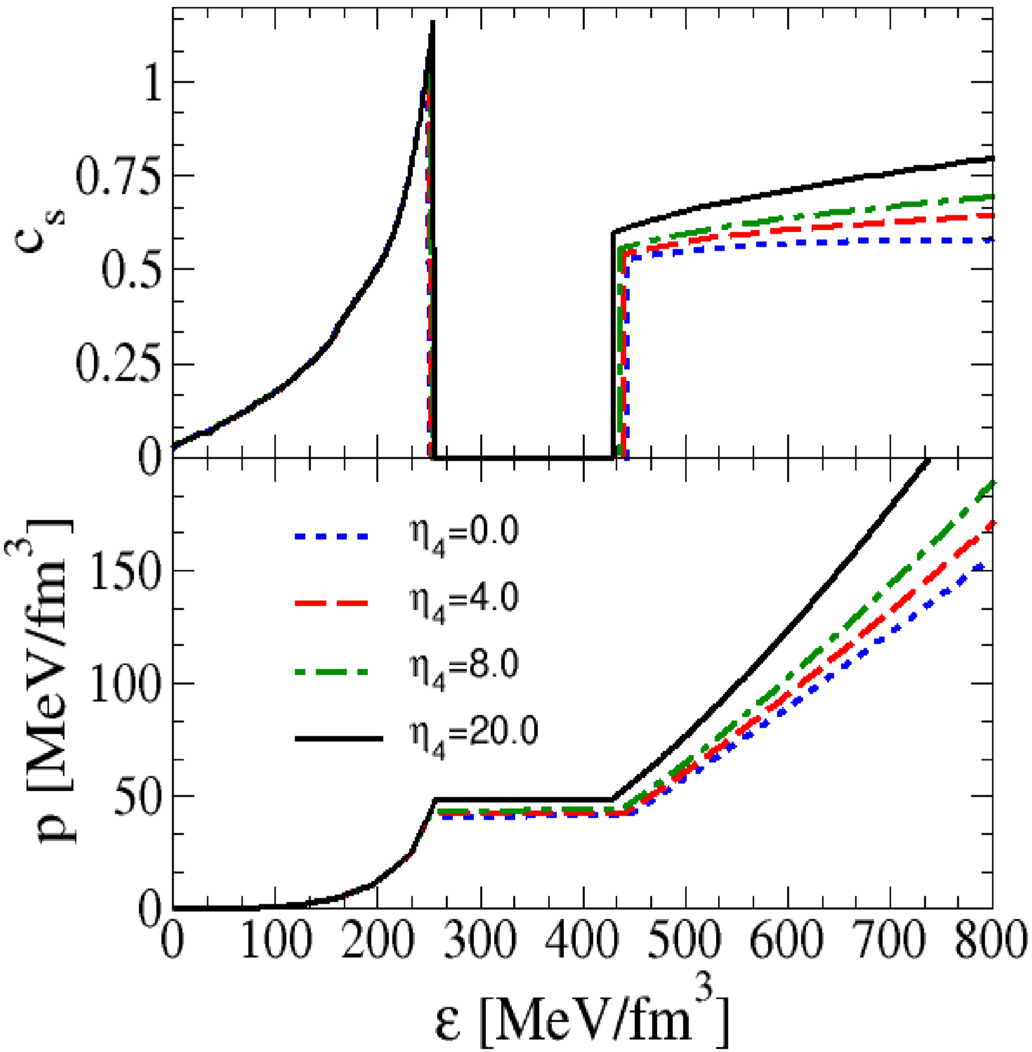


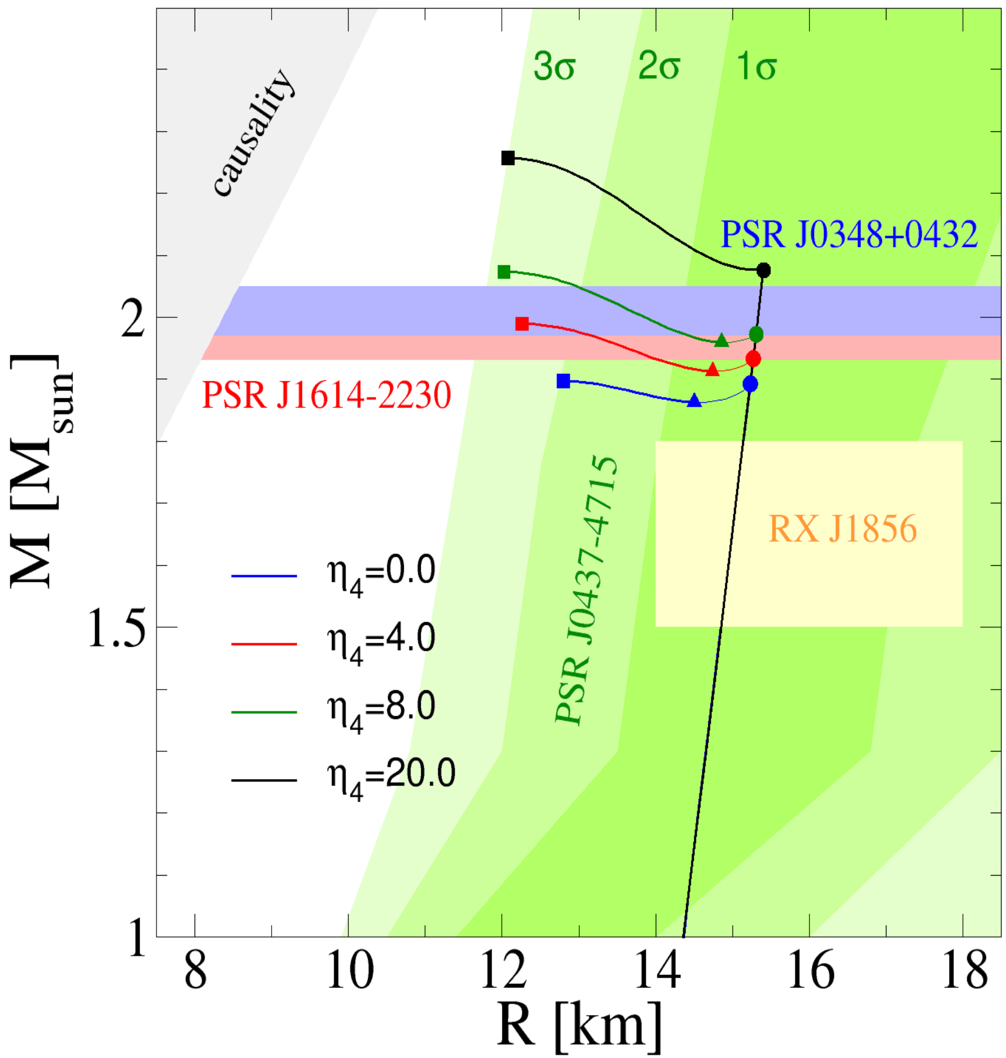
S. Benic, D.B., D. Alvarez, T. Fischer: “Hybrid EoS supporting high-mass twin stars”, in preparation (2014)

2.3. Hybrid EoS - Results

Mass-radius sequences:
- vertical hadronic branch
- horizontal hybrid branch

Observable !!

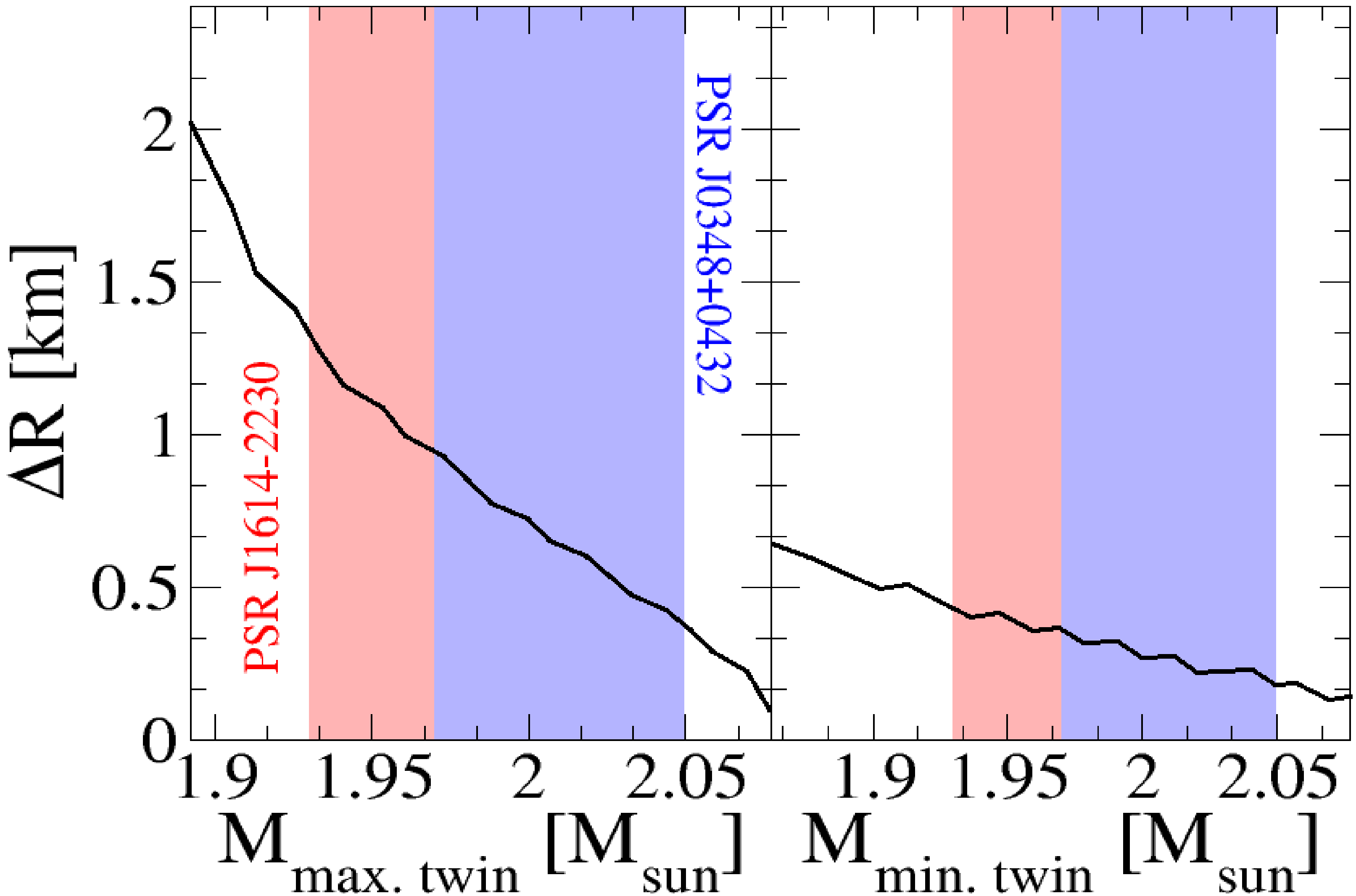




S. Benic, D.B., D. Alvarez, T. Fischer: "Hybrid EoS supporting high-mass twin stars", in preparation (2014)

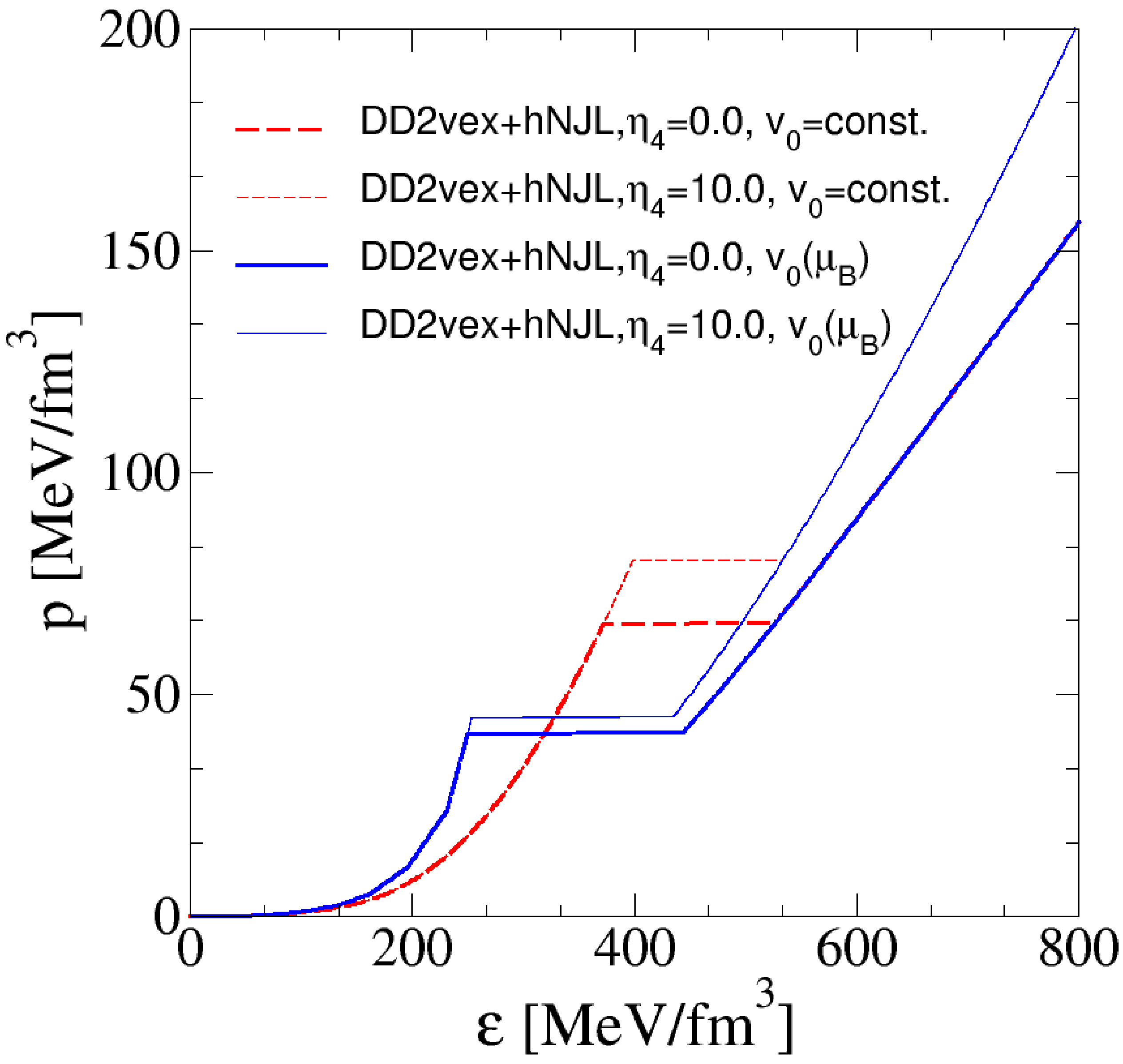
2.3. Hybrid EoS - Results

Observable: Radius difference of high-mass twin stars !!



S. Benic, D.B., D. Alvarez, T. Fischer: “Hybrid EoS supporting high-mass twin stars”, in preparation (2014)

2.3. Hybrid EoS - Results

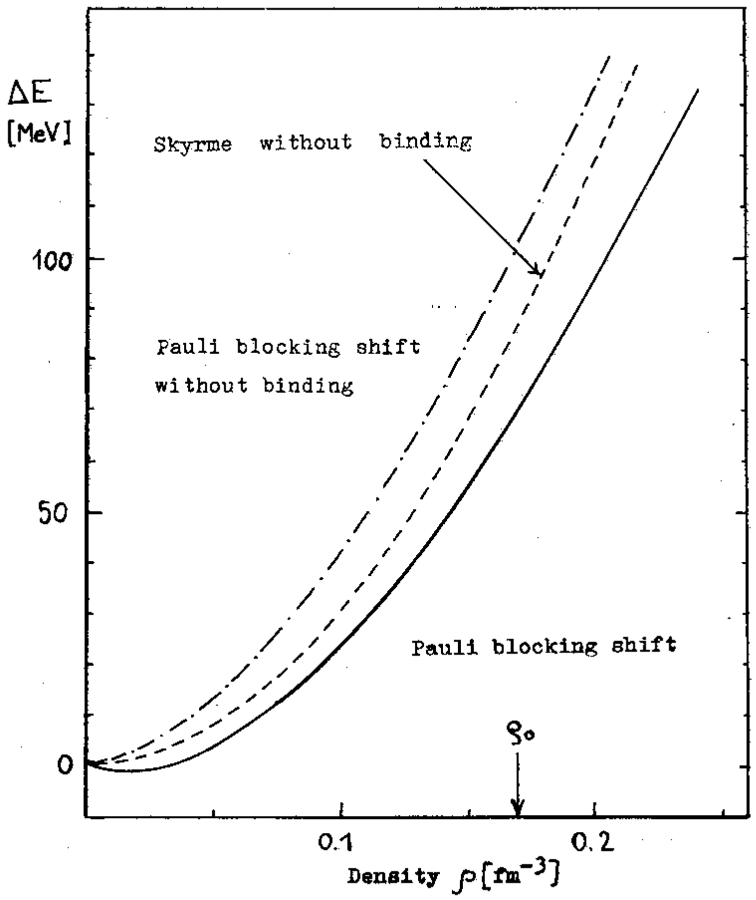


Essential for twin stars: density-dependent EVA !!

Constant EVA: No twins !!

S. Benic, D.B., D. Alvarez, T. Fischer: "Hybrid EoS supporting high-mass twin stars", in preparation (2014)

3. Density-dependent EVA: Quark Pauli Blocking !



Density dependent nucleon radius from Virial theorem:

$$\langle r^2 \rangle / \langle r^2 \rangle_0 = 1 + m b^2 \left(\frac{a_1}{6} \rho + \frac{a_2}{8} \rho^{5/3} \right)$$

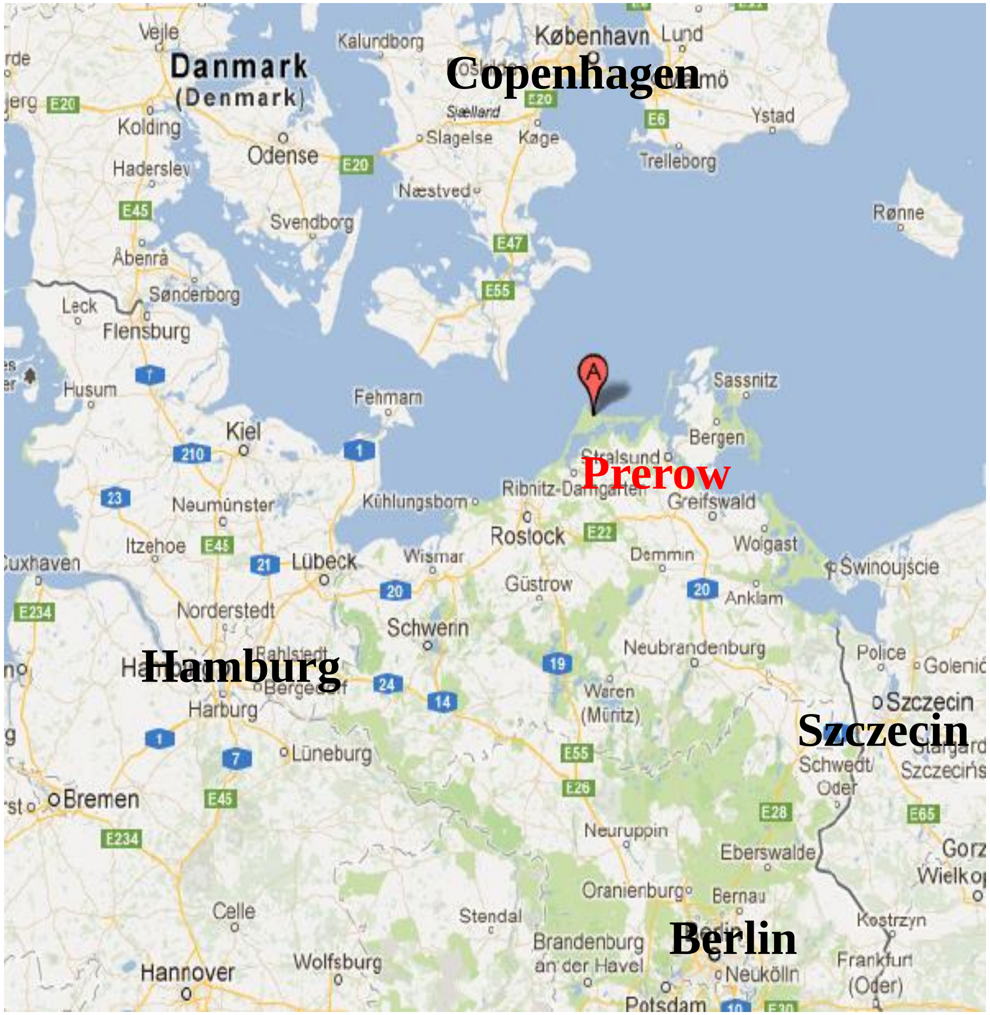
$$\Delta E_{nn'}^{Pauli} = \frac{1}{N_{nn'}} \langle \Phi_{nn'} | H | \Phi_{nn'} \rangle - E_n - E_{n'} \, ,$$

$$\Phi_{nn'}(1 \ldots 6) = \left(1 - \sum_{i=1}^{3} P_{i,i+3}\right)(1 - P_{nn'}) \Psi_n(123) \Psi_{n'}(456)$$

$$\Delta E_{\nu P_F}^{Pauli} = \frac{5}{8\sqrt{3\pi}} \frac{b}{m} \left\{ -P_F^3 + \frac{1054}{225} b^2 P_F^5 \right\}$$

D.B., G. Roepke: “Pauli blocking for hadrons in nuclear matter …”, Dubna Preprint E2-88-77 (1988)

CSQCD IV: Prerow, Sept. 26-30, 2014



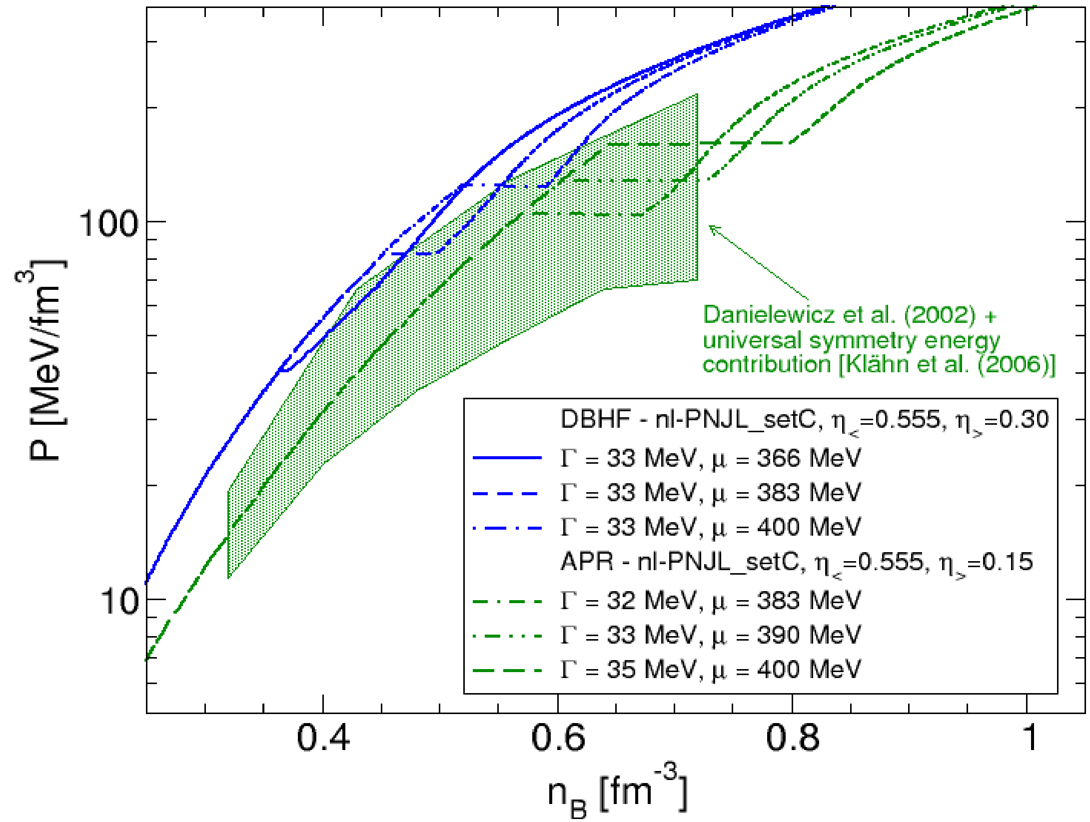
Website:

http://www.ift.uni.wroc.pl/~csqcdiv

Proving the CEP with Compact Stars

David Blaschke (University of Wroclaw, Poland & JINR Dubna, Russia)

1. Goal: Find 1st order PT
2. Observation: M & R
3. Theory: QCD based EoS



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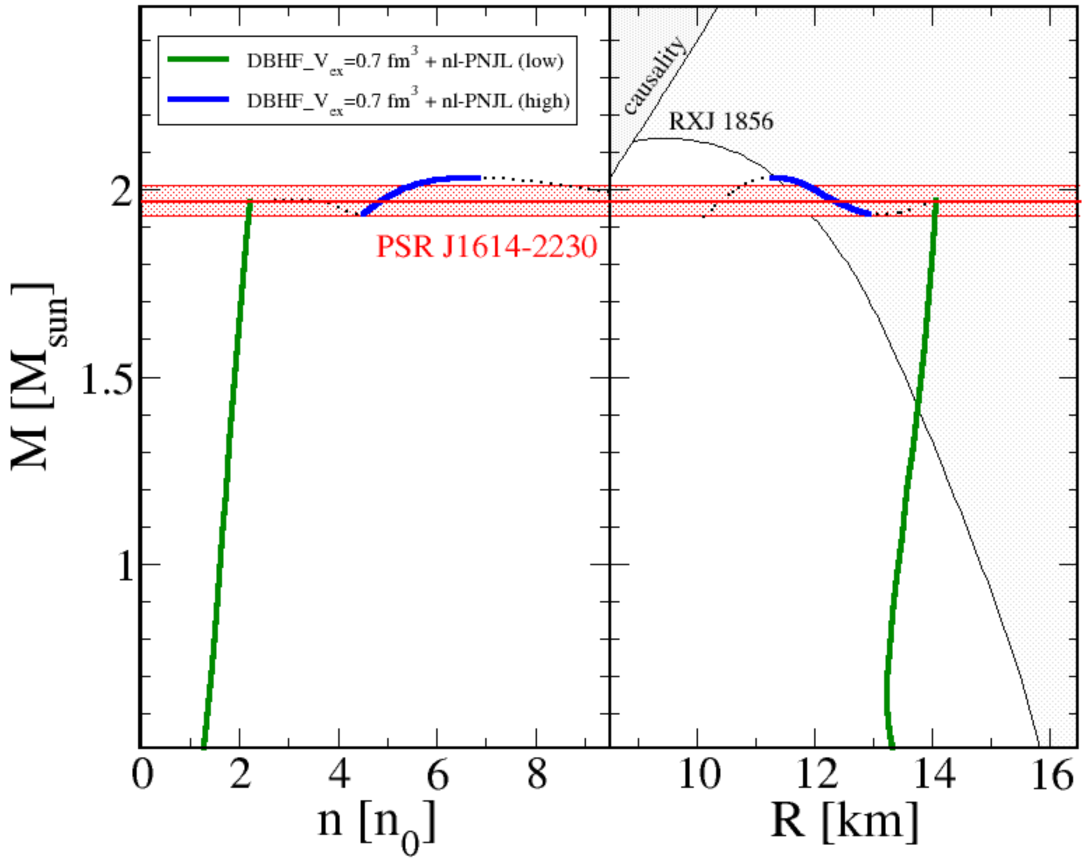

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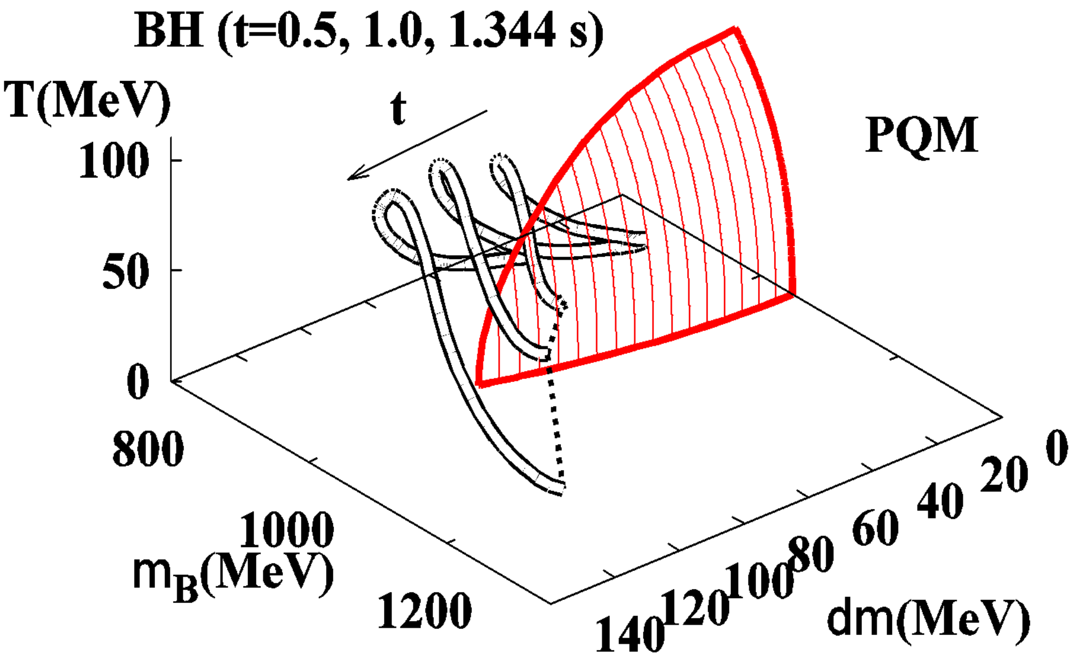

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4. Holy Grail: Twins !
5. Hot: BH formation
6. Future: LOFT, SKA, ...

JINR DUBNA · RFBR · Narodowe Centrum Nauki · Uniwersytet Wrocławski

Goal 1: Measure the cold EoS !

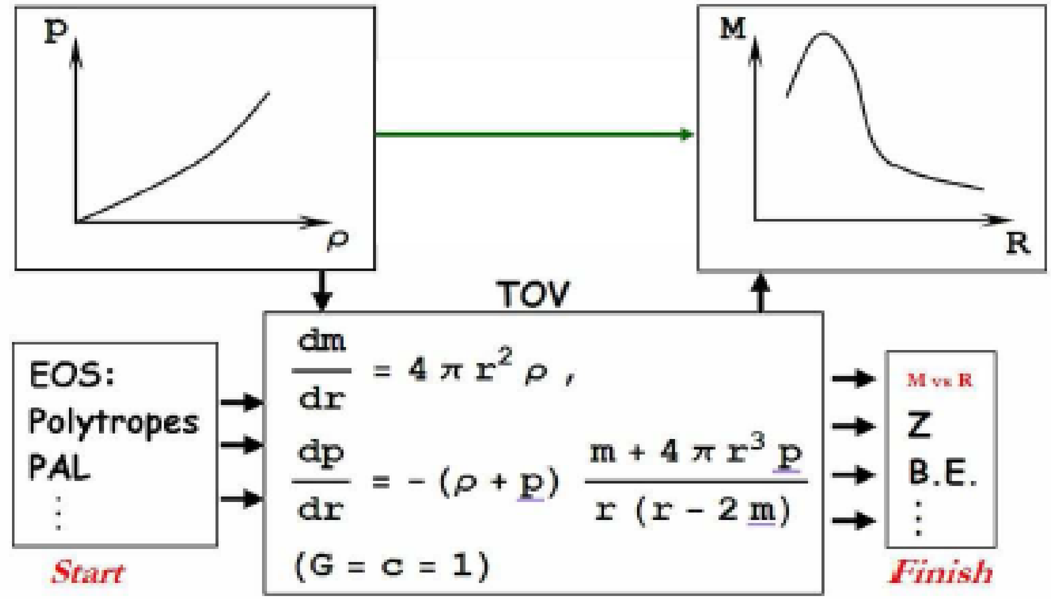
Direct approach:

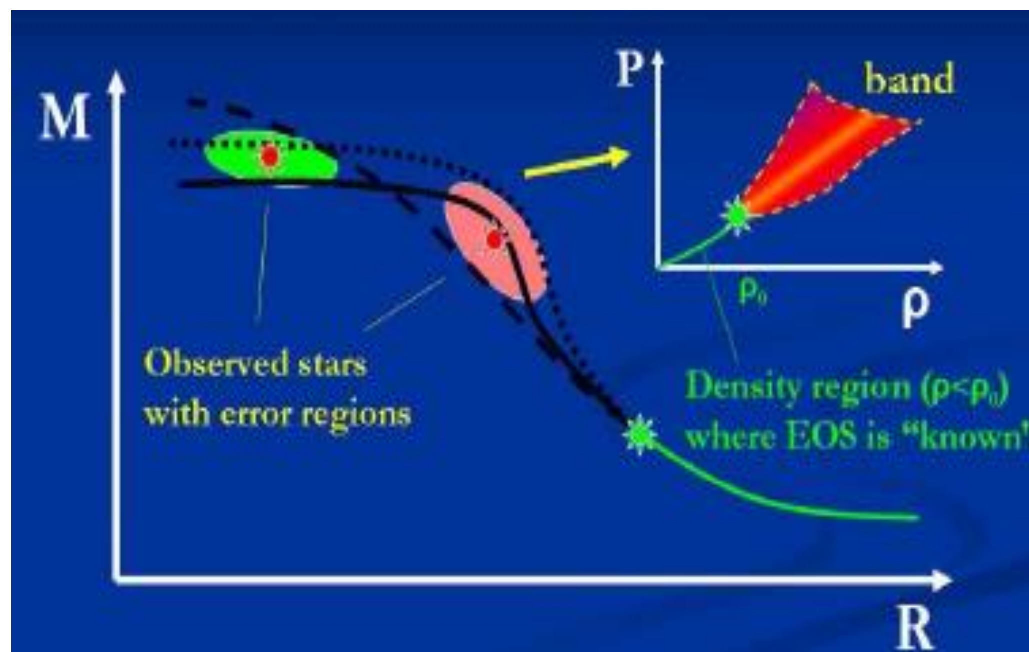
EoS is given as P(ρ)
→ solve the TOV Equation
 to find M(R)

Idea: Invert the approach

Given M(R) → find the EoS

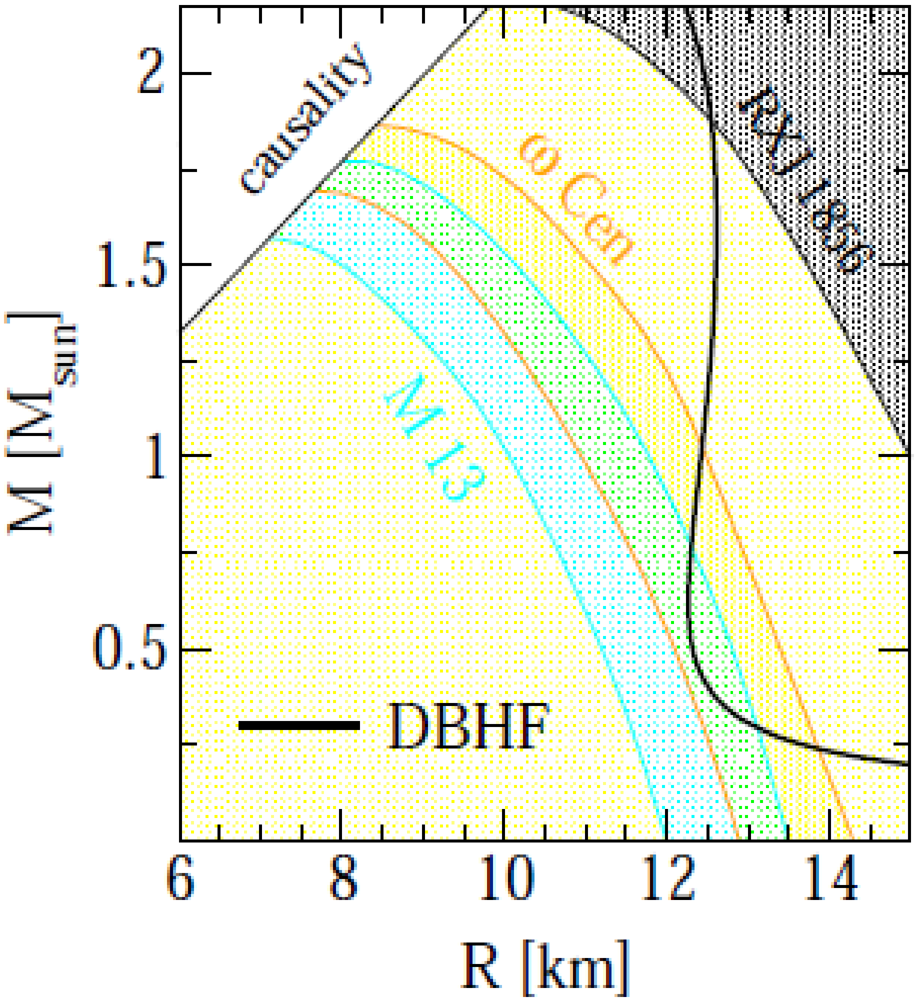
Bayesian analysis





Plots: M. Prakash, Talk Hirschegg 2009

Measure masses and radii of CS!



- Distance measured
- Spectrum measured (ROSAT, XMM, Chandra)
- Luminosity measured

→ effective temperature T_∞

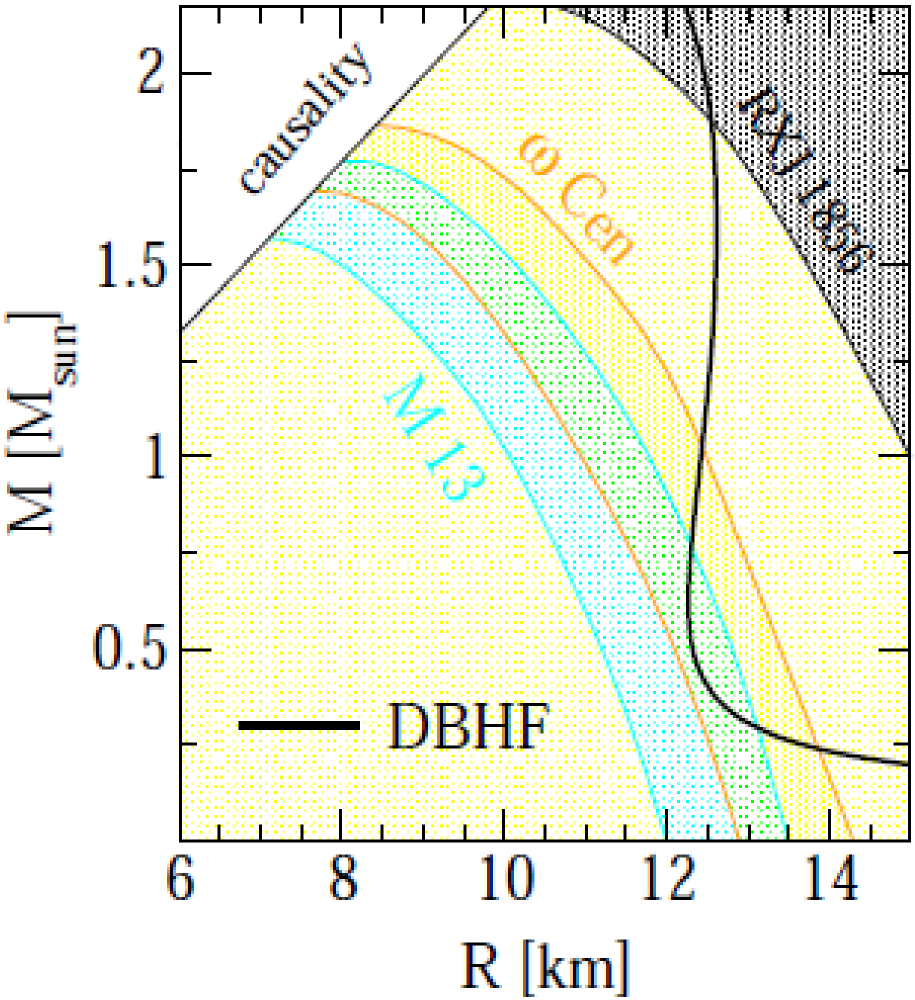
→ photospheric radius

$$R_\infty = R/\sqrt{1 - R/R_S}\ , \quad R_S = 2GM/R$$

Object	R_∞ [km]	Reference
RXJ 1856	16.8	Trümper et al. (2004)
ωCen	13.6 $\pm$0.3	Gendre et al. (2003)
M13	12.8 $\pm$0.4	Gendre et al. (2004)

Lower limit from RXJ 1856 incompatible with ω Cen and M13 ?

Measure masses and radii of CS!



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Lower limit from RXJ 1856 incompatible with ω Cen and M13 ?

... unless the latter sources emit X-rays from "hot spots" → lower limit on R

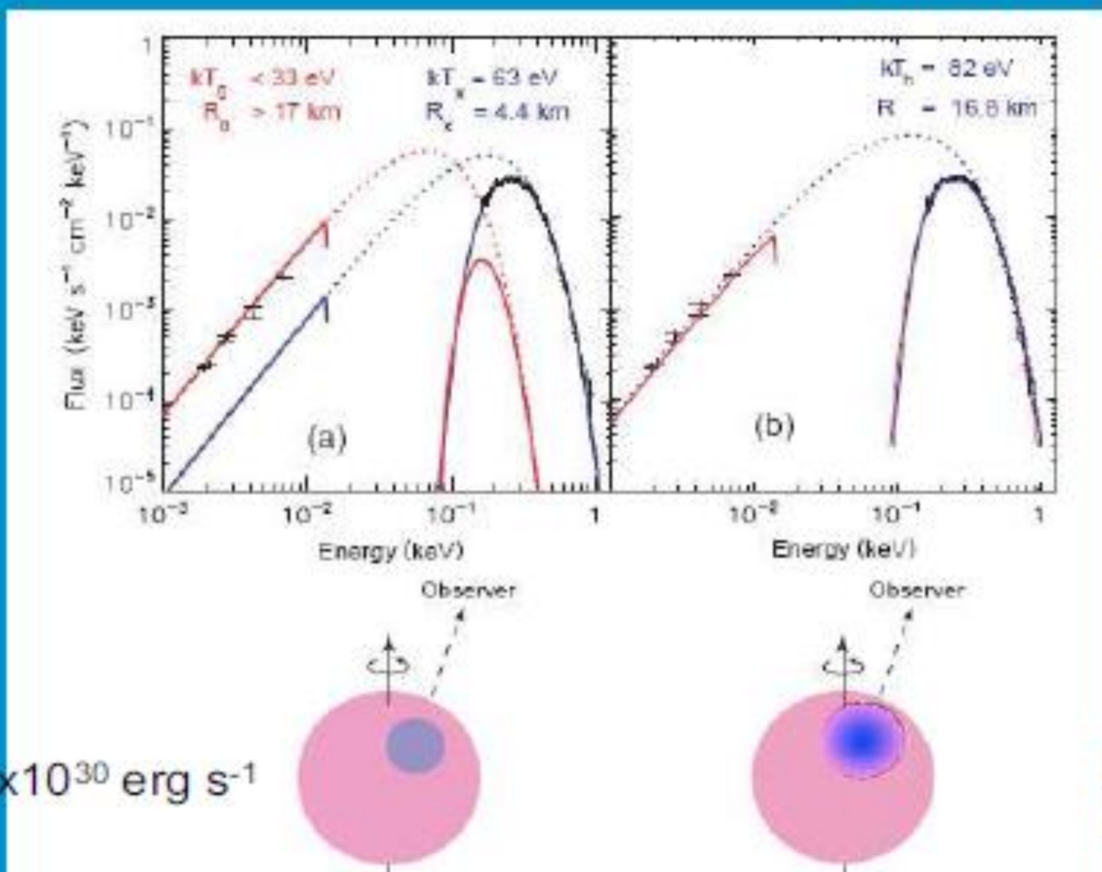
The lesson learned from RX J1856

blackbody fits to the optical and X-ray spectra of RX J1856.5-3754 (Trümper,2004)

radius determination ⇒ EoS ⇒ state of matter at high densities

two-component model

model with continuous T-distribution



completely featureless X-ray spectrum: condensed surface? ⇒ strong B?

L_x =5.4x10^{30} erg s^{-1}

pulsed fraction < 1% ⇒ line of sight || rotation axis?

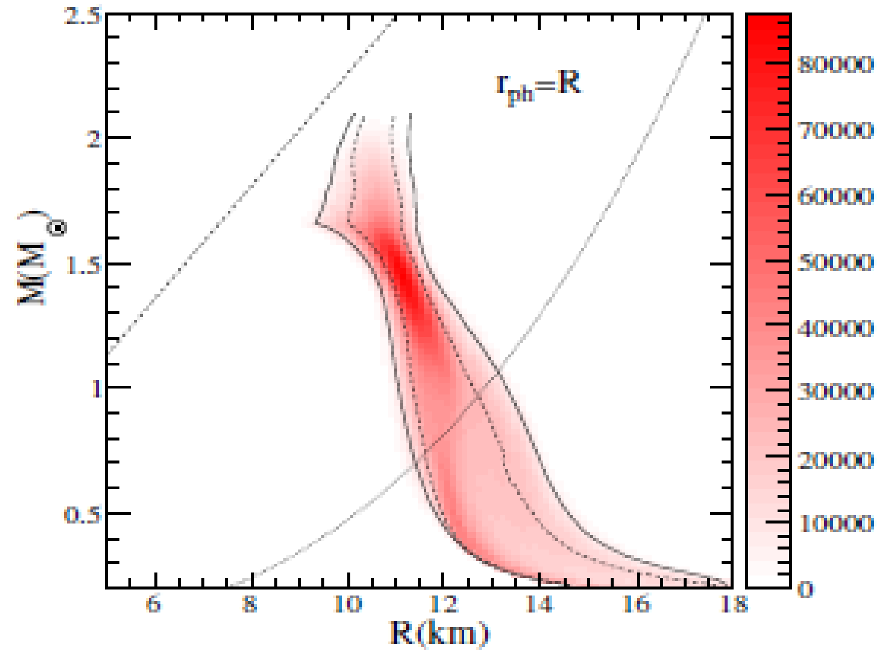
X-ray emitting region is a “hot spot”, J. Trumper et al., Nucl. Phys. Proc. Suppl. 132 (2004) 560

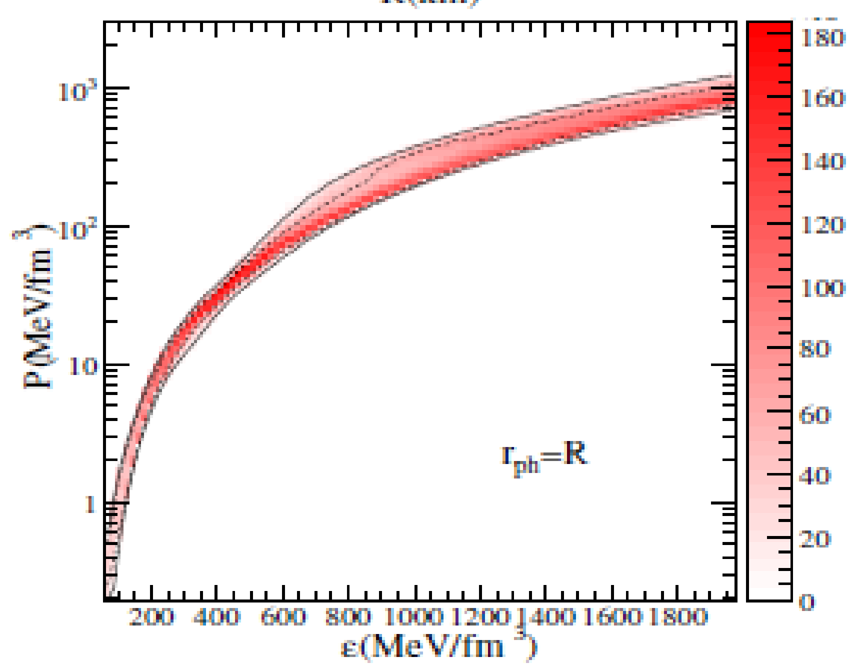
Goal 1: Measure the cold EoS !

Bayesian TOV analysis:
Steiner, Lattimer, Brown, ApJ 722 (2010) 33

Most Probable Values for Masses and Radii for Neutron Stars Constrained to Lie on One Mass Versus Radius Curve

Object	$M\ (M_\odot)$	R (km)	$M\ (M_\odot)$	R (km)
	$r_{ph} = R$		$r_{ph} \gg R$	
4U 1608–522	$1.52^{+0.22}_{-0.18}$	$11.04^{+0.53}_{-1.50}$	$1.64^{+0.34}_{-0.41}$	$11.82^{+0.42}_{-0.89}$
EXO 1745–248	$1.55^{+0.12}_{-0.36}$	$10.91^{+0.86}_{-0.65}$	$1.34^{+0.450}_{-0.28}$	$11.82^{+0.47}_{-0.72}$
4U 1820–30	$1.57^{+0.13}_{-0.15}$	$10.91^{+0.39}_{-0.92}$	$1.57^{+0.37}_{-0.31}$	$11.82^{+0.42}_{-0.82}$
M13	$1.48^{+0.21}_{-0.64}$	$11.04^{+1.00}_{-1.28}$	$0.901^{+0.28}_{-0.12}$	$12.21^{+0.18}_{-0.62}$
ω Cen	$1.43^{+0.26}_{-0.61}$	$11.18^{+1.14}_{-1.27}$	$0.994^{+0.51}_{-0.21}$	$12.09^{+0.27}_{-0.66}$
X7	$0.832^{+1.19}_{-0.051}$	$13.25^{+1.37}_{-3.50}$	$1.98^{+0.10}_{-0.36}$	$11.3^{+0.95}_{-1.03}$





Caution:
If optical spectra are not measured, the observed X-ray spectrum may not come from the entire surface But from a hot spot at the magnetic pole!
J. Trumper, Prog. Part. Nucl. Phys. 66 (2011) 674

Such systematic errors are not accounted for in Steiner et al. → M(R) is a lower limit → softer EoS

Goal 1: Measure the cold EoS !

Bayesian TOV analysis:
Steiner, Lattimer, Brown, ApJ 722 (2010) 33

Most Probable Values for Masses and Radii for Neutron Stars Constrained to Lie on One Mass Versus Radius Curve

Object	M ($M_\odot$)	R (km)	M ($M_\odot$)	R (km)
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4U 1608–522	$1.52^{+0.22}_{-0.18}$	$11.04^{+0.53}_{-1.50}$	$1.64^{+0.34}_{-0.41}$	$11.82^{+0.42}_{-0.89}$
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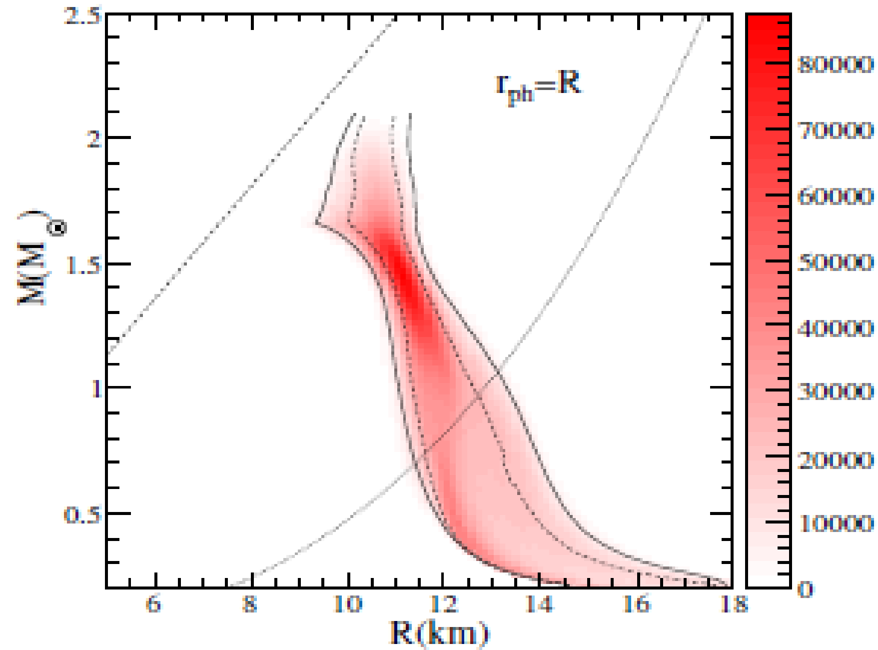

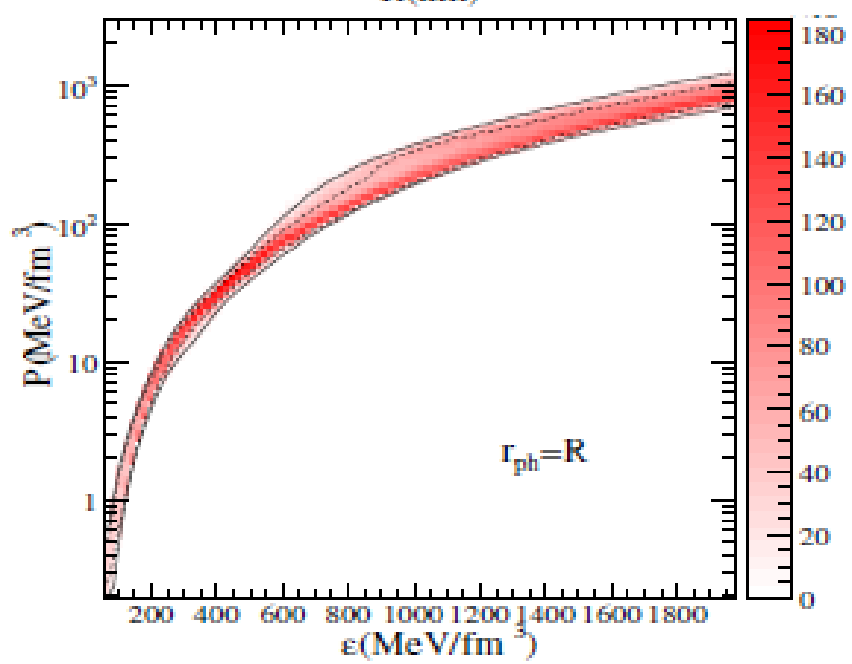

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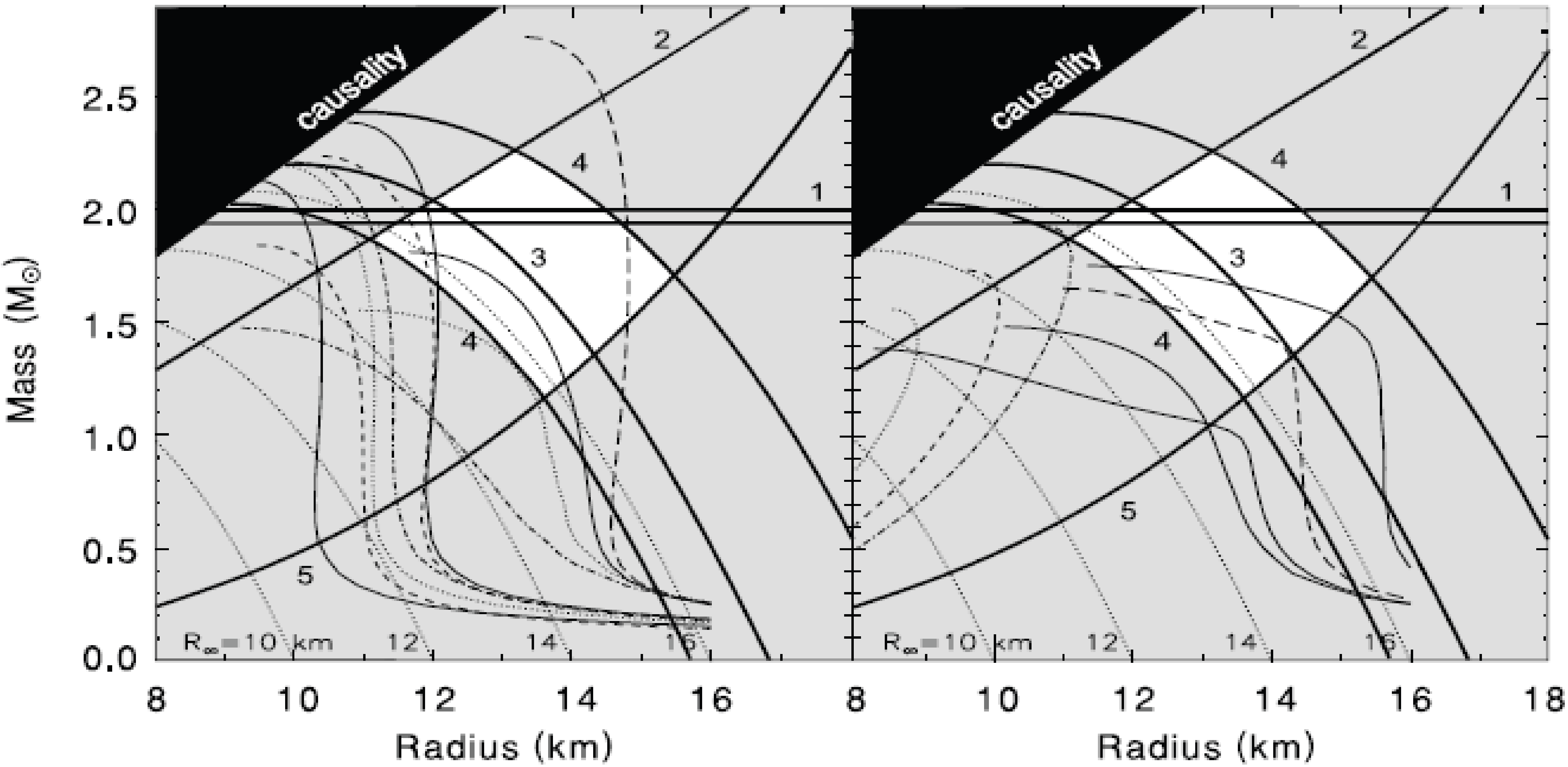
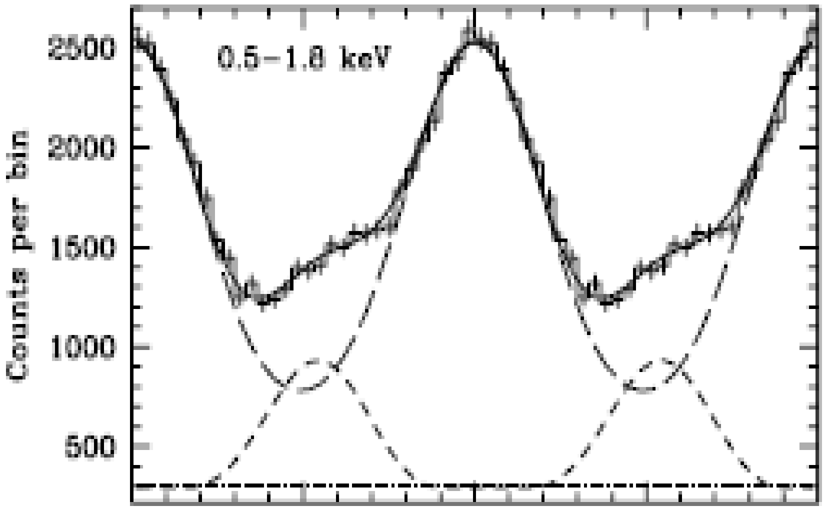
Which constraints can be trusted ?



1 – Largest mass J1614 – 2230 (Demorest et al. 2010)
2 – Maximum gravity XTE 1814 – 338 (Bhattacharyya et al. (2005)
3 – Minimum radius RXJ 1856 – 3754 (Trumper et al. 2004)
4 – Radius, 90% confidence limits LMXB X7 in 47 Tuc (Heinke et al. 2006)
5 – Largest spin frequency J1748 – 2446 (Hessels et al. 2006)

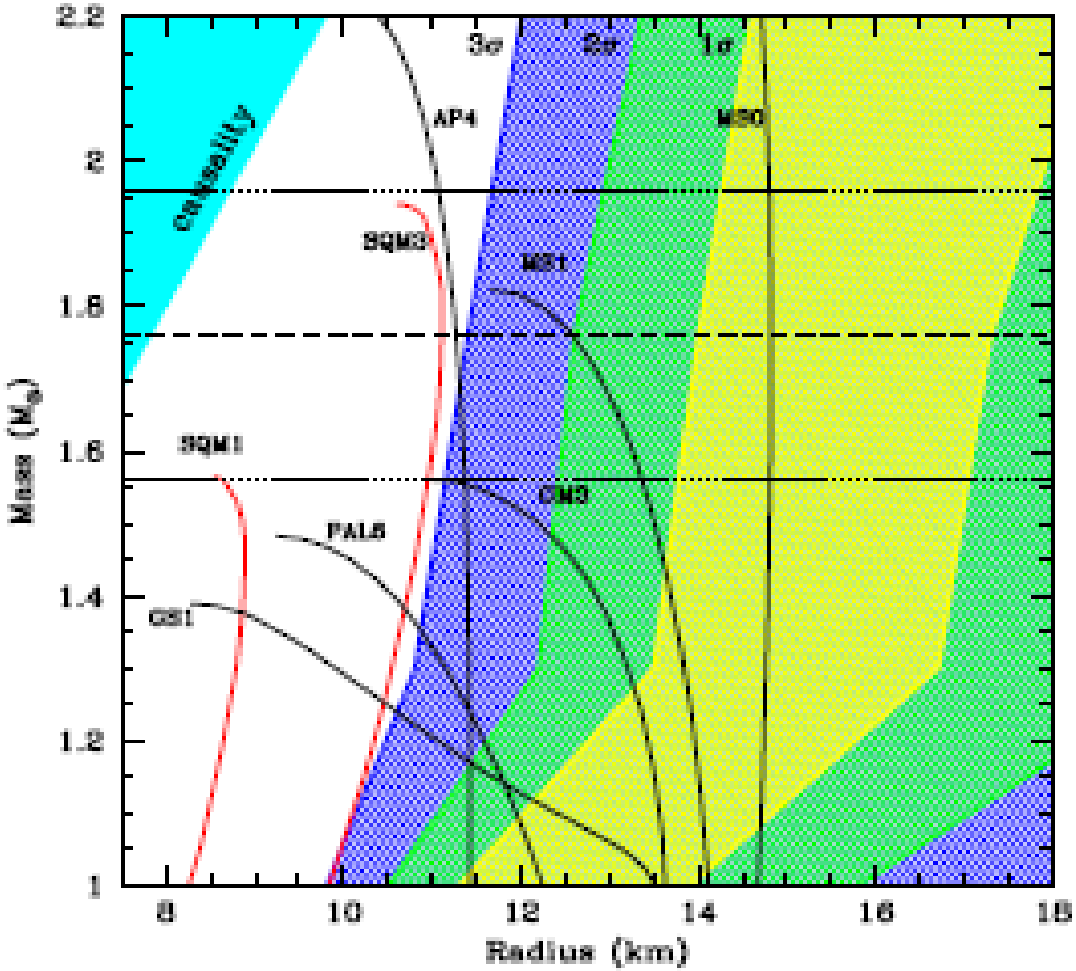
Which constraints can be trusted ?

Nearest millisecond pulsar PSR J0437 – 4715 revisited by XMM Newton
Distance: d = 156.3 +/- 1.3 pc
Period: P= 5.76 ms, dot P = 10^-20 s/s, field strength B = 3x10^8 G



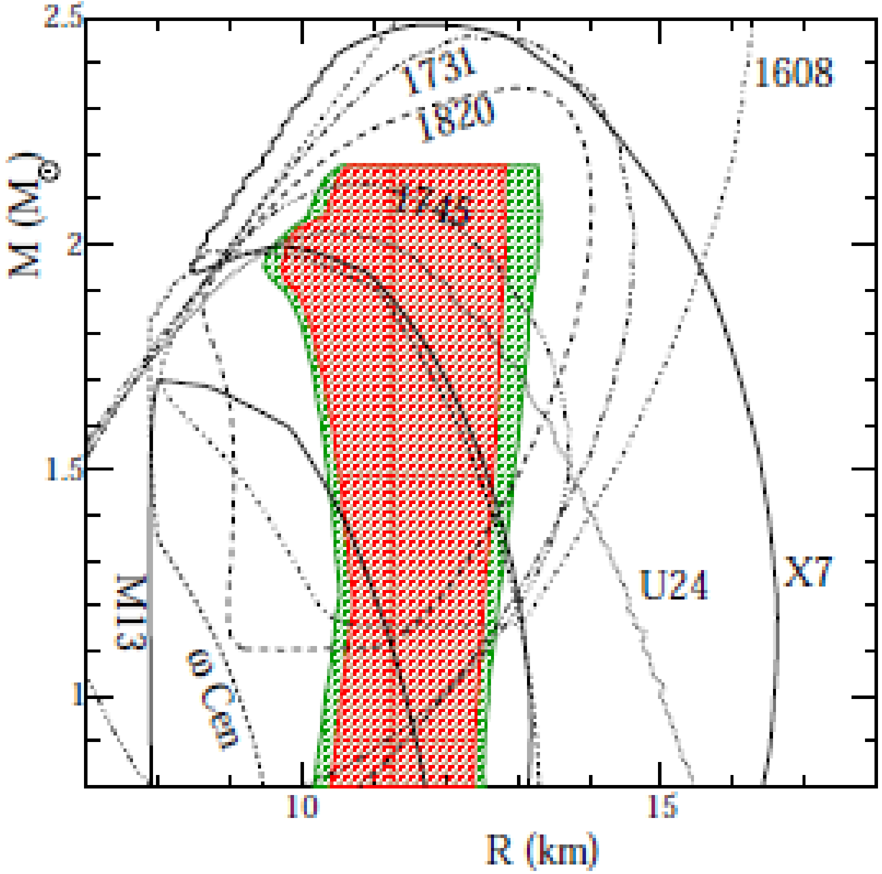
Three thermal component fit
R > 11.1 km (at 3 sigma level)
M = 1.76 M_sun

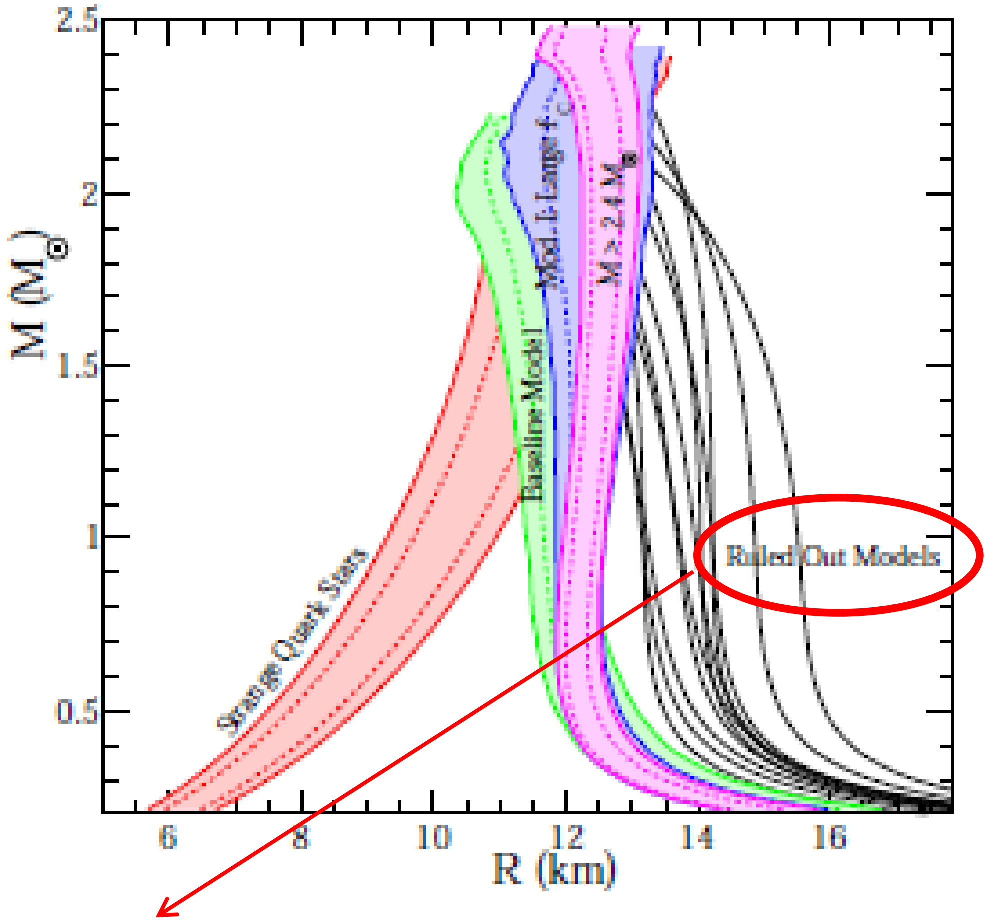
S. Bogdanov, arxiv:1211.6113 (2012)



Which constraints require caution ?

A. Steiner, J. Lattimer, E. Brown, ApJ Lett. 765 (2013) L5



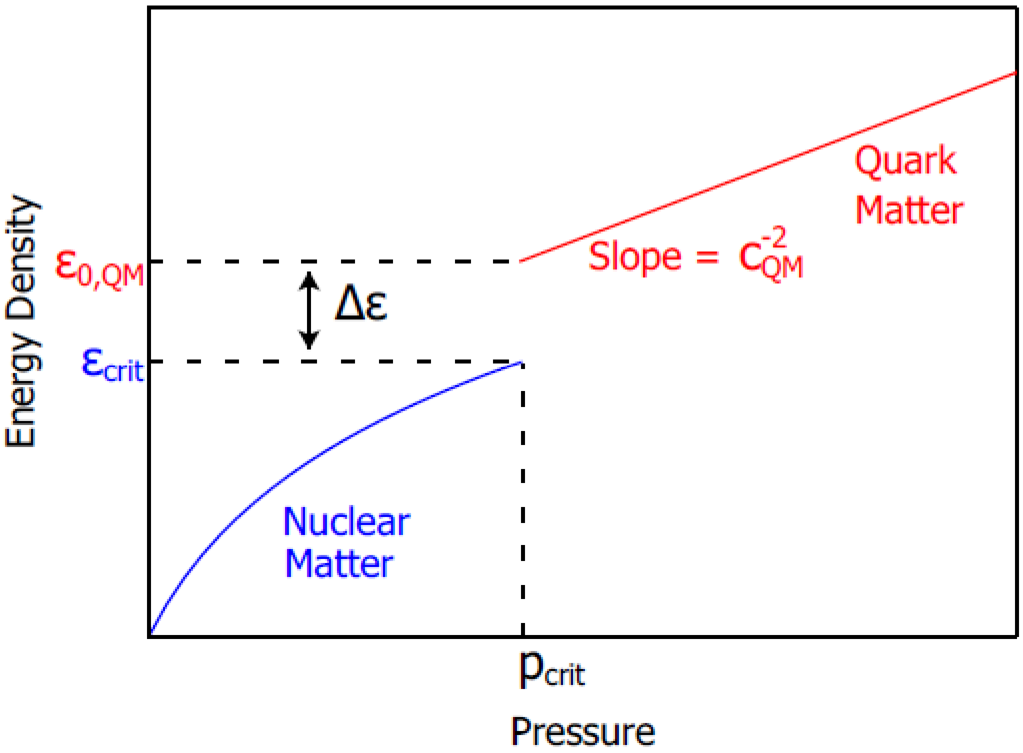


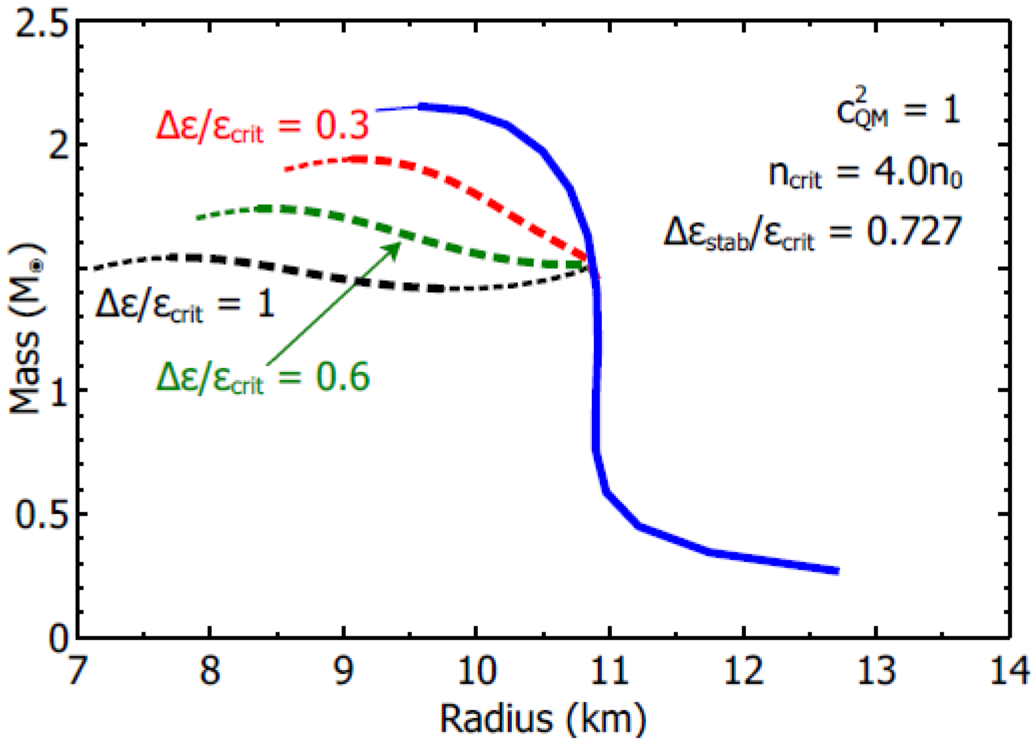
“Ruled out models” - too strong a conclusion!
M(R) constraint is a lower limit, which is itself included in that from RX J1856, which is one of the best known sources.

Goal 2: Be lucky – detect a 1st order PT

Alford, Han, Prakash, arxiv:1302.4732

First order PT can lead to a stable branch of hybrid stars with quark matter cores which, depending on the size of the “latent heat” (jump in energy density), can even be disconnected from the hadronic one by an unstable branch → **“third family of CS”**.

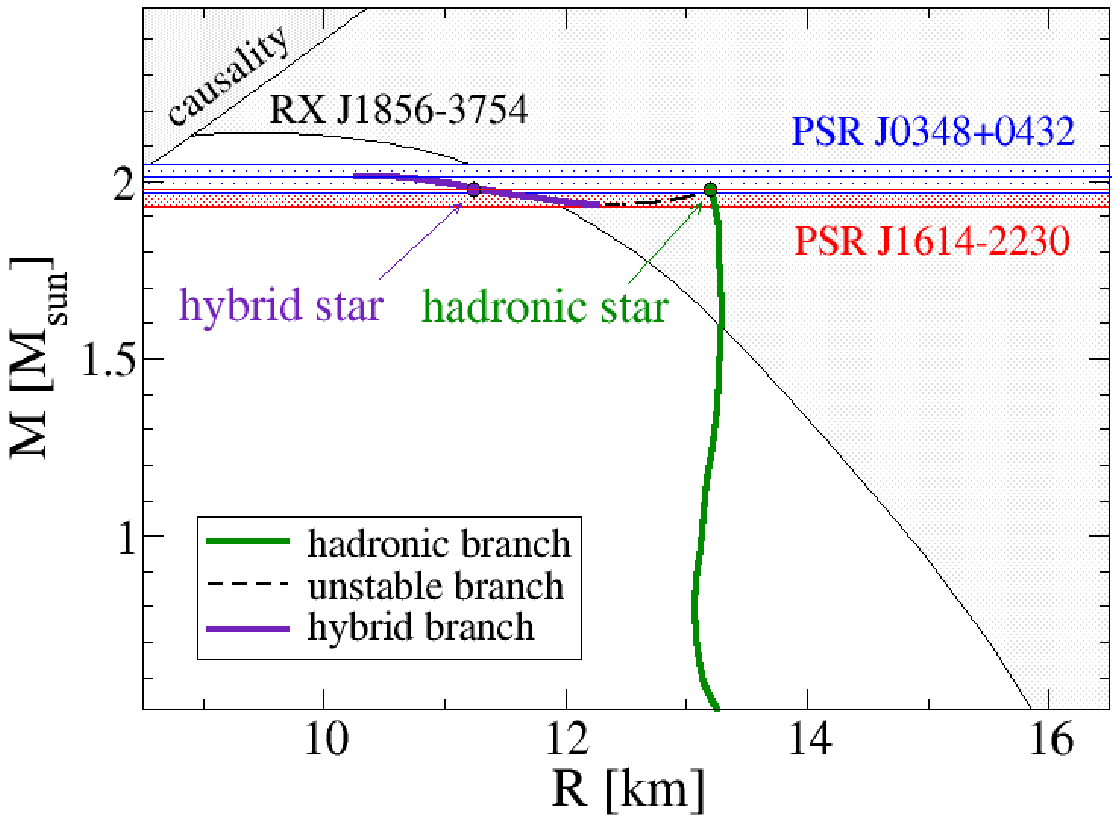


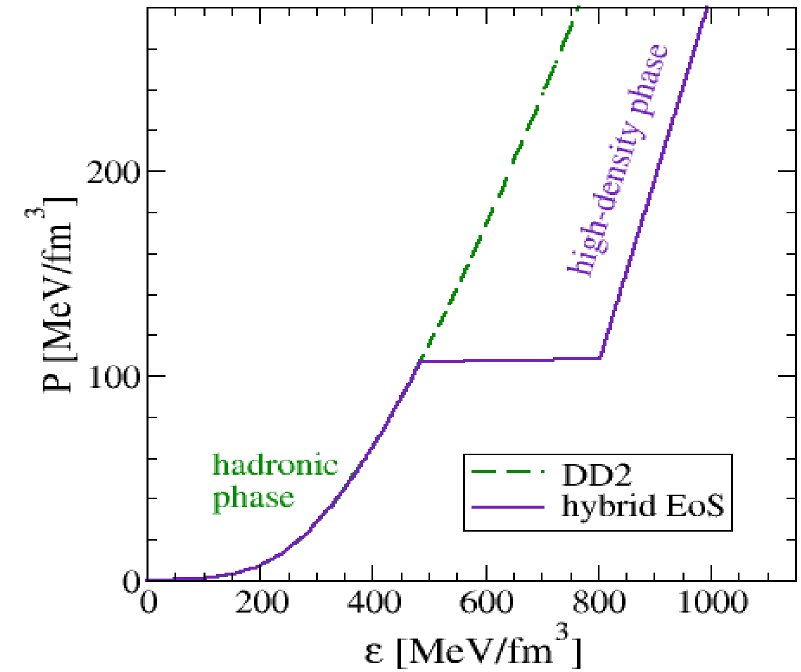


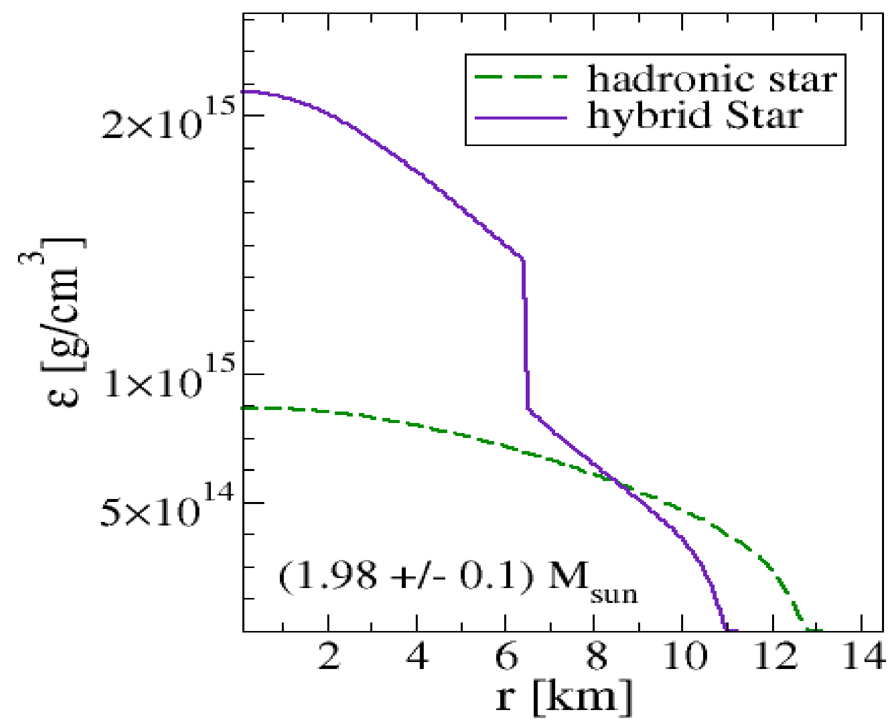
Measuring two **disconnected populations** of compact stars in the M-R diagram would be the **detection of a first order phase transition** in compact star matter and thus the indirect proof for the existence of a **critical endpoint (CEP) in the QCD phase diagram**!

Goal 2: Observe High-Mass Twin Stars

Twins prove exitence of **disconnected populations** (third family) in the M-R diagram
Consequence of a **first order phase transition**
Question: Do twins prove the 1st order phase trans.?







Alvarez & Blaschke, arxiv:1304.7758

A QCD-based hybrid EoS - nonlocal PNJL model

DB, Alvarez Castillo, Benic, Contrera, Lastowiecki, arxiv:1302.6275 (2012)

$$\mathscr{L} = \bar{q}(i\not{D} - m_0)q + \mathscr{L}_{\rm int} + \mathscr{U}(\Phi) ,$$

$$\mathscr{L}_{\rm int} = -\frac{G_S}{2}\left[j_S(x)j_S(x) + j_P(x)j_P(x) - j_p(x)j_p(x)\right] - \frac{G_V}{2} j_V(x)j_V(x),$$

$$j_a(x) = \int d^4z\, g(z)\, \bar{q}\left(x + \frac{z}{2}\right)\Gamma_a\, q\left(x - \frac{z}{2}\right), \quad a = S,P,V , \quad (\Gamma_S, \Gamma_P, \Gamma_V) = (\mathbb{1}, i\gamma_5\vec{\tau}, \gamma_0)$$

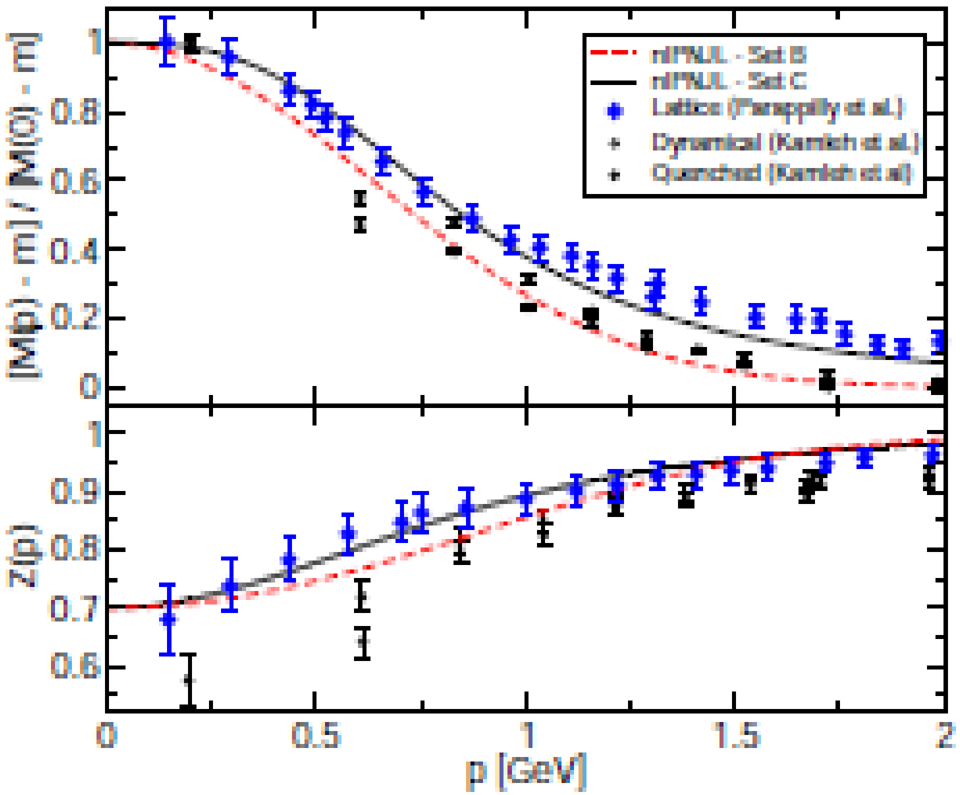
$$j_p(x) = \int d^4z\, f(z)\, \bar{q}\left(x + \frac{z}{2}\right)\frac{i\overleftrightarrow{\not\partial}}{2\,\kappa_p}\, q\left(x - \frac{z}{2}\right), \quad u(x')\overleftrightarrow{\partial} v(x) = u(x')\partial_x v(x) - \partial_{x'} u(x')v(x).$$

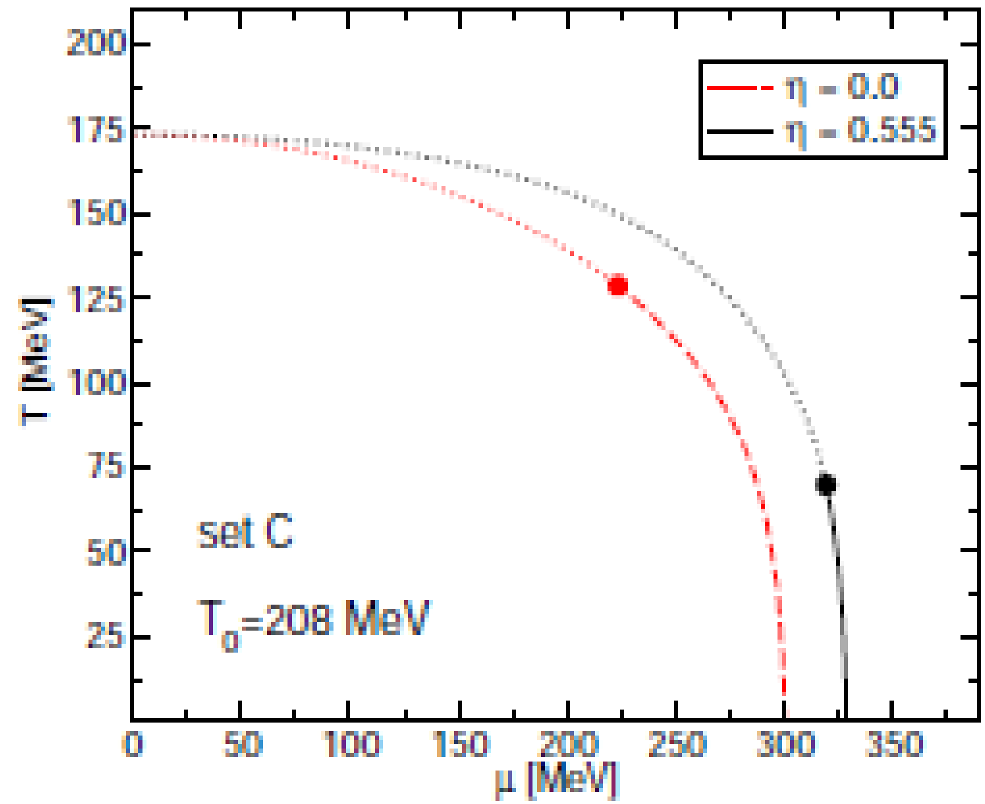
$$\mathscr{U}(\Phi, T, \mu) = (a_0 T^4 + a_1\mu^4 + a_2 T^2\mu^2)\Phi^2 + a_3 T_0^4 \ln(1 - 6\Phi^2 + 8\Phi^3 - 3\Phi^4) ,$$

$$\Omega^{\rm MFA} = -4T\sum_{n,c}\int\frac{d^3\vec{p}}{(2\pi)^3}\ln\left[\frac{(\tilde{\rho}^c_{n,\vec{p}})^2 + M^2(\rho^c_{n,\vec{p}})}{Z^2(\rho^c_{n,\vec{p}})}\right] + \frac{\sigma_1^2 + \kappa_p^2\,\sigma_2^2}{2\,G_S} - \frac{\omega^2}{2G_V} + \mathscr{U}(\Phi, T) ,$$

$$M(p) = Z(p)\left[m + \sigma_1\, g(p)\right], \quad Z(p) = \left[1 - \sigma_2\, f(p)\right]^{-1}. \quad \tilde{\mu} = \mu - \omega\, g(p)\, Z(p) .$$

A QCD-based hybrid EoS



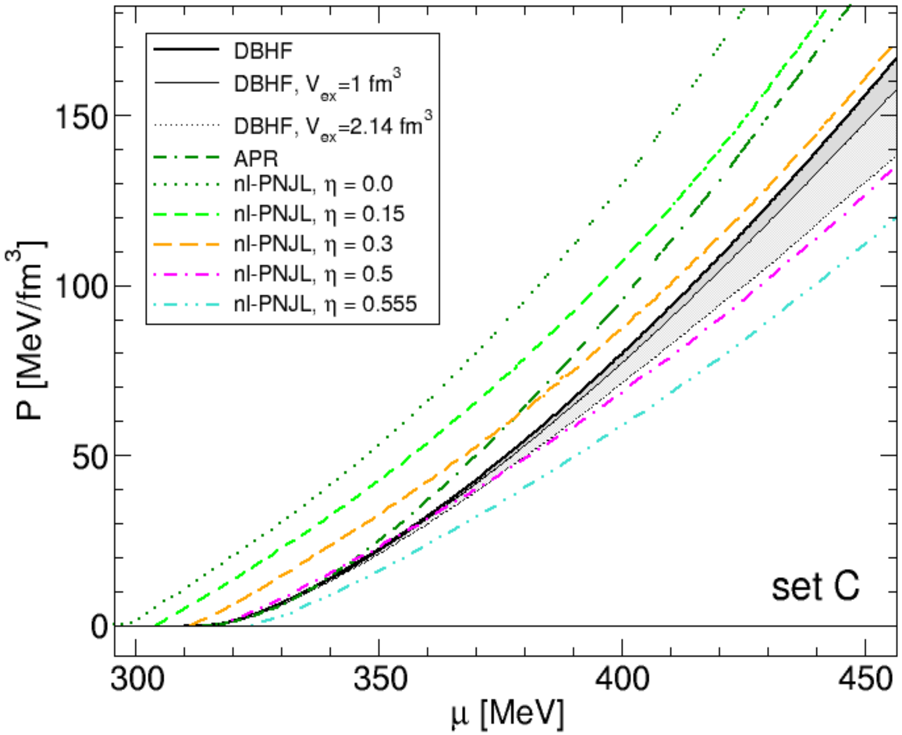


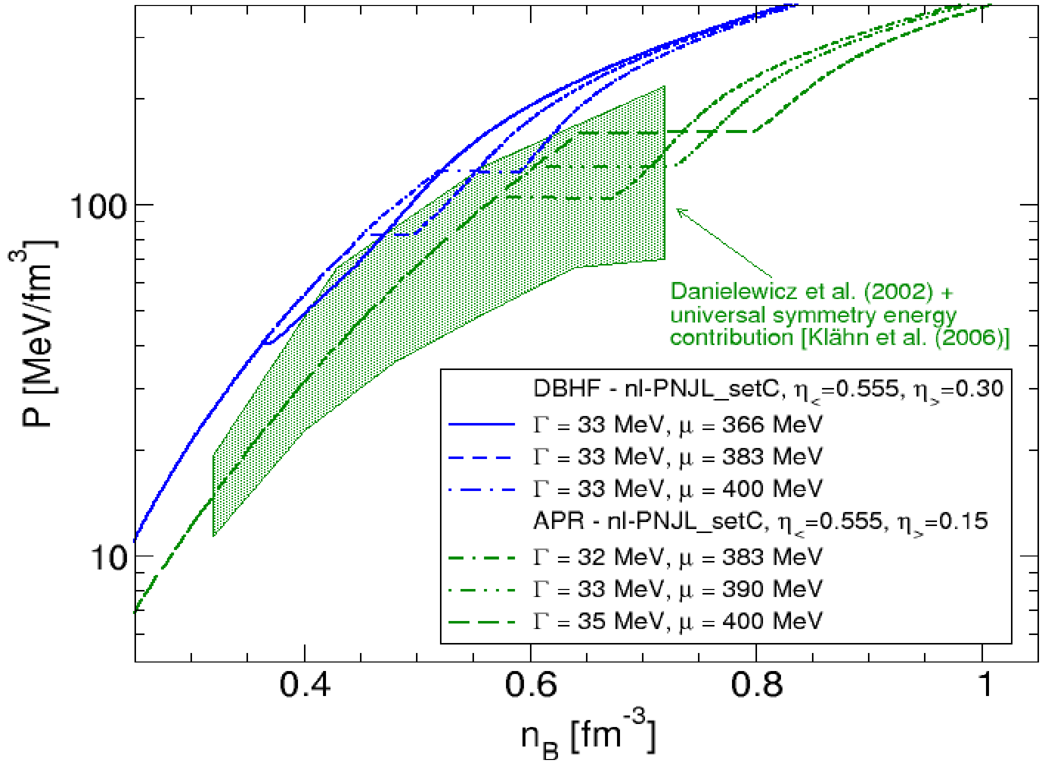
- Formfactors of the nonlocal chiral quark model fixed by comparison with M(p) and Z(p) from lattice QCD calculations of the quark propagator [Parapilly et al. PRD 73 (2006)

- Vector coupling strength adjusted to describe the slope of the pseudocritical temperature In accordance with lattice QCD [Kaczmarek et al., PRD 83 (2011) 014504]

- CEP does not vanish !! Controversial discussion, see Hell et al., arxiv:1212.4017 (2012)

A QCD-based hybrid EoS





- for strong vector coupling nuclear matter is stable at low densities
- for small vector coupling quark matter is stable at high densities
- for intermediate couplings → masquerade problem [Alford et al. ApJ 629 (2005) 969]

Here:

(A) Maxwell construction

$$P_Q(\mu_c) = P_H(\mu_c)$$

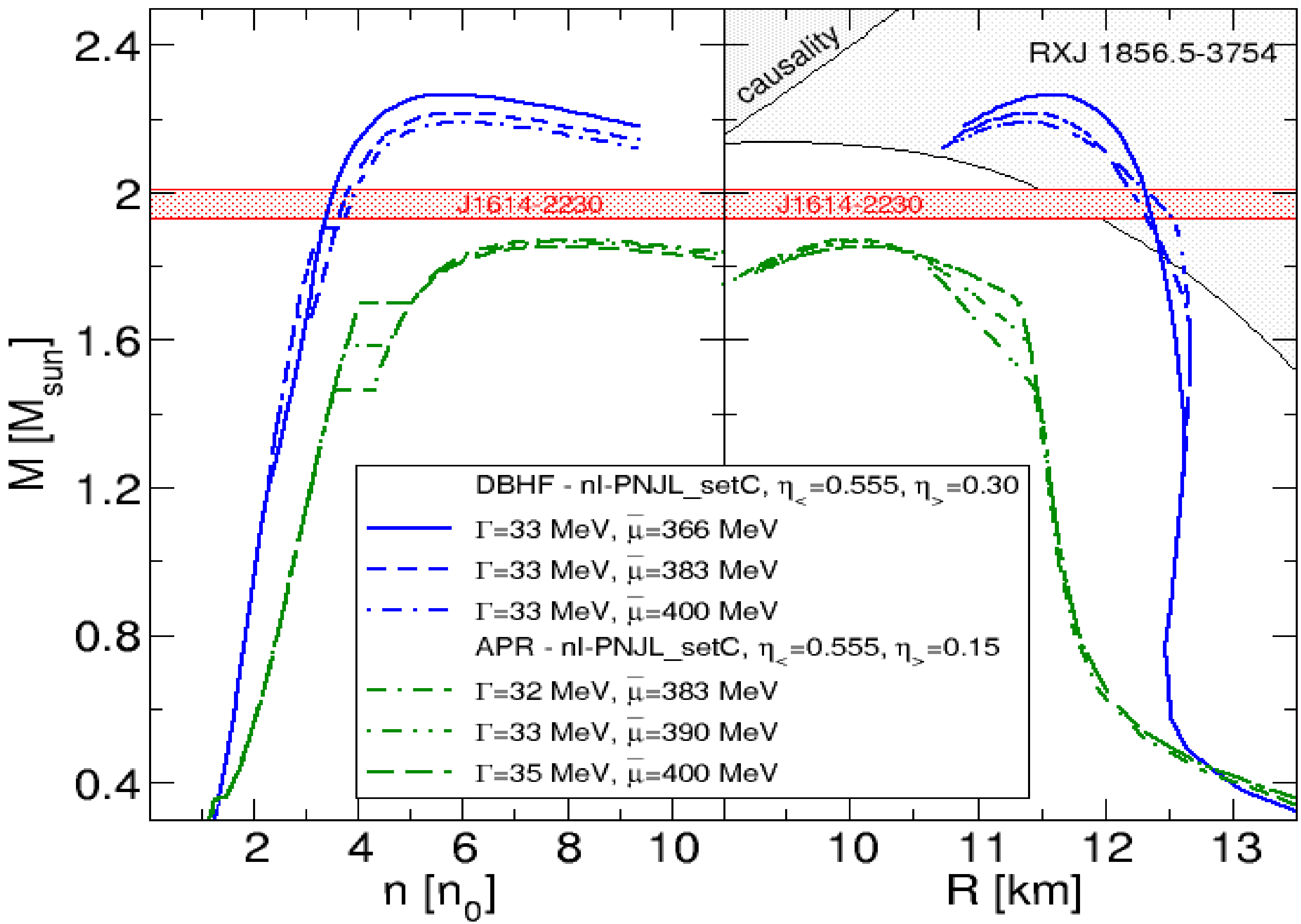
H = DBHF, APR; Q = nl-PNJL

(B) mu-dependent vector coupling:

$$P_Q(\mu) = P(0,\mu;\eta_<)\, f_<(\mu) + P(0,\mu;\eta_>)\, f_>(\mu)\ ,$$

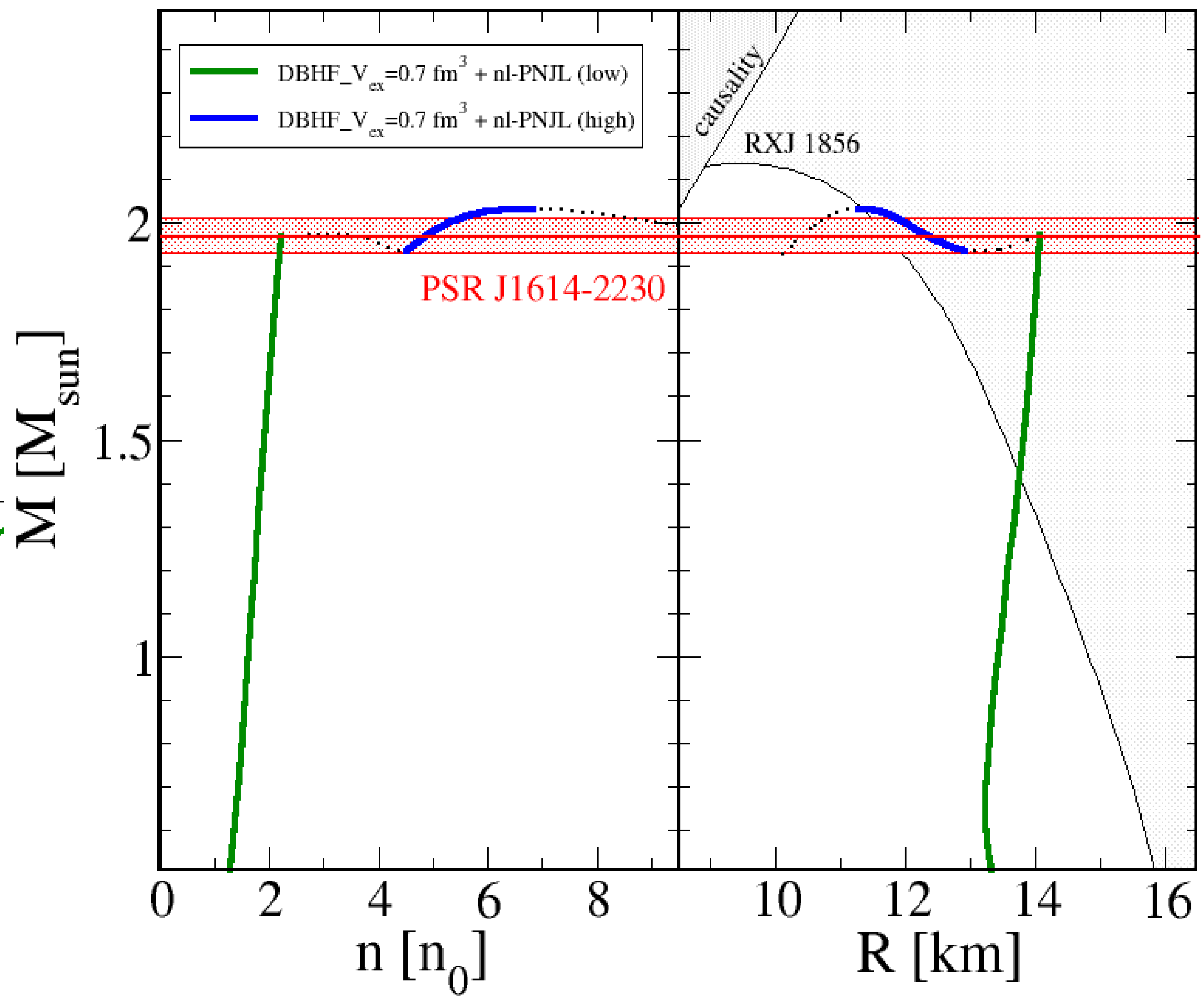
$$f_{\lessgtr}(\mu) = \frac{1}{2}\left[1 \mp \tanh\left(\frac{\mu-\bar{\mu}}{\Gamma}\right)\right]\ .$$

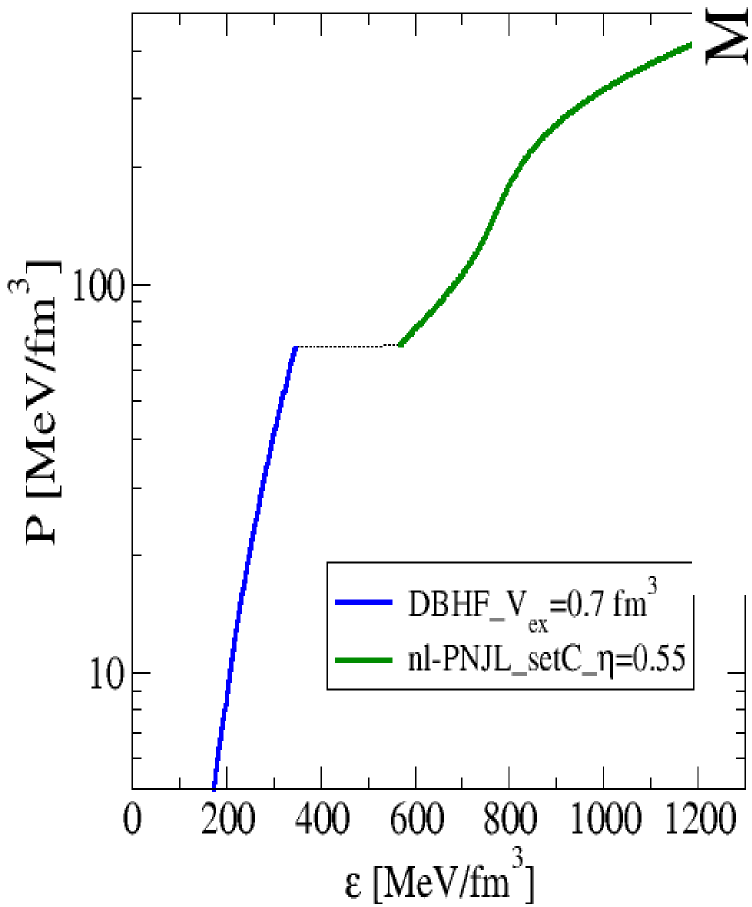
Result 1: hybrid stars fulfill Demorest and RXJ1856



DB, Alvarez Castillo, Benic, Contrera, Lastowiecki, arxiv:1302.6275 (2012)

Result 2:
High mass twins
are possible !





SUMMARY:

- excluded volume (quark Pauli blocking) in DBHF
- high-density quark matter slightly stiffer eta_v=0.25
- the scaled energy density jump (0.65) fulfills the twin condition of the schematic model by Alford et al. (2013)

→ Find the disconnected star branches !!

Main Problem:
Measure Compact Star Radii!

Gravitational binding: double pulsar J0737-3039

Double Pulsar System J0737-3039

Pulsar A $P^{(A)} = 22.7$ ms, $M^{(A)} \approx 1.338 M_\odot$

Pulsar B $P^{(B)} = 2.77$ s, $M^{(B)} = 1.249 \pm 0.001 M_\odot$ (record!)

Progenitor ONeMg white dwarf, driven hydrodyn. unstable by e^- captures on Mg & Ne; no mass-loss during collapse

Observational constraint for $M(M_N)$ from PSR J0737-3039:

- observed NSs gravitational mass (remnant star) $M^{(B)} = 1.248 - 1.250 M_\odot$
- critical baryon mass for ONeMg white dwarf $M_N^{(B)} = 1.366 - 1.375 M_\odot$

Theory: $M(M_N)$ characteristic for remnants EoS

$M = 4\pi \int_0^R \mathrm{d}r r^2 \varepsilon(r)$;

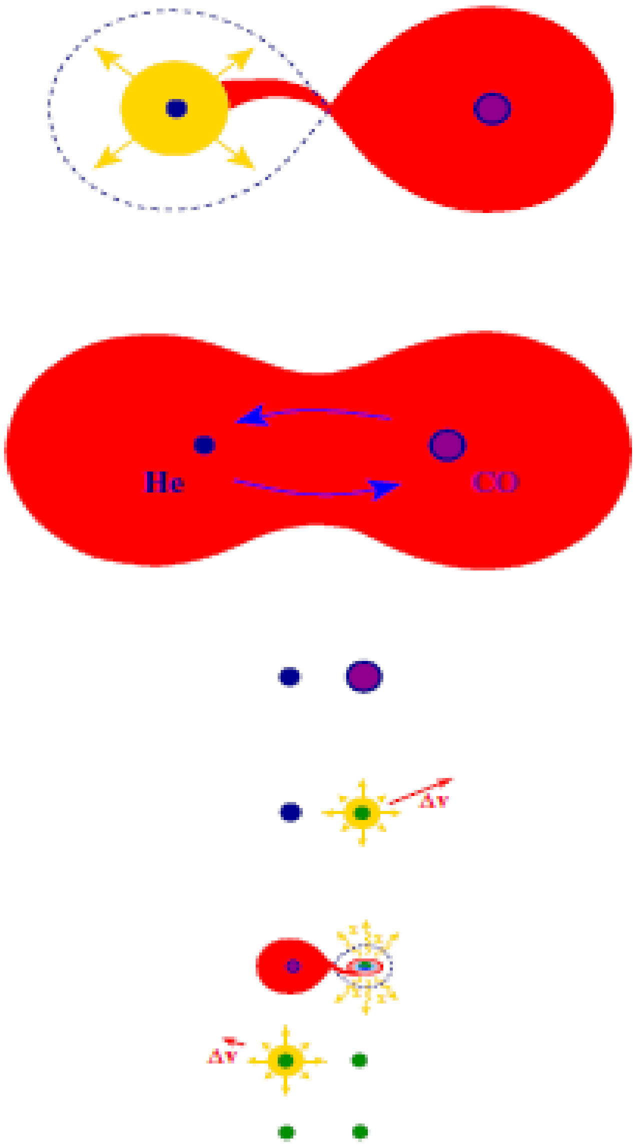
$M_N = u N_B = 4\pi u \int_0^R \mathrm{d}r \frac{r^2 n(r)}{\sqrt{1-2GM(r)/r}}$

(conversion of baryon number to mass by $u = 931.5$ MeV)

P. Podsiadlowski et al., Mon. Not. Roy. Astron. Soc. **361**, 1243 (2005)

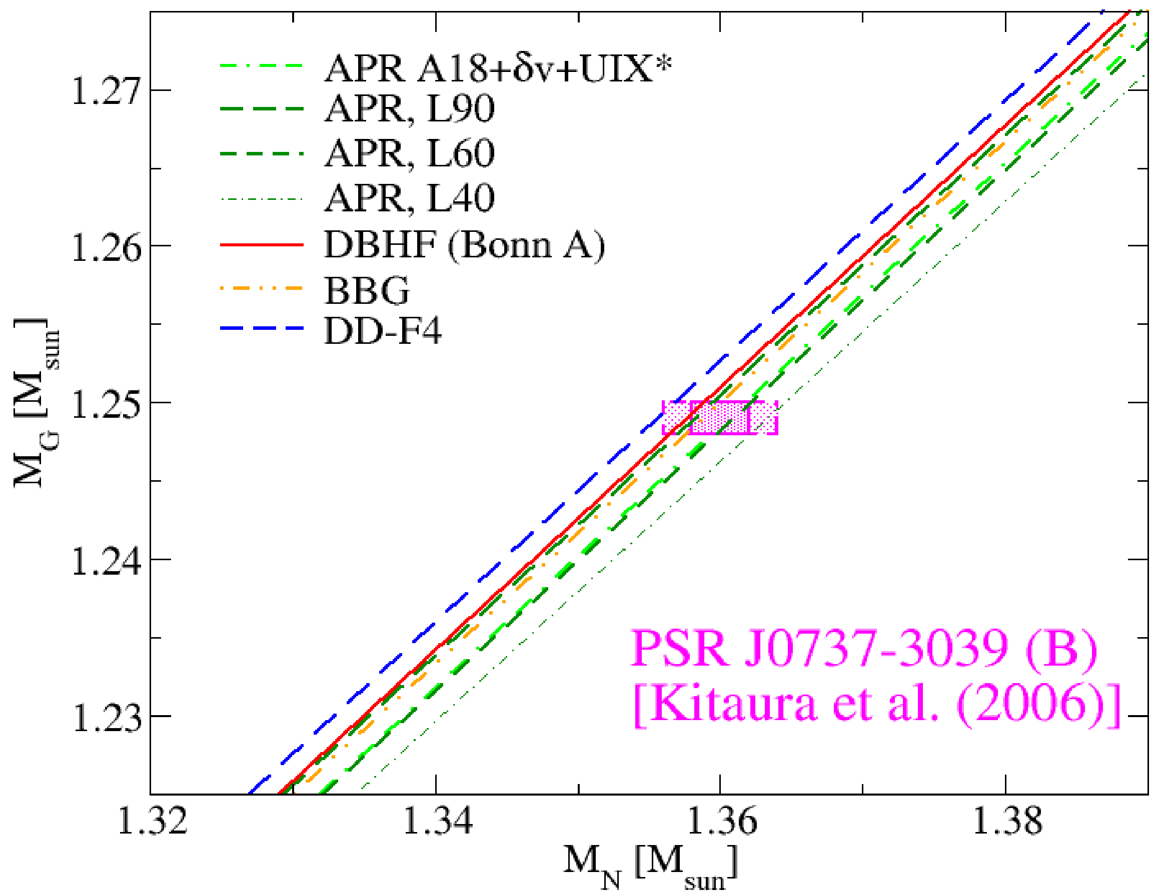
EoS constraint: double pulsar J0737-3039

Double core scenario:



Dewi et al., MNRAS (2006)

Baryon mass vs. gravitational mass - constraint or consistency check?

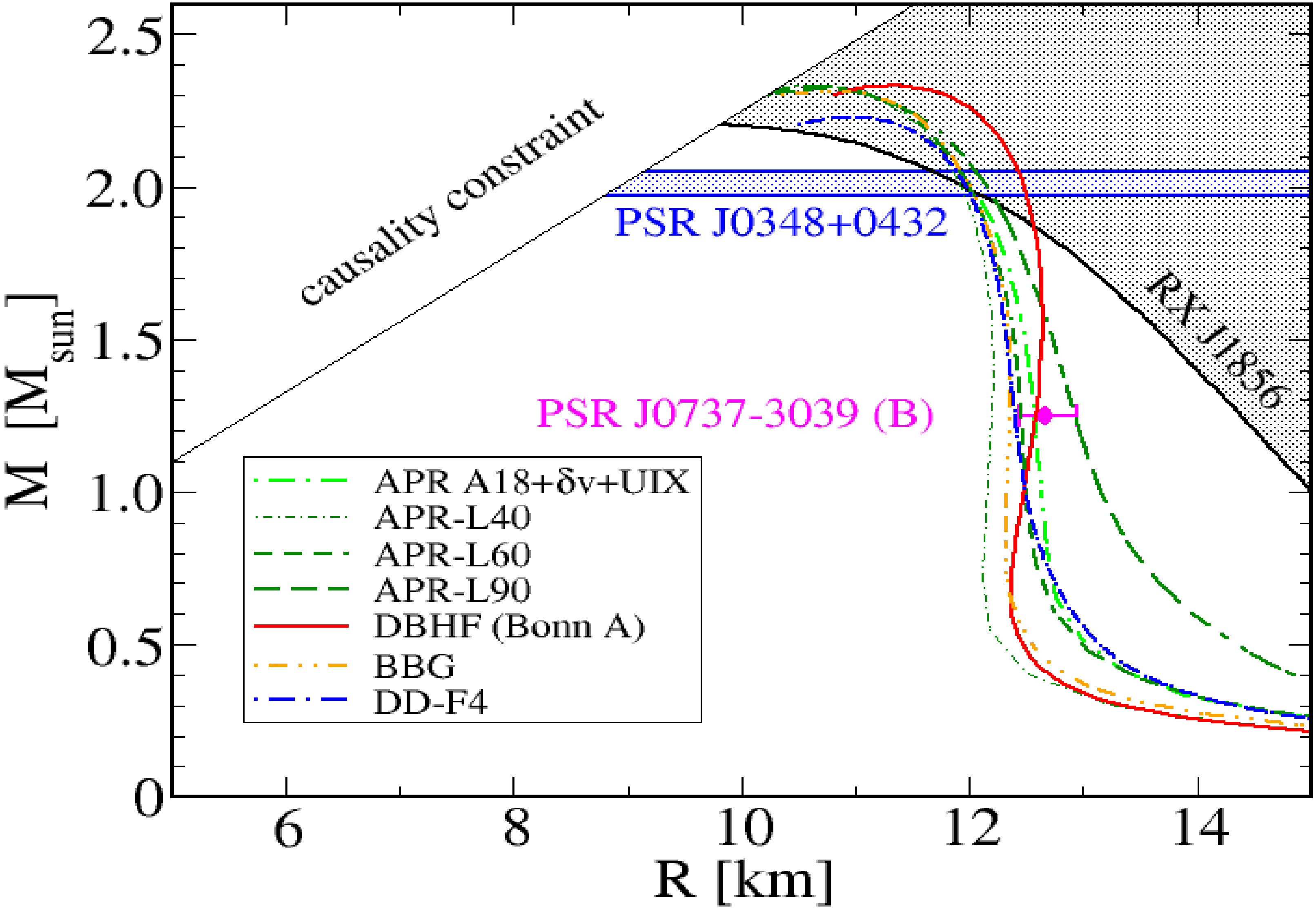


Podsiadlowski et al., MNRAS 361 (2005) 1243

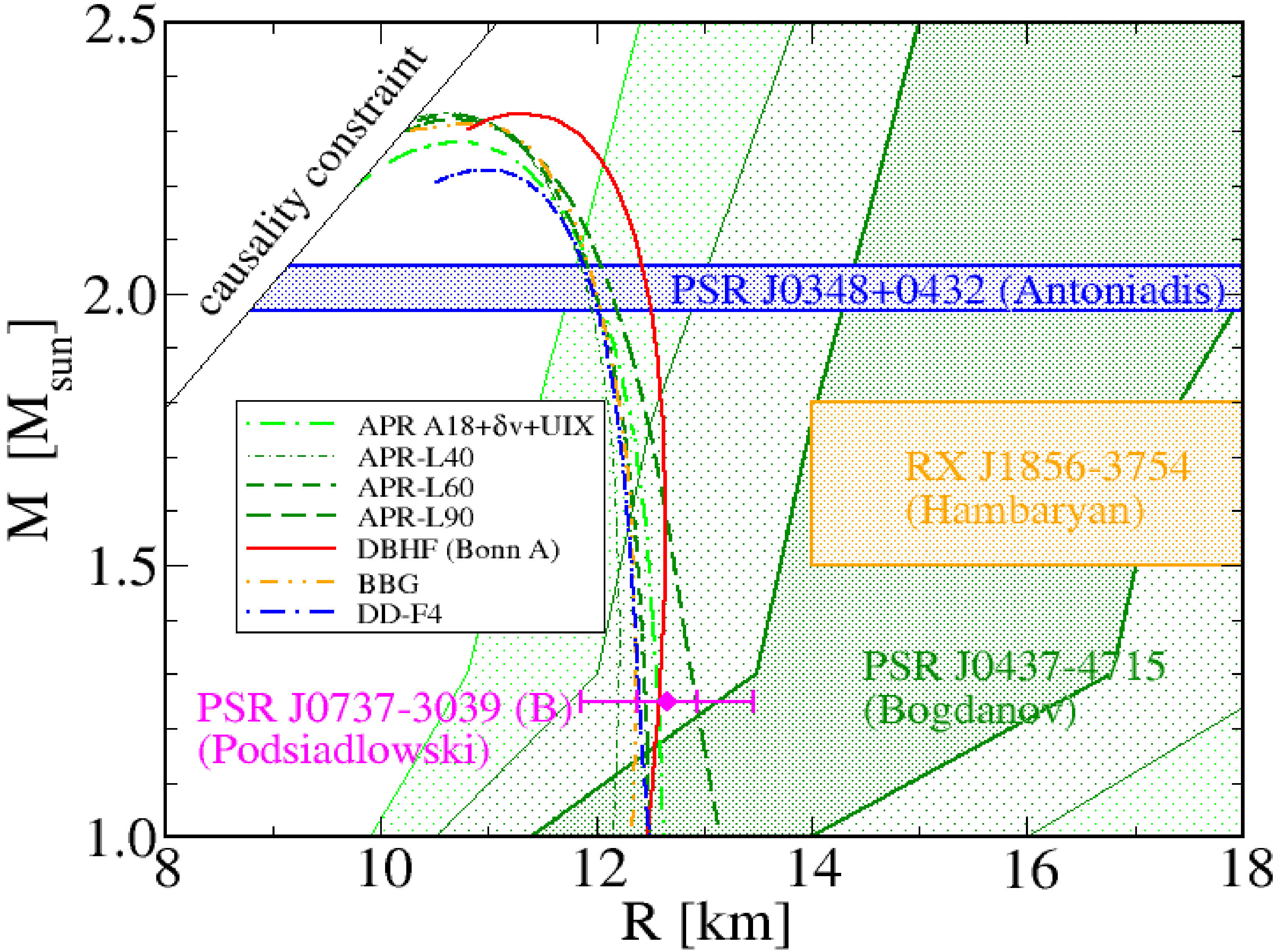
Kitaura, Janka, Hillebrandt, A& A (2006); [astro-ph/0512065]

D.B., T. Klähn, F. Weber, CBM Physics Book (2008)

Double pulsar: mass & radius ?!

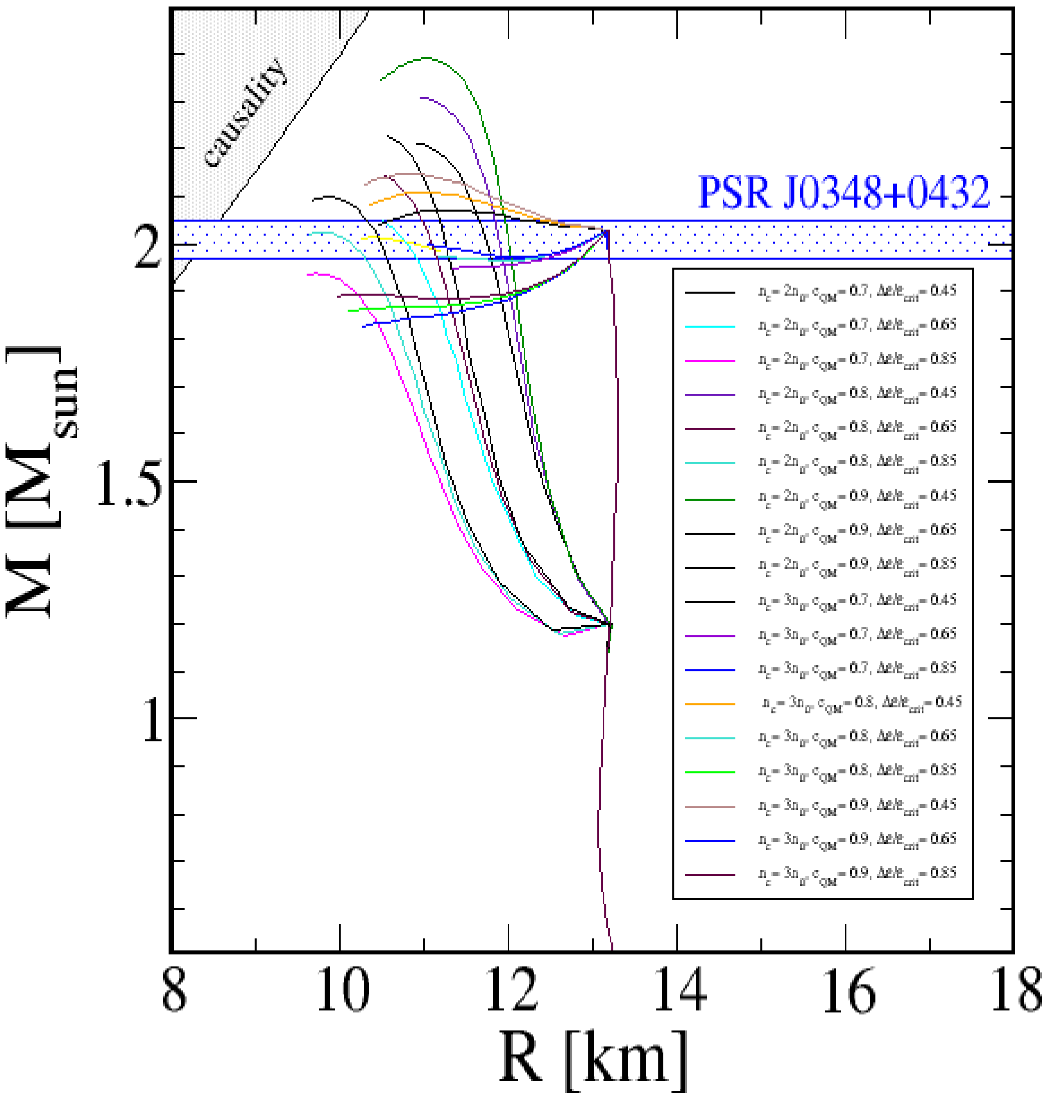


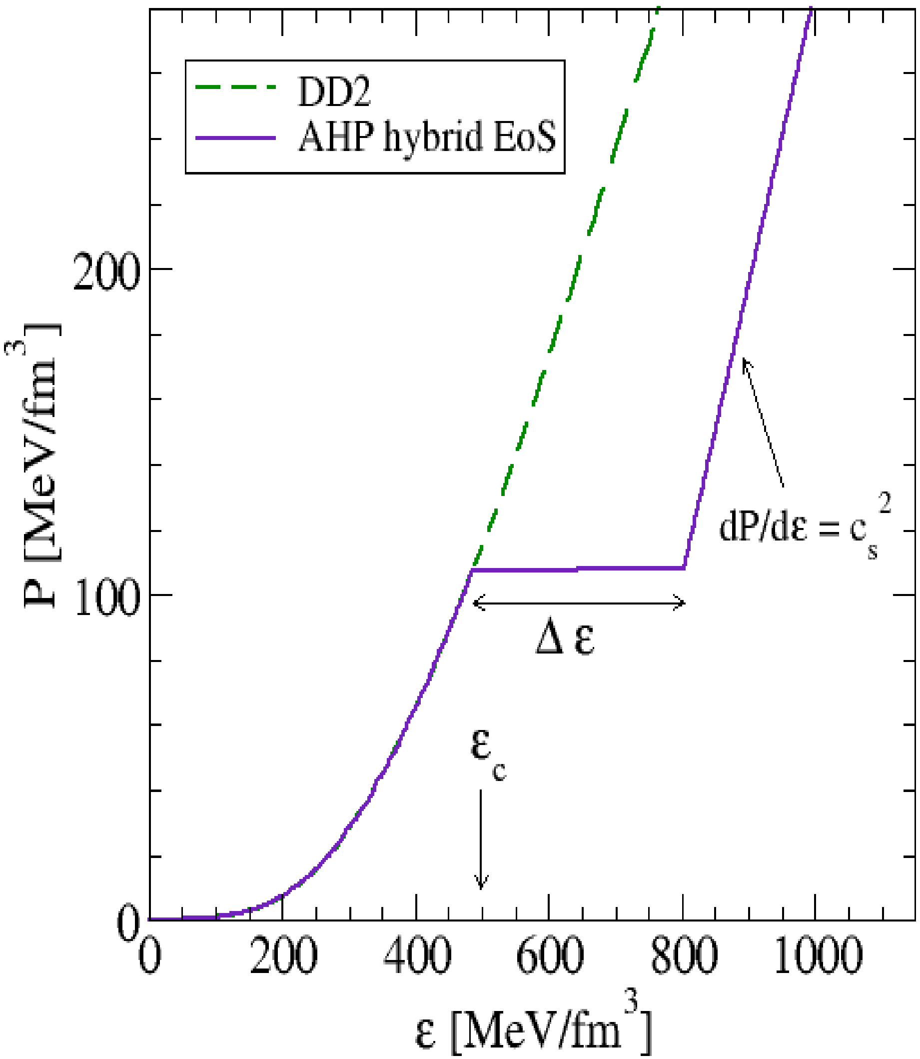
Disjunct M-R constraints for Bayesian analysis !



Alvarez, Ayriyan, Blaschke, Grigorian, … (work in progress, 2013)

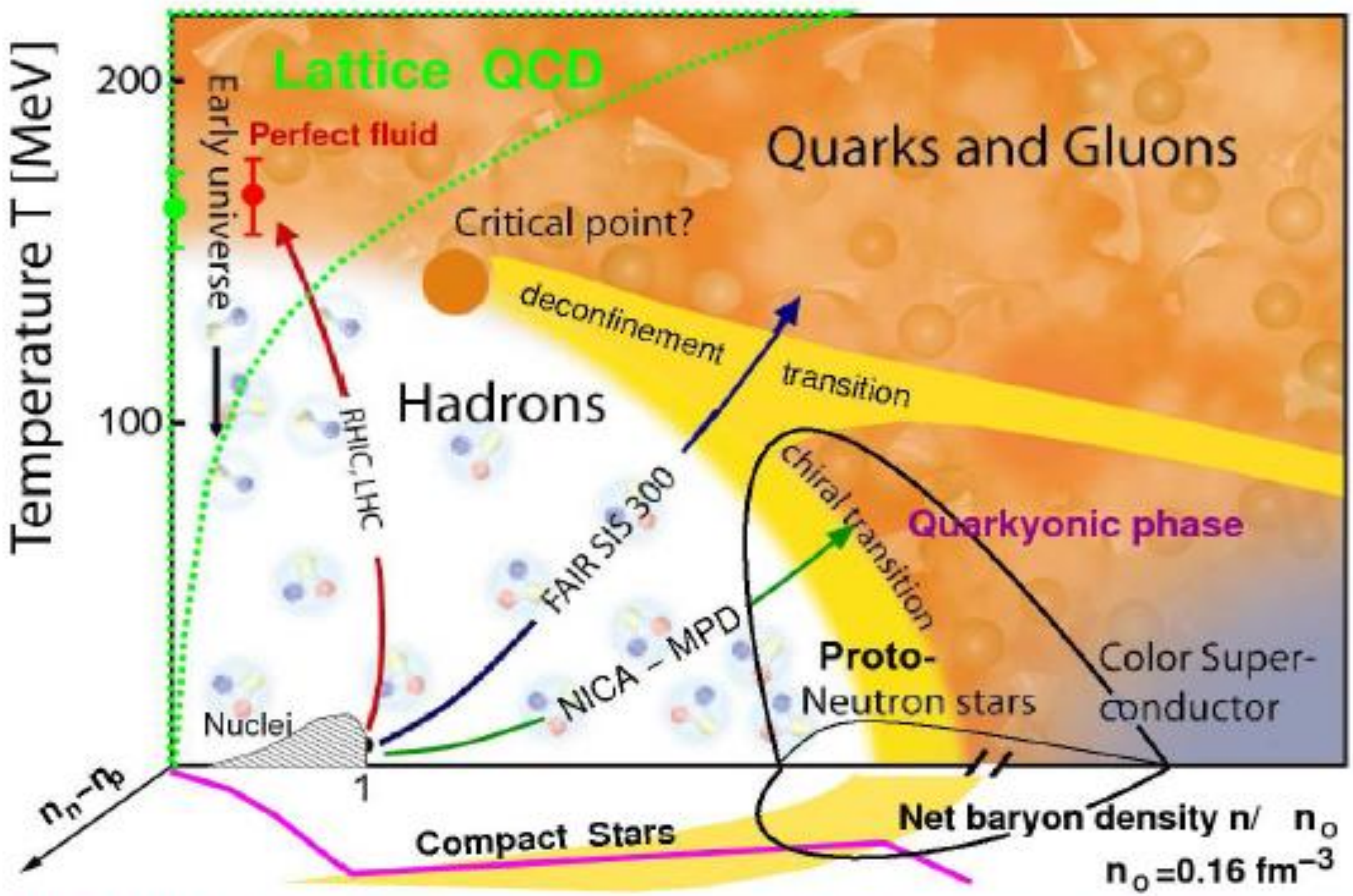
Disjunct M-R constraints for Bayesian analysis !





Alvarez, Ayriyan, Blaschke, Grigorian, … (work in progress, 2013)

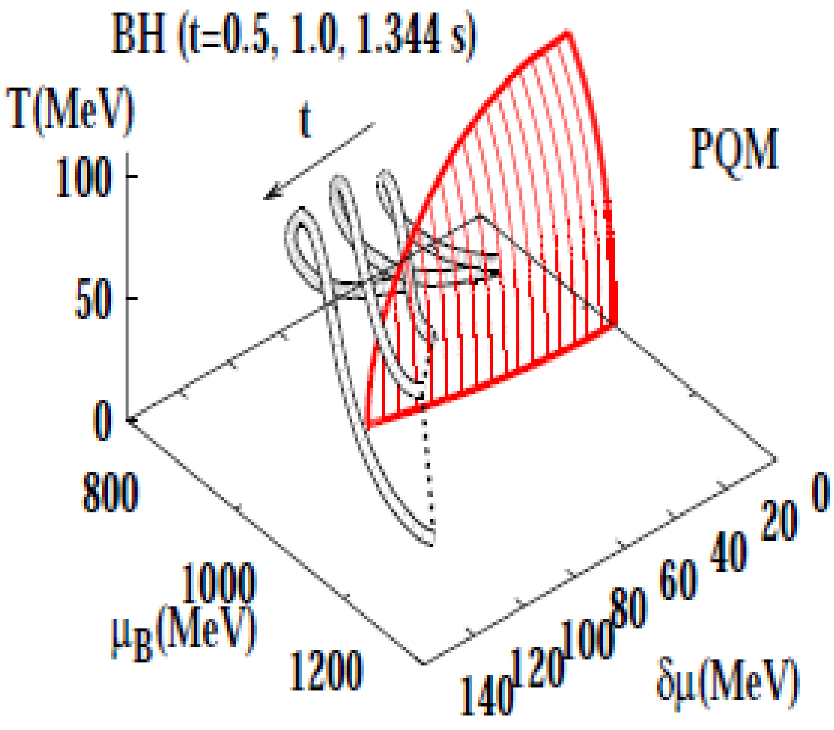
How to probe the line of CEP's in Astrophysics?

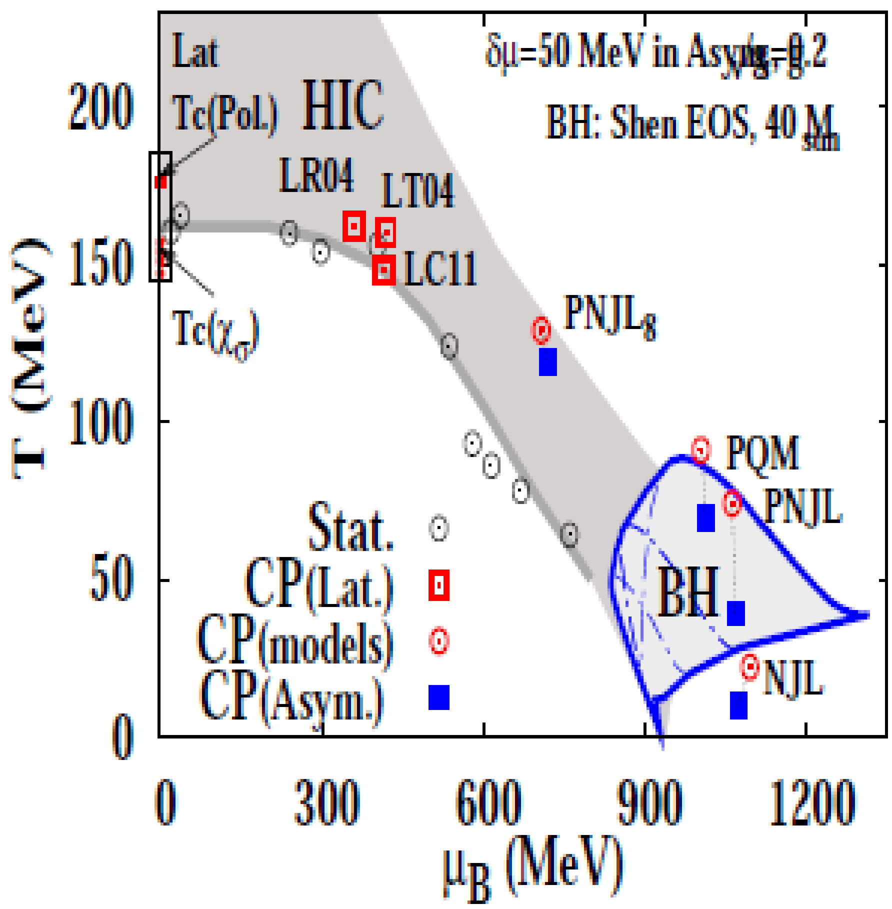


NICA White Paper, http://theor.jinr.ru/twiki-cgi/view/NICA/WebHome

How to probe the line of CEP's in Astrophysics?

→ by sweeping ("flyby") the critical line in SN collapse and BH formation



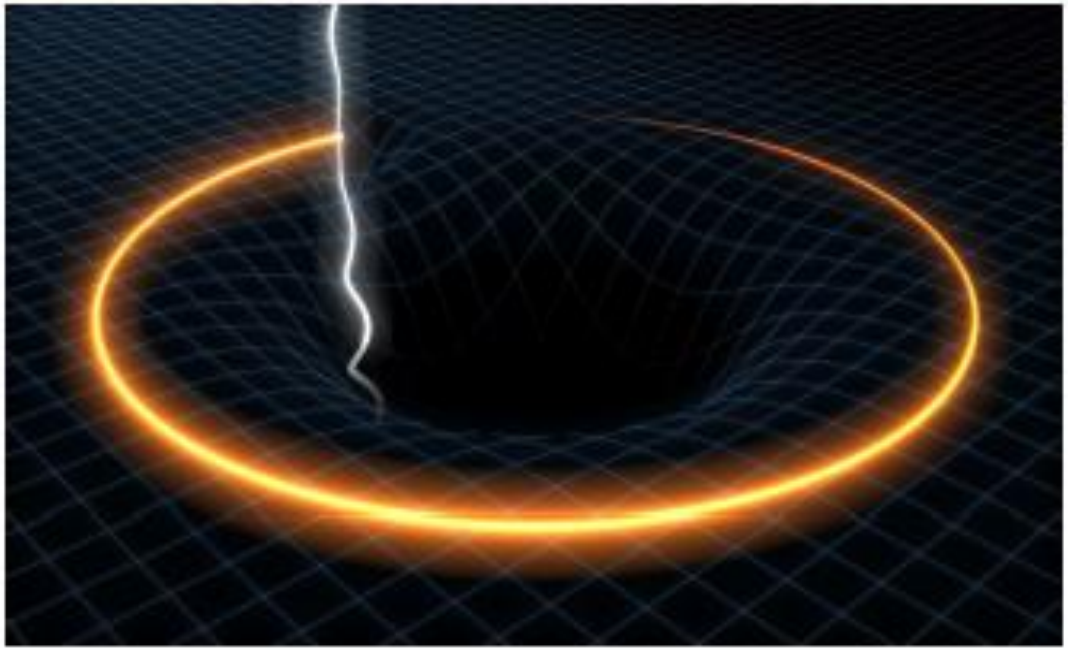


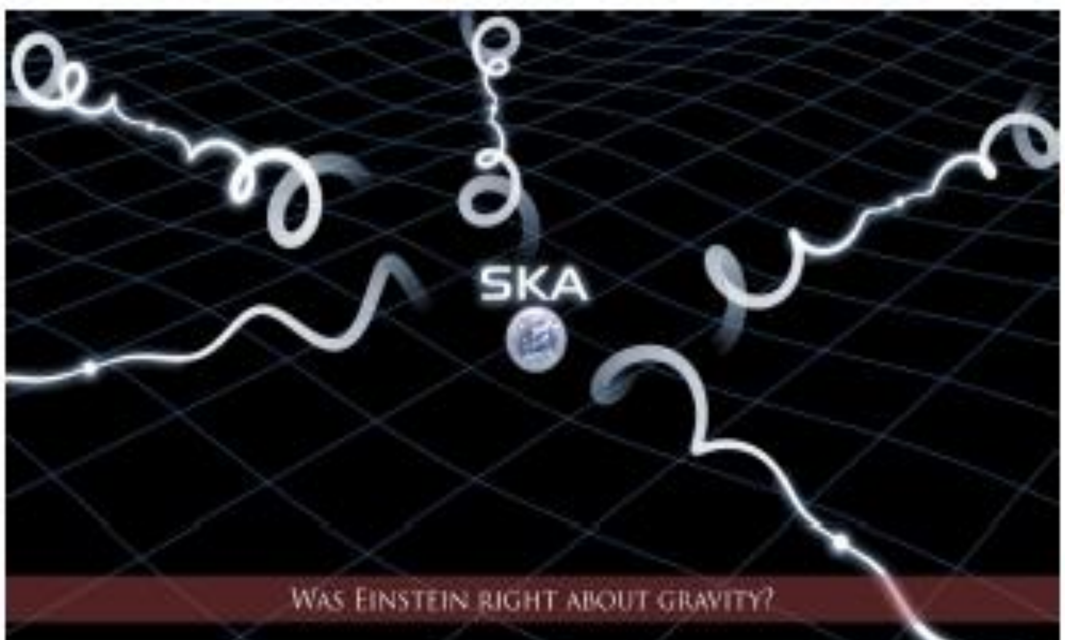
A. Ohnishi, H. Ueda, T. Nakano, M. Ruggieri, K. Sumiyoshi, Phys. Lett. B **704**, (2011) 284.

Perspectives for new Instruments?

The Future: SKA - Square Kilometer Array

THE FUTURE: SKA - SQUARE KILOMETER ARRAY





SKA Facts:

- The dishes of the SKA will produce 10 times the global internet traffic
- The data collected by the SKA in a single day would take nearly two million years to playback on an ipod
- The SKA will be so sensitive that it will be able to detect an airport radar on a planet 50 light years away

Discovery Potential:

- Find a Pulsar - Black Hole Binary
- Constrain Einstein Gravity
- Gravitational waves

LOFT - the Large Observatory For x-ray Timing

Main Science Objective of the LOFT MIssion:
Study of matter in ultradense environments and under strong gravity

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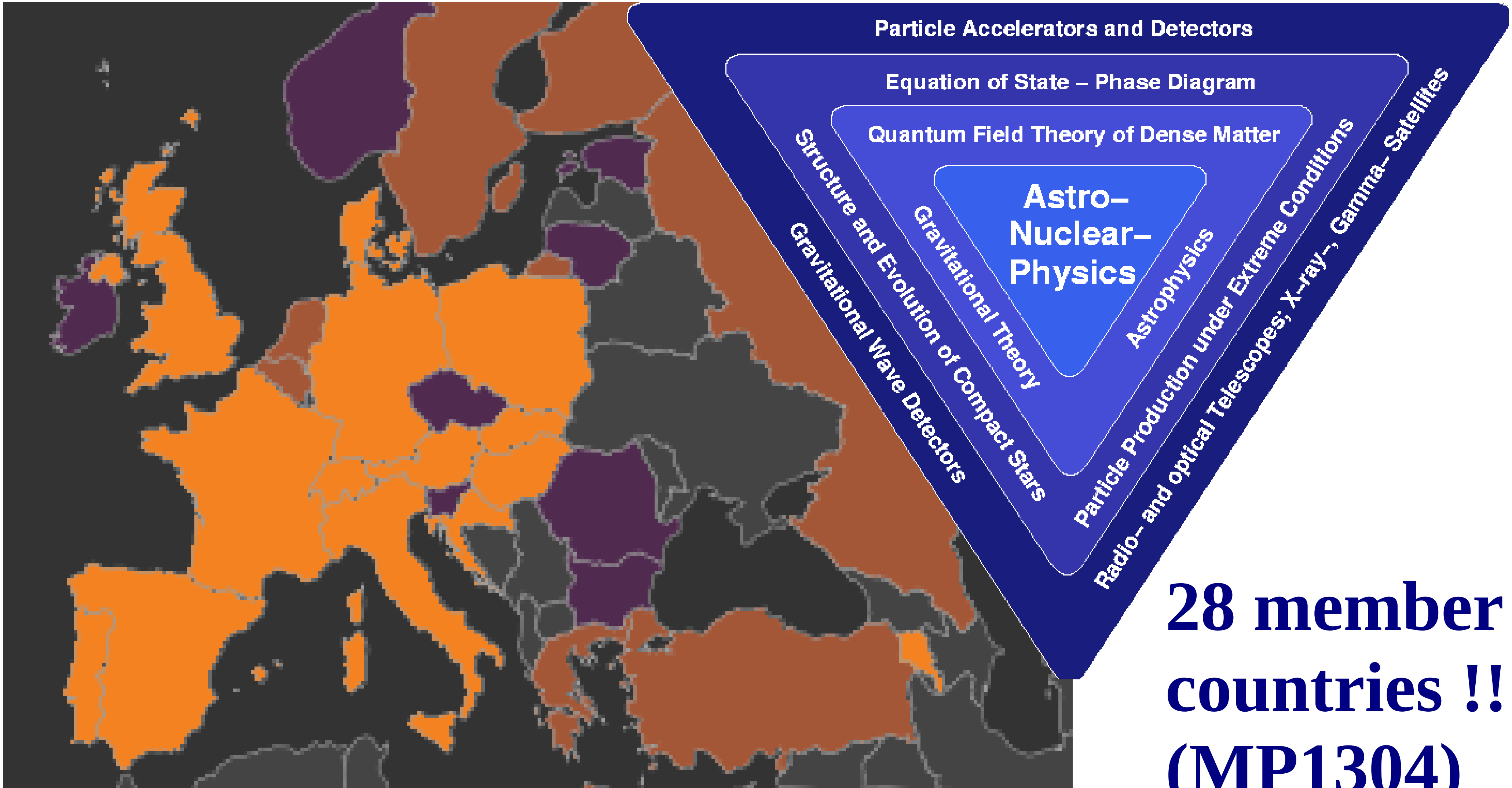

28 member countries !! (MP1304)

New compstar **!**

cost
EUROPEAN COOPERATION
IN SCIENCE AND TECHNOLOGY

Kick-off: Brussels, November 25, 2013