Elastic electron-deuteron scattering in the Bethe-Salpeter approach

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Motivation

Elastic *eD* scattering

- Deuteron - the simplest two-nucleon bound system with strong interaction
- EM structure of nucleons
- Investigations in high-energy region are of great interest in the context of future experiments at being upgraded JLab facilities
- Such investigations require relativistic approaches, for instance Bethe-Salpeter approach

Elastic eD scattering

Scattering cross section

$$\frac{d\sigma}{d\Omega'} = \left(\frac{d\sigma}{d\Omega'}\right)_{\rm Mott}\left[A(q^2) + B(q^2)\tan^2\frac{\theta}{2}\right],$$

$$\left(\frac{d\sigma}{d\Omega'}\right)_{\rm Mott} = \frac{\alpha^2\cos^2\theta/2}{4E^2(1+2E/M_d\sin^4\theta/2)},$$

where θ is the electron scattering angle, M_d is the deuteron mass, E_e is the incident electron energy.
Deuteron structure functions $A(q^2)$ and $B(q^2)$

$$A(q^2) = F_{\rm C}^2(q^2) + \frac{8}{9}\eta^2 F_{\rm Q}^2(q^2) + \frac{2}{3}\eta F_{\rm M}^2(q^2)$$

$$B(q^2) = \frac{4}{3}\eta(1+\eta)F_{\rm M}^2(q^2)$$

where $\eta = -q^2/4M_d^2 = Q^2/4M_d^2$

Deuteron current parametrization

The deuteron current matrix element parametrization
(due to P- and T-parity conservation and gauge invariance)

$$\langle D'\mathcal{M}'|J_\mu|D\mathcal{M}\rangle = -e\xi^*_{\alpha\ \mathcal{M}'}(P')\ \xi_{\beta\ \mathcal{M}}(P)$$

$$\times\left[(P'+P)_\mu\left(g^{\alpha\beta}F_1(q^2) - \frac{q^\alpha q^\beta}{2M_d^2}F_2(q^2)\right) - (q^\alpha g^\beta_\mu - q^\beta g^\alpha_\mu)G_1(q^2)\right]$$

$\xi_{\mathcal{M}}(P)$ and $\xi^*_{\mathcal{M}'}(P')$ are the polarization 4-vectors of the initial and final deuteron, respectively.

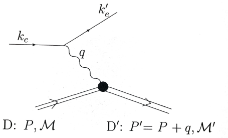
Form factors $F_{1,2}(q^2)$, $G_1(q^2)$ are related to functions $F_{\mathrm{C}}(q^2)$, $F_{\mathrm{Q}}(q^2)$ and $F_{\mathrm{M}}(q^2)$ by the equations

$$F_{\mathrm{C}} = F_1 + \frac{2}{3}\eta\left[F_1 + (1+\eta)F_2 - G_1\right]$$

$$F_{\mathrm{Q}} = F_1 + (1+\eta)F_2 - G_1$$

$$F_{\mathrm{M}} = G_1$$

Relativistic impulse approximation (RIA)



Deuteron current matrix element

$$\langle D'\mathcal{M}'|J_\mu^{RIA}|D\mathcal{M}\rangle =$$

$$ie\int \frac{d^4k}{(2\pi)^4}\left\{\bar{\chi}_{1\mathcal{M}'}(P',k')\Gamma_\mu^{(S)}(q)\chi_{1\mathcal{M}}(P,k)(P\cdot\gamma/2 - k\cdot\gamma + m)\right\}$$

$\chi^{1\mathcal{M}}(P,k)$ - the BS amplitude of the deuteron, $P' = P + q$ and $k' = k + q/2$.

The vertex of γNN interaction

$$\Gamma_\mu^{(S)}(q) = \gamma_\mu F_1^{(S)}(q^2) - \frac{\gamma_\mu q\cdot\gamma - q\cdot\gamma\gamma_\mu}{4m}F_2^{(S)}(q^2)$$

is chosen to be the form factor on mass shell.

The isoscalar form factors of the nucleon

$$F_{1,2}^{(S)}(q^2) = (F_{1,2}^{(p)}(q^2) + F_{1,2}^{(n)}(q^2))/2$$

Partial-wave decomposition

$|aM\rangle \equiv |\pi, {}^{2S+1}L_J^{\rho} M\rangle$

BS amplitude in the rest frame $\chi_{1M}(p; P_{(0)}) = \sum_a \mathcal{Y}_M^a(\mathbf{p})\ \phi_a(p_0, |\mathbf{p}|)$,

$\phi_a(p_0, |\mathbf{p}|)$ is the amplitude radial parts.

The spin-angular momentum functions:

$$\mathcal{Y}_M^a(\mathbf{p}) = i^L \sum_{m_L m_S m_1 m_2 \rho_1 \rho_2} C^{S_\rho \rho}_{\frac{1}{2}\rho_1 \frac{1}{2}\rho_2} C^{JM}_{Lm_L Sm_S} C^{Sm_S}_{\frac{1}{2} m_1 \frac{1}{2} m_2} \times \\ \times Y_{Lm_L}(\mathbf{p}) u^{\rho_1\,(1)}_{m_1}(\mathbf{p}) u^{\rho_2\,(2)}_{m_2}(-\mathbf{p})$$

S - the total spin

L - the orbital angular momentum

J - the total angular momentum with the projection M

ρ and π - the relative-energy and spatial parity

$u_m^{\rho=\pm,1/2}$ - Dirac spinors ($u^{\rho=+1/2} \equiv u,\ u^{\rho=-1/2} \equiv v$)

Y_{Lm_L} - the spherical harmonics

$C^{j\ m}_{j_1 m_1 j_2 m_2}$ - Clebsch-Gordan coefficients

Bethe-Salpeter formalism

Bethe-Salpeter equation for the amplitude (deuteron)

$$\chi(p;P) = \frac{i}{4\pi^3} S_2(p;P) \int d^4p'\, V(p,p';P)\,\chi(p';P)$$

p', p - the relative four-momenta
P - the total four-momentum

$V(p',p;P)$ - the interaction kernel

$S_2^{-1}(p;P) = \left(\frac{1}{2}\, P\cdot\gamma + p\cdot\gamma - m\right)^{(1)} \left(\frac{1}{2}\, P\cdot\gamma - p\cdot\gamma - m\right)^{(2)}$
free two-particle Green function

The partial-wave decomposed equation for amplitude

$$\phi_a(p_0,|\mathbf{p}|) = \frac{i}{4\pi^3} \sum_{bc} S_{ab}(p_0,|\mathbf{p}|;s) \int_{-\infty}^{+\infty} dk_0 \int_0^{\infty} \mathbf{k}^2 d|\mathbf{k}|\, V_{bc}(p_0,|\mathbf{p}|;k_0,|\mathbf{k}|;s)\,\phi_c(k_0,|\mathbf{k}|)$$

Solution

Separable ansatz for kernel

$$V_{a'a}(p_0', |\mathbf{p}'|; p_0, |\mathbf{p}|; s) = \sum_{m,n=1}^{N} \lambda_{mn}^{r[a'a]}(s) g_m^{[a']}(p_0', |\mathbf{p}'|) g_n^{[a]}(p_0, |\mathbf{p}|)$$

Solution for the BS amplitude: $\phi_a(p_0, |\mathbf{p}|) = \frac{g_a(p_0, |\mathbf{p}|)}{(\sqrt{s}/2 - E_{\mathbf{p}} + i\epsilon)^2 - p_0^2}$
solution for radial parts of the vertex function:

$$g_a(p_0, |\mathbf{p}|) = \sum_{i,j=1}^{N} \lambda_{ij}(s) g_i^{[a]}(p_0, |\mathbf{p}|) c_j(s)$$

with coefficients

$$c_i(s) - \sum_{k,j=1}^{N} h_{ik}(s) \lambda_{kj}(s) c_j(s) = 0$$

where functions $h_{ij}(s) = -\frac{i}{4\pi^3} \sum_a \int dk_0 \int \mathbf{k}^2 d|\mathbf{k}| \frac{g_i^{[a]}(k_0, |\mathbf{k}|) g_j^{[a]}(k_0, |\mathbf{k}|)}{(\sqrt{s}/2 - E_{\mathbf{k}} + i\epsilon)^2 - k_0^2}$

GRAZ II kernel

Graz II covariant kernel, rank III

$$g_1^{(\mathrm{S})}(p_0,|\mathbf{p}|) = \frac{1-\gamma_1(p_0^2-\mathbf{p}^2)}{(p_0^2-\mathbf{p}^2-\beta_{11}^2)^2},$$

$$g_2^{(\mathrm{S})}(p_0,\mathbf{p}) = -\frac{(p_0^2-\mathbf{p}^2)}{(p_0^2-\mathbf{p}^2-\beta_{12}^2)^2},$$

$$g_3^{(\mathrm{D})}(p_0,|\mathbf{p}|) = \frac{(p_0^2-\mathbf{p}^2)(1-\gamma_2(p_0^2-\mathbf{p}^2))}{(p_0^2-\mathbf{p}^2-\beta_{21}^2)(p_0^2-\mathbf{p}^2-\beta_{22}^2)^2},$$

$$g_1^{(\mathrm{D})}(p_0,|\mathbf{p}|) = g_2^{(\mathrm{D})}(p_0,|\mathbf{p}|) = g_3^{(\mathrm{S})}(p_0,|\mathbf{p}|) \equiv 0.$$

Nucleon form factors models

The vertex of γNN interaction: $\Gamma_\mu^{(\mathrm{S})}(q) = \gamma_\mu F_1^{(\mathrm{S})}(q^2) - \frac{\gamma_\mu q\cdot\gamma - q\cdot\gamma\gamma_\mu}{4m} F_2^{(\mathrm{S})}(q^2)$

- the original dipole fit for the proton and neutron form factors (DFF)

$$\boxed{\mu_{p,n}\frac{G_E^p}{G_M^{p,n}} = 1} \qquad F_d = (1 + Q^2/0.71)^{-2}$$

$$G_\mathrm{E}^\mathrm{p} = F_d, \qquad G_\mathrm{M}^\mathrm{p} = \mu_\mathrm{p} G_\mathrm{E}^\mathrm{p},$$
$$G_\mathrm{E}^\mathrm{n} = 0, \qquad G_\mathrm{M}^\mathrm{n} = \mu_\mathrm{n} G_\mathrm{E}^\mathrm{p};$$

- the modified dipole fit (MDFF) $\boxed{\mu_p\frac{G_E^p}{G_M^{p,n}} = 1 - 0.13(Q^2 - 0.04)}$

$$G_\mathrm{E}^\mathrm{p} = (1 - 0.13(Q^2 - 0.04))F_d, \qquad G_\mathrm{M}^\mathrm{p} = \mu_\mathrm{p} F_d,$$
$$G_\mathrm{E}^\mathrm{n} = -\frac{\mu_n\tau}{1 + 5.6\tau}F_d, \qquad G_\mathrm{M}^\mathrm{n} = \mu_\mathrm{n} F_d;$$

S. Galster et al., Nucl. Phys. B 32, 221 (1971).

Nucleon form factors models

- nine-resonance U&A model of the nucleon has 12 free parameters.
  C. Adamuščín, E. Bartoš, S. Dubnička, A.Z. Dubničková, Nucl. Phys. Proc. Suppl. **245**, 69 (2013).

- the relativistic harmonic oscillator

$$
\begin{aligned}
I^{(3)} &= \frac{1}{(1+Q^2/2m^2)^2} \times \exp \frac{1}{2\cdot 0.42}\frac{-Q^2}{1+Q^2/2m^2}, \\
G_{\mathrm{E}}^{\mathrm{p}} &= I^{(3)}, \\
G_{\mathrm{E}}^{\mathrm{n}} &= Q^2/2m^2 I^{(3)}, \\
\frac{G_{\mathrm{M}}^{\mathrm{p}}}{\mu_{\mathrm{p}}} &= \frac{G_{\mathrm{M}}^{\mathrm{n}}}{\mu_{\mathrm{n}}} = I^{(3)}.
\end{aligned}
$$

V.V. Burov *et al.*, Europhys.Lett. 24, 443 (1993)

Analytic structure

Deuteron EM current matrix element

$$\langle D'\mathcal{M}'|J_\mu^{RIA}|D\mathcal{M}\rangle = \frac{i}{(2\pi)^4}\int dp_0 p^2 dpd(cos\theta)\times$$

$$\times\left\{\sum_k A^{(k)}(p_0,p,cos\theta)_\mu^{ll'MM'} F_{(k)} g^{l'}(p_0',p')S_+^{(1)}S_{++}g^l(p_0,p)\right\}$$

Poles structure

propagator $S_+^{(1)}$:

$p_0 =$

$-\frac{1}{2}(1+4\eta)M \pm\sqrt{\mathbf{p}^2+m^2+4\sqrt{\eta(1+\eta)}M|\mathbf{p}|cos\theta+4\eta(1+\eta)M^2} \mp i\epsilon$

functions $g_i^{(L)}$:

$p_0 = -\eta M \pm\sqrt{\mathbf{p}^2+\beta_i^2+2\sqrt{\eta(1+\eta)}M|\mathbf{p}|cos\theta+\eta(1+\eta)M^2} \mp i\epsilon$

CALCULATION

Cauchy theorem

$$i\int_{-\infty}^{\infty} f dp_0 + i\int_{+i\infty}^{-i\infty} f dp_0 = -2\pi\sum_k Res_k(f, p_0 = \bar{p}_0^k)$$

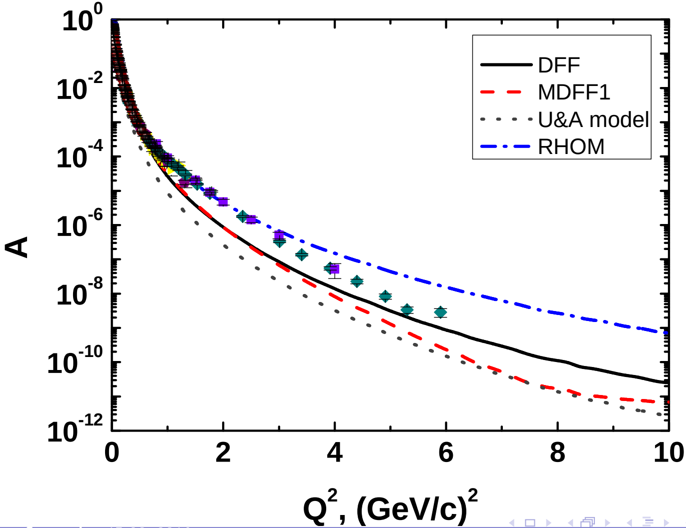
Wick rotation $\longmapsto$ $\mathrm{p}_0 = ip_4$

$$i\int_{-\infty}^{\infty} f dp_0 = \int_{-\infty}^{\infty} f dp_4 - 2\pi\sum_k \theta(Q^2 - Q_k^2) Res_k(f, p_0 = \bar{p}_0^k)$$

Threshold:

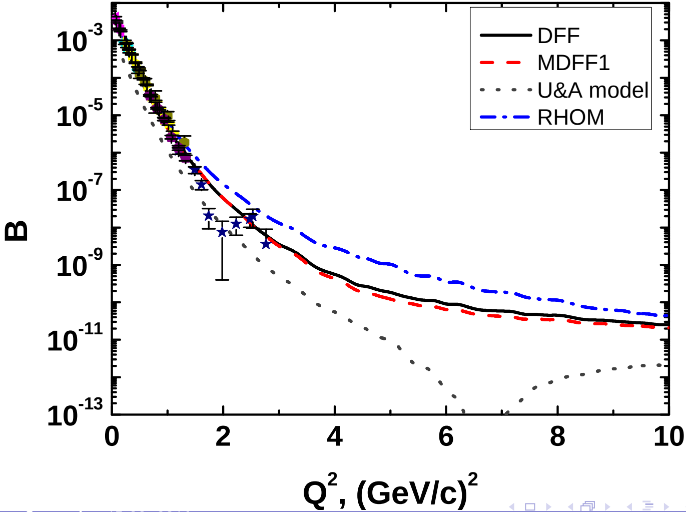
$\mathrm{Q}_1^2 = 0,004(\frac{GeV}{c})^2$
$\mathrm{Q}_2^2 = 1,181(\frac{GeV}{c})^2$
$\mathrm{Q}_3^2 = 1,735(\frac{GeV}{c})^2$
$\mathrm{Q}_4^2 = 3,914(\frac{GeV}{c})^2$
$\mathrm{Q}_5^2 = 5,963(\frac{GeV}{c})^2$

RESULTS

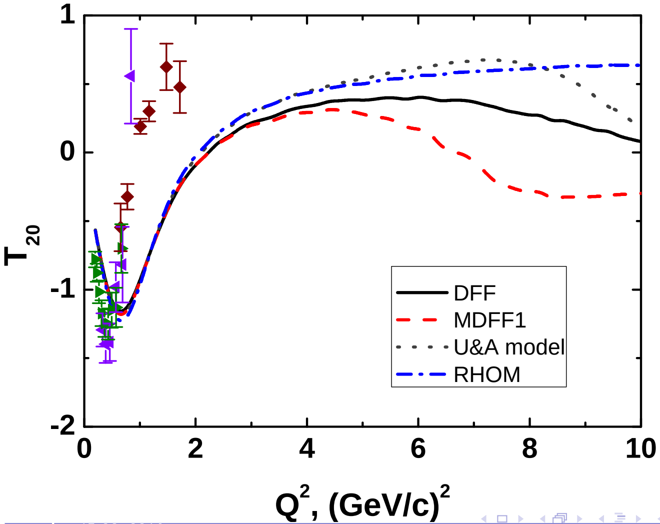


DFF
MDFF1
U&A model
RHOM
A
10^{0}
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{-12}
0
2
4
6
8
10
Q^2, $(GeV/c)^2$

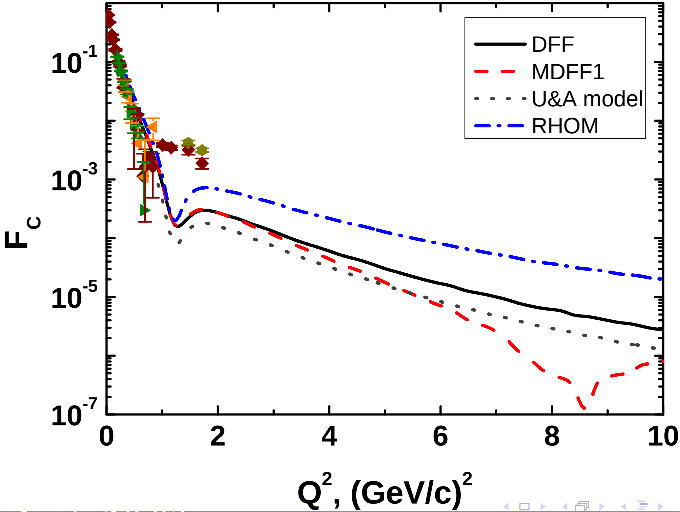
RESULTS


DFF
MDFF1
U&A model
RHOM
B
10^{-3}
10^{-5}
10^{-7}
10^{-9}
10^{-11}
10^{-13}
0
2
4
6
8
10
Q^2, (GeV/c)^2

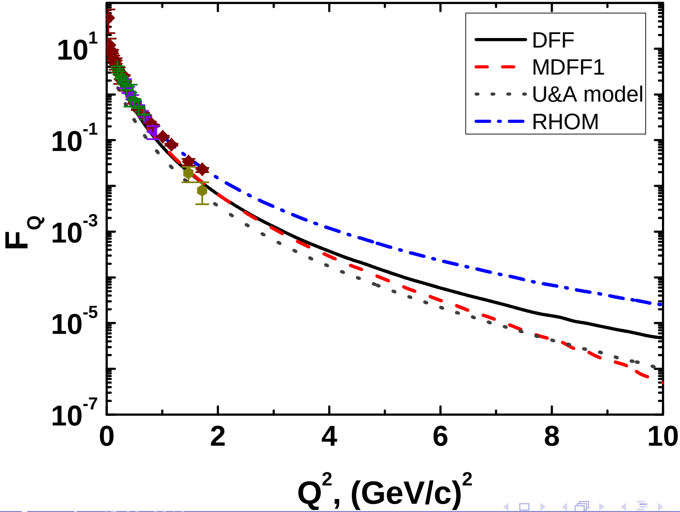
RESULTS



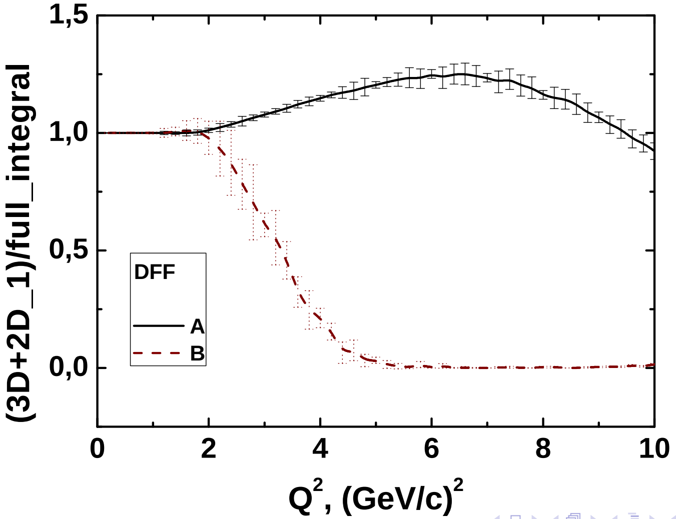
RESULTS



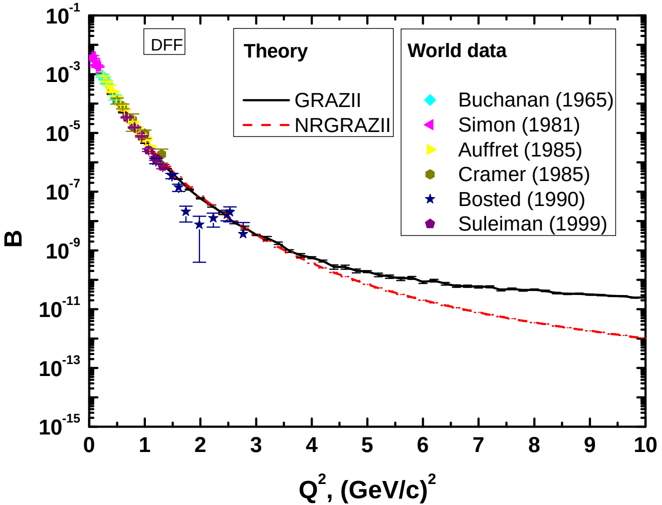
RESULTS


DFF
MDFF1
U&A model
RHOM
F_Q
10^{1}
10^{-1}
10^{-3}
10^{-5}
10^{-7}
0
2
4
6
8
10
Q^2, (GeV/c)^2

RESULTS



RESULTS



DFF
Theory
GRAZII
NRGRAZII
World data
Buchanan (1965)
Simon (1981)
Auffret (1985)
Cramer (1985)
Bosted (1990)
Suleiman (1999)
B
10^-1
10^-3
10^-5
10^-7
10^-9
10^-11
10^-13
10^-15
0 1 2 3 4 5 6 7 8 9 10
Q^2, (GeV/c)^2

Conclusions

- Calculations in high-energy region require relativistic approaches (Bethe-Salpeter equation, light-front dynamics, equal-time equation and so on)
- Calculations in the Bethe-Salpeter approach should take into account contribution from all poles of BS vertex function
- Experimental data at high energies are necessary to understand prediction of different nucleon EM form factors
- It is important to take into account effects of relativistic P-states in deuteron, interaction currents and off-shell nucleon effects

Reference A.V. Bekzhanov, S.G. Bondarenko, V.V. Burov, JETP Lett. **99**, 613 (2014)