Ts. Baatar[1], A.I.Malakhov[2], B.Otgongerel[1], G.Sharkhuu[1]

[1]Institute of Physics and Technology, MAS, Mongolia

[2]Joint Institute for Nuclear Research, Dubna, Russia

# A particle emission region size in multiparticle production process

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## Abstract

In this paper we have obtained the formula which determines the particle emission region size (r~1/Q) and corresponding distributions are shown for the secondary negative pions and protons from $\pi^{-}C$ interactions at 40 GeV/c.

## 1. Introduction

An investigation of multiparticle production processes of the secondary particles produced in hadron-hadron (hh), hadron-nucleus (hA) and nucleus-nucleus (AA) interactions at high energies and large momentum transfers is very important for understanding the strong interaction mechanism and inner quark-gluon structure of nuclear matter.

A determination of the particle production size of the secondary particles of above mentioned interactions is one of the key problems in the studies of high energy physics. A correct estimation of this parameter (r) gives us the possibility to calculate the volume, mass and energy densities and to establish the main characteristics of the phase transition process, to distinguish the different mechanisms of the secondary particle's production and so on.

During the last years the collective phenomena such as the cumulative particle production, the production of nuclear matter with high densities, the phase transition from the hadronic matter to the quark-gluon plasma state, the state of the color superconductivity is widely discussed in the literature [1-7].

According to the different ideas and models, if exist these phenomena in the nature then they should be reflected to the dynamics of interaction process and will be observed in hA and AA interactions at high energies and large momentum transfers. We would like to stress that in hA and AA collisions, in difference from hh collisions, the secondary particles may be produced in the regions kinematically not allowed to hh interactions.

## 2. A determination of the particle emission region size

The cumulative number $n_c$ in the fixed target experiment is determined by the next formula [1],

$$n_c = \frac{(P_a \cdot P_c)}{(P_a \cdot P_b)} = \frac{E_c - \beta_a \cdot P_c^{||}}{m_p} \cong \frac{E_c - P_c^{||}}{m_p} \qquad (1)$$

Where $P_a$, $P_b$ and $P_c$ are the four dimensional momenta of incident, target and considering secondary particles, $E_c$ and $P_c^{||}$ are the energy and longitudinal momentum of the secondary particle, $\beta_a$ is the velocity of the incident particle and $m_p$ is the proton mass.

This variable $n_c$ is connected with the four momentum transfer t by the next formula,

$$t = -Q^2 = -(P_a - P_c)^2 \cong S_{\pi^- p} \cdot n_c \qquad (2)$$

Where $S_{\pi^- p}$ is the total energy square of $\pi^- p$ interaction.

A determination of the particle emission region size r of the secondary particles is very important in the study of multiparticle production dynamics.

It is well known that the particle emission region size r is inversely proportional to the momentum transfer Q

$$r \sim \frac{1}{Q} \quad (3)$$

Supposing that the dependence (3) is correct and then including a coefficient of proportionality $k_0$ we obtain the next equation,

$$r = \frac{k_0}{Q} = \frac{k_0}{\sqrt{S_{hN} \cdot n_c}} \quad (4)$$

Now we choose the condition $n_c = 1$, this procedure gives us the possibility to simplify the formula (4) and to determine the value of the parameter r corresponding to this condition ($n_c = 1$). We note that the condition $n_c = 1$ means that the value of the target mass which is required for producing of the given secondary particle is equal to proton mass $m_p$. In this case the formula (4) is written in the next form,

$$k_0 = r \cdot \sqrt{S_{hN}} \quad (5)$$

We note that as mentioned above at $n_c = 1$, the parameter r is equal to the Compton wavelength of proton $\lambda_c^P$, in other words,

$$r = \lambda_c^P = \frac{1}{m_p} = 0.21 fm \quad (6)$$

Inserting the formula (6) to equation (5) the coefficient of proportionality $k_0$ is determined by the next formula,

$$k_0 = \lambda_c^P \cdot \sqrt{S_{hN}} = \frac{\sqrt{S_{hN}}}{m_p}, \qquad \tan\alpha = \frac{1}{0.938} = 1.066\ or\ (\alpha = 46.8°) \qquad (7)$$

Fig .1 shows the dependence of the parameter $k_0$ on $\sqrt{S_{hN}}$. From Fig.1 one can see that the dependence of the parameter $k_0$ on $\sqrt{S_{hN}}$ is described by the linear law with $\tan\alpha = 1.066$. This dimensionless number $k_0$ changing with $\sqrt{S_{hN}}$ may be called the running energy constant. This parameter indicates that how many times the total energy $\sqrt{S_{hN}}$ of the experiment larger than proton mass $m_p$.

Receiving the formula (7) for the parameter $k_0$, of course, we have the possibility to obtain the formula which determines the particle emission region size r.

Inserting the formula (7) to the formula (4) we have been obtained the next formula for the parameter r,

$$r = \frac{1}{\sqrt{n_c} \cdot m_p} = \frac{\lambda_c^P}{\sqrt{n_c}} = \frac{0.21 fm}{\sqrt{n_c}} \qquad (8)$$

From the formula (8) one can see that the parameter r is fully determined by the Compton wavelength of proton $\lambda_c^P$ and the cumulative number $n_c$. So the determination of the numerical value of the variable $n_c$ gives us the possibility to obtain the numerical value of the particle emission region size r for the every secondary particle produced in collisions of interacting particles and nuclei at high energies.

From formula (4) the parameter $k_0$ is written in the next form

$$k_0 = r_i \sqrt{S_{hN} \cdot n_c^i}$$

The dependence of the parameter $k_0$ on the variable $n_c$ was shown on Fig.2. From this figure we see that the parameter $k_0$ do not depend on the variable $n_c$ at fixed energy $\sqrt{S_{hN}}$.

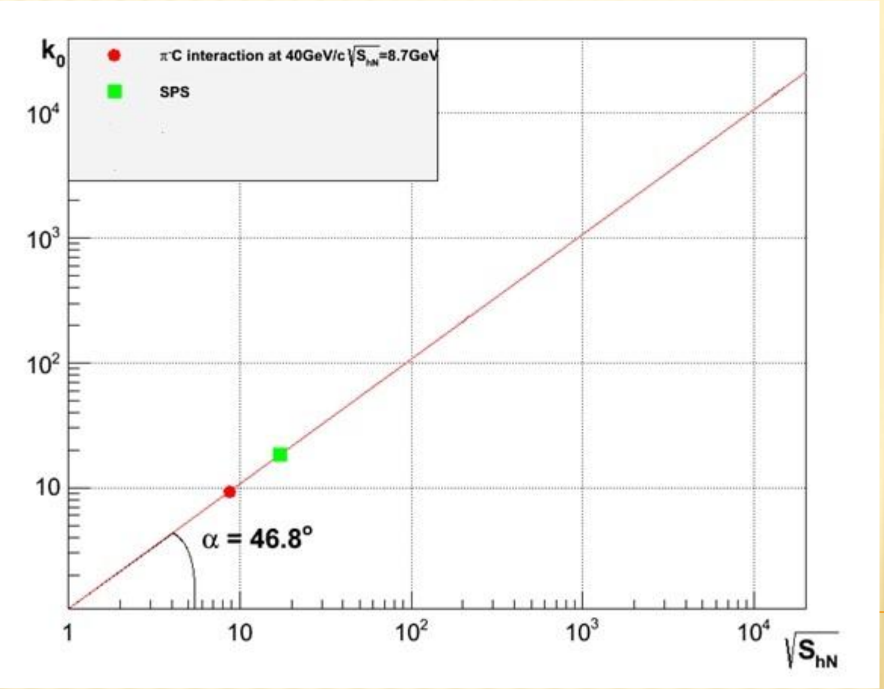



Fig. 1. shows the dependence of the parameter $k_0$ on $\sqrt{S_{hN}}$

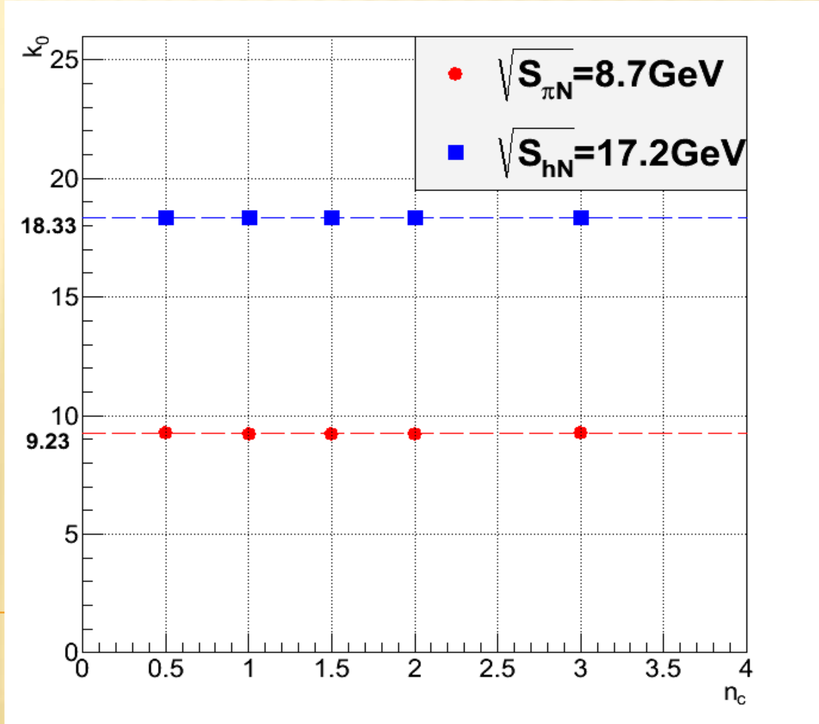



Fig.2. Dependence of the parameter $k_0$ on the variable $n_c$

The dependence of the parameter $r^2$ on the variable $n_c$ calculated by the formula (8) is presented on Fig.3. This dependence do not depend on the type of the secondary particles.

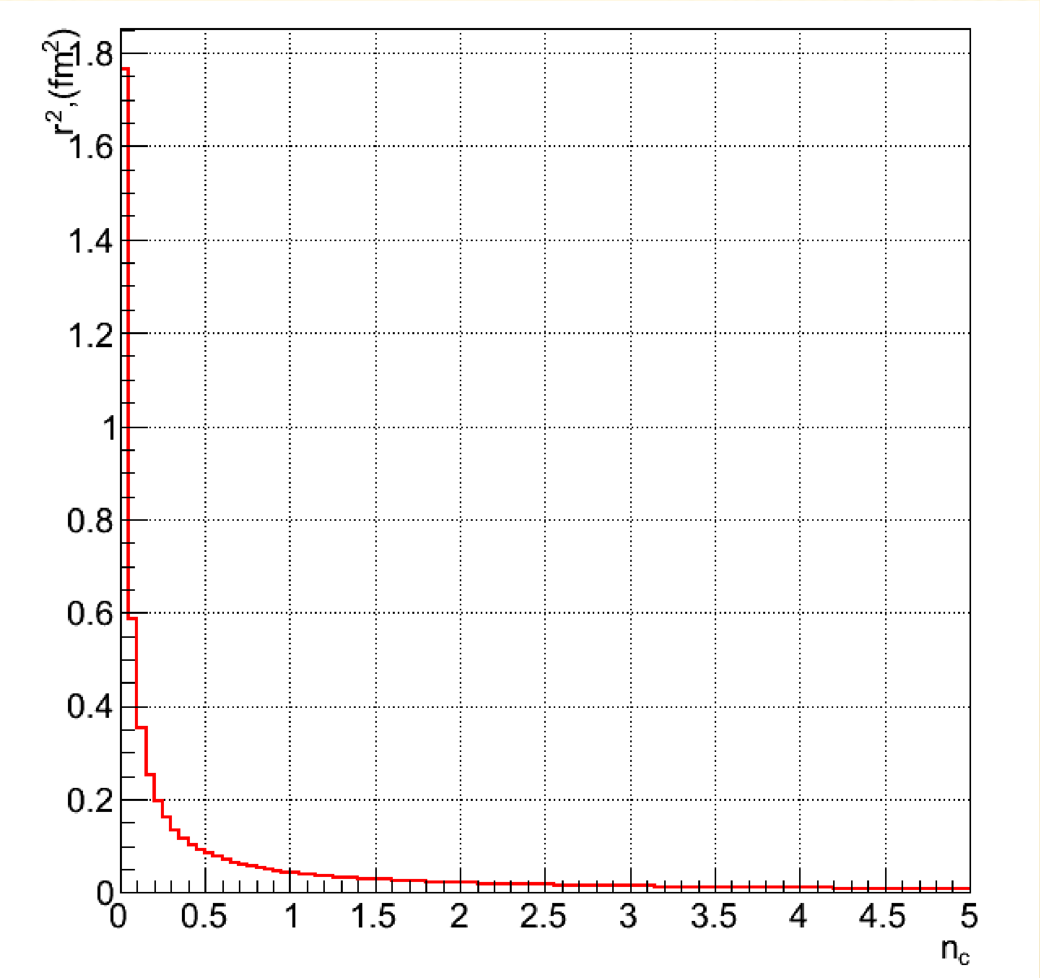



Fig.3. Dependence of the parameter $r^2$ on $n_c$

## 3. In the case of $\pi^{-}C$ interactions at 40 GeV/c

### 3.1 $\pi^{-} + C \rightarrow \pi^{-} + X$ analysis

We would like to stress that the leading $\pi^{-}$-mesons may give some effect to the experimental result. So we are considered for $\pi^{-}$-mesons two cases with and without leading particles. The angular distribution of $\pi^{-}$-mesons with momentum $P_{\pi^-} > 20\ GeV/c$ was shown on Fig.4. From this distribution $\pi^{-}$-mesons with momentum $P_{\pi^-} > 20\ GeV/c$ and scattering angle $\theta < 4^{0}$ are regarded as leading particles. 1182 $\pi^{-}$-mesons are excluded from total 30162 pions as leading particles.

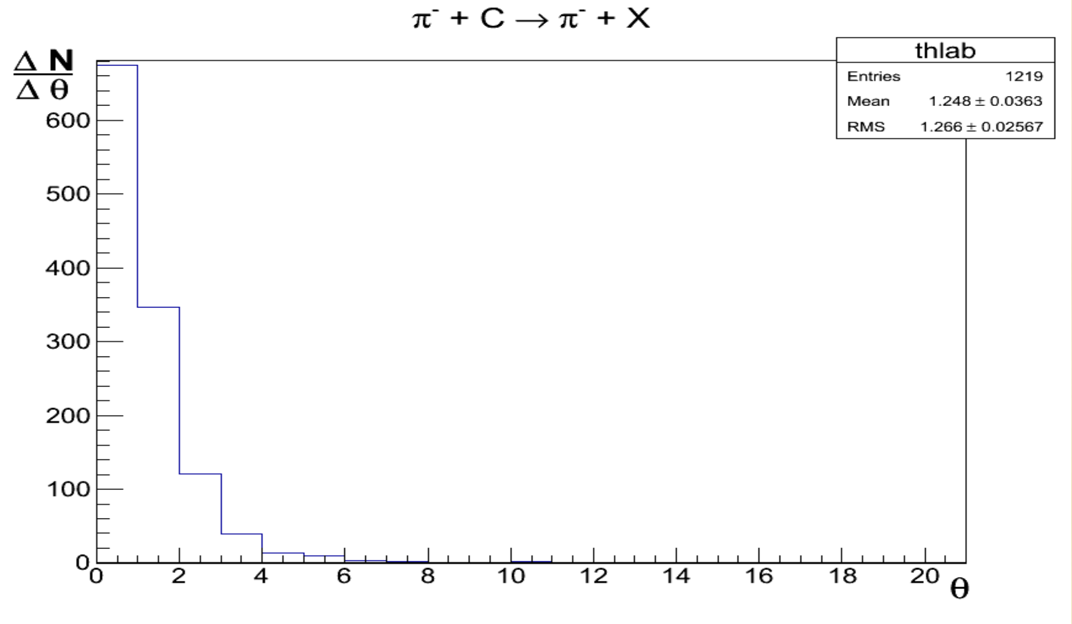
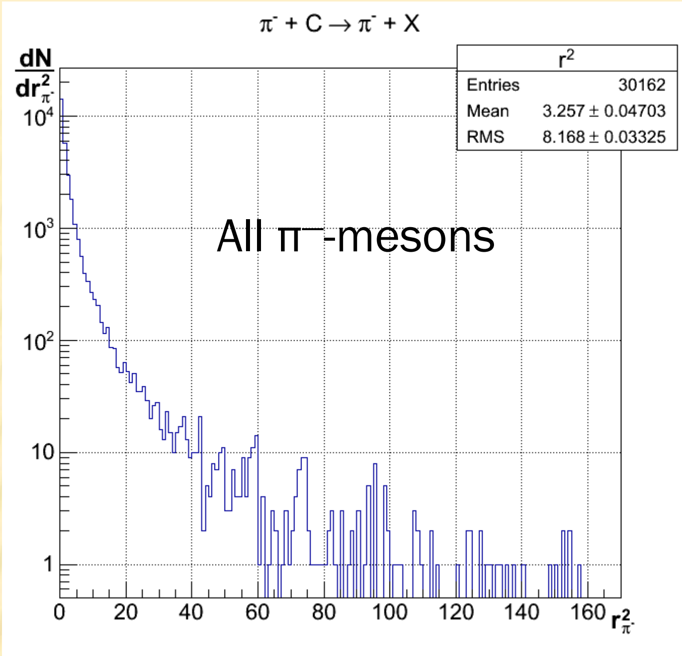
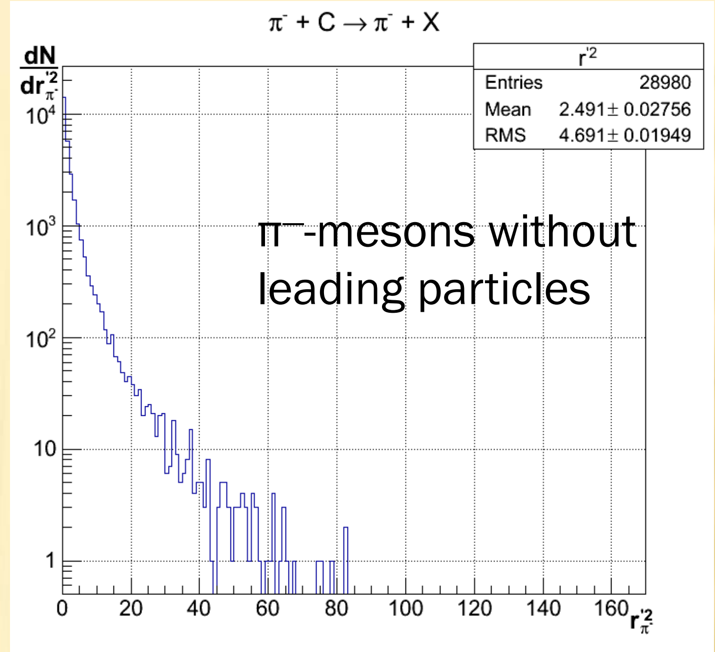



Fig. 4. Angular distribution of π⁻-mesons with momentum $P_{\pi^-} > 20\ GeV/c$

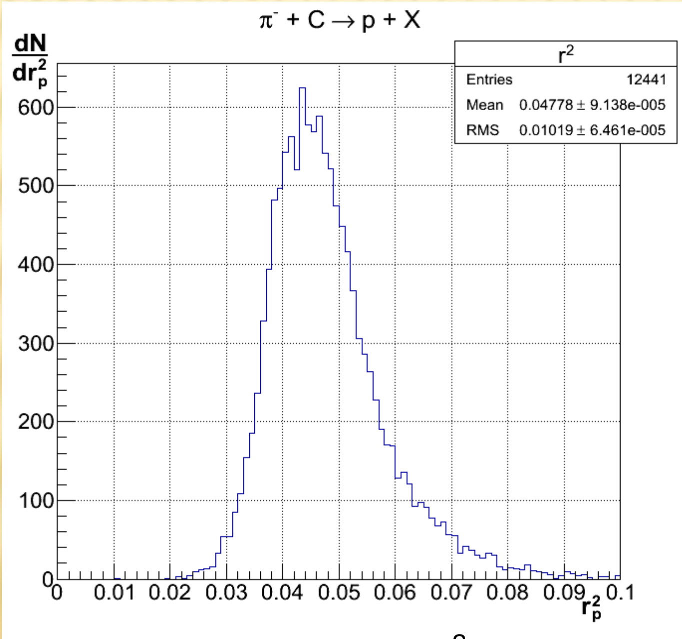
a)




b)




c)




$$\langle r\rangle_{\pi^-} = \sqrt{<r^2>_{\pi^-}} = (1.805 \pm 0.026)\ fm$$

$$\langle r'\rangle_{\pi^-} = \sqrt{<(r')^2>_{\pi^-}} = (1.578 \pm 0.017)\ fm$$

$$\langle r\rangle_{p} = \sqrt{<r^2>_{p}} = (0.218 \pm 0.002)\ fm$$

$$\frac{\langle r\rangle_{\pi^-}}{\langle r\rangle_p} = \frac{1.805}{0.218} = 8.2$$

$$\frac{\langle r'\rangle_{\pi^-}}{\langle r\rangle_p} = \frac{1.578}{0.218} = 7.2$$

Fig. 5 a, b, c. Distribution on $r^2$ for $\pi^-$-mesons (a, b) and protons (c) from $\pi^-C$ interactions at 40 GeV/c

The experimental distributions on the parameter r$^2$ calculated by the formula (8) for all $\pi^-$-mesons $r_{\pi^-}^2$, $\pi^-$-mesons without leading particles $(r'_{\pi^-})^2$ and protons $r_p^2$ from $\pi^- C$ interactions at 40 GeV/c are shown on Fig.5 (a, b, c). The next average values are obtained:

$$\langle r \rangle_{\pi^-} = \sqrt{< r^2 >_{\pi^-}} = (1.805 \pm 0.026)\ fm$$

$$\langle r' \rangle_{\pi^-} = \sqrt{< (r')^2 >_{\pi^-}} = (1.578 \pm 0.017)\ fm$$

$$\langle r \rangle_p = \sqrt{< r^2 >_p} = (0.218 \pm 0.002)\ fm$$

$$\frac{\langle r \rangle_{\pi^-}}{\langle r \rangle_p} = \frac{1.805}{0.218} = 8.2\ ,\ \frac{\langle r' \rangle_{\pi^-}}{\langle r \rangle_p} = \frac{1.578}{0.218} = 7.2$$

From this analysis we see that the average value of the parameter $\langle r \rangle_{\pi^-}$ for $\pi^-$-mesons is 7÷8 times greater than the case for protons $\langle r \rangle_p$. We note that ratios of $\langle r \rangle_{\pi^-}$, $\langle r' \rangle_{\pi^-}$ to $\langle r \rangle_p$ obtained within our approach is in reasonable consistence with a theoretical prediction referred in [7] as follows:

$$\lambda_c^{\pi} \cong 7 \cdot \lambda_c^{P}$$

Where $\lambda_c^{\pi}$ and $\lambda_c^{P}$ are the Compton wave lengths of pion and proton.

Average values of the parameters $< r^2 >_{\pi^-}$, $< r >_{\pi^-}$ for the all secondary $\pi^-$ - mesons and for $\pi^-$ -mesons in three different reqions on the variable $n_c$ are presented in Table 1.

Corresponding values without leading $\pi^-$ -mesons are presented in Table 2.

Table 1

| $\pi^- + C \to \pi^- + X$ | | | | |
|---|---|---|---|---|
| | All $\pi^-$-mesons | $\pi^-$-mesons with $n_c \leq 0.07$ | $\pi^-$-mesons $0.07 < n_c < 0.5$ | $\pi^-$-mesons $n_c > 0.5$ |
| $N_{\pi^-}$ | 30162 | 19702 | 9879 | 581 |
| $\langle r^2 \rangle_{\pi^-} (fm^2)$ | 3.122 ± 0.041 | 4.614 ± 0.059 | 0.331 ± 0.002 | 0.06146 ± 0.0009 |
| $\langle r \rangle_{\pi^-} (fm)$ | 1.769 ± 0.023 | 2.148 ± 0.028 | 0.575 ± 0.003 | 0.2479 ± 0.0036 |

Table 2

| $\pi^- + C \to \pi^- + X$ | | | | |
|---|---|---|---|---|
| | $\pi^-$-mesons without leading particles | $\pi^-$-mesons with $n_c \leq 0.07$ | $\pi^-$-mesons $0.07 < n_c < 0.5$ | $\pi^-$-mesons $n_c > 0.5$ |
| $N_{\pi^-}$ | 28980 | 18522 | 9877 | 581 |
| $\langle r'^2 \rangle_{\pi^-} (fm^2)$ | 2.491 ± 0.0276 | 3.72 ± 0.040 | 0.331 ± 0.002 | 0.06146 ± 0.0009 |
| $\langle r' \rangle_{\pi^-} (fm)$ | 1.5783 ± 0.017 | 1.929 ± 0.021 | 0.575 ± 0.003 | 0.2479 ± 0.0036 |

## 3.2 $\pi^-$-mesons in three different $n_c$ regions.

The dependence of the effective temperature T on the variable $n_c$ is presented on Fig.6. This dependence was taken from the publication [1]. From this figure we see that with increasing $n_c$ the effective temperatures T in the beginning are increasing until $n_c \lesssim 0.07$, and then in the $n_c \simeq (0.07 \div 0.5)$ interval the parameter T remains practically constant on the level $T \simeq 0.220 \div 0.230$ GeV and then from the point $n_c > 0.5$ again increases.

The corresponding values of the parameter r at breaking points of the dependence are shown by arrows on this figure.

The proton charge radius usually is determined from the elastic electron-proton scattering [9] as follows:

$$R_P = 0{,}8775 \pm 0{,}0051\ fm$$

The value of the parameter r calculated in by formula. (8) at the first critical point ($n_c$=0,071) which gives the beginning of the plateau on parameter T is obtained equal to 0.79 fm (see Fig.6). We note that this value is compatible with the charge radius of proton mentioned above. So, if the secondary $\pi^-$ - mesons are produced at the smaller or approximately equal distances to the charge radius of proton then, these particles begin to actively participate in the equilibrium process (or phase transition).

The plateau on parameter T is continued to $< r >_{\pi^-} \leq 0{,}297\ fm$, so the negative pions produced at the distances $< r >_{\pi^-} \geq 0{,}297\ fm$ give the main contribution to the pure quark-gluon plasma state.

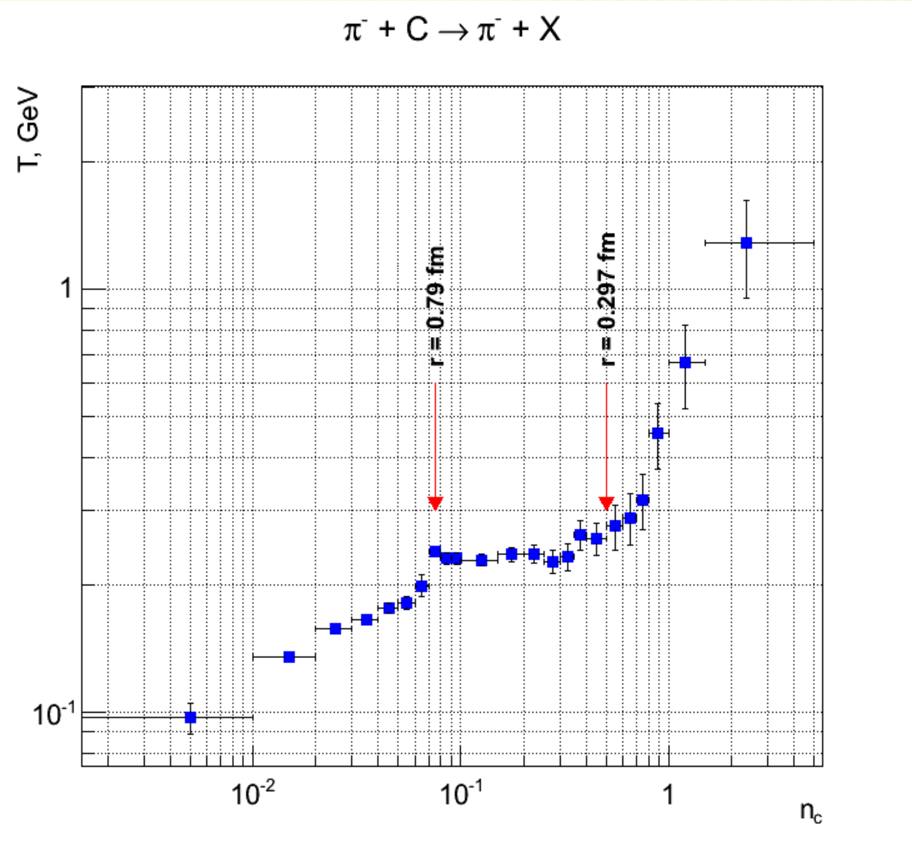
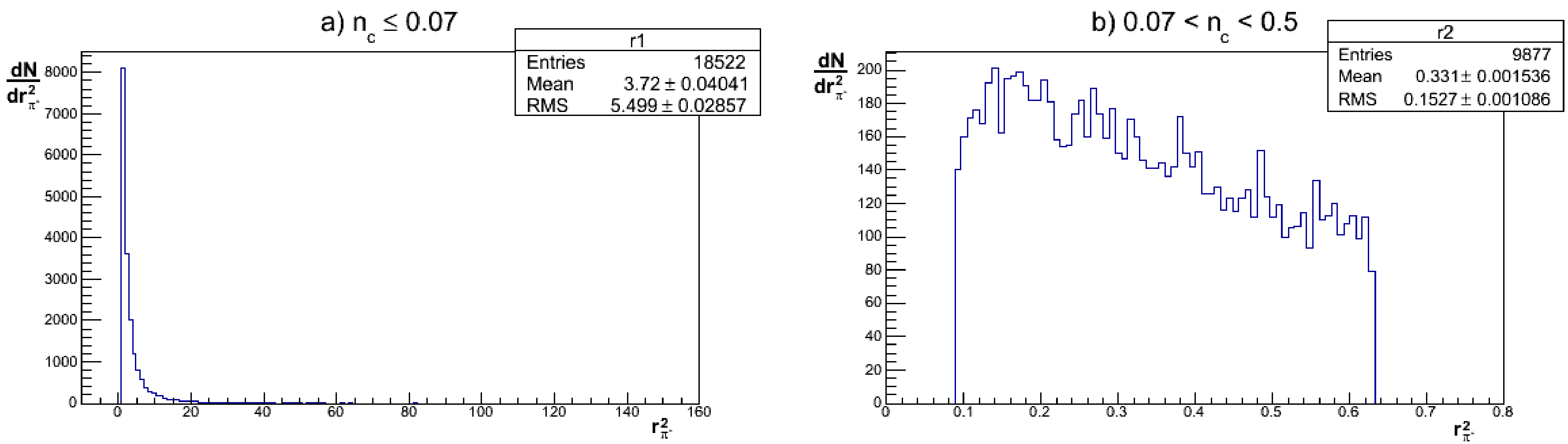
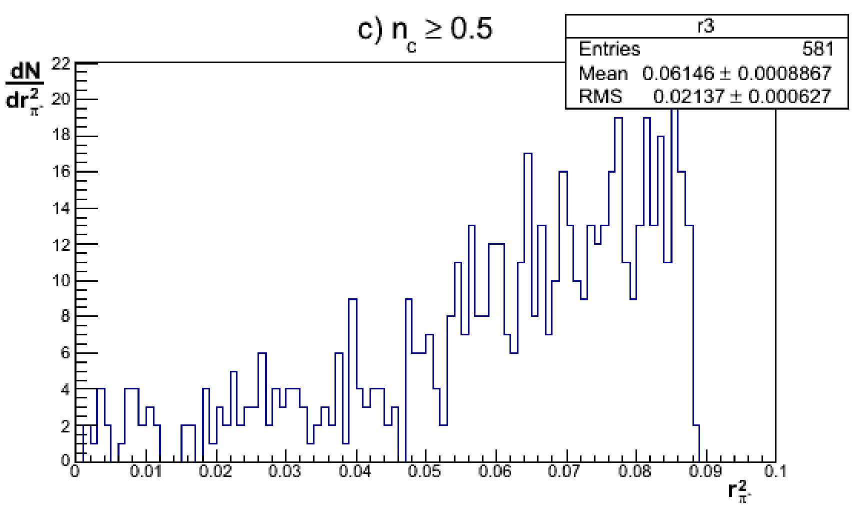



Fig. 6. The effective temperature T of the secondary $\pi^-$-mesons as a function of the variable $n_c$ This figure was taken from paper [1]

Fig.7 a, b, c shows distributions on parameter $r^2$ for $\pi^-$-mesons in the above mentioned three different phase transition regions. We would like to stress that the particle production region size r is essentially different in every region.




$$\sqrt{\langle {r_1}^2 \rangle_{\pi^-}} = (1.929 \pm 0.021)\ fm$$

$$\sqrt{\langle {r_2}^2 \rangle_{\pi^-}} = (0.575 \pm 0.003)\ fm$$

$$\sqrt{\langle {r_3}^2 \rangle_{\pi^-}} = (0.2479 \pm 0.0036)\ fm$$

Fig. 7 a, b, c. Distribution on $r^2$ for $\pi^-$-mesons in 3 different phase transition regions

## 3.3 $\pi^{-} + C \rightarrow P + X$ analysis

Now we consider the proton case.

Fig. 8 shows the dependence of the effective temperature T on the variable $n_c$ for the secondary protons from $\pi^{-}C$ interactions. From this figure we see that with increasing $n_c$, this dependence is divided into two parts, where temperature T is remained practically constant on the level $T \simeq 50$ MeV in the $n_c \cong 0.5 \div 1.2$ interval and T is increased in the region $n_c > 1.2$.

The corresponding values of the parameter r on the breaking points of the dependence are shown by arrows on this figure.

We note that the numerical values of the parameter T which gives the plateau for $\pi^{-}$- mesons (~230 MeV) and for protons (~50MeV) are essentially different but the behaviors of the dependence of parameter T on the variable $n_c$ are the same in both cases.

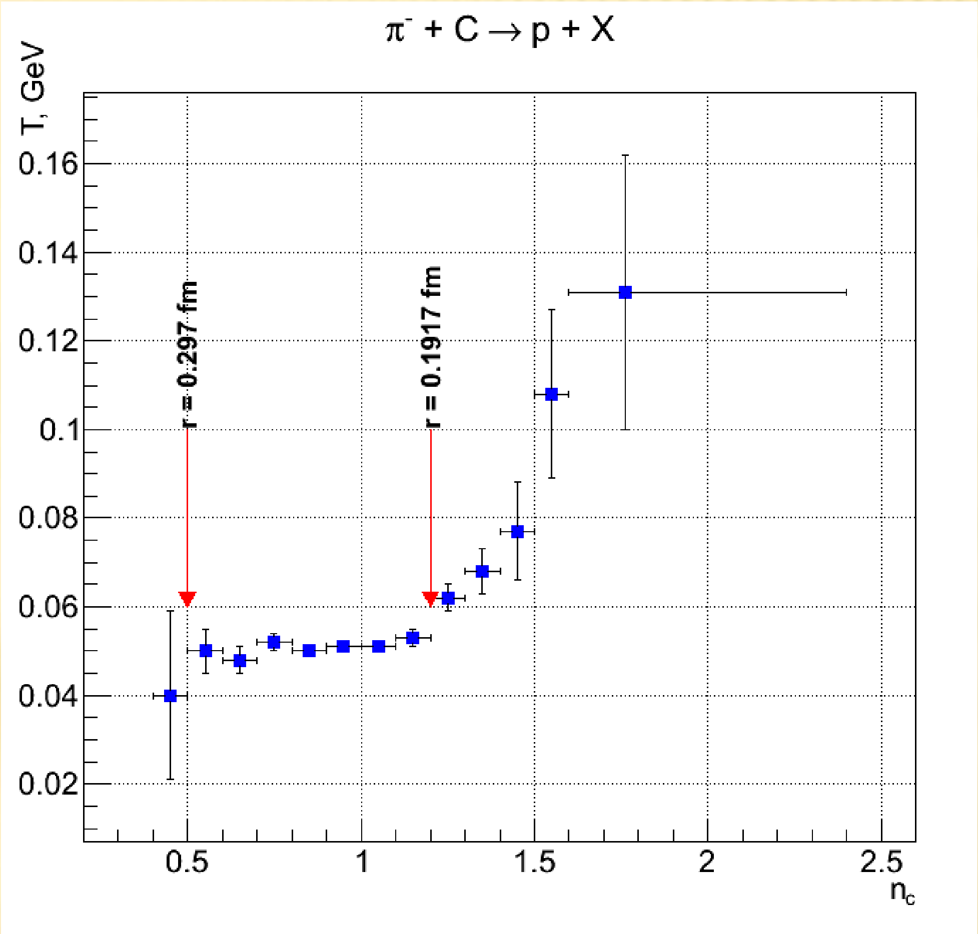



Fig. 8. The effective temperature T of the secondary protons as a function of the variable $n_c$. This figure was taken from paper [1]

Fig. 9 a, b shows distributions on parameter $r^2$ in above mentioned two regions. We would like to note that the protons from $\pi^-C$ interactions at 40 GeV/c are produced at comparatively small values of the parameter $r \approx$ 0.22 fm and the numerical values of those parameters are slightly different in these two regions.

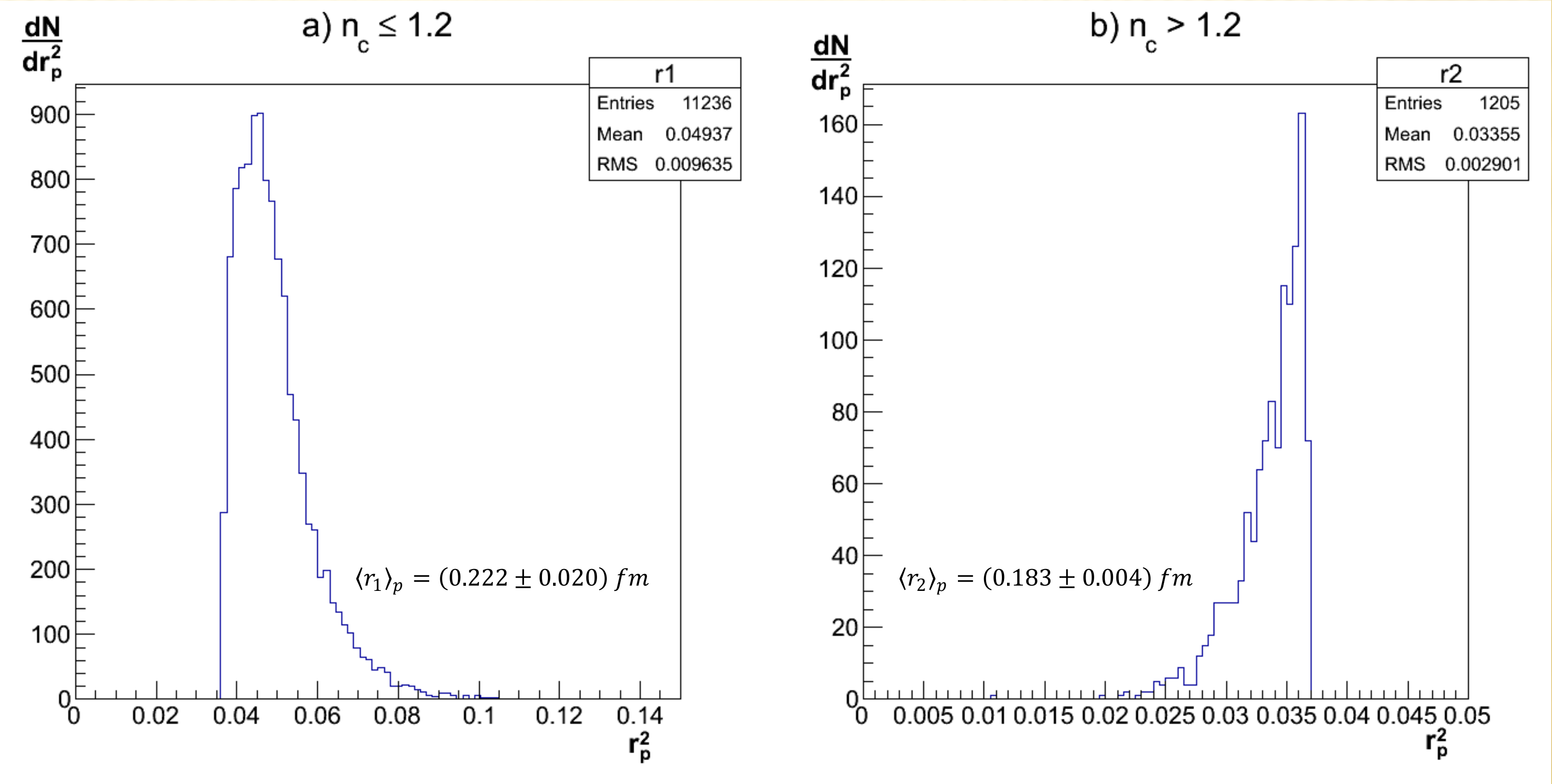



Fig. 9 a, b. Distributions on the parameter $r^2$ for protons in different $n_c$ regions

Average values of the parameters $< r^2 >_P$, $< r >_P$ for all protons and protons in two different regions on the variable $n_c$ are given in Table 3.

We would like to note that average values of the parameter $< r >$ are essentially different not only for all $\pi^-$ -mesons and protons, but also for $\pi^-$ -mesons and protons produced in different $n_c$ regions corresponding to different phase.

Table 3

| $\pi^- + C \to p + X$ | | | |
|---|---|---|---|
| | All protons | Protons with $n_c \le 1.2$ | Protons with $n_c > 1.2$ |
| $N_p$ | 12441 | 11236 | 1205 |
| $\langle r^2 \rangle_p (fm^2)$ | 0.04778 ± 0.00009 | 0.04937 ± 0.0046 | 0.03355 ± 0.00008 |
| $\langle r \rangle_p (fm)$ | 0.21858 ± 0.00041 | 0.2222 ± 0.0020 | 0.1832 ± 0.0004 |

## 3.4 Average values of the proton momentum $< P >_P$ on the variable $n_c$

The distribution on the variable $n_c$ for protons from $\pi^-C$ interactions at 40 GeV/c is presented on Fig.10. From this figure we see that the maximum of the distribution is at $n_c \approx 1$ and ~60% of protons are produced in the region $n_c \leq 1$ and ~40% of protons are produced in the region $n_c > 1$. So 40% of protons are produced in the cumulative particle production region, $n_c > 1$.

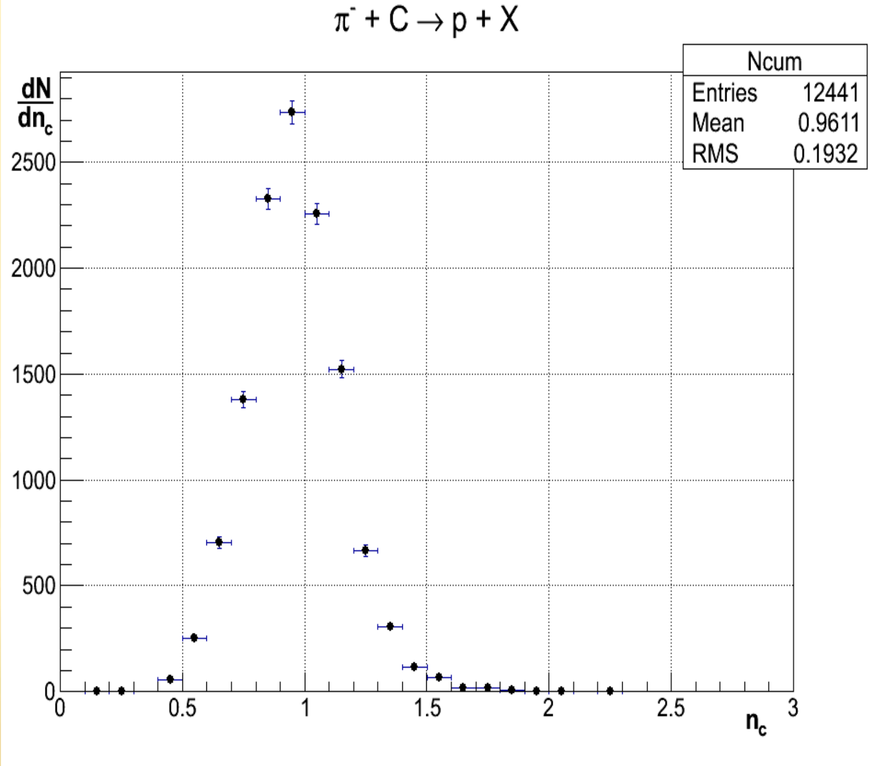



Fig. 10. Cumulative number ($n_c$) distribution of protons. This figure was taken from paper [1]

Fig 11 presents the dependence of the average momentum of protons as a function of variable $n_c$. With increasing $n_c$ average values of the momentum $< P >_P$ are decreased and reach the minimum at $n_c \approx 1$ and then in the cumulative particle production region ($n_c > 1$) are essentially increased. In addition to this we note that average values of the transverse momentum square $< P_t^2 >$ (see [1]) and the effective temperature T (Fig.7) are remained practically constant in the region $r \lesssim \lambda_c^p = 0.21 fm$, but in the region $r > \lambda_c^p$ we see the reversed feature, in other words, $\langle P \rangle_p$, $\langle P_t^2 \rangle$ and T are essentially increased. So, the essentially different features of the above mentioned characteristics $\langle P \rangle_p$, $\langle P_t^2 \rangle$ and T in these two regions ($r \lesssim \lambda_c^p$ and $r > \lambda_c^p$) as mentioned in the previous paper [1] indicate about the particle production different mechanism.

The numerical value of the parameter r at $n_c \approx 1$ was shown by arrow on this figure.

From the other hand side, it is well known that if a particle is localized in the region $r < \lambda_c^P = 0.21 fm$, then this particle is regarded as a "not point like object" and its interaction should be described by Quantum field Theory (QFT)[7]. So, the essentially different behaviors of the proton's characteristics indicate about the separation of protons produced as a result of soft and hard processes.

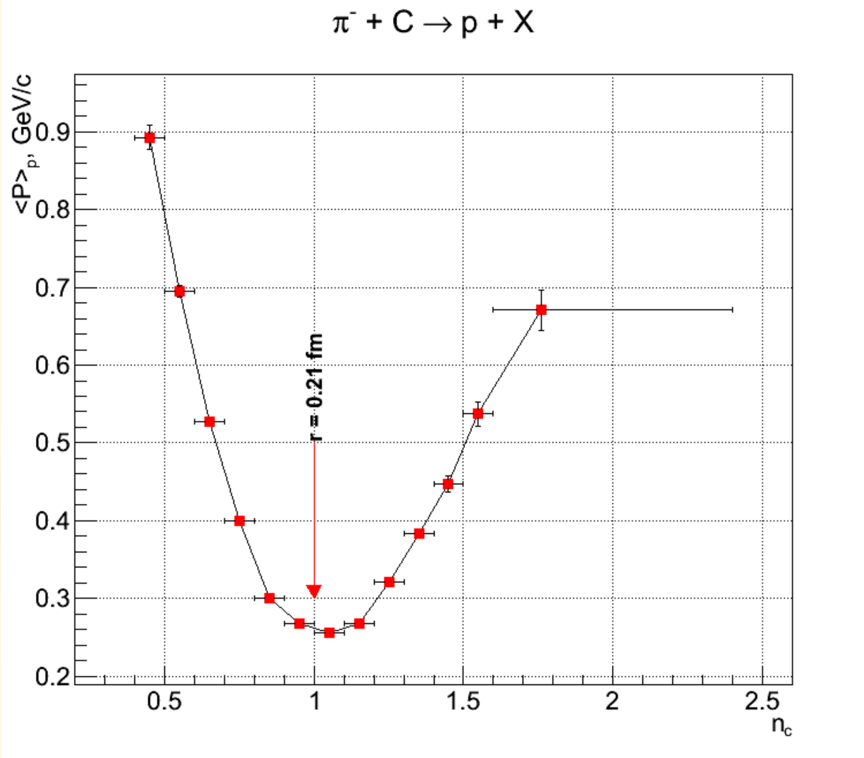



Fig. 11. The average values of the momentum of the secondary protons as a function of the variable $n_c$. This figure was taken from paper [1]

**Conclusion**

In this paper we have obtained the formula which determines the particle emission region size $r$.

$$r = \frac{1}{\sqrt{n_c} \cdot m_p} = \frac{\lambda_c^P}{\sqrt{n_c}} = \frac{0.21 fm}{\sqrt{n_c}}$$

The distribution on the parameter r calculated by the formula (8) and their average values $\sqrt{< r >^2}$ are presented for the all secondary $\pi^-$-mesons and protons and also for $\pi^-$-mesons and protons corresponding to the different phase transition regions from $\pi^-C$ interactions at 40 GeV/c.

**Aknowledgement**

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