Factorization theorem and hard semi-inclusive processes with transverse polarizations

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Dedicated to A.V. Efremov on occasion on his 80th anniversary

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Factorization theorem, in a nutshell

Factorization theorem states that the short (hard) and long (soft) distance dynamics can be separated out provided large Q^2, *i.e.*

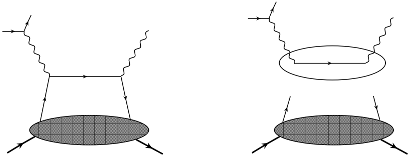
$$T_{\mu\nu} = \int d^4k \, \mathrm{tr}\big[E_{\mu\nu}(k)\Phi(k)\big] \overset{Q^2\to\infty}{\Longrightarrow} \int dx \, \mathrm{tr}\big[E_{\mu\nu}(x)\,\Phi(x)\big] + \mathcal{O}(1/Q^2)$$

where $E_{\mu\nu}$ implies the product of propagators, while

$$\Phi(k) = -\int d^4z \, e^{i(k-\Delta/2)\cdot z} \langle p'|\psi(z)\,\bar{\psi}(0)|p\rangle \,,$$

$$\Phi(x) = \int d^4k \, \delta(x - k\cdot n)\,\Phi(k)\,.$$

Schematically, this corresponds to



or, an alternative expression reads

$$\text{Amplitude} = \{\text{Hard part (pQCD)}\} \otimes \{\text{Soft part (npQCD)}\}\,,$$

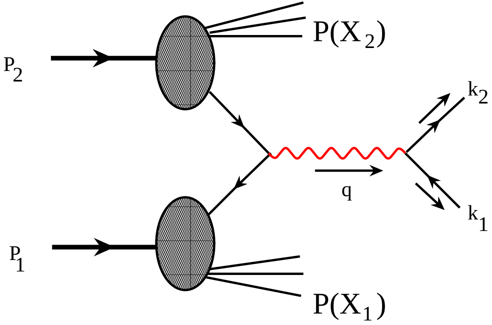
where both hard and soft parts are independent of each other, UV- and IR-renormalizable and, finally, parton distributions must possess the universality property.

Drell-Yan process

We study

$$N^{(\uparrow\downarrow)}(p_1) + N(p_2) \to \gamma^*(q) + X(P_X) \to \ell(l_1) + \bar{\ell}(l_2) + X(P_X),$$

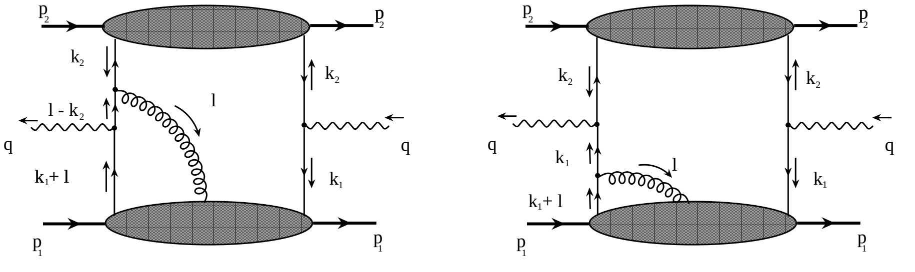
where $l_1 + l_2 = q$ has a large mass squared ($q^2 = Q^2$).



The cross-sections reads (kinematics: $p_1 \sim n^{*\,+}$, $p_2 \sim n^-$)

$$d\sigma = (dP.S.)^2\ \mathcal{L}_{\mu\nu}\mathcal{W}^{GI}_{\mu\nu}\,,$$

where $\mathcal{L}_{\mu\nu}$ is a lepton tensor, and $\mathcal{W}^{GI}_{\mu\nu}$ – the QED gauge invariant hadron tensor.



a) b)

Single Spin Asymmetry

Any SSA are defined as

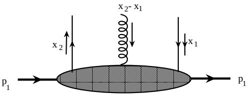
$$\text{SSA} \sim d\sigma^{(\uparrow)} - d\sigma^{(\downarrow)} \sim \mathcal{L}_{\mu\nu}\, H_{\mu\nu}\,.$$

In our case, we deal with the unpolarized leptons, *i.e.* $\mathcal{L}_{\mu\nu} \in \Re\text{e}$. Therefore, the hadron tensor $H_{\mu\nu}$ should also be real one, *i.e.* $H_{\mu\nu} \in \Re\text{e}$, provided, at the same time, one of hadrons is transversely polarized. Usually, it is possible if

$$H^{(a)}_{\mu\nu} \sim \Im\text{m}\,[\text{Hard}] \otimes \left\{ \langle p_1, S_T | \mathcal{O}(\bar{\psi}, \psi, A) | S_T, p_1 \rangle \overset{\mathcal{F}}{\sim} i\varepsilon_{\alpha\beta S_T p_1} \Phi \right\},$$

$$H^{(b)}_{\mu\nu} \sim \text{Hard} \otimes \left\{ \langle p_1, S_T | \mathcal{O}(\bar{\psi}, \psi, A) | S_T, p_1 \rangle \overset{\mathcal{F}}{\sim} i\varepsilon_{\alpha\beta S_T p_1} \Im\text{m}\,[\Phi] \right\}.$$

However, for the pure real B^V -function $\in \Re$e, which parametrizes



$$\langle p_1, S^T|\bar{\psi}(\lambda_1\tilde{n})\,\gamma_\beta\, gA^T_\alpha(\lambda_2\tilde{n})\,\psi(0)|S^T, p_1\rangle =$$
$$i\varepsilon_{\beta\alpha S^T p_1}\int dx_1\, dx_2\, e^{ix_1\lambda_1 + i(x_2 - x_1)\lambda_2}\, B^V(x_1, x_2)\,,$$

the diagram (b) does NOT contribute to the SSA.

As a result, we are faced to a problem with QED gauge invariance and, therefore, with the factorization breaking.

The inference on B^V -function $\in \Re e$ is based on the solution of the differential equation (within the gauge: $A^+ = 0$)

$$\partial^+ A_T^\alpha = G_T^{+\,\alpha}\,,$$

one can get that

$$\begin{aligned} A^\mu(z) &= \int_{-\infty}^{\infty} d\omega^- \theta(z^- - \omega^-) G^{+\mu}(\omega^-) + A^\mu(-\infty) \\ &= -\int_{-\infty}^{\infty} d\omega^- \theta(\omega^- - z^-) G^{+\mu}(\omega^-) + A^\mu(\infty)\,. \end{aligned}$$

Inserting the above-mentioned presentations into the corresponding m.e., we thus obtain that

$$\Phi^{\alpha}_{A}(x_1, x_2) = \delta(x_1 - x_2)\Phi^{\alpha}_{A(-\infty)}(x_1) + \frac{(-i)\,\Phi^{\alpha}_{G}(x_1, x_2)}{x_2 - x_1 - i\epsilon}\,,$$

and

$$\Phi^{\alpha}_{A}(x_1, x_2) = \delta(x_1 - x_2)\Phi^{\alpha}_{A(+\infty)}(x_1) + \frac{(-i)\,\Phi^{\alpha}_{G}(x_1, x_2)}{x_2 - x_1 + i\epsilon}\,.$$

Calculation the plus and minus combinations leads to

$$\Phi^{\alpha}_{A}(x_1, x_2) = \frac{1}{2}\Phi^{\alpha}_{A}(x_1, x_2) + \frac{1}{2}\Phi^{\alpha}_{A}(x_1, x_2) =$$
$$\frac{1}{2}\delta(x_1 - x_2)\Big\{\Phi^{\alpha}_{A(-\infty)}(x_1) + \Phi^{\alpha}_{A(+\infty)}(x_1)\Big\} +$$
$$\frac{\mathcal{P}}{x_2 - x_1}(-i)\Phi^{\alpha}_{G}(x_1, x_2)$$

and

$$0 = \Phi^{\alpha}_{A}(x_1, x_2) - \Phi^{\alpha}_{A}(x_1, x_2) =$$
$$\delta(x_1 - x_2)\Big\{\Phi^{\alpha}_{A(+\infty)}(x_1) - \Phi^{\alpha}_{A(-\infty)}(x_1)\Big\} -$$
$$2i\,\pi\,\delta(x_1 - x_2)(-i)\Phi^{\alpha}_{G}(x_1, x_2)\,.$$

So, this ambiguity ultimately gives us the standard-used representation:

$$B^V(x_1, x_2) = \frac{\mathcal{P}}{x_1 - x_2} T(x_1, x_2)\,,$$

$$T(x_1, x_2) \stackrel{\mathcal{F}}{\sim} \langle \bar{\psi}\, \gamma_\beta\, \tilde{n}_\nu G_{\nu\alpha}\, \psi \rangle \qquad T(x, x) \neq 0\,.$$

provided the asymmetric boundary condition for gluons:

$$B^V_{A(\infty)}(x) = -B^V_{A(-\infty)}(x)$$

Thus, for the considered DY, a pure real $B^V(x_1, x_2)$ will lead to the problem with QED gauge invariance which means factorization breaking.

Way of solution

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Actually, the B^V-function is not the real one. Indeed,

the sign of $i\epsilon$ in the propagator of the h. p. $\Longrightarrow$
θ-function in the gluon field repres. $\Longrightarrow$
the contour gauge for gluons which demands

$$g(x) \equiv [x, x_0] = Pexp\Big\{ ig \int\limits_{\mathbb{P}(x_0,x)} d\omega \cdot A(\omega) \Big\} = 1 \; (\forall x \in \mathbb{R}^4),$$

where the final point at the minus infinity changes the fixed “startingpoint x_0, whereas the point z changes the point x.

For an arbitrary path connecting z and $-\infty$, one has

$$A^{\rm ax}_\mu(z) = [z,-\infty]^{-1}\,A_\mu(z)\,[z,-\infty] + \frac{i}{g}[z,-\infty]^{-1}\partial_\mu[z,-\infty]$$
$$= \int\limits_{-\infty}^{z} d\omega_\alpha \frac{\partial\omega_\beta}{\partial z_\mu}\,G_{\alpha\beta}(\omega) + A_\mu(-\infty)\,.$$

Choosing now the path in the form of the straight line:

$$\omega_\alpha(v)\Big|_z^{-\infty} = z_\alpha - \tilde{n}_\alpha \lim_{\epsilon\to 0}\frac{1-e^{-\epsilon v}}{\epsilon}\Big|_0^{\infty},$$

we arrive at

$$A^\mu(z) = \int\limits_{-\infty}^{\infty} d\omega^-\theta(z^- - \omega^-)G^{+\mu}(\omega^-) + A^\mu(-\infty)\,.$$

Roughly speaking, the hard and soft parts are NOT fully independent:

$$\text{Causal Prescrip. Hard Part} \stackrel{\text{C. g.}}{\Longleftrightarrow} \text{Pole Prescrip. Soft Part}$$

Despite of this, factorization still works owing to the universal property of *B*-function.

All these fix (due to the T-reversal inv-ce, $B^V_{A(-\infty)}(x) = 0$)

$$B^V(x_1, x_2) = \frac{T(x_1, x_2)}{x_1 - x_2 + i\epsilon} + \delta(x_1 - x_2) B^V_{A(-\infty)}(x_1)\,,$$

which leads to the non-zero contribution from the diagram (b).

Conclusions for DY:

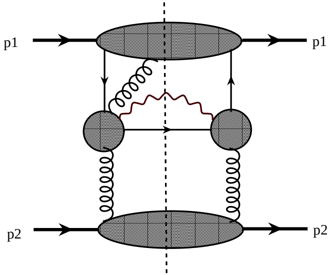
$$\textbf{ISI} \Rightarrow \frac{1}{\ell^+ - i\epsilon} \Rightarrow [z^-, -\infty^-] \Rightarrow \text{b.c. } A_\mu(-\infty) \Rightarrow \frac{T(x_1, x_2)}{x_1 - x_2 + i\epsilon} \Rightarrow \textbf{GI}$$

Direct Photon Production in hadron collisions

We now dwell on the direct photon production in two hadron collisions:

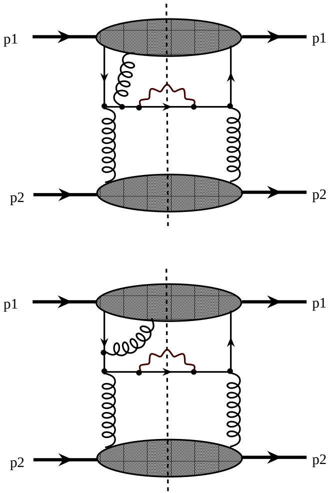
$$N^{(\uparrow\downarrow)}(p_1) + N(p_2) \to \gamma(q) + X(P_X)\,.$$

where $q^2 = Q^2$ is relatively large. The cross-section $d\sigma$ is defined by the hadron tensor as



QCD gauge invariance

To study the QCD gauge invariance, we consider the following diagrams:



The quark-gluon correlator reads

$$\Phi^{\perp}_{\rho}(k_1,\ell) = -\int (d^4\eta_1\, d^4 z) e^{-ik_1\eta_1 - i\ell z} \langle p_1|\bar{\psi}(0)\gamma^+\psi(\eta_1)A^{\perp}_{\rho}(z)|p_1\rangle$$
$$= -\varepsilon^{\perp}_{\rho}\int (d^4\eta_1) e^{-ik_1\eta_1} \langle p_1|\bar{\psi}(0)\gamma^+\psi(\eta_1)a^+(\ell)|p_1\rangle\,.$$

Factorization procedure gives us

$$\Phi^{\perp}_{\rho}(x_1,x_2) = \int (d^4k_1\, d^4\ell)\delta(x_1 - k_1 n)\delta(x_{21} - \ell n)\Phi^{\perp}_{\rho}(k_1,\ell) =$$
$$-\varepsilon^{\perp}_{\rho}\int (d\lambda_1) e^{-ix_1\lambda_1} \langle p_1|\bar{\psi}(0)\gamma^+\psi(\lambda_1 n) \int (d^4\ell)\delta(x_{21} - \ell n)a^+(\ell)|p_1\rangle\,.$$

- For checking of the QCD gauge invariance, we make a replacement: $\hat{\varepsilon}^{\perp} \Rightarrow \hat{\ell}_L$ in the diagrams.

Initial and Final states interactions

In the process we consider, we have both ISI and FSI:

$$\textbf{ISI} \Rightarrow \frac{1}{\ell^+ - i\epsilon} \Rightarrow [z^-, -\infty^-] \Rightarrow \text{b.c. } A_\mu(-\infty) \Rightarrow \frac{T(x_1, x_2)}{x_1 - x_2 + i\epsilon}$$

$$\textbf{FSI} \Rightarrow \frac{1}{\ell^+ + i\epsilon} \Rightarrow [+\infty^-, z^-] \Rightarrow \text{b.c. } A_\mu(+\infty) \Rightarrow \frac{T(x_1, x_2)}{x_1 - x_2 - i\epsilon}$$

QCD gauge invariance: final stage

$$\overline{W^{(1)}} \sim \mathbf{C}_2 \frac{1}{x_1} \int dx_2 \, \frac{x_2 - x_1}{x_2} \, \frac{T(x_1, x_2)}{x_1 - x_2 - i\epsilon} \, ,$$

$$\overline{W^{(2)}} \sim \mathbf{C}_2 \frac{1}{x_1} \int dx_2 \, \frac{1}{x_2} \, \frac{T(x_1, x_2)}{x_1 - x_2 - i\epsilon} \, ,$$

$$\overline{W^{(3)}} \sim \mathbf{C}_1 \frac{1}{x_1^2} \int dx_2 \, \frac{T(x_1, x_2)}{x_1 - x_2 + i\epsilon} \, ,$$

$$\overline{W^{(4)}} \sim \mathbf{C}_3 \frac{1}{x_1^2} \int dx_2 \, \frac{T(x_1, x_2)}{x_1 - x_2 + i\epsilon} \, ,$$

where $\mathbf{C}_i$ are corresponding colour factors. After calculation of imaginary parts, we get

$$\boxed{+\mathbf{C}_2 - \mathbf{C}_1 - \mathbf{C}_3 = -[t^a, t^b]\, t^b\, t^a - if^{abc}\, t^c\, t^a\, t^b = 0}$$

Conclusions and Discussions

- Drell-Yan process: (i) It is mandatory to include a contribution of the extra diagram which naively does not have an imaginary part; (ii) This additional contribution emanates from the complex gluonic pole prescription in the representation of the twist 3 correlator $B^V(x_1, x_2)$ which, in its turn, is directly related to the complex pole prescription in the quark propagator forming the hard part of the corresponding hadron tensor; (iii)The causal prescription in the quark propagator, involved in the hard part of the diagram on Fig.(a), selects from the physical axial gauges the contour gauge.

- Direct Photon Production: In contact to DY, this process includes both **ISI** and **FSI** that leads to the different gluonic pole prescriptions in the diagrams under our consideration; (ii) In turn, the different gluonic pole prescriptions ensure the QCD gauge invariance.
- We observed the universality breaking, which spoils the standard factorization. However, the factorization procedure we proposed can still be applied for calculations.