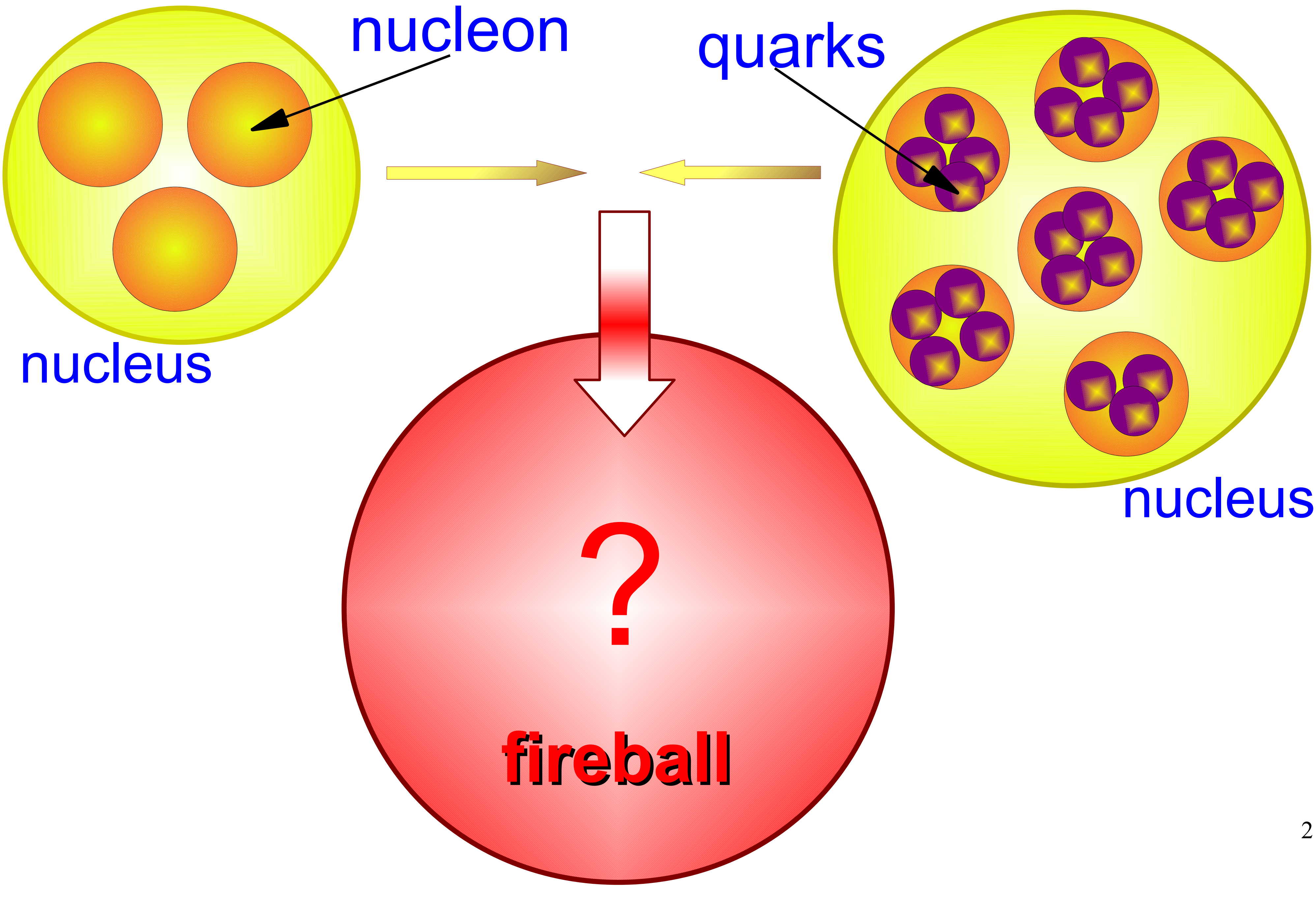
Models of Mixed Quark-Hadron Matter

V.I. Yukalov and E.P. Yukalova

Joint Institute for Nuclear Research

Dubna, 141980 Russia

Nuclear or Heavy – Ion Collisions



Local Equilibrium

Experimental fireball lifetime

$$t_{\exp} \sim 10^{-22}\,\mathrm{s}$$

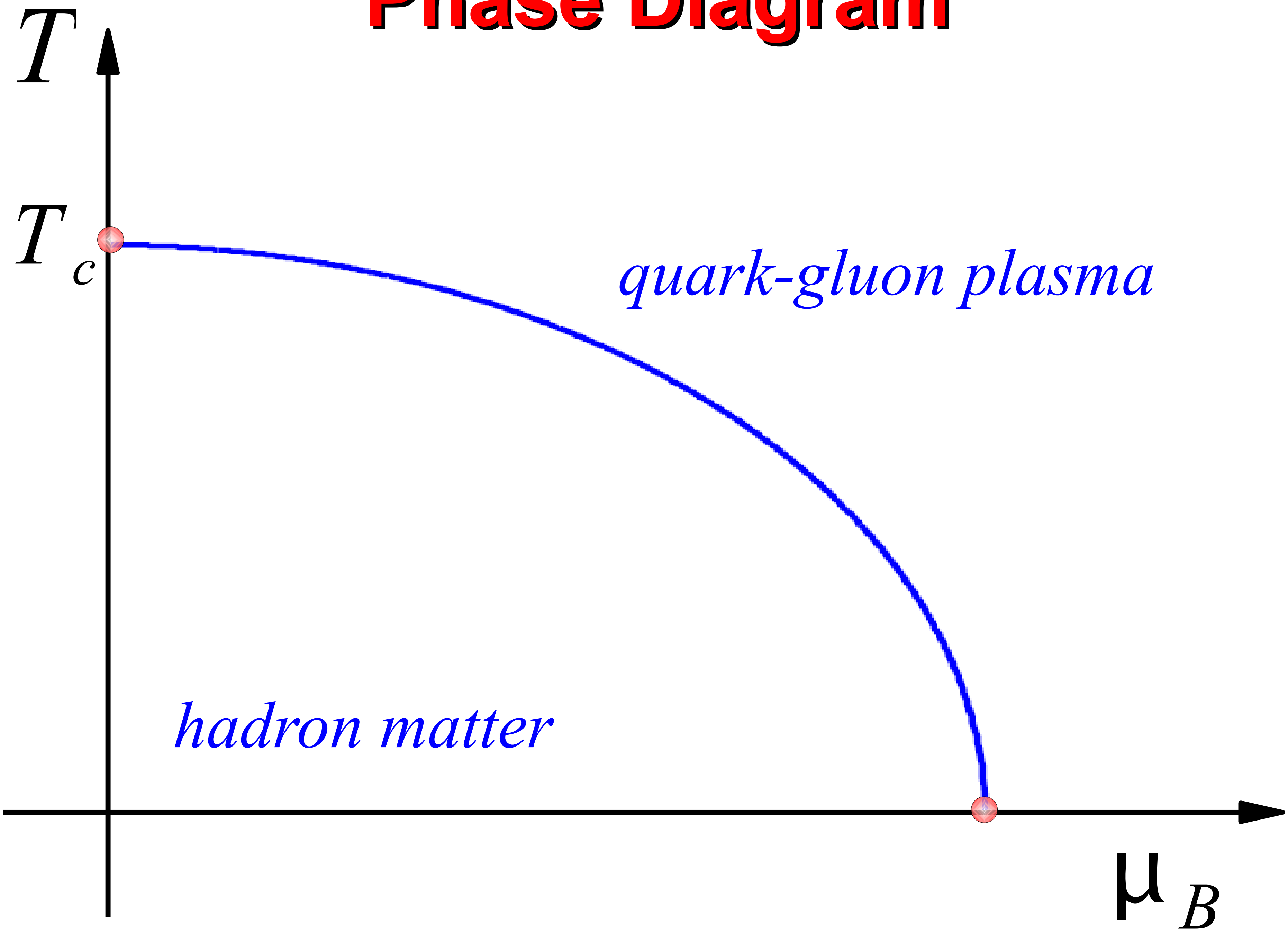
Local equilibration time

$$t_{loc} \sim 10^{-23}\,\mathrm{s}$$

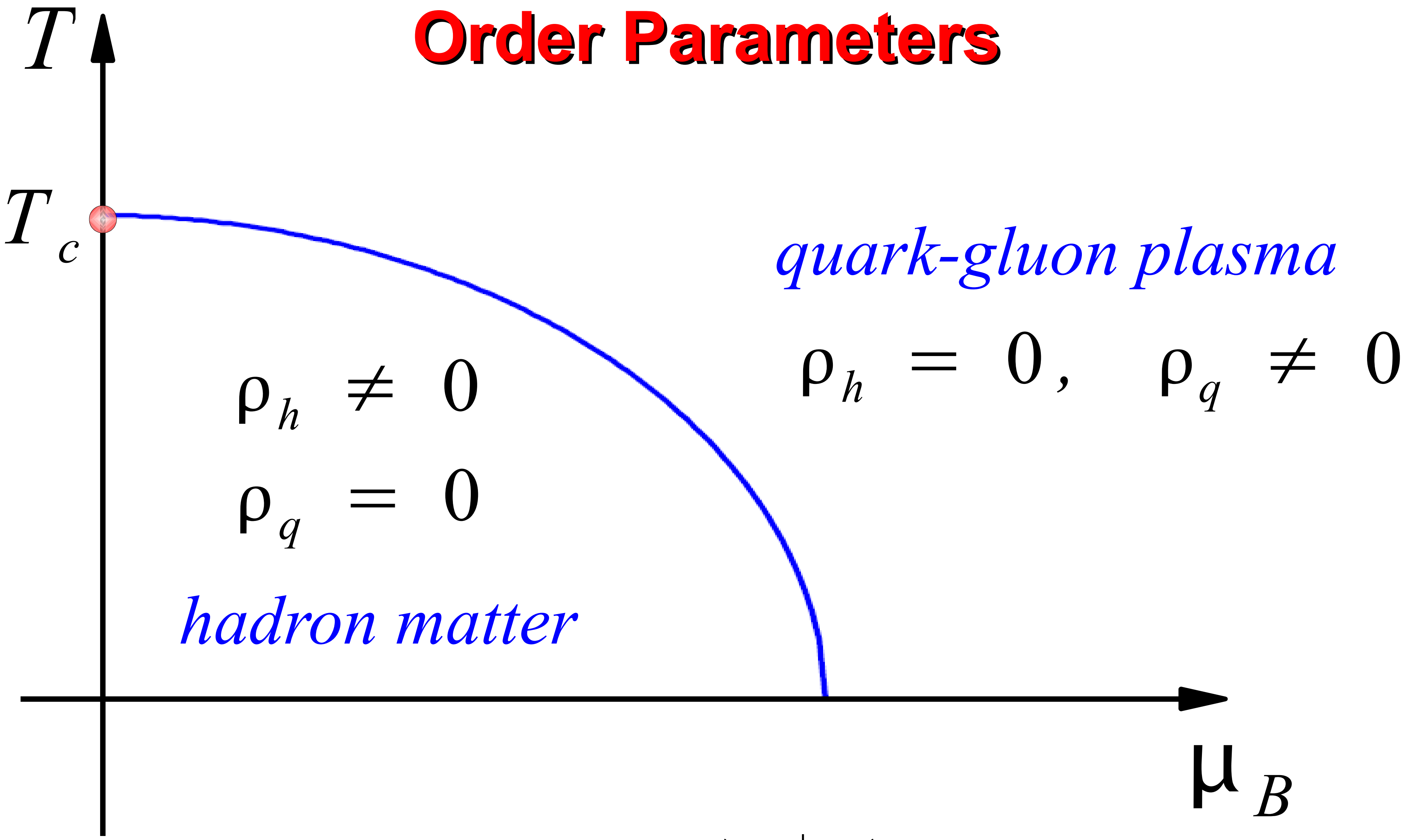
Equilibration feasible

$$t_{loc} \ll t_{\exp}$$

Phase Diagram



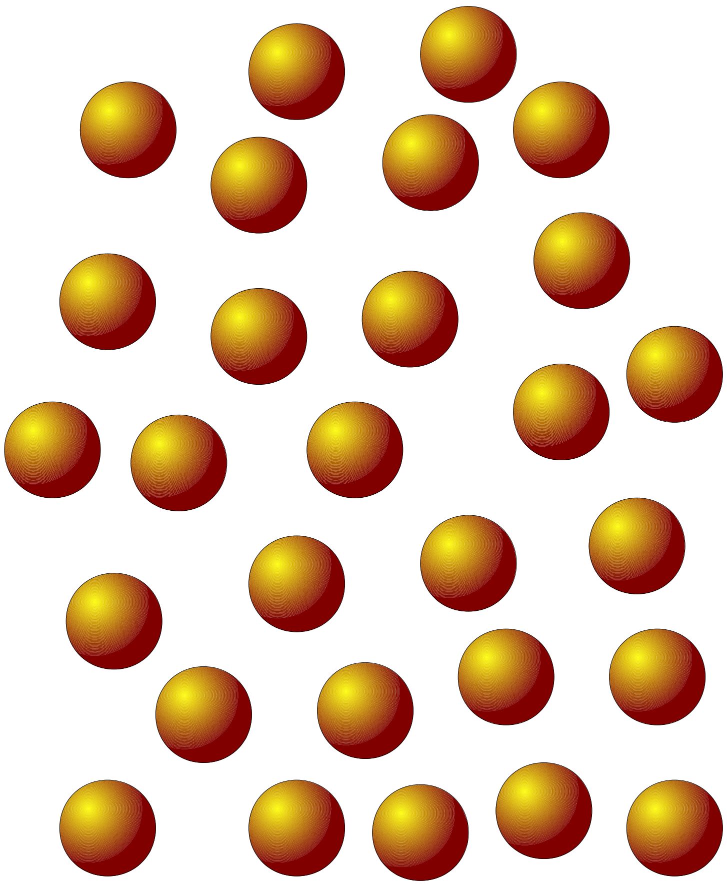
Order Parameters



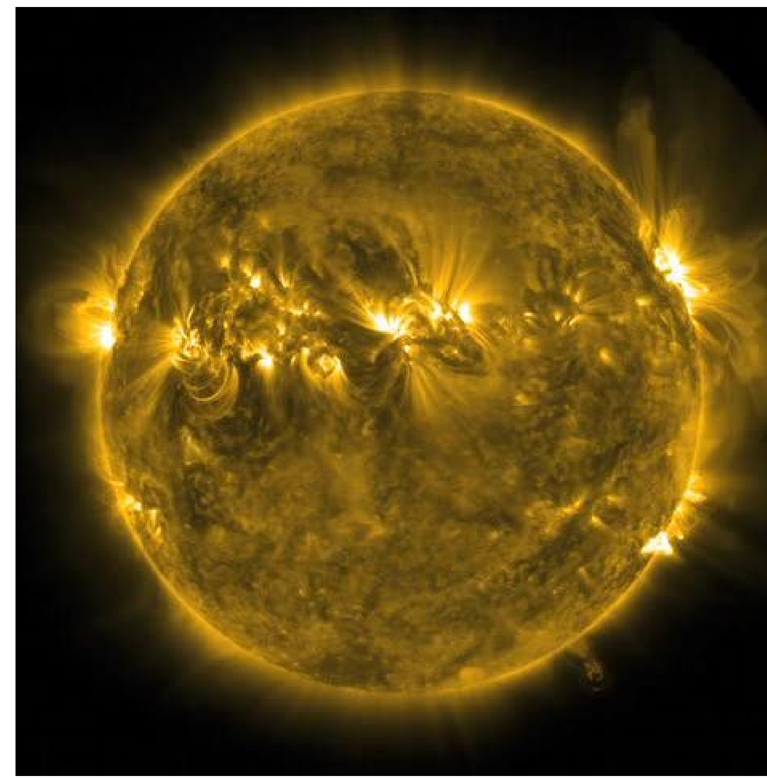
hadron density: $\rho_h = \langle h^+ h \rangle$

quark density: $\rho_q = \langle q^+ q \rangle$

Mixed Phase



quark-gluon plasma



?

mixed
phase

hadron matter

$\rho\ ,\ T$

Thermodynamic variables

Particle density ρ_i

Pressure P

Barion density

$$\rho_B = \sum_i B_i \rho_i = \frac{\partial P}{\partial \mu_B}$$

Strangeness density

$$\rho_S = \sum_i S_i \rho_i = \frac{\partial P}{\partial \mu_S}$$

Chemical potential

$$\mu_i = \mu_B B_i + \mu_S S_i$$

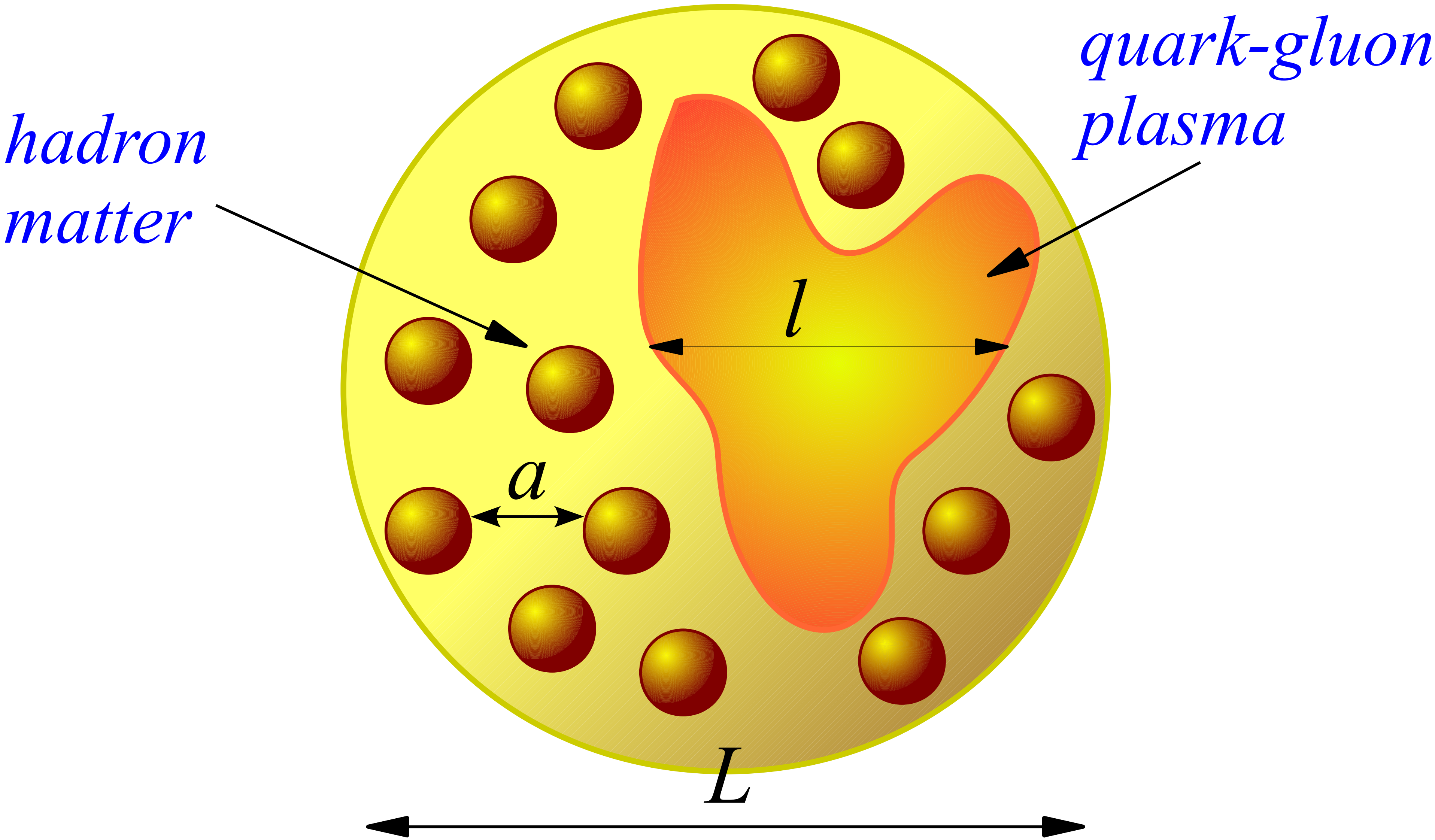
Naive Picture

Gas bubble

Water

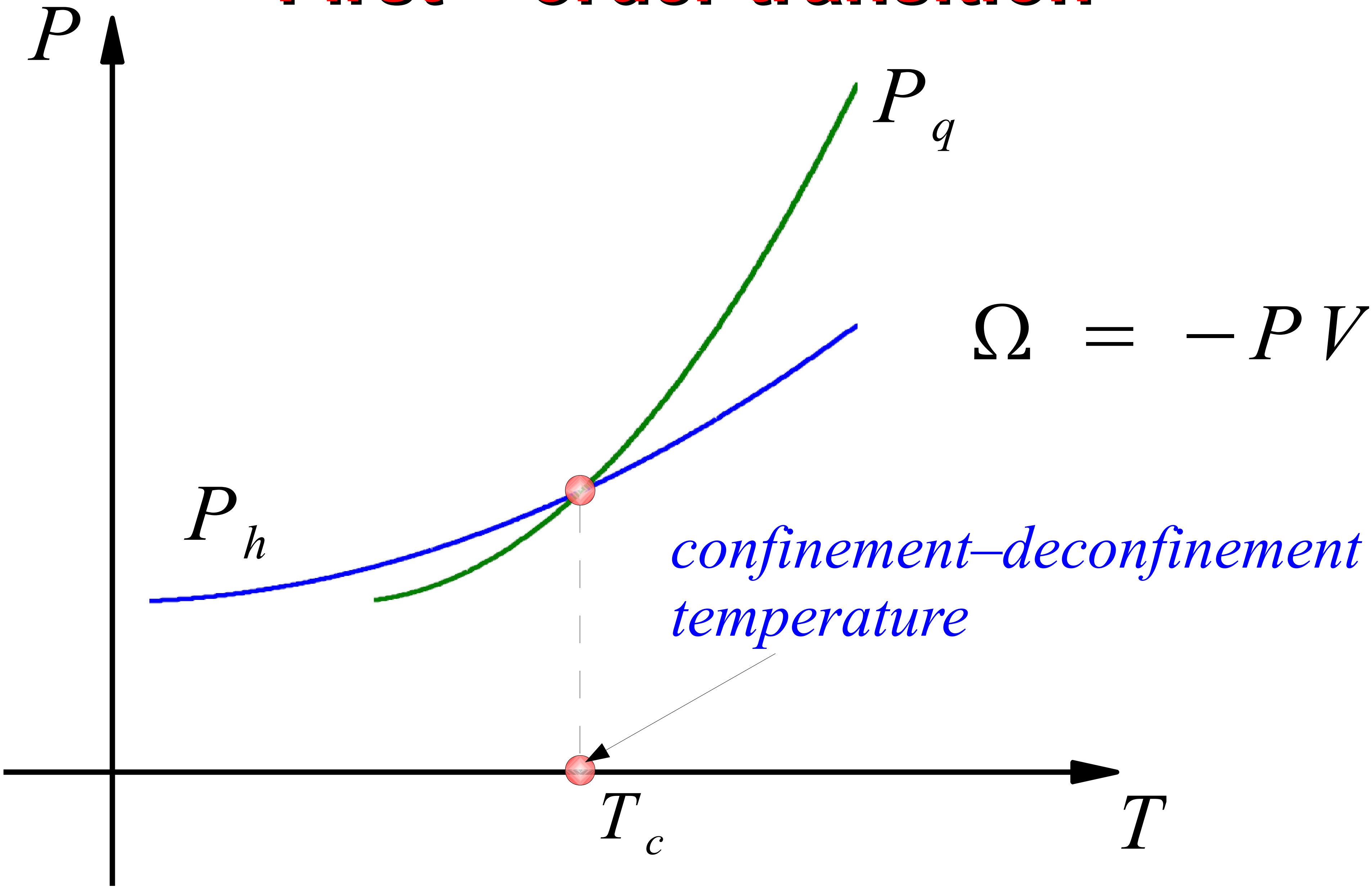
Adiabatic heating

Macroscopis Mixture



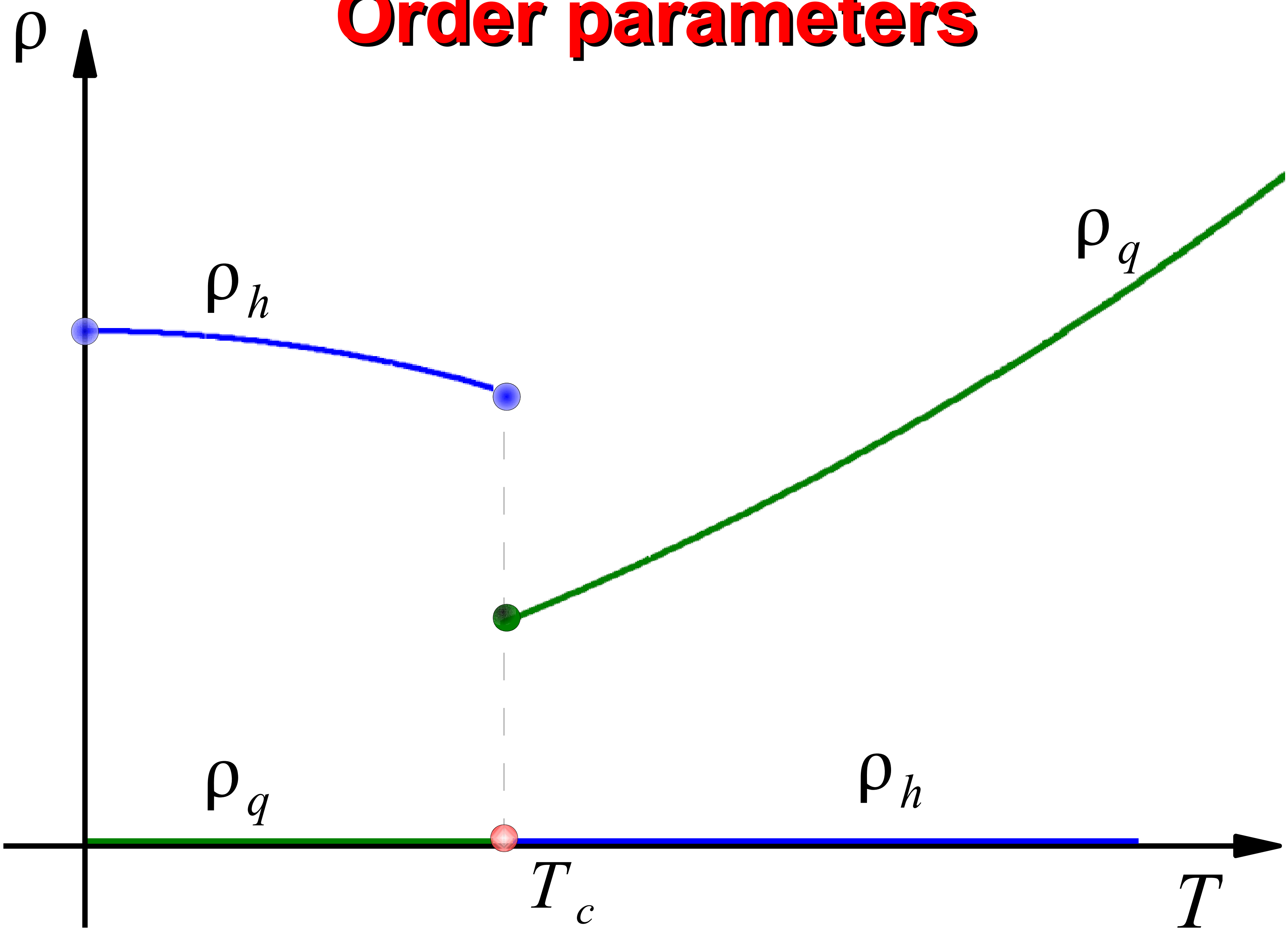
$$a \ll l \sim L$$

First – order transition



$$\Omega = -PV$$

Order parameters



Fixed strangennes

Fixed ρ_S

$$P = P(T, \mu_B)$$

$$\rho_B = \frac{\partial P}{\partial \mu_B} = \rho_B(T, \mu_B)$$

$$\mu_B = \mu_B(T, \rho_B)$$

Transition temperature

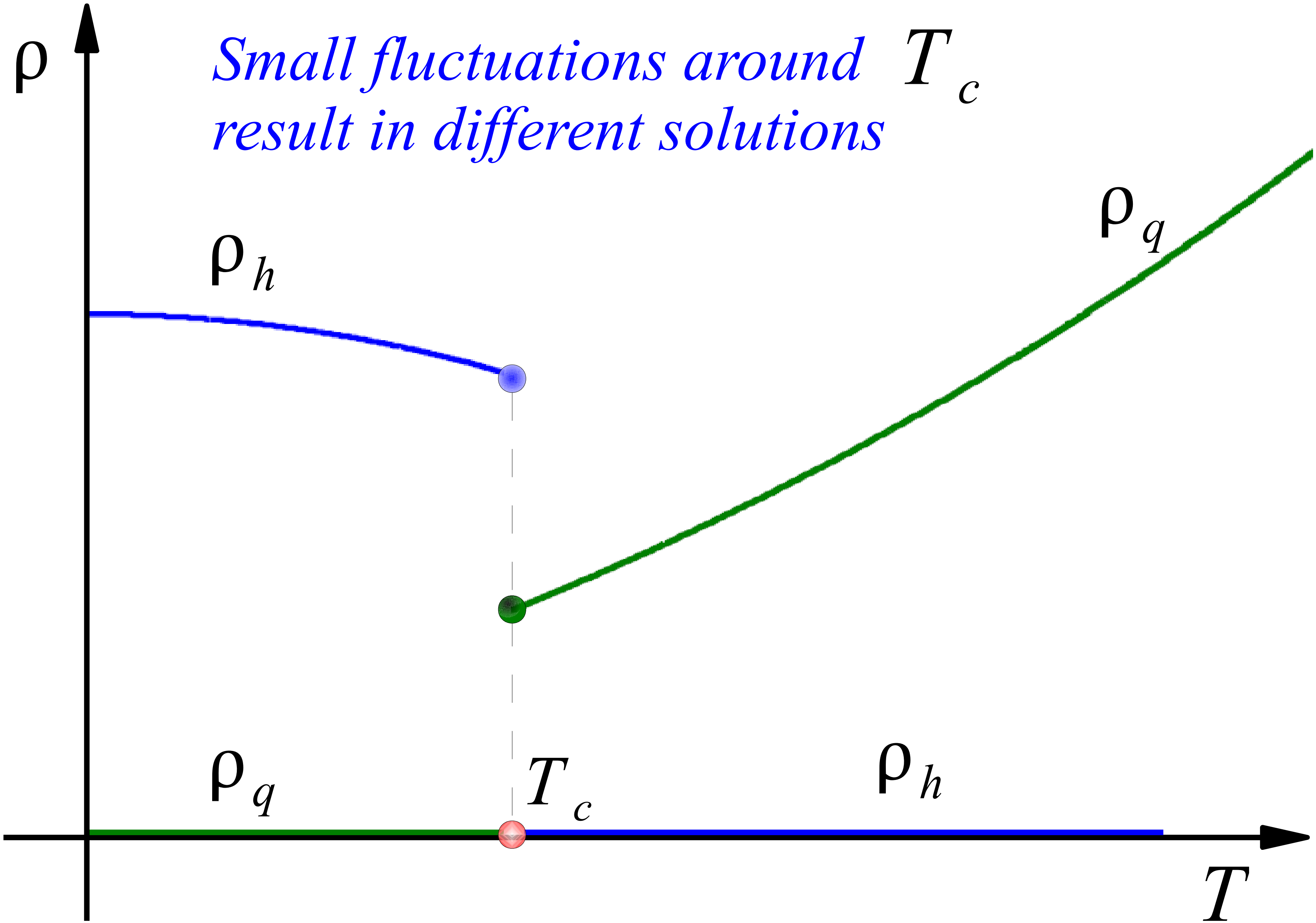
$$P_h(T_c, \mu_B) = P_q(T_c, \mu_B), \qquad T_c = T_c(\mu_B)$$

$$\rho_{Bh} = \frac{\partial P_h}{\partial \mu_B} = \rho_{Bh}(T, \mu_B), \qquad \mu_B = \mu_h(T, \rho_{Bh})$$

$$\rho_{Bq} = \frac{\partial P_q}{\partial \mu_B} = \rho_{Bq}(T, \mu_B), \qquad \mu_B = \mu_q(T, \rho_{Bq})$$

$$\mu_h(T_c, \rho_{Bh}) = \mu_q(T_c, \rho_{Bq}), \qquad T_c = T_c(\rho_{Bh}, \rho_{Bq})$$

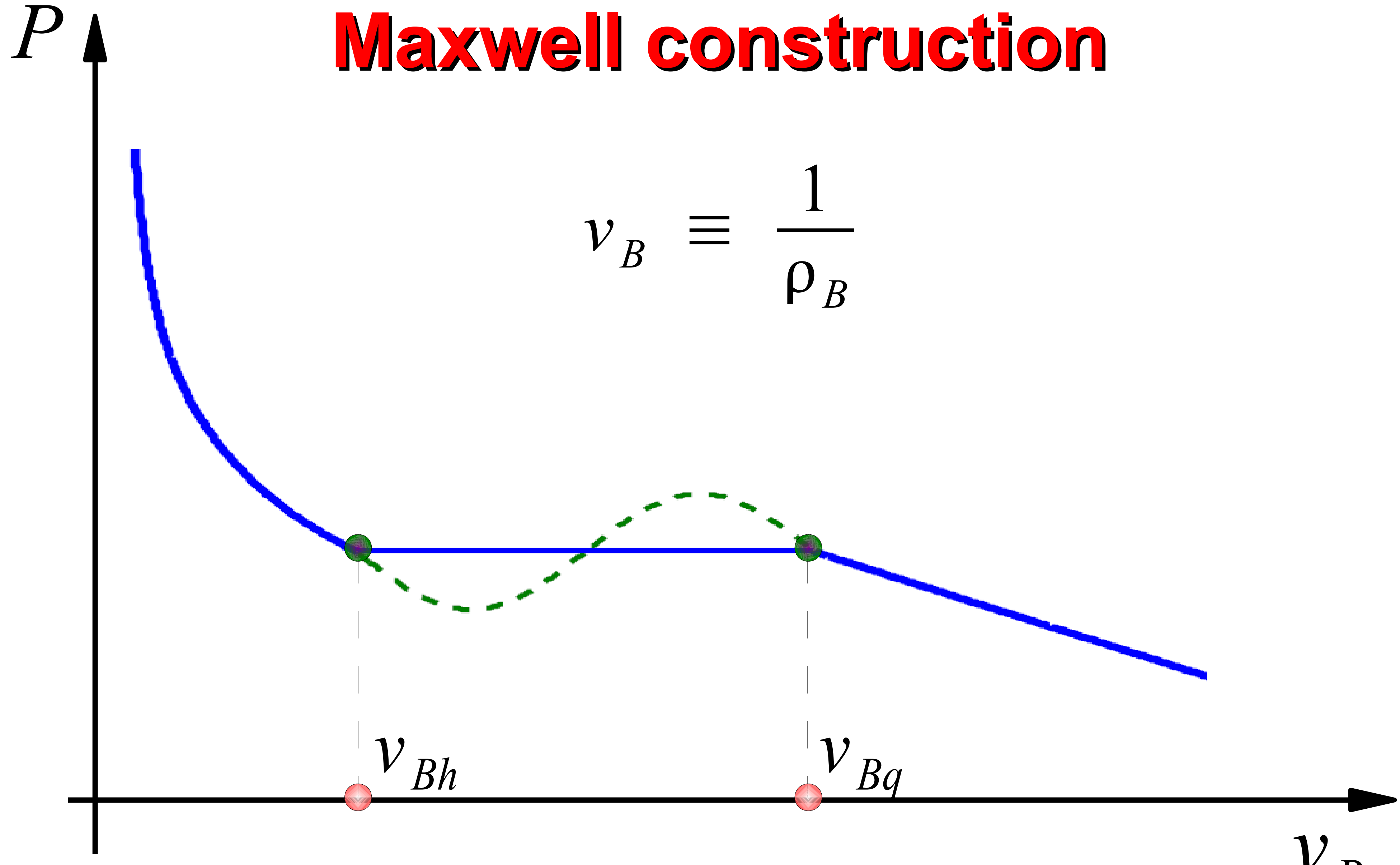
Phase – transition point is the point of instability



Under first – order deconfinement transition, mixed quark – hadron phase exists only at T_c *, where it is unstable*

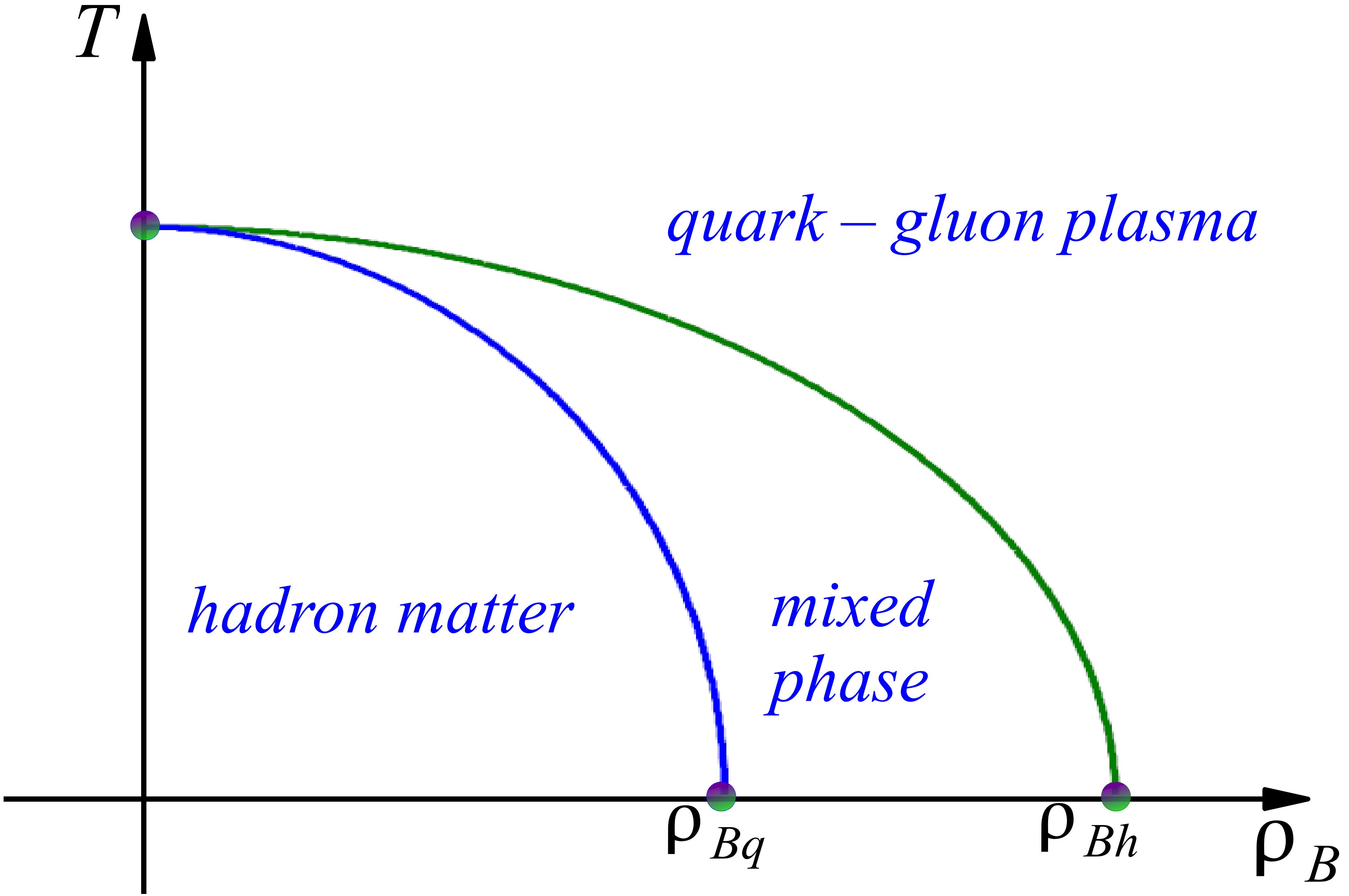
Maxwell construction

$$v_B \equiv \frac{1}{\rho_B}$$

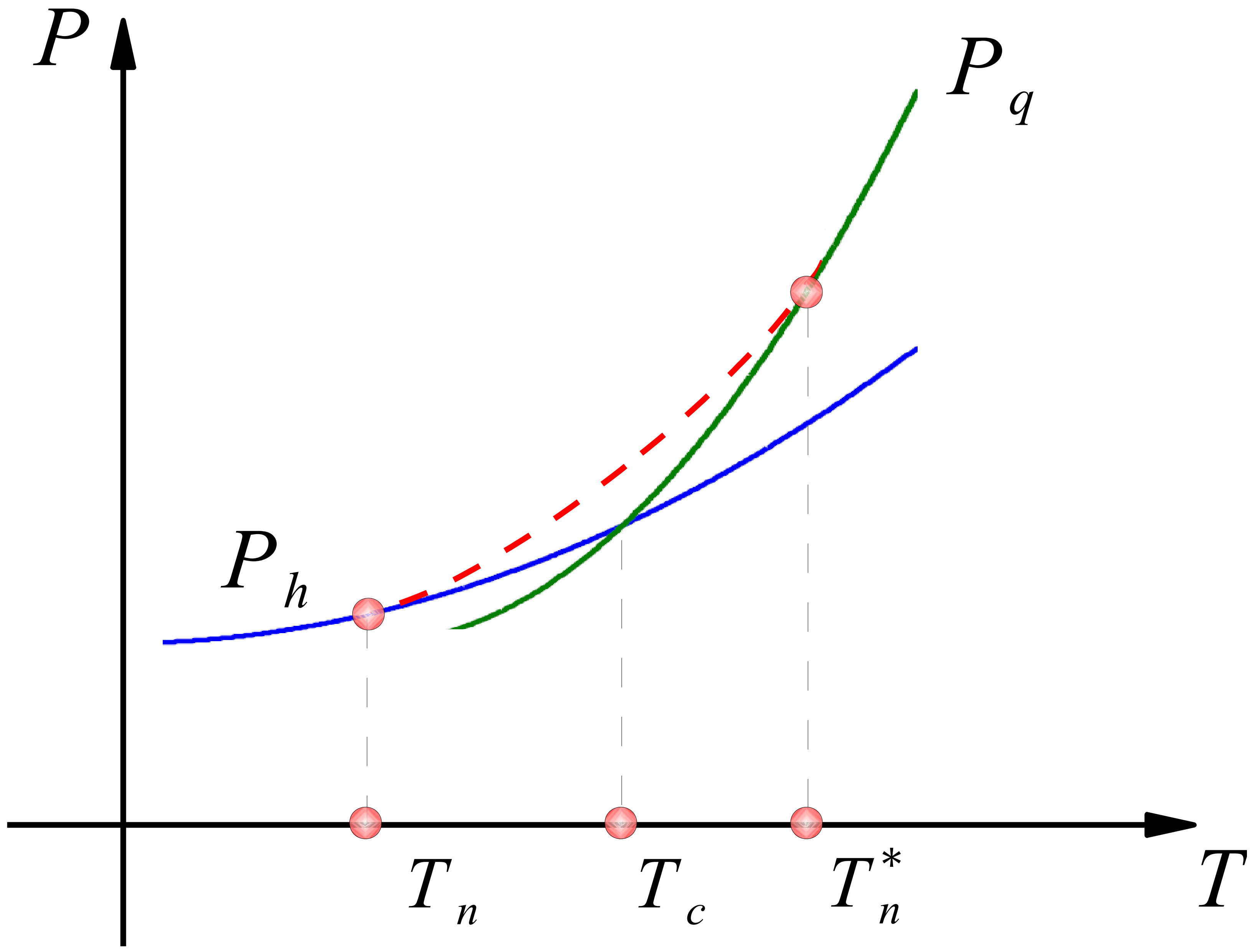


$$\frac{\partial P}{\partial v_B} = 0 \qquad (v_{Bh} < v_B < v_{Bq})$$

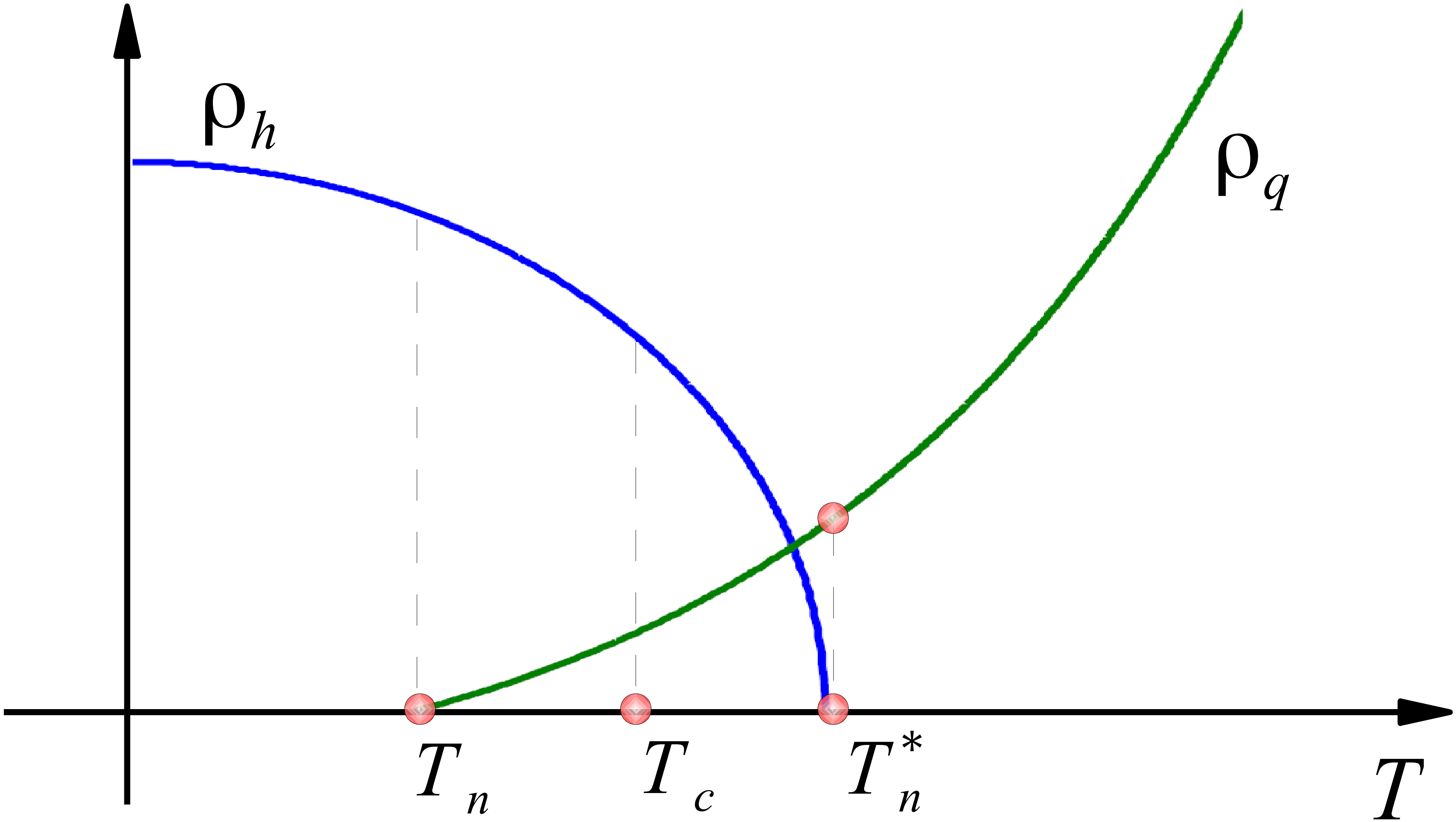
Mixed – phase region



Smoothed pressure



Extended order parameters



Mixed phase exists

$$T_n < T < T_n^*$$

Mixed quark – hadron phase is unstable

On coexistence line

$$\frac{\partial P}{\partial v_B} = 0$$

Compressibility

$$\kappa_T = -\frac{1}{v_B}\left(\frac{\partial P}{\partial v_B}\right)^{-1}$$

$$\kappa_T \to \infty \qquad (T_n < T < T_n^*)$$

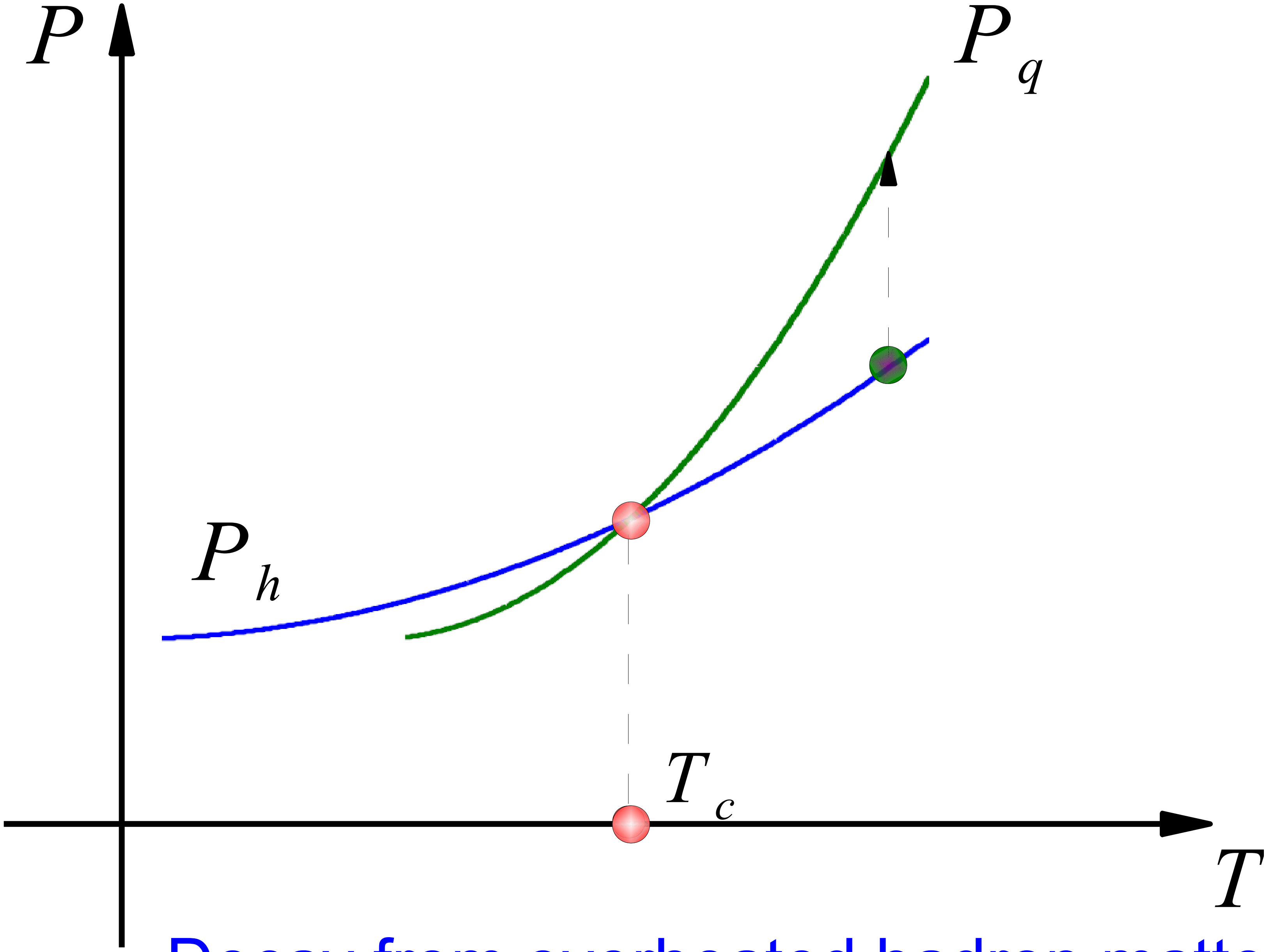
Arbitrary weak pressure fluctuations cause the system explosion

Macroscopic equilibrium mixed quark – hadron state is unstable !

Lifetime

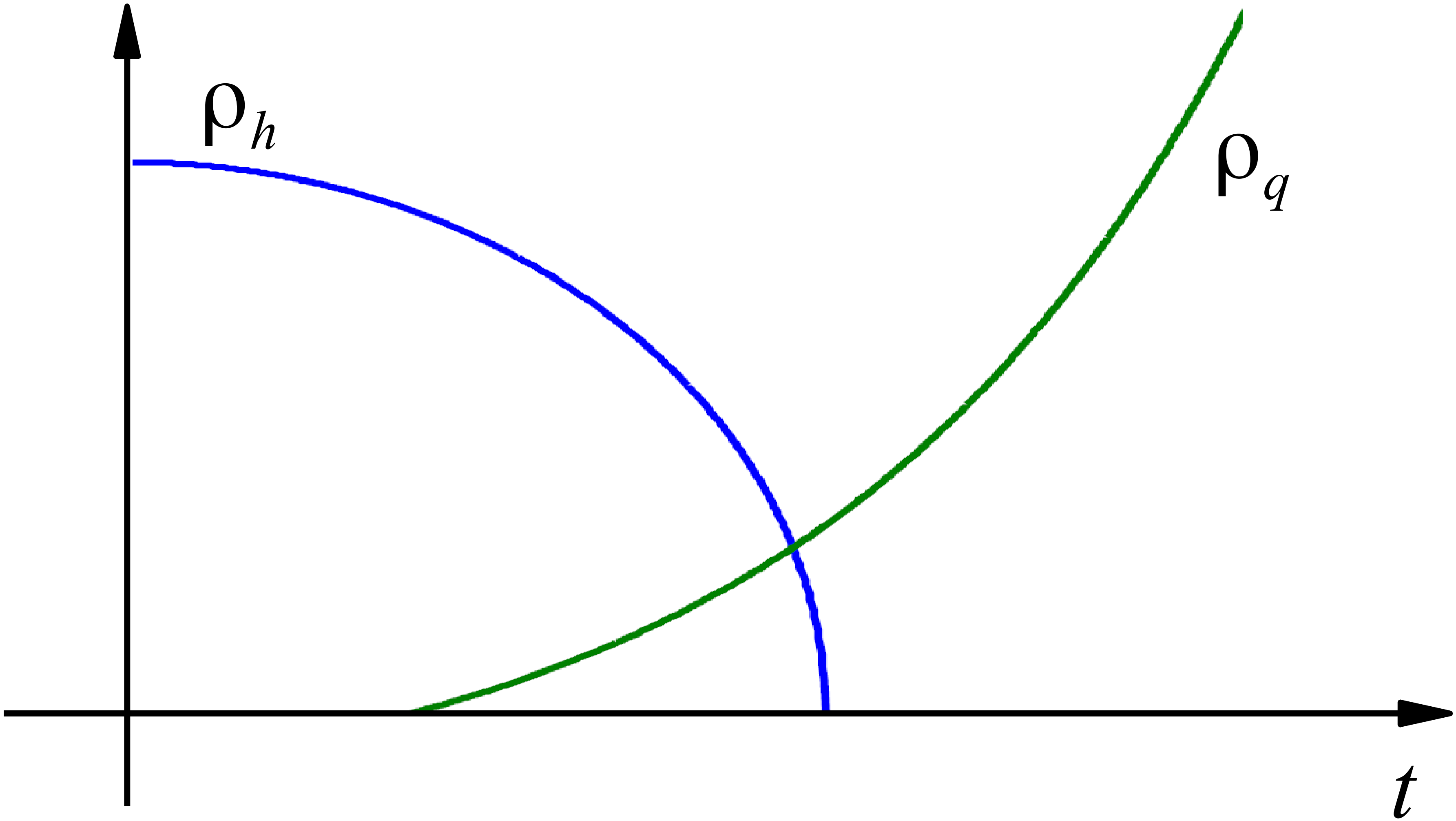
$$t_{mix} \sim \frac{1}{\kappa_T} \rightarrow 0$$

Nonequilibrium quark – hadron phase



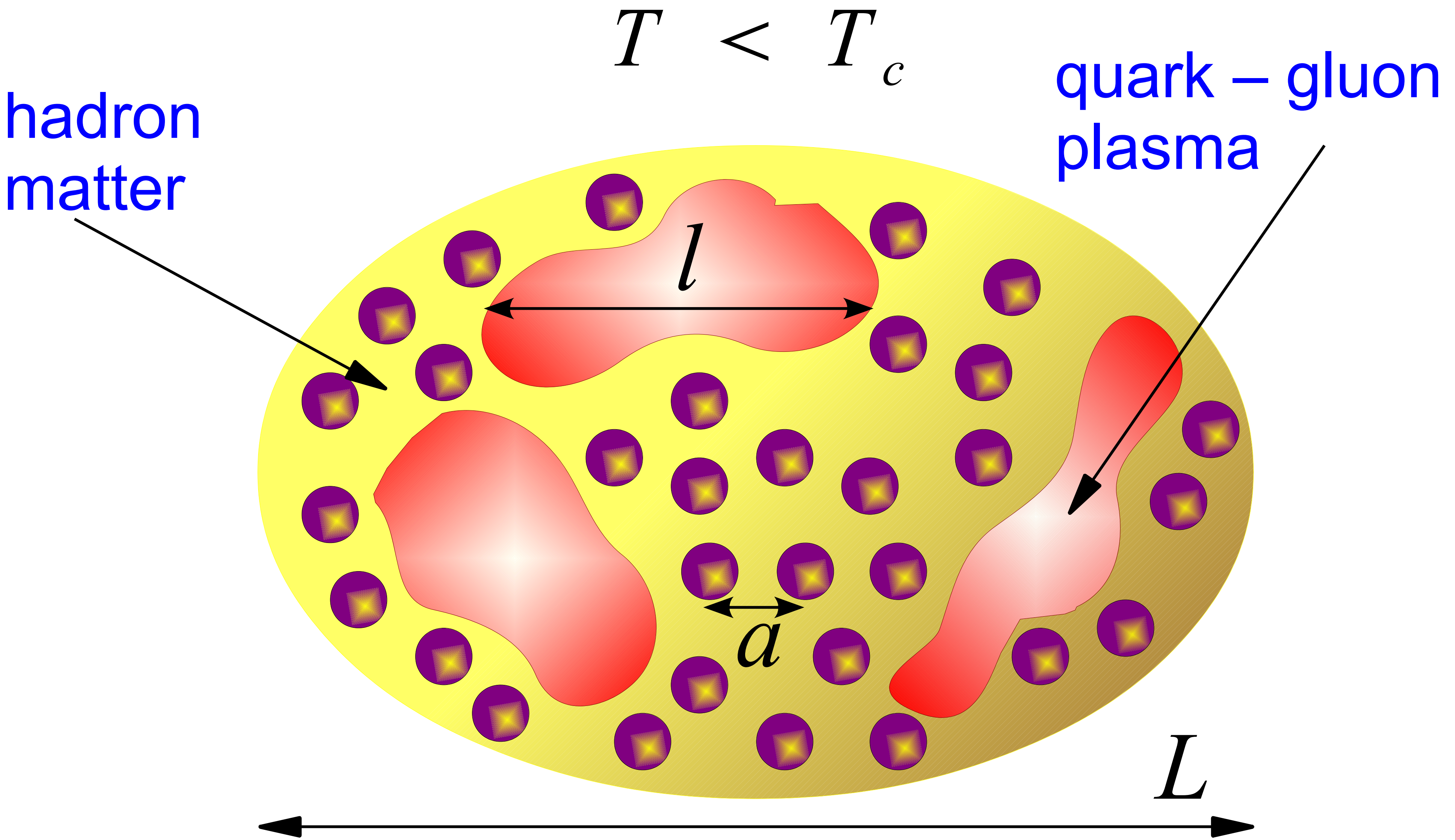
Decay from overheated hadron matter

Temporal mixture

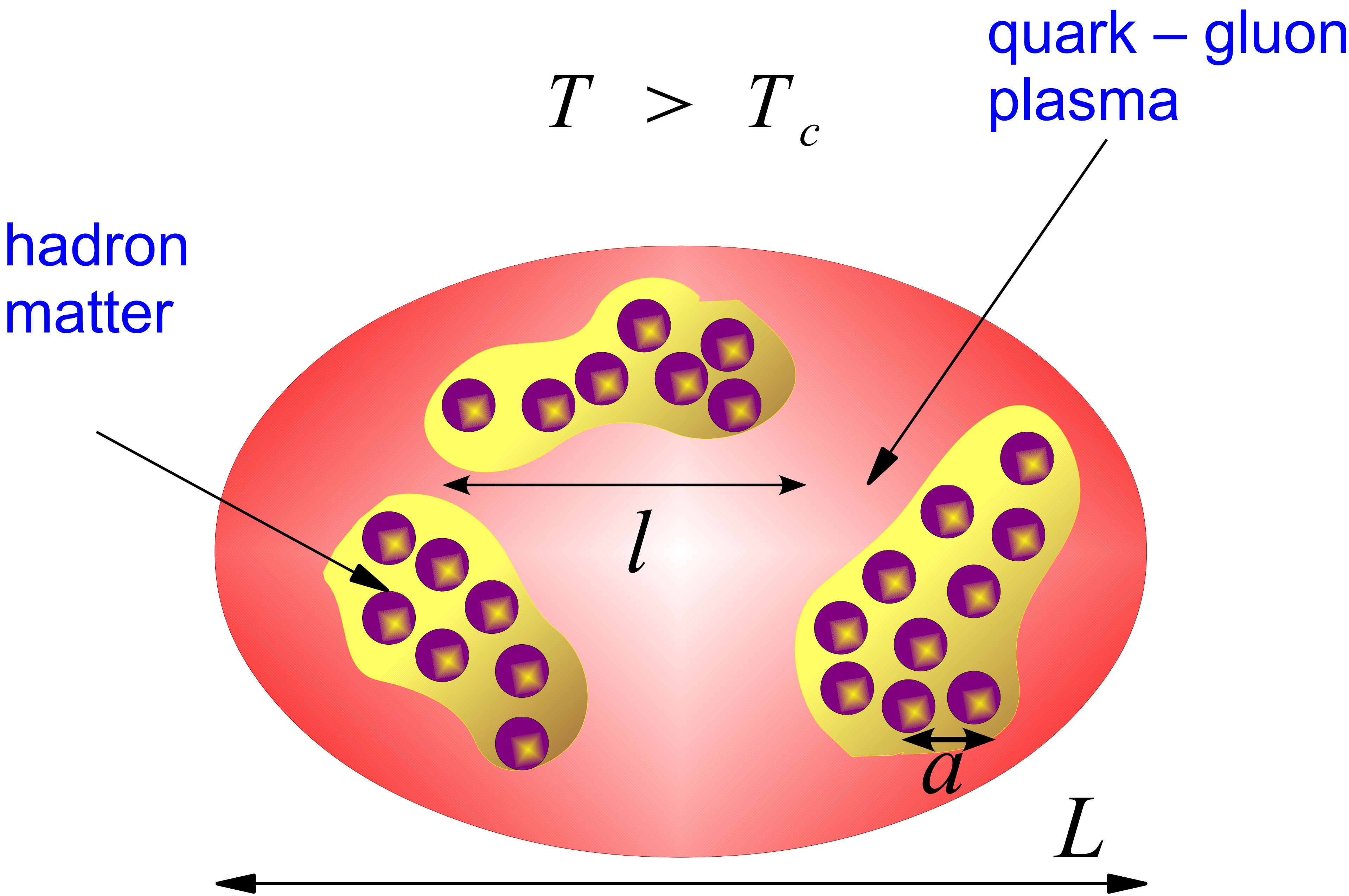


Needs nonequilibrium description

Mesoscopic mixed quark – hadron phase

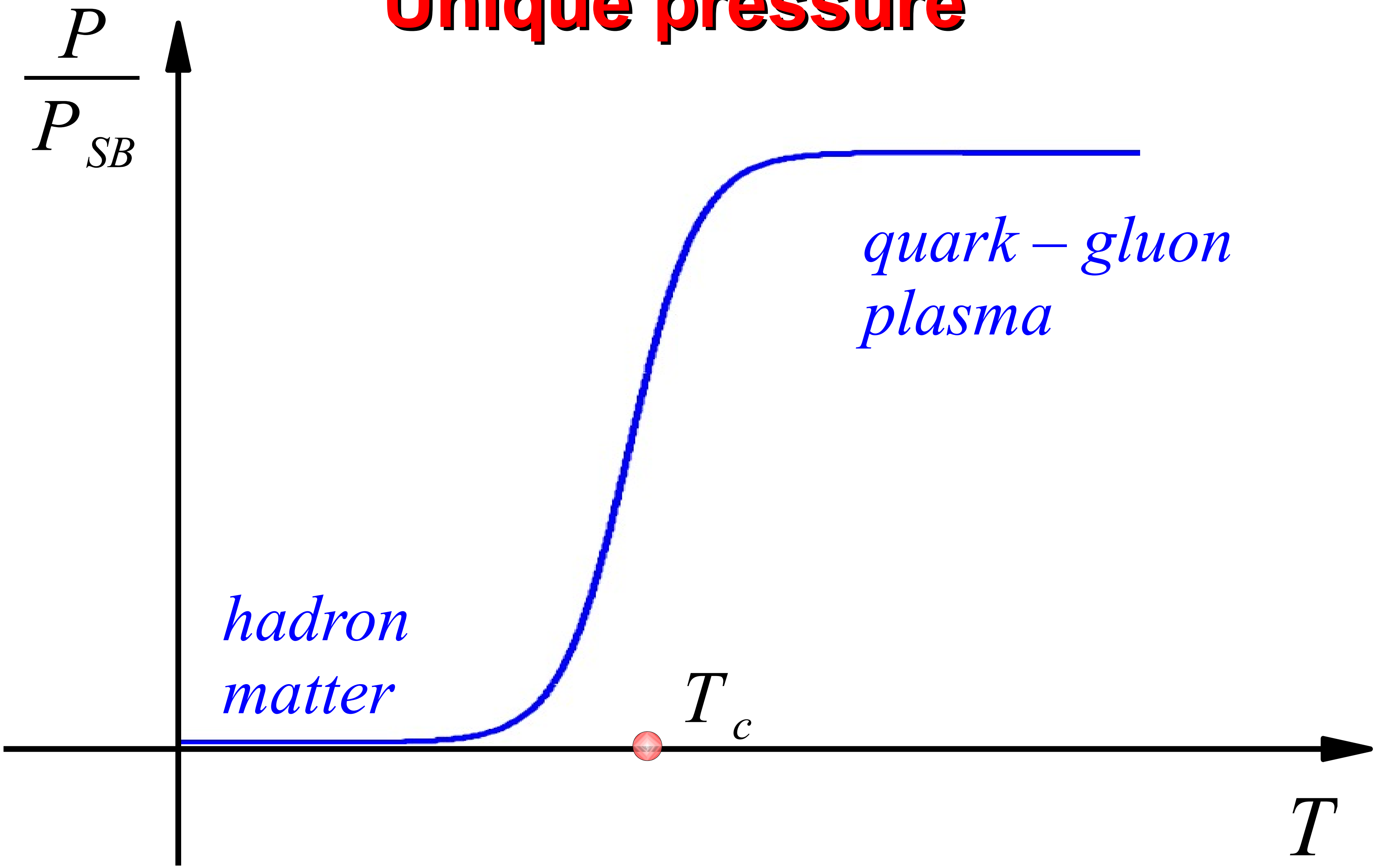


$$a \ll l \ll L$$



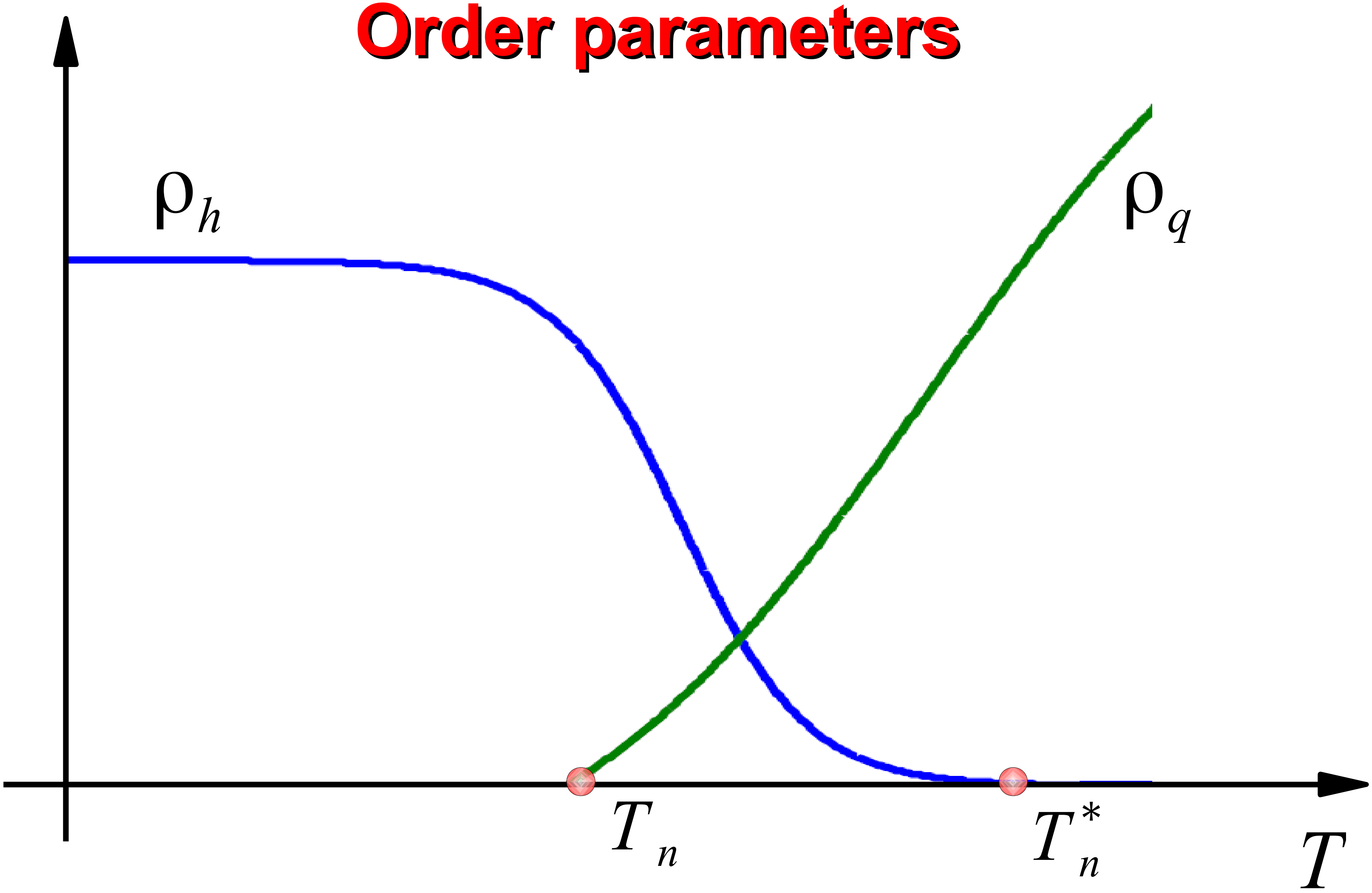
$$a \ll l \ll L$$

Unique pressure



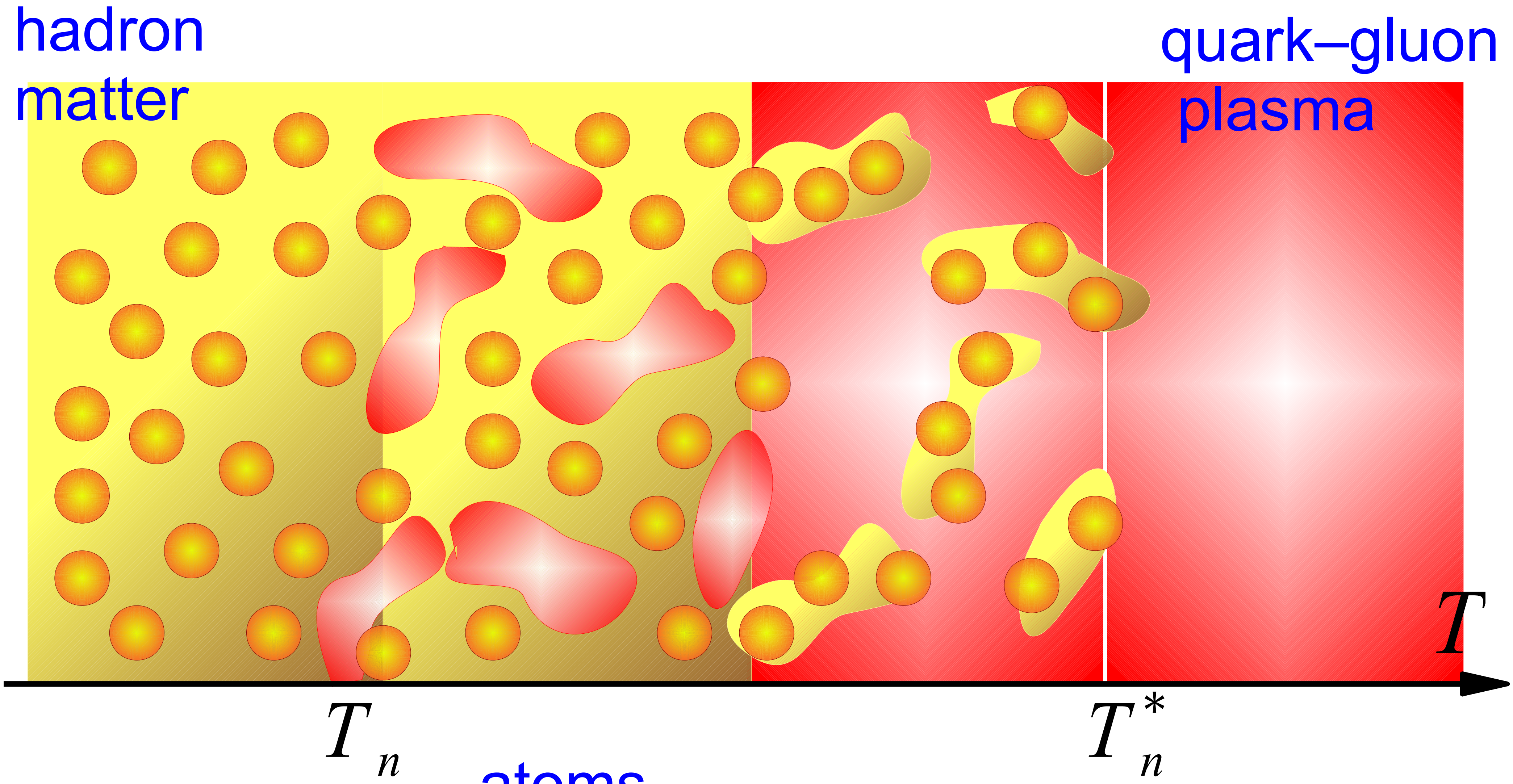
$$\mathrm{sup}_T \frac{\partial P}{\partial T} \rightarrow T_c$$

Order parameters



mixture: $T_n < T < T_n^*$

Deconfinement transition



atoms → atoms, ions, electrons → ions + electrons

Main features of mesoscopic mixed phase

1. Local equilibrium

Germ lifetime

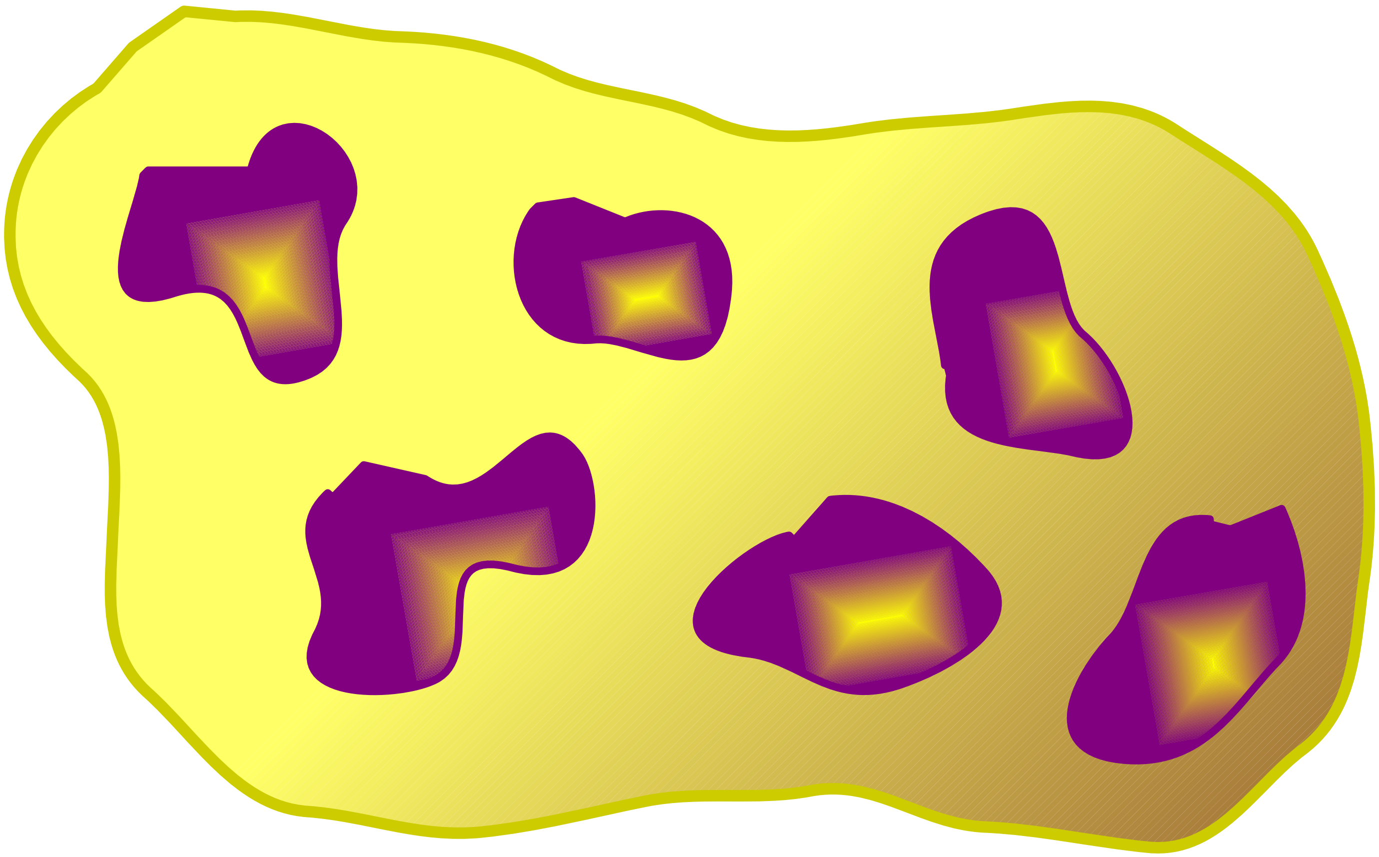
$$t_g$$

Local equilibration time

$$t_{loc}$$

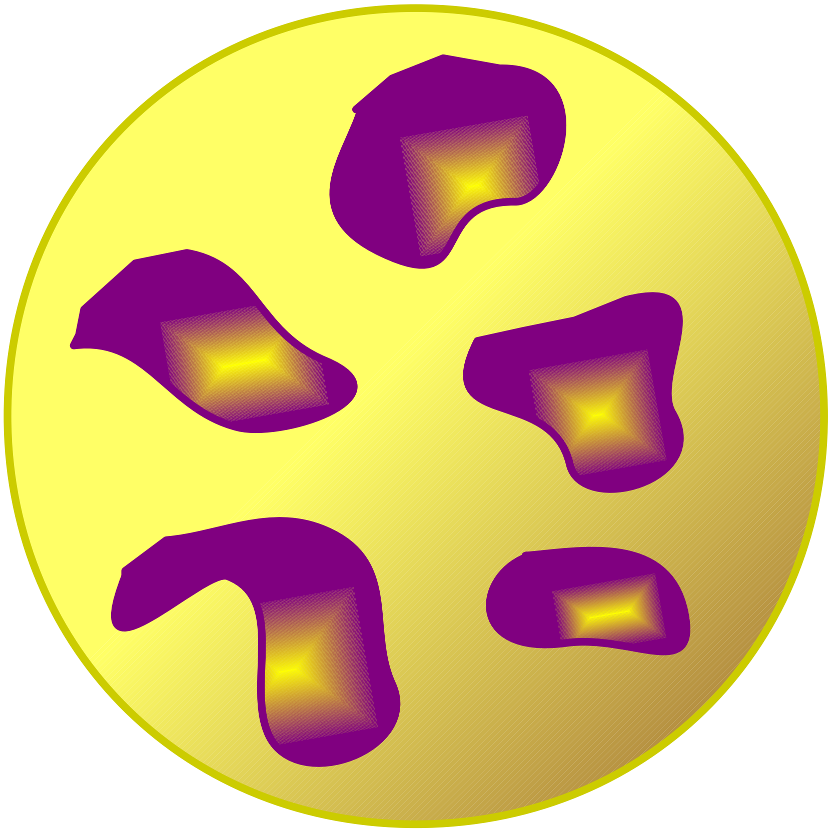
$$t_{loc} \ll t_g$$

2. Random spatial distribution at a snapshot

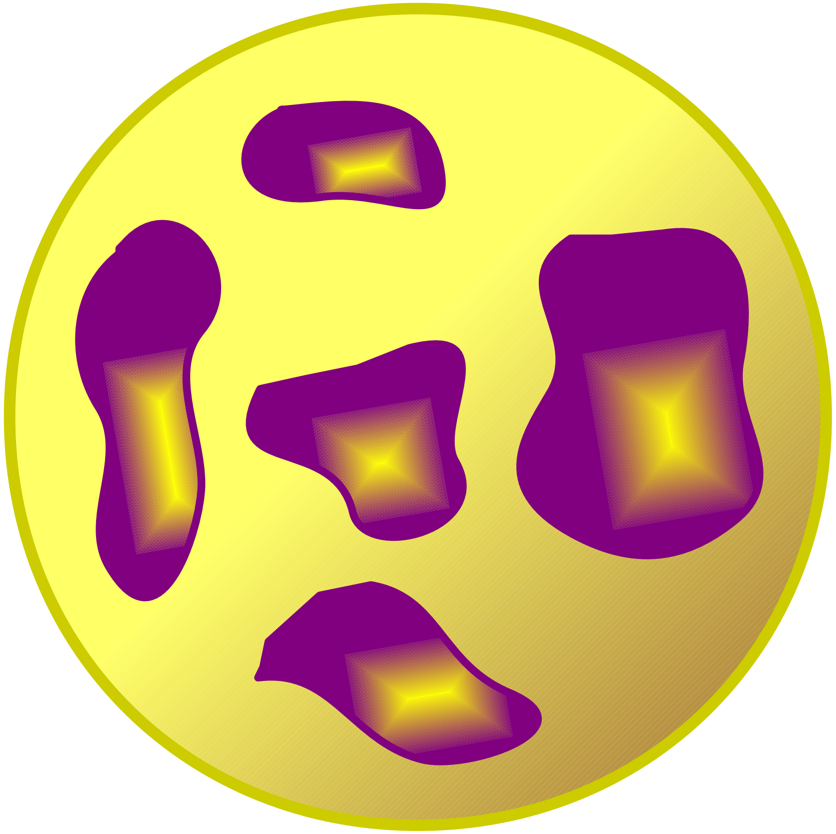


no ordered structure

3. Random positions in different experiments



1 experiment



2 experiment

4. Mesoscopic typical size of germs

Mean interparticle distance

$$a$$

Typical germ size

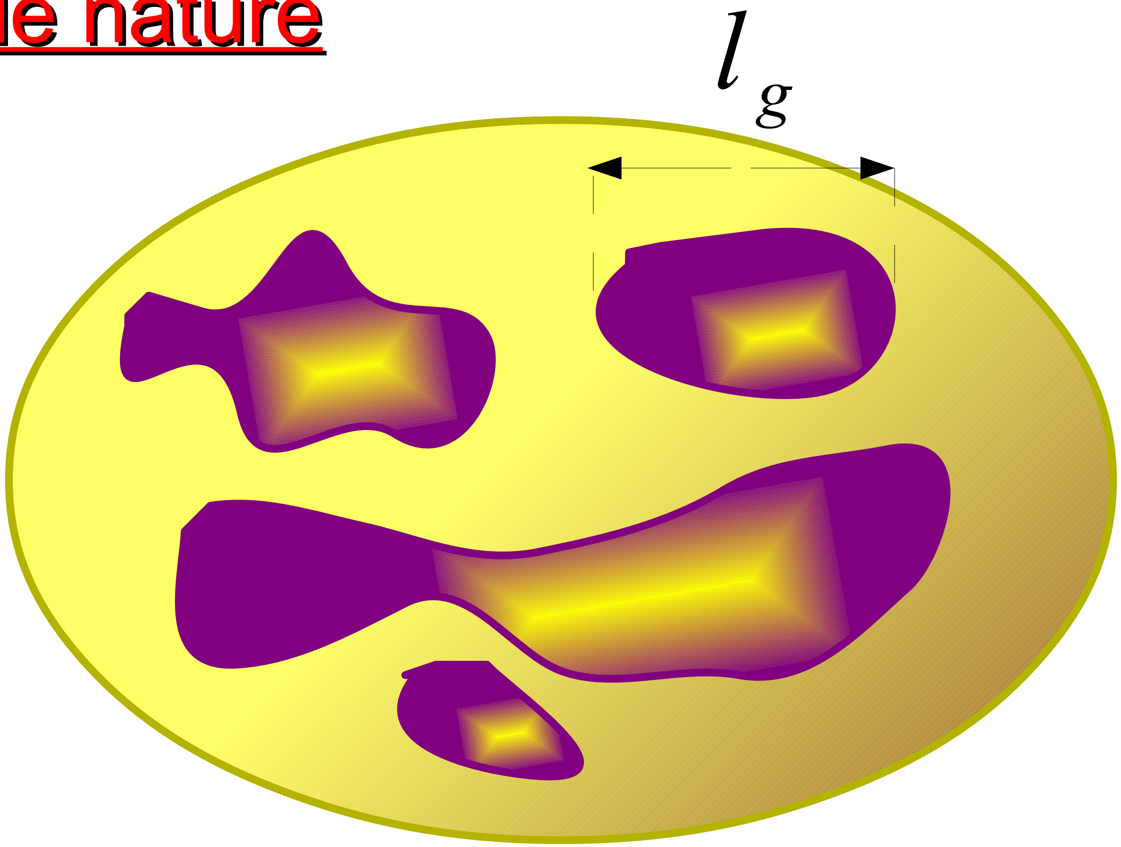
$$l_g$$

Fireball size

$$L$$

$$a \ll l_g \ll L$$

5. Multiscale nature



Various shapes

Various sizes in a dense interval: $l_g \in [\, l_g^{min}, l_g^{max} \,]$

Mesoscopic dense interval:

$$a \ll l_g^{min} < l_g^{max} \ll L$$

Phase separation

Gibbs equimolecular separating surface

$$\mathbb{V} = \mathbb{V}_1 \bigcup \mathbb{V}_2$$

$$V = V_1 + V_2$$

$$N = N_1 + N_2$$

Manifold indicator function

$$\xi_\nu(\vec{r}) = \begin{cases} 1, & \vec{r} \in \mathbb{V}_\nu \\ 0, & \vec{r} \notin \mathbb{V}_\nu \end{cases}$$

Quasiequilibrium ensemble

$$\{\mathcal{H},\ \hat{\rho}(\xi)\}$$

Space of states

$$\mathcal{H} \equiv \mathcal{H}_1 \bigotimes \mathcal{H}_2$$

$\mathcal{H}_\nu$ weighted Hilbert space

$$\mathrm{Tr} \int \hat{\rho}(\xi)\, \mathcal{D}\xi \;=\; 1$$

Averaging over random configurations

Representative ensemble

Energy Hamiltonian $\hat{H}(\xi)$

$$E = \mathrm{Tr}\int \hat{\rho}(\xi)\hat{H}(\xi)\,\mathcal{D}\xi$$

Constraint operators $\hat{C}_i(\xi)$

$$C_i = \mathrm{Tr}\int \hat{\rho}(\xi)\hat{C}_i(\xi)\,\mathcal{D}\xi$$

Shannon information

$$\mathrm{Tr}\int \hat{\rho}(\xi)\ln\hat{\rho}(\xi)\,\mathcal{D}\xi$$

Informational functional

$$I[\hat{\rho}(\xi)] = \mathrm{Tr}\int \hat{\rho}(\xi)\ln\hat{\rho}(\xi)\,\mathcal{D}\xi \;+$$

$$+\lambda_0\left[\mathrm{Tr}\int \hat{\rho}(\xi)\,\mathcal{D}\xi - 1\right] \;+$$

$$+\beta\left[\mathrm{Tr}\int \hat{\rho}(\xi)\hat{H}(\xi)\,\mathcal{D}\xi - E\right] \;+$$

$$+\sum_i \lambda_i\left[\mathrm{Tr}\int \hat{\rho}(\xi)\hat{C}_i(\xi)\,\mathcal{D}\xi - C_i\right]$$

Principle of minimal information

Statistical operator

$$\hat{\rho}(\xi) = \frac{1}{Z} \exp\{-\beta H(\xi)\}$$

Grand Hamiltonian

$$H(\xi) = \hat{H}(\xi) - \sum_i \mu_i \hat{C}_i(\xi)$$

$$\beta = \frac{1}{T}, \qquad \mu_i \equiv -\lambda_i T$$

Partition function

$$Z = \mathrm{Tr} \int \exp\{-\beta H(\xi)\} \mathcal{D}\xi$$

Effective Hamiltonian

$$\tilde{H}$$

$$\int \exp\{-\beta H(\xi)\} \mathcal{D}\xi = \exp(-\beta \tilde{H})$$

Partition function

$$Z = \mathrm{Tr}\, e^{-\beta \tilde{H}}$$

Effective temperature

$$T = \frac{1}{\beta}$$

Phase probabilities

$$w_\nu = \int \xi_\nu(\vec{r})\,\mathcal{D}\xi$$

$$w_1 + w_2 = 1$$

Grand potential

$$\Omega = -T \ln \mathrm{Tr}\, e^{-\beta \tilde{H}}$$

Equation for w_ν

$$\frac{\partial \Omega}{\partial w_\nu} = 0$$

Stability condition

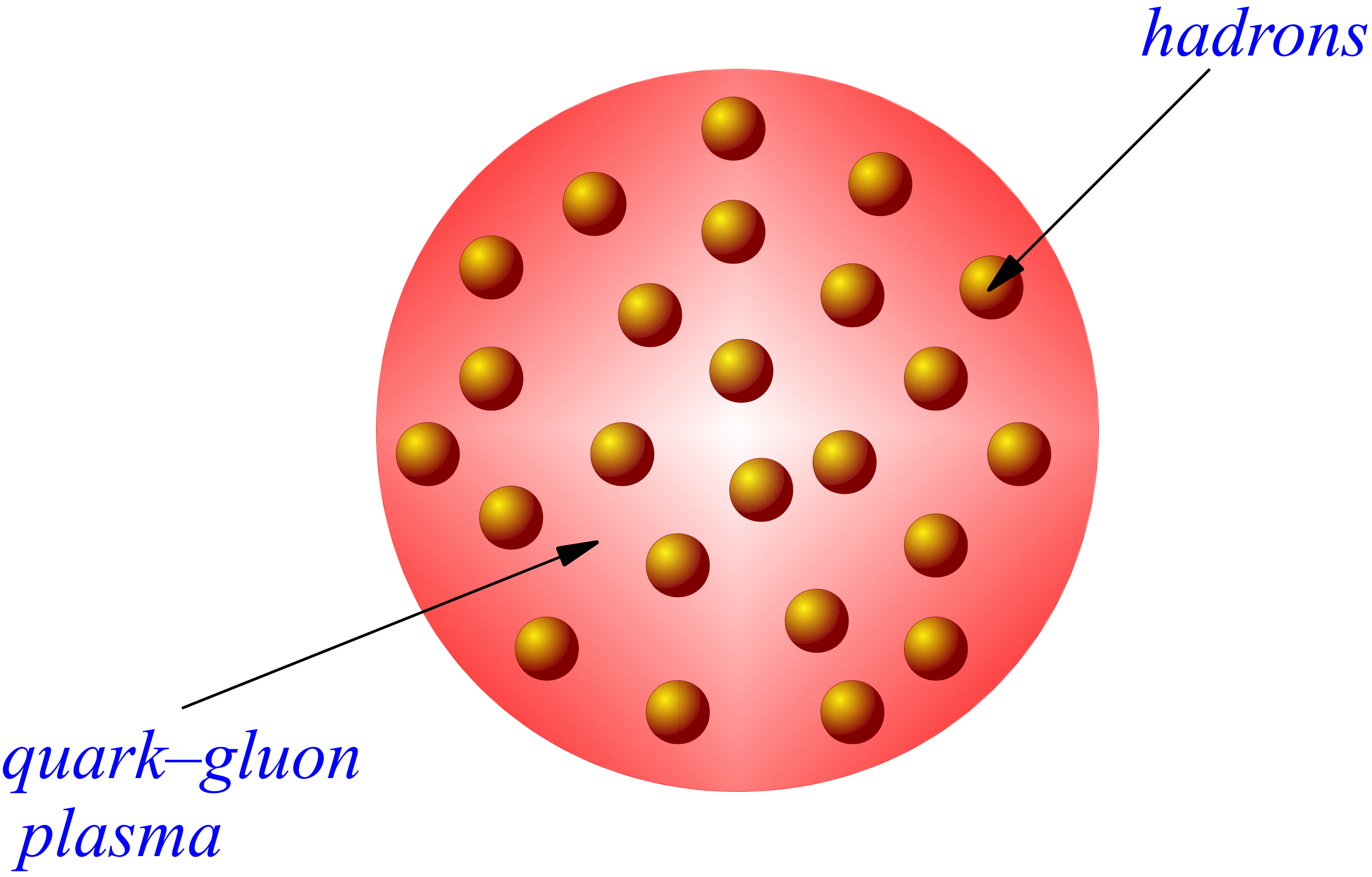
$$\frac{\partial^2 \Omega}{\partial w_\nu^2} > 0$$

Mesoscopic mixture is stable

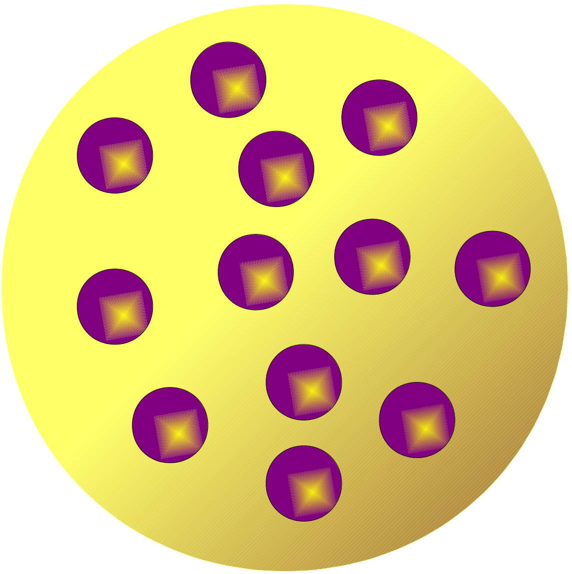
Pressure is uniquely defined

Deconfinement is a sharp crossover

Microscopic Mixture



Clustering Matter



n — quark cluster

B_i — barion number

S_i — strangeness

z_i — compositeness

Quark: $z_q = 1$

Meson: $z_m = 2$

Nucleon: $z_N = 3$

Mixture space of states

Fock space of i – clusters

$$\mathcal{F}_i$$

Mixture space

$$\mathcal{M} = \bigotimes_i \mathcal{F}_i$$

Cluster densities

Cluster momentum distribution

$$\langle\, a_i^{+}(\vec{k})\, a_i(\vec{k})\,\rangle \;=\; \zeta_i\, n_i(\vec{k})$$

Degeneracy factor

$$\zeta_i$$

Density of i – clusters

$$\rho_i \;=\; \zeta_i \int n_i(\vec{k})\, \frac{d\,\vec{k}}{(2\pi)^3}$$

Total density

$$\rho \;=\; \sum_i z_i \rho_i$$

Cluster weights

$$w_i \equiv \frac{z_i \rho_i}{\rho}$$

Properties

$$0 \leq w_i \leq 1, \qquad \sum_i w_i = 1$$

$$w_i = w_i(\rho_B, \rho_S, T)$$

Statistical Correctness

i – cluster Hamiltonian $\hat{H}_i$

System energy Hamiltonian

$$\hat{H} = \sum_i \hat{H}_i + \frac{1}{2} \sum_{i \neq j} \hat{H}_{ij}$$

$$\hat{H} = \hat{H}(\{\rho_i\}, T)$$

Grand Hamiltonian

$$H = \hat{H} - \sum_i \mu_i \hat{N}_i + CV$$

Correctness conditions

$$\left\langle \frac{\partial H}{\partial \rho_i} \right\rangle = 0, \qquad \left\langle \frac{\partial H}{\partial T} \right\rangle = 0$$

$$\frac{\partial C}{\partial \rho_i} = -\frac{1}{V} \left\langle \frac{\partial \hat{H}}{\partial \rho_i} \right\rangle$$

$$\frac{\partial C}{\partial T} = -\frac{1}{V} \left\langle \frac{\partial \hat{H}}{\partial T} \right\rangle$$

$$C = C(\{\rho_i\}, T)$$

Thermodynamic relations

$$P = -\frac{\Omega}{V} = -\frac{\partial \Omega}{\partial V}$$

$$E = \frac{1}{V}\langle \hat{H} \rangle = T\frac{\partial P}{\partial T} - P + \mu_B \rho_B + \mu_S \rho_S$$

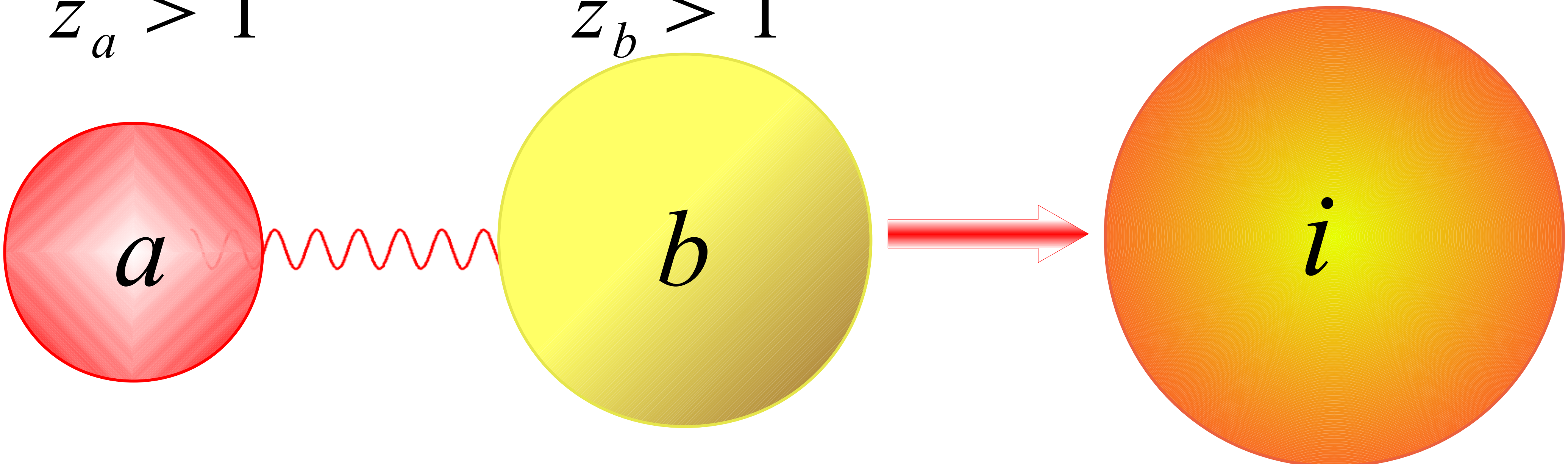
$$S = \frac{\partial P}{\partial T} = \frac{1}{T}\left(E + P - \mu_B \rho_B - \mu_S \rho_S \right)$$

$$\rho_i = \frac{1}{V}\langle \hat{N}_i \rangle = \frac{\partial P}{\partial \mu_i}$$

$$\rho_B = \sum_i B_i \rho_i = \frac{\partial P}{\partial \mu_B}, \qquad \rho_S = \sum_i S_i \rho_i = \frac{\partial P}{\partial \mu_S}$$

Cluster reactions

$z_a > 1$ $z_b > 1$



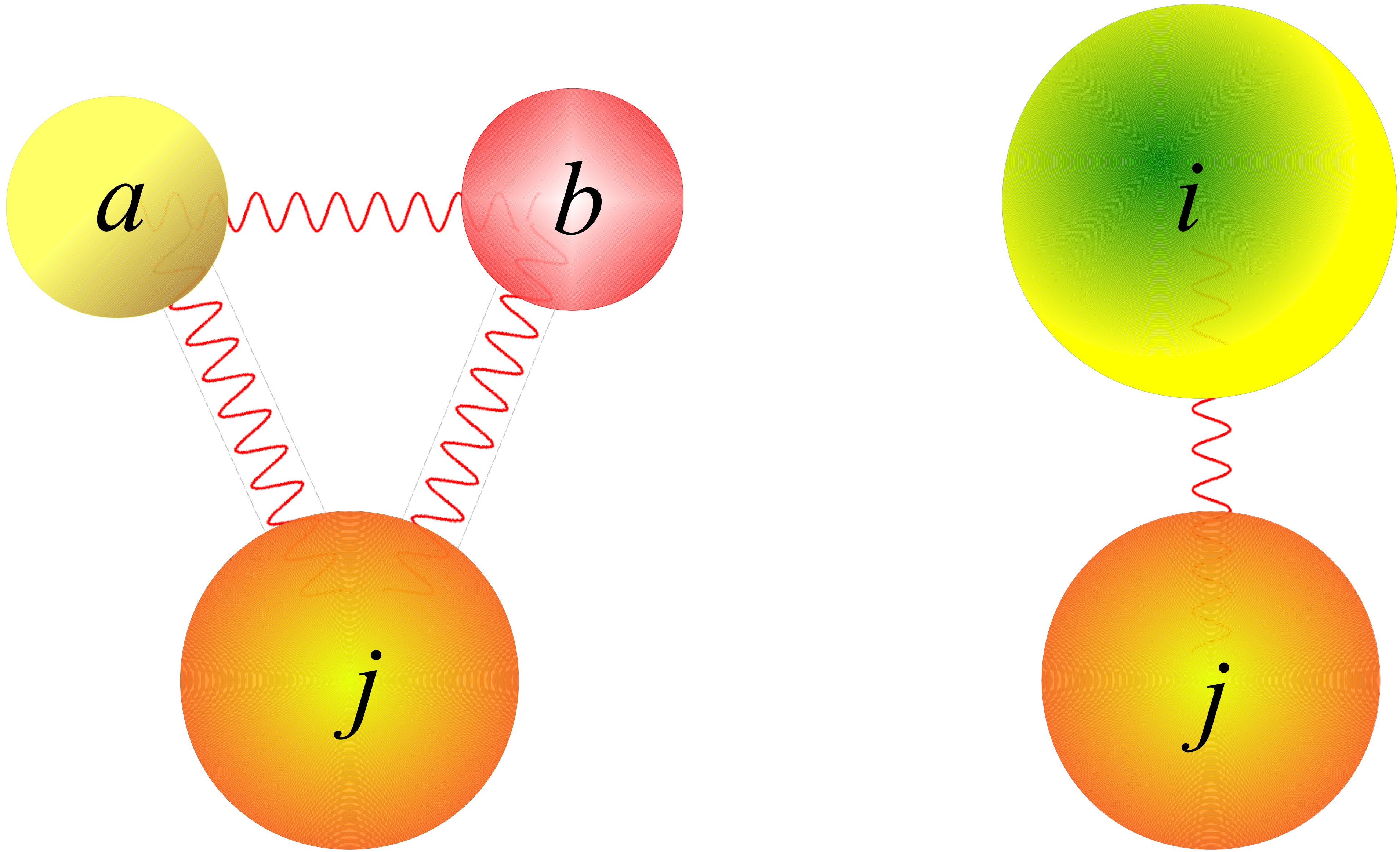
Conservation of compositeness

$$z_a + z_b = z_i$$

Conservation of mass

$$m_a + m_b + \Phi_{ab} = m_i$$

Φ_{ab} interaction energy



$$m_a + m_b + m_j + \Phi_{ab} + \Phi_{aj} + \Phi_{bj} = m_i + m_j + \Phi_{ij}$$

Potential scaling

$$\frac{\Phi_{ij}}{Z_i Z_j} = \frac{\Phi_{ab}}{Z_a Z_b}$$

$$\Phi_{ij} = \frac{Z_i Z_j}{9} \Phi_{NN}$$

Nucleon interaction energy

$$\Phi_{NN}$$

Particles

Gluons, glueballs

Quarks, multiquark clusters

Unflavored mesons:

$$\pi^{+} \quad \pi^{-} \quad \pi^{0} \quad \eta \quad \rho^{+} \quad \rho^{-} \quad \rho^{0} \quad \omega$$

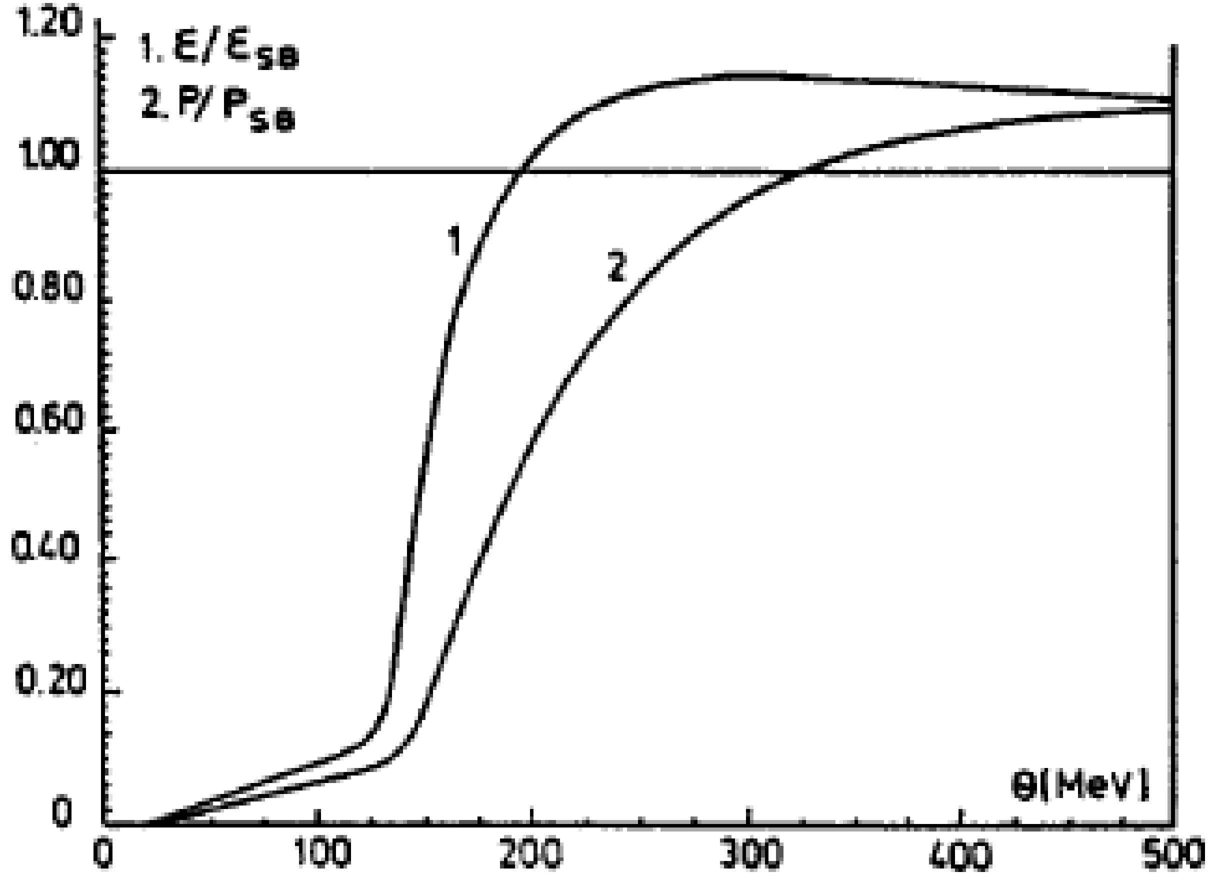
Strange mesons:

$$K^{+} \quad K^{-} \quad K^{0} \quad \overline{K}^{0}$$

Baryons

$$n \quad \overline{n} \quad p \quad \overline{p} \quad \Delta \quad \overline{\Delta}$$

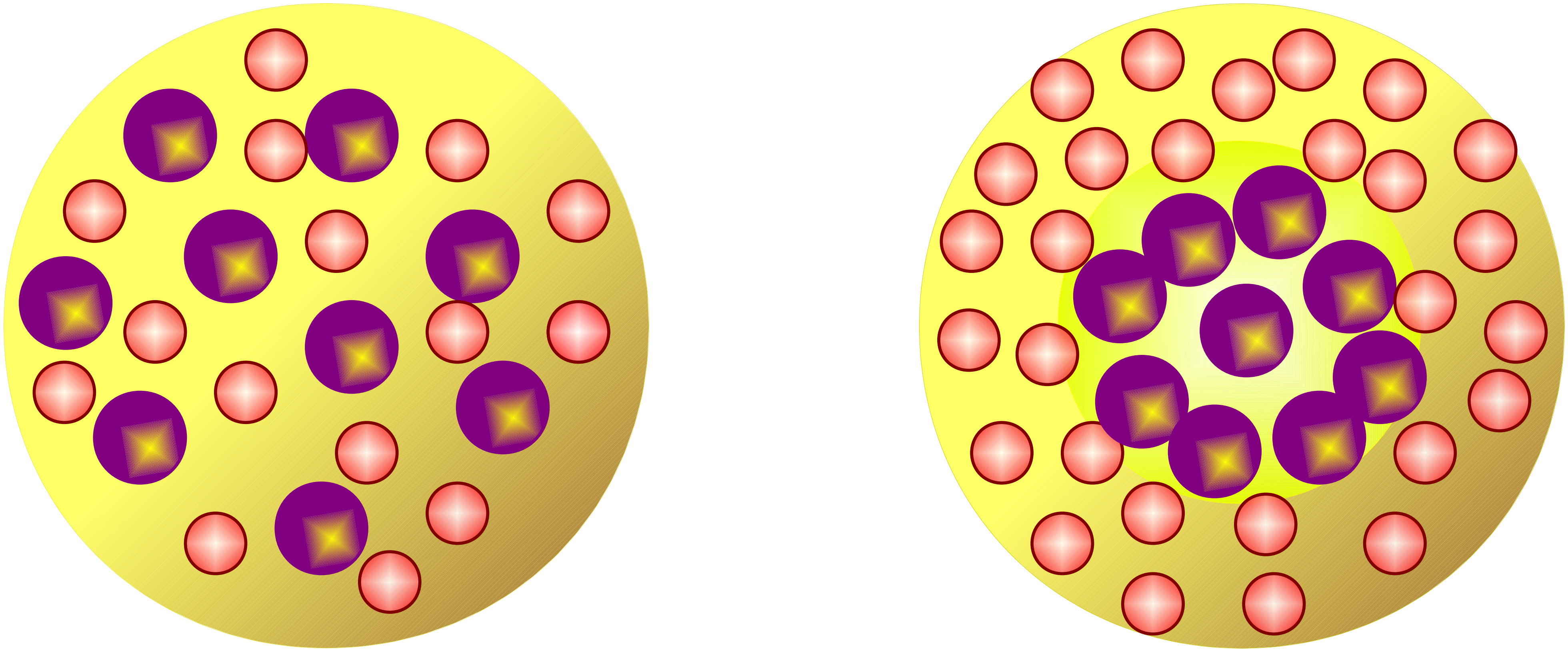
Typical behavior of energy and pressure



Relative energy and pressure for the mixed quark-gluon-meson system.

Crossover transition at $T_c \approx 170\,\mathrm{MeV}$

Stratification instability



Clusters of first kind: N_1

Clusters of second kind: N_2

$$V = V_1 + V_2, \quad N = N_1 + N_2$$

Stability condition

$$F_{mix} < F_{sep}$$

$$\Phi_{ij} \equiv \int V_{ij}(\vec{r})\, g_{ij}(\vec{r})\, d\vec{r}$$

$$\Phi_{12} < \sqrt{\Phi_{11}\Phi_{12}} + \frac{T\,V}{N_1 N_2} \Delta S_{mix}$$

$$\Delta S_{mix} = -k_B \left(N_1 \ln \frac{N_1}{N} + N_2 \ln \frac{N_2}{N} \right)$$

No stratification under potential scaling

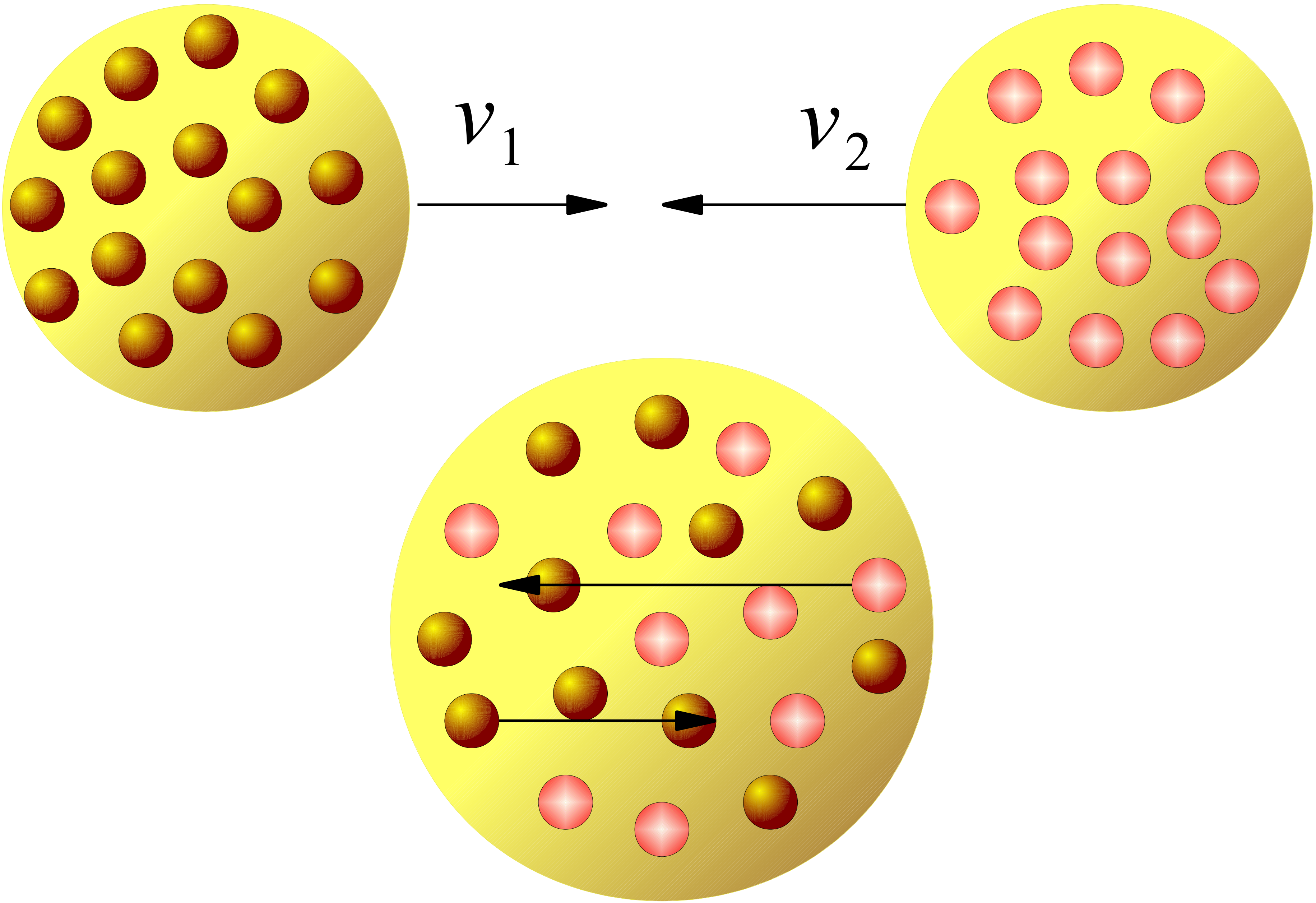
$$\Phi_{12} = \frac{z_1 z_2}{9} \Phi_{NN}$$

$$\Phi_{11} = \frac{z_1^2}{9} \Phi_{NN}, \qquad \Phi_{22} = \frac{z_2^2}{9} \Phi_{NN}$$

Always valid

$$\Delta S_{mix} > 0$$

Counterflow instability



Counterflow instability

$$\vec{v} = \vec{v}_2 - \vec{v}_1$$

$$v > v_c$$

$$v_c = \sqrt{\frac{\rho_2}{m_2 \Phi_{11}} \left(\Phi_{11} \Phi_{22} - \Phi_{12}^2 \right)}$$

Potential scaling: $v_c \rightarrow 0$

Conclusion

- Macroscopic mixed quark-hadron state is unstable. *Deconfinement is the first-order phase transition*.
- Mesoscopic mixed phase is stable. *Deconfinement is rather a sharp crossover*.
- Microscopic mixed phase is stable. *Deconfinement is a sharp crossover*.
- Mutual motion of cluster components can lead to *counterflow instability*.