THE DEPENDENCE OF THE NUMBER OF POMERONS ON THE IMPACT PARAMETER AND THE LONG-RANGE RAPIDITY CORRELATIONS IN *pp* COLLISIONS

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Dubna, 12 September 2012

Outline

- ♢ Distribution of strings in the transverse plane in AA and pp interactions
- ♢ Event-by-event fluctuations of the number of cut pomerons in non-diffractive *pp* collisions
- ♢ Comparison with quasi-eikonal and Regge approaches
- ♢ String fusion effects
- ♢ The cellurar version of SFM
- ♢ p_t-n correlation correlation function

Distribution of strings in the transverse plane

AA interactions

$$\langle N_{str}(b)\rangle \sim \langle N_{coll}^{in}(b)\rangle = AB\frac{\sigma_{NN}^{in}}{\sigma_{AB}(b)}\int T_A(\vec{s}-\vec{b}/2)\,T_B(\vec{s}+\vec{b}/2)d^2\vec{s} \quad (1)$$

$\sigma_{AB} = \int \sigma_{AB}(b)\ d^2\vec{b}$ - production cross section

$T_A(\vec{s}) = \int\limits_{-\infty}^{+\infty} \rho_A(\vec{s},z)dz$ - nuclear profile function

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$$w_{str}(\vec{s},\vec{b}) \equiv d\langle N_{str}(b)\rangle/d^2\vec{s}$$

$$w_{str}(\vec{s},\vec{b}) \sim w_{coll}^{in}(\vec{s},\vec{b}) = AB\frac{\sigma_{NN}^{in}}{\sigma_{AB}(b)}T_A(\vec{s}-\vec{b}/2)\,T_B(\vec{s}+\vec{b}/2) \qquad (2)$$

V.V. Vechernin, R.S. Kolevatov Phys. of Atom.Nucl. **70** (2007) 1797; 1858.

Distribution of strings in the transverse plane

pp interactions

$$w_{str}(\vec{s},\vec{b}) \sim T(\vec{s}-\vec{b}/2)\,T(\vec{s}+\vec{b}/2)/\sigma_{pp}(b) \qquad (3)$$

$\sigma_{pp} = \int \sigma_{pp}(b)\ d^2\vec{b}$ - non-diffractive pp cross section

$T(\vec{s}) = \int\limits_{-\infty}^{+\infty} \rho(\vec{s},z)\ dz$ - parton profile function of nucleon

$$\rho(r) = \frac{1}{\pi^{3/2}\alpha^3} e^{-r^2/\alpha^2}\ , \qquad T(s) = \frac{e^{-s^2/\alpha^2}}{\pi\alpha^2}\ , \qquad (4)$$

$$w_{str}(\vec{s},\vec{b}) \sim e^{-(\vec{s}+\vec{b}/2)^2/\alpha^2} e^{-(\vec{s}-\vec{b}/2)^2/\alpha^2}/\sigma_{pp}(b) = e^{-2s^2/\alpha^2} e^{-b^2/2\alpha^2}/\sigma_{pp}(b)$$

b–s factorization $\Rightarrow$

$$\langle N_{str}(b)\rangle \sim e^{-b^2/2\alpha^2}/\sigma_{pp}(b) \qquad (5)$$

Dependence of the average number of cut pomerons on the impact parameter in non-diffractive *pp* collisions

$N_{str} = 2N, \quad N$ - the number of cut pomerons in a given event

$$\boxed{\langle N(b) \rangle \sim e^{-b^2/2\alpha^2} / \sigma_{pp}(b)}$$

Event-by-event fluctuations of the number of cut pomerons

$\boxed{P(N,b) = e^{-\overline{N}(b)}\overline{N}(b)^N/N!}$ -Poisson, $P(0,b) = e^{-\overline{N}(b)}$

$\boxed{\widetilde{P}(N,b) = P(N,b)/[1-P(0,b)]}$ -modified Poisson, $\sum_{N=1}\widetilde{P}(N,b) = 1$

$$\langle N(b)\rangle = \sum_{N=1} N\,\widetilde{P}(N,b) = \overline{N}(b)/[1-P(0,b)] \qquad (6)$$

$$\sigma_{pp}(b) = 1 - P(0,b) = 1 - e^{-\overline{N}(b)} \qquad (7)$$

$$\langle N(b)\rangle = \overline{N}(b)/\sigma_{pp}(b) \quad \text{by slide 5} \Rightarrow$$

$$\boxed{\overline{N}(b) = N_0 e^{-b^2/2\alpha^2}} \qquad \boxed{\langle N(b)\rangle = \overline{N}(b)/[1-\exp(-\overline{N}(b))]}$$

Integration over impact parameter - min.bias *pp* collisions

$$f(b) \equiv \sigma_{pp}(b)/\sigma_{pp} \ , \qquad \int f(b)\, d^2\vec{b} = 1 \tag{8}$$

$$\langle N \rangle = \int \langle N(b) \rangle\, f(b)\, d^2\vec{b} = \int \overline{N}(b)\, d^2\vec{b}/\sigma_{pp} = 2\pi\alpha^2 N_0/\sigma_{pp} \tag{9}$$

$$\langle N^2 \rangle = \int \langle N^2(b) \rangle\, f(b)\, d^2\vec{b} = \pi\alpha^2 N_0(N_0+2)/\sigma_{pp} = \langle N \rangle\, (N_0+2)/2 \tag{10}$$

$$D_N \equiv \langle N^2 \rangle - \langle N \rangle^2 \qquad \text{we have used that} \quad \overline{N}(b) = N_0 e^{-b^2/2\alpha^2}$$

$$\sigma_{pp} = \int \sigma_{pp}(b)\, d^2 b = \int [1 - P(0,b)]\, d^2 b = \int [1 - \exp(-\overline{N}(b))]\, d^2 b \tag{11}$$

$$\sigma_{pp} = 2\pi\alpha^2 [E_1(N_0) + \gamma + \ln N_0] \qquad \sigma_{pp} = \pi b_{max}^2 n_{sim}(N \neq 0)/n_{sim}$$

$$E_1(z) = \int_1^\infty e^{-zt}\, \frac{dt}{t} \ , \qquad \gamma = 0.577...$$

Probability to have N cut pomerons in a non-diffractive pp collision

$$w_N = \int \widetilde{P}(N,b)\, f(b)\, d^2\vec{b} = \frac{1}{\sigma_{pp}} \int P(N,b)\, d^2\vec{b}\,, \tag{12}$$

$$w_N = \frac{1}{\sigma_{pp} N!} \int e^{-\overline{N}(b)} (\overline{N}(b))^N\, d^2\vec{b} = \frac{2\pi}{\sigma_{pp} N!} \int_0^\infty e^{-\overline{N}(b)} (\overline{N}(b))^N\, b\, db\,. \tag{13}$$

Recall that $\overline{N} = \overline{N}(b) = N_0 e^{-b^2/2\alpha^2}$ and hence $\alpha^2 d\overline{N}/\overline{N} = -b\, db$. Then

$$w_N = \frac{2\pi\alpha^2}{\sigma_{pp} N!} \int_0^{N_0} e^{-\overline{N}}\, \overline{N}^{N-1}\, d\overline{N}\,, \tag{14}$$

what can be calculated explicitly

$$w_N = \frac{2\pi\alpha^2}{\sigma_{pp} N!} \left[\Gamma(N) - \Gamma(N, N_0)\right]\,, \tag{15}$$

where $\Gamma(N)$ and $\Gamma(N, N_0)$ - gamma and incomplete gamma functions.

Probability to have N cut pomerons in a non-diffractive pp collision

At an integer N it can be reduced to

$$w_N = \frac{2\pi\alpha^2}{\sigma_{pp} N}\left[1 - e^{-N_0}\sum_{k=0}^{N-1} N_0^k/k!\right] = \frac{\sigma_N}{\sigma_{pp}}$$

where we have introduced the σ_N by

$$\sigma_N \equiv \frac{2\pi\alpha^2}{N}\left[1 - e^{-N_0}\sum_{k=0}^{N-1} N_0^k/k!\right]$$

The direct summing gives

$$\sum_{N=1}^{\infty} \sigma_N = 2\pi\alpha^2[Ei(1, N_0) + \gamma + \ln N_0] = \sigma_{pp}$$

where σ_{pp} is the non-diffractive pp cross section (see slide 7).

Comparison with quasi-eikonal and Regge approaches

Now we see that our formula for the σ_N coincides with the well known result for the cross-section σ_N of N cut-pomeron exchange, obtained in the quasi-eikonal and Regge approaches:

$$\sigma_N = \frac{4\pi\lambda}{C\,N}\left[1 - e^{-z}\sum_{k=0}^{N-1} z^k/k!\right]$$

where

$$z = \frac{2\gamma C}{\lambda}\exp(\Delta\xi)\ , \quad \lambda = R^2 + \alpha'\xi\ , \quad \xi = \ln(s/1GeV^2)\ .$$

Here Δ and α' are the residue and the slope of the pomeron trajectory. The parameters γ and R characterize the coupling of the pomeron trajectory with initial hadrons. The quasi-eikonal parameter C is related to the small-mass diffraction dissociation of incoming hadrons.

K.A. Ter-Martirosyan Phys. Lett. B **44**, 377 (1973).
A.B. Kaidalov, K.A. Ter-Martirosyan Yad. Fiz. **39**, 1545 (1984); **40**, 211 (1984).
V.A. Abramovsky, V.N. Gribov, O.V. Kancheli Yad. Fiz. **18**, 595 (1973).

Comparison with the quasi-eikonal and Regge approaches

This enables to connect the parameters N_0 and α of our model with the parameters of the pomeron trajectory and its couplings to hadrons. Comparing we have

$$N_0 = z = \frac{2\gamma C}{\lambda}\exp(\Delta\xi) \ , \quad \alpha = \sqrt{\frac{2\lambda}{C}} \ , \quad \lambda = R^2 + \alpha'\xi \qquad (16)$$

The numerical values of the parameters are taken from the paper:
G.H.Arakelyan,A.Capella,A.B.Kaidalov,Yu.M.Shabelski Eur.Phys.J.C**26**,81(2002)

$$\Delta = 0.139, \quad \alpha' = 0.21 \ GeV^{-2},$$
$$\gamma = 1.77 \ GeV^{-2}, \quad R^2 = 3.18 \ GeV^{-2}, \quad C = 1.5 \ ,$$

which gives for example:
α=0.51fm and N_0=3.38 at $\sqrt{s}$=60GeV
α=0.60fm and N_0=9.02 at $\sqrt{s}$=7000GeV

String fusion effects

local fusion (overlaps)

M.A. Braun, C. Pajares Eur.Phys.J. **C16**, 349, (2000)

$$\langle n\rangle_k = \mu_0\sqrt{k}\ S_k/\sigma_0\ , \qquad \langle p_t^2\rangle_k = p_0^2\sqrt{k}\ , \qquad k = 1,2,3,... \quad (17)$$

global fusion (clusters)

M.A. Braun, F. del Moral, C. Pajares, Phys.Rev. **C65**, 024907, (2002)

$$\langle p_t^2\rangle_{cl} = p_0^2\sqrt{k_{cl}}\ , \qquad \langle n\rangle_{cl} = \mu_0\sqrt{k_{cl}}S_{cl}/\sigma_0\ , \qquad k_{cl} = k\sigma_0/S_{cl} \quad (18)$$

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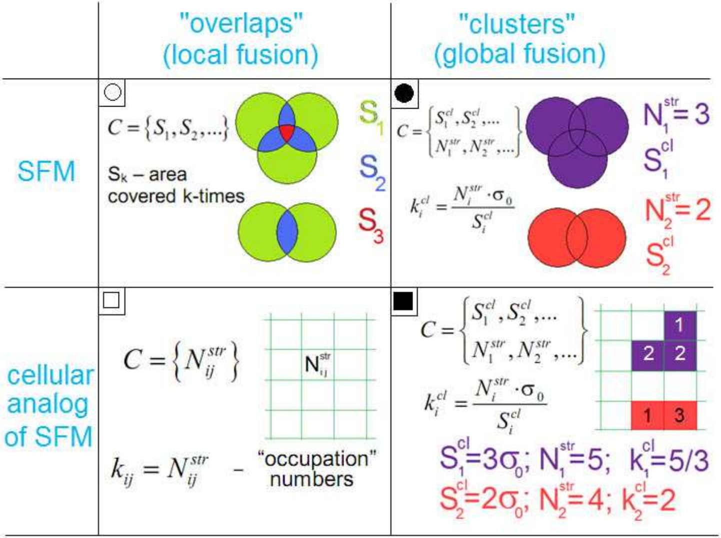
the cellurar version of SFM

Vechernin V.V., Kolevatov R.S., hep-ph/0304295; hep-ph/0305136

Braun M.A., Kolevatov R.S., Pajares C., Vechernin V.V. Eur.Phys.J. C32 (2004) 535.

String fusion effects



Monte-Carlo algorithm for long-range correlation functions

mean multiplicity and transverse momentum:

$$\langle n\rangle = \sum_C w(C)\langle n\rangle_C \ , \qquad \langle p_t\rangle = \sum_C w(C)\langle p_t\rangle_C \tag{19}$$

$$\sum_C w(C)\ \ldots = \frac{1}{n_{sim}} \sum_{sim}\ \ldots$$

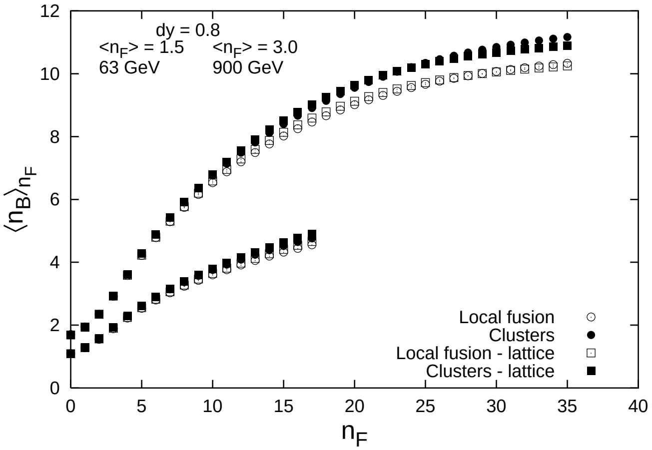
p_t-n correlation correlation function (regression):

$$\langle p_t\rangle_n = \frac{\sum_C w(C)\ \langle p_t\rangle_C\ P_C(n)}{\sum_C w(C)\ P_C(n)} \tag{20}$$

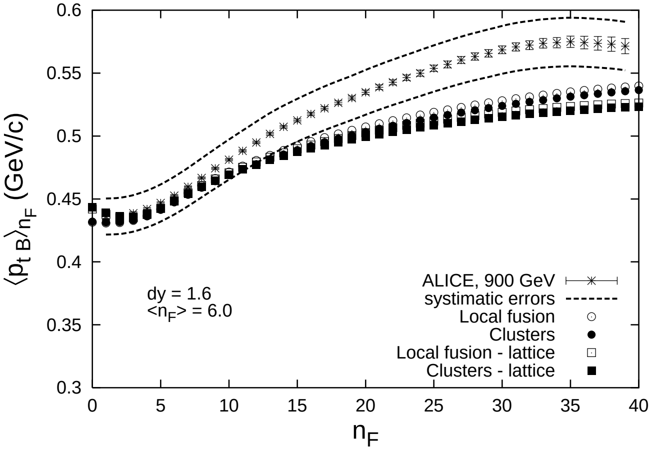
$$P_C(n) = P_{\langle n\rangle_C}(n) = e^{-\langle n\rangle_C} \frac{(\langle n\rangle_C)^n}{n!} \tag{21}$$

Braun M.A., Pajares C., Vechernin V.V. Phys. Lett. B493 (2000) 54
Vechernin V.V., Kolevatov R.S., hep-ph/0304295; hep-ph/0305136
Vechernin V.V., Kolevatov R.S. Phys. of Atom.Nucl. **70** (2007) 1797; 1858

$n - n$ correlation



$p_t - n$ correlation



ALICE collaboration Phys. Lett. B693 (2010) 53

Conclusions

- The simple model which enables to take into account the effect of colour string fusion in *pp* interactions is suggested..
- The parameters of the model are connected with the parameters of the pomeron trajectory and its couplings to hadrons.
- On the base of the model the MC algorithm which enables to calculate the long-range correlation functions between multiplicities and between the average transverse momentum and the multiplicity in *pp* collisions at different energies is developed.