Combined Analysis of Processes $\pi\pi \to \pi\pi, K\overline{K}, \eta\eta$ and J/ψ Decays and Parameters of Scalar Mesons

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The problem of interpretation of scalar mesons is tightly related to the most profound topics in particle physics which concern the QCD vacuum. Parameters of the scalar mesons, their nature and status of some of them are still not well settled (K.Nakamura et al. (PDG), J.Phys. G**37** (2010) 075021; J.Beringer et al. (PDG), PR D**86** (2012) 010001). E.g., applying our model-independent method in the 3-channel analyses of multi-channel $\pi\pi$ scattering (Yu.S.Surovtsev, P.Bydžovský, R.Kamiński, M.Nagy, PR D**81** (2010) 016001; Yu.S.Surovtsev, P.Bydžovský, V.E.Lyubovitskij, PR D**85** (2012) 036002) we have obtained parameters of the $f_0(600)$ and $f_0(1500)$ which differ considerably from results of analyses utilizing other methods (mainly based on dispersion relation and Breit-Wigner approaches). Reasons for this difference should be understood because our method of analysis is based only on the demand for analyticity and unitarity of amplitude using a uniformization procedure. The construction of the amplitude is essentially free from any dynamical (model) assumptions utilizing only the *mathematical* fact that a local behaviour of analytic functions determined on the Riemann surface is governed by the nearest singularities on all corresponding sheets.
In view of indicated circumstances, related to parameters and status of scalar mesons, there are known problems as to determining their QCD nature and assignment to the quark-model configurations.

Recent reviews: C.Amsler, N.A.Törnqvist, Phys.Rep. **389** (2004) 61; D.V. Bugg, Phys.Rep. **397** (2004) 257; F.E.Close, N.A.Törnqvist, J.Phys. G**28** (2002) R249; E.Klempt, A.Zaitsev, Phys.Rep. **454** (2007) 1.

We present results of the coupled-channel analysis of data on isoscalar S-wave processes $\pi\pi \to \pi\pi, K\overline{K}, \eta\eta$ and on decays $J/\psi \to \pi\pi, K\overline{K}$ for studying f_0-mesons.

First we explain quite transparently that it is impossible to obtain correct parameters for broad scalar resonances analyzing only the $\pi\pi$ scattering data as, e.g., in papers: I.Caprini et al., PRL **96** (2006) 132001; R.Garcia-Martin et al., PRL **107** (2011) 072001. To this end we considered the $\pi\pi$ scattering and $\pi\pi \to K\overline{K}$ in the 2-channel approach. The thing is that in our above-indicated 3-channel analyses, we were enforced to construct a 4-sheeted model of the initial 8-sheeted Riemann surface. This was achieved by neglecting the $\pi\pi$-threshold branch-point which means that we have considered the nearest to the physical region semi-sheets of the initial Riemann surface. This is in the line with our approach of a consistent account of the nearest singularities on all relevant sheets. *The 2-channel analysis utilizes the full Riemann surface and is, therefore, free of this approximation and of any suppositions.*

Analyzing only the $\pi\pi$ scattering data when using a 2-channel uniformizing variable, we have obtained practically the same values for the resonance parameters as in the above-indicated dispersion relation analyses (I.Caprini et al., PRL **96** (2006) 132001; R.Garcia-Martin et al., PRL **107** (2011) 072001).

However our approach has revealed flaws of the only $\pi\pi$ scattering data analysis which are cured in the combined 2-channel analysis of the $\pi\pi \to \pi\pi, K\overline{K}$ data successfully performed. Then the resonance parameters are inevitably changed, obtaining the new values closer to those in our previous 3-channel analysis, confirming, thus, a plausibility of our assumptions in the 3-channel calculations. Moreover, some remaining pseudo-background in the combined 2-channel analysis of the $\pi\pi \to \pi\pi, K\overline{K}$ indicates apparently that it is necessary some more to allow for at least the $\eta\eta$ threshold. Therefore we have performed also the combined analysis of the $\pi\pi \to \pi\pi, K\overline{K}, \eta\eta$ using a 3-channel uniformizing variable and adding data on decays $J/\psi \to \pi\pi, K\overline{K}$ from Mark III (W.Lockman, Hadron'89, Proceedings, p.109), from DM2 (A.Falvard et al., PR D**38** (1988) 2706), and from BES (M.Ablikim et al., PL B **607** (2005) 243). The latter reduces considerably the number of possible scenarios which are possible when describing only the multi-channel $\pi\pi$ scattering where some spread in resonance parameters was obtained.
In Conclusions some spectroscopic implications from results of the analysis are discussed.

Model-independent approach to the 2- and 3-channel $\pi\pi$ scattering

Our model-independent method which essentially utilizes a uniformizing variable can be used only for the 2-channel case and under some conditions for the 3-channel one. Only in these cases we obtain a simple symmetric (easily interpreted) picture of the resonance poles and zeros of the S-matrix on the uniformization plane. The 2- or 3-channel S-matrix is determined on the 4- or 8-sheeted Riemann surface, respectively. The matrix elements S_{ij}, where $i,j=1,2,3$ denote channels, have the right-hand cuts along the real axis of the s complex plane (s is the invariant total energy squared), starting with the channel thresholds s_i ($i=1,2,3$), and the left-hand cuts. The Riemann-surface sheets are numbered according to the signs of analytic continuations of the square roots $\sqrt{s-s_i}\ \ (i=1,2,3)$ as follows:

	I	II	III	IV	V	VI	VII	VIII
Im$\sqrt{s-s_1}$	$+$	$-$	$-$	$+$	$+$	$-$	$-$	$+$
Im$\sqrt{s-s_2}$	$+$	$+$	$-$	$-$	$-$	$-$	$+$	$+$
Im$\sqrt{s-s_3}$	$+$	$+$	$+$	$+$	$-$	$-$	$-$	$-$

Resonance representations on the Riemann surface are obtained using formulas (D.Krupa, V.A.Meshcheryakov, Yu.S.Surovtsev, NC A**109** (1996) 281), expressing analytic continuations of the S-matrix elements to all sheets in terms of those on the physical (I) sheet that have only the resonances zeros (beyond the real axis), at least, around the physical region. Then, starting from the resonance zeros on sheet I, one can obtain an arrangement of poles and zeros of resonance on the whole Riemann surface.

Process	I	II	III	IV	V	VI	VII	VIII
$1 \to 1$	S_{11}	$\frac{1}{S_{11}}$	$\frac{S_{22}}{D_{33}}$	$\frac{D_{33}}{S_{22}}$	$\frac{\det S}{D_{11}}$	$\frac{D_{11}}{\det S}$	$\frac{S_{33}}{D_{22}}$	$\frac{D_{22}}{S_{33}}$
$1 \to 2$	S_{12}	$\frac{iS_{12}}{S_{11}}$	$\frac{-S_{12}}{D_{33}}$	$\frac{iS_{12}}{S_{22}}$	$\frac{iD_{12}}{D_{11}}$	$\frac{-D_{12}}{\det S}$	$\frac{iD_{12}}{D_{22}}$	$\frac{D_{12}}{S_{33}}$
$2 \to 2$	S_{22}	$\frac{D_{33}}{S_{11}}$	$\frac{S_{11}}{D_{33}}$	$\frac{1}{S_{22}}$	$\frac{S_{33}}{D_{11}}$	$\frac{D_{22}}{\det S}$	$\frac{\det S}{D_{22}}$	$\frac{D_{11}}{S_{33}}$
$1 \to 3$	S_{13}	$\frac{iS_{13}}{S_{11}}$	$\frac{-iD_{13}}{D_{33}}$	$\frac{-D_{13}}{S_{22}}$	$\frac{-iD_{13}}{D_{11}}$	$\frac{D_{13}}{\det S}$	$\frac{-S_{13}}{D_{22}}$	$\frac{iS_{13}}{S_{33}}$
$2 \to 3$	S_{23}	$\frac{D_{23}}{S_{11}}$	$\frac{iD_{23}}{D_{33}}$	$\frac{iS_{23}}{S_{22}}$	$\frac{-S_{23}}{D_{11}}$	$\frac{-D_{23}}{\det S}$	$\frac{iD_{23}}{D_{22}}$	$\frac{iS_{23}}{S_{33}}$
$3 \to 3$	S_{33}	$\frac{D_{22}}{S_{11}}$	$\frac{\det S}{D_{33}}$	$\frac{D_{11}}{S_{22}}$	$\frac{S_{22}}{D_{11}}$	$\frac{D_{33}}{\det S}$	$\frac{S_{11}}{D_{22}}$	$\frac{1}{S_{33}}$

In Table, the superscript I is omitted to simplify the notation, $\det S$ is the determinant of the 3×3 S-matrix on sheet I, $D_{\alpha\beta}$ is the minor of the element $S_{\alpha\beta}$, that is, $D_{11} = S_{22}S_{33} - S_{23}^2$, $D_{22} = S_{11}S_{33} - S_{13}^2$, $D_{33} = S_{11}S_{22} - S_{12}^2$, $D_{12} = S_{12}S_{33} - S_{13}S_{23}$, $D_{23} = S_{11}S_{23} - S_{12}S_{13}$, etc.

In the 2-channel case, *3 types* of resonances are obtained corresponding to a pair of conjugate zeros on sheet I only in S_{11} – the type (**a**), only in S_{22} – (**b**), and simultaneously in S_{11} and S_{22} – (**c**).

In the 3-channel case, we obtain *7 types* of resonances corresponding to 7 possible situations when there are resonance zeros on sheet I only in S_{11} – (**a**); S_{22} – (**b**); S_{33} – (**c**); S_{11} and S_{22} – (**d**); S_{22} and S_{33} – (**e**); S_{11} and S_{33} – (**f**); S_{11}, S_{22} and S_{33} – (**g**). The resonance of every type is represented by the pair of complex-conjugate clusters (of poles and zeros on the Riemann surface).

A necessary and sufficient condition for existence of the multi-channel resonance is its representation by one of the types of pole clusters. A main model-independent contribution of resonances is given by the pole clusters and possible remaining small (model-dependent) contributions of resonances can be included in the background. This is confirmed further by the obtained simple description of the background.

The cluster type is related to the nature of state. *E.g.*, if we consider the $\pi\pi$, $K\overline{K}$ and $\eta\eta$ channels, then a resonance, coupled relatively more strongly to the $\pi\pi$ channel than to the $K\overline{K}$ and $\eta\eta$ ones is described by the cluster of type (**a**). In the opposite case, it is represented by the cluster of type (**e**) (say, the state with the dominant $s\bar{s}$ component). The glueball must be represented by the cluster of type (**g**) (of type (**c**) in the 2-channel consideration) as a necessary condition for the ideal case.

Whereas cases (**a**), (**b**) and (**c**) can be related to the resonance representation by Breit-Wigner forms, cases (**d**), (**e**), (**f**) and (**g**) practically are lost at the Breit-Wigner description.

One can formulate *a model-independent test as a necessary condition* to distinguish a bound state of colorless particles (*e.g.*, a $K\overline{K}$ molecule) and a $q\bar{q}$ bound state (D.Morgan, M.R.Pennington, PR D**48** (1993) 1185; D.Krupa, V.A.Meshcheryakov, Yu.S.Surovtsev, Nuovo Cim. A**109** (1996) 281). In the 1-channel case, the existence of the particle bound-state means the presence of a pole on the real axis under the threshold on the physical sheet.

<u>In the 2-channel case</u>, existence of the bound-state in channel 2 ($K\overline{K}$ molecule) that, however, can decay into channel 1 ($\pi\pi$ decay), would imply the presence of the pair of complex conjugate poles on sheet II under the second-channel threshold without the corresponding shifted pair of poles on sheet III.

<u>In the 3-channel case</u>, the bound state in channel 3 ($\eta\eta$) that, however, can decay into channels 1 ($\pi\pi$ decay) and 2 ($K\overline{K}$ decay), is represented by the pair of complex conjugate poles on sheet II and by the pair of shifted poles on sheet III under the $\eta\eta$ threshold without the corresponding poles on sheets VI and VII.

According to this test, earlier we rejected interpretation of the $f_0(980)$ as the $K\overline{K}$ molecule because this state is represented by the cluster of type (**a**) in the 2-channel analysis of processes $\pi\pi \to \pi\pi, K\overline{K}$ and, therefore, does not satisfy the necessary condition to be the $K\overline{K}$ molecule (D.Krupa, V.A.Meshcheryakov, Yu.S.Surovtsev, NC A**109** (1996) 281).

It is convenient to use the Le Couteur-Newton relations (K.J.LeCouteur, Proc.Roy.Soc. A**256** (1960) 115; R.G.Newton, J.Math.Phys. **2** (1961) 188; M.Kato, Ann.Phys. **31** (1965) 130). They express the S-matrix elements of all coupled processes in terms of the Jost matrix determinant $d(\sqrt{s-s_1},\cdots,\sqrt{s-s_n})$ that is a real analytic function with the only branch-points at $\sqrt{s-s_i}=0$.

The important branch points, corresponding to the thresholds of the coupled channels and to the crossing ones, are taken into account in the proper uniformizing variable.

For the data on multi-channel $\pi\pi$ scattering we used the results of phase analyses which are given for phase shifts of the amplitudes $\delta_{\alpha\beta}$ and for the modules of the S-matrix elements $\eta_{\alpha\beta}=|S_{\alpha\beta}|$ $(\alpha,\beta=1,\ 2,\ 3)$:

$$S_{\alpha\alpha}=\eta_{\alpha\alpha}e^{2i\delta_{\alpha\alpha}}, \qquad S_{\alpha\beta}=\eta_{\alpha\beta}e^{i\phi_{\alpha\beta}}.$$

If below the third threshold there is the 2-channel unitarity then the relations

$$\eta_{11}=\eta_{22},\ \ \eta_{12}=(1-{\eta_{11}}^2)^{1/2},\ \ \phi_{12}=\delta_{11}+\delta_{22}$$

are fulfilled in this energy region.

For the $\pi\pi$ scattering, the data from the threshold to 1.89 GeV are taken from J.R.Batley et al, EPJ C**54** (2008) 411; Hyams et al., NP B**64** (1973) 134; **100** (1975) 205 (1975); A.Zylbersztejn et al., PL B**38** (1972) 457; P.Sonderegger, P.Bonamy, in *Proc. 5th Intern. Conf. on Elem. Part., Lund, 1969*, paper 372; J.R.Bensinger et al., PL B**36** (1971) 134; J.P.Baton et al., PL B**33** (1970) 525, 528; P.Baillon et al., PL B**38** (1972) 555; L.Rosselet et al., PR D**15** (1977) 574; A.A.Kartamyshev et al., Pis'ma v ZhETF **25** (1977) 68; A.A.Bel'kov et al., Pis'ma v ZhETF **29** (1979) 652.
For $\pi\pi \to K\overline{K}$, practically all the accessible data are used (W.Wetzel et al., NP B**115** (1976) 208; V.A.Polychronakos et al., PR D**19** (1979) 1317; P.Estabrooks, PR D**19** (1979) 2678 ; D.Cohen et al., PR D**22** (1980) 2595; G.Costa et al., NP B**175** (1980) 402; A.Etkin et al., PR D**25** (1982) 1786).
For $\pi\pi \to \eta\eta$, we used data for $|S_{13}|^2$ from the threshold to 1.72 GeV (F.Binon et al., NC A**78** (1983) 313).

Here the combined 2- and 3-channel analyses of the coupled processes are carried out assuming that in the 1500-MeV region two states – wide and narrow – exist.

In the 2-channel analysis of processes $\pi\pi \to \pi\pi, K\overline{K}$ we applied the uniformizing variable which takes into account, in addition to the $\pi\pi$- and $K\overline{K}$-threshold branch-points, the left-hand branch-point at $s = 0$, related to the $\pi\pi$ crossed channels:

$$v = \frac{\sqrt{(s-s_1)s_2} + \sqrt{(s-s_2)s_1}}{\sqrt{s(s_2-s_1)}} \quad (s_1 = 4m_\pi^2 \text{ and } s_2 = 4m_K^2).$$

It maps the 4-sheeted Riemann surface with two unitary cuts and the left-hand cut onto the v-plane.

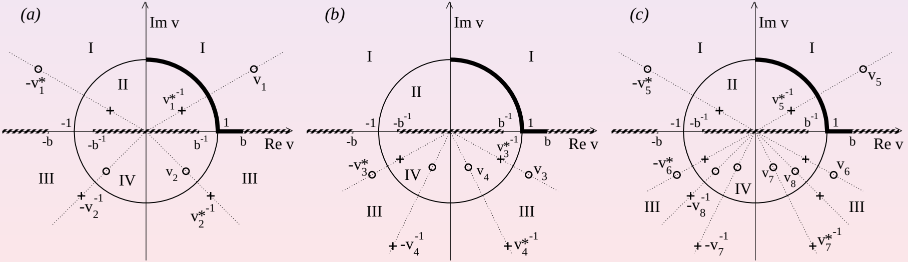


Figure: Representation of resonances of type (**a**), (**b**) and (**c**) on the uniformization v-plane in S_{11}.

On the v-plane, $S_{11}(v)$ has no cuts; $S_{12}^2(v)$ and $S_{22}(v)$ do have the cuts which arise from the left-hand cut on the s-plane, starting at $s=4(m_K^2-m_\pi^2)$. Further the contribution of the left-hand cut is approximated by a pole

$$d_L = v^{-4}(1-(p-i\sqrt{1-p^2})v)^4(1+(p+i\sqrt{1-p^2})v)^4 \qquad (p=0.903\pm 0.0004).$$

The 4th power is stipulated by the following model-independent arguments. First, a pole on the real s-axis on the physical sheet in S_{22} is accompanied by a pole on sheet II at the same s-value. On the v-plane this implies the pole of 2nd order (and also zero of the same order, symmetric to the pole with respect to the real axis). Second, for the s-channel process $\pi\pi \to K\overline{K}$, the crossing u- and t-channels are the $\pi - K$ and $\overline{\pi} - K$ scattering (exchanges in these channels give contributions on the left-hand cut). This results in the additional doubling of the multiplicity of the indicated pole on the v-plane. Therefore, the contribution of the left-hand cut is approximated as the fourth-power pole on the real s-axis on sheet I in the sub-$K\overline{K}$-threshold region.

On the v-plane, the Le Couteur-Newton relations are

$$S_{11} = \frac{d(-v^{-1})}{d(v)}, \quad S_{22} = \frac{d(v^{-1})}{d(v)}, \quad S_{11}S_{22} - S_{12}^2 = \frac{d(-v)}{d(v)}.$$

Now $d(v) = d_{res} d_L d_{bg}$ does not possess already branch points.

The $d_{res}(v)$ represents the contribution of resonances.

$$d_{res} = v^{-M} \prod_{n=1}^{M} (1 - v_n^* v)(1 + v_n v),$$

where M is the number of pairs of the conjugate zeros.
The background part is

$$d_{bg} = \mathrm{exp}[-i \sum_{n=1}^{3} \frac{\sqrt{s - s_n}}{2m_n} (\alpha_n + i\beta_n)]$$

with

$$\alpha_n = a_{n1} + a_{n\eta} \frac{s - s_\eta}{s_\eta} \theta(s - s_\eta) + a_{n\sigma} \frac{s - s_\sigma}{s_\sigma} \theta(s - s_\sigma) + a_{nv} \frac{s - s_v}{s_v} \theta(s - s_v)$$

$$\beta_n = b_{n1} + b_{n\eta} \frac{s - s_\eta}{s_\eta} \theta(s - s_\eta) + b_{n\sigma} \frac{s - s_\sigma}{s_\sigma} \theta(s - s_\sigma) + b_{nv} \frac{s - s_v}{s_v} \theta(s - s_v)$$

where s_η and s_σ are the $\eta\eta$ and $\sigma\sigma$ thresholds, respectively; s_v is a combined threshold of $\eta\eta'$, $\rho\rho$ and $\omega\omega$ channels in the vicinity of 1.5 GeV. From the analysis: $s_\sigma = 1.6558 \text{ GeV}^2$, $s_v = 2.1293 \text{ GeV}^2$.

Initially analyzing only the $\pi\pi$ scattering, we achieved an excellent description of the data for δ_{11} and modulus of the S-matrix element (the total $\chi^2/\text{NDF} = 171.715/(189-29) \approx 1.07$). Obtained parameters of resonances largely coincide with the values cited as estimations of the PDG, though a negative phase-shift in the background on the $\pi\pi$ threshold arises.

Table: Pole clusters for resonances on the complex $\sqrt{s}$-plane in the analysis of only $\pi\pi$-scattering. Pole positions $\sqrt{s_r} = \mathrm{E}_r - i\Gamma_r/2$ in MeV are shown.

Sheet		II	III	IV
$f_0(600)$	E_r	447.5±5.9	492.7±36.0	
	$\Gamma_r/2$	267.0±6.5	307.8±16.5	
$f_0(980)$	E_r	1001.1±3.7	979.1±12.0	
	$\Gamma_r/2$	20.3±2.6	38.5±7.1	
$f_0(1370)$	E_r		1375.8±51.5	1301.1±47.9
	$\Gamma_r/2$		179.5±36.5	224.0±49.3
$f_0(1500)$	E_r		1498.8±39.3	1503.7±45.1
	$\Gamma_r/2$		51.8±43.3	56.5±39.4
$f_0'(1500)$	E_r	1511.4±11.2	1499.8±104.3 1509.1±119.4	1505.9±38.5
	$\Gamma_r/2$	200.5±11.0	310.5±58.8 241.0±63.8	168.1±40.6
$f_0(1710)$	E_r		1700.3±31.2	1720.1±32.2
	$\Gamma_r/2$		58.6±26.4	64.9±30.1

In the analysis, the $f_0(600)$ and $f_0(980)$ are described by the clusters of type (**a**); $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$, type (**b**); $f_0'(1500)$, type (**c**).

The received background parameters are: $\underline{a_{11} = -0.0895 \pm 0.0030}$, $a_{1\eta} = 0.04 \pm 0.03$, $a_{1\sigma} = 0.0 \pm 0.8$, $a_{1v} = 0.0 \pm 0.7$, $b_{11} = 0.0 \pm 0.007$, $b_{1\eta} = 0.0 \pm 0.01$, $b_{1\sigma} = 0.0 \pm 0.02$, $b_{1v} = 0.054 \pm 0.036$.

For the $f_0(600)$ the found pole on sheet II coincides practically with the value $450 - i275$ MeV which was found in the dispersive $\pi\pi$-scattering data analyses which considered the region from the threshold to 1400÷1500 MeV (R.García-Martín et al., PR D**83** (2011) 074004; G.Colangelo et al., NP B**603** (2001) 125 (2001); B. Ananthanarayan et al., Phys.Rept. **353** (2001) 207).
The parameters of the $f_0(1370)$, narrow $f_0(1500)$ and $f_0(1710)$ are very near to the estimations of the PDG.
More remarkable differences were only for the mass of $f_0(980)$ (1001 MeV against 980±10 MeV of PDG tables of 2010) and the existence the wide $f_0'(1500)$.
However, the mass of $f_0(980)$ slightly above 1 GeV was also obtained in other works which analyzed the $\pi\pi$ scattering (e.g. R.García-Martín et al., PRL **107** (2011) 072001). In the quite recent PDG tables of 2012, for the mass of $f_0(980)$ there is indicated already the estimation 990±20 MeV (J.Beringer et al. (PDG), PR D**86** (2012) 010001.

The obtained $\pi\pi$ scattering length $a_0^0 = 0.222 \pm 0.008\ m_{\pi^+}^{-1}$ is also in the very good agreement with the experimental results and with the ChPT calculations.

However, this analysis displays two important flaws:

1. The negative phase-shift in the background beginning from the $\pi\pi$ threshold ($\underline{a_{11} = -0.0895}$) is necessary for a successful description of the data. This should not be the case because, in the uniformizing variable, we have taken into account the left-hand branch-point at $s = 0$ which gives a main contribution to the $\pi\pi$ background below the $K\overline{K}$ threshold. Other possible contributions of the left-hand cut related with exchanges by the nearest mesons – the ρ-meson and the $f_0(600)$ – practically obliterate each other (Yu.S.Surovtsev et al., EPJ A**15** (2002) 409) because vector and scalar particles contribute with the opposite signs due to the gauge invariance.
2. The description of the data on reaction $\pi\pi \to K\overline{K}$, using the same parameters of resonances as in the $\pi\pi$ channel, is satisfactory only for the phase shift ϕ_{12} which is due to the approximation of the left-hand cut in S_{12} and S_{22}, beginning at $s = 4(m_K^2 - m_\pi^2)$, by the 4th-power pole on the v-plane. The module of the S-matrix element η_{12} is described well only from the $K\overline{K}$ threshold up to about 1.15 GeV as it should be due to the two-channel unitarity. Above this energy the description fails even qualitatively.

We conclude: If the data are consistent, for obtaining more correct parameters of wide resonances the combined analysis of data on coupled processes $\pi\pi \to \pi\pi, K\overline{K}$ is needed.

Further that analysis is performed. The resonances are described by pole clusters of the same types as in the analysis only of the $\pi\pi$ scattering: the $f_0(600)$ and $f_0(980)$ are described by the clusters of type (**a**); $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$, type (**b**); $f_0'(1500)$, type (**c**).

The data for the $\pi\pi$ scattering below 1 GeV admit two satisfactory solutions for the phase shift – A and B – which mutually differ mainly in the pole position on sheet II for the $f_0(600)$. The total χ^2/NDF is $416.887/(312-40) \approx 1.53$ for the A-solution and ≈ 1.44 for the B-solution.

The obtained background parameters for the A-solution are: $\underline{a_{11} = 0.0 \pm 0.003}$, $a_{1\eta} = -0.1004 \pm 0.0301$, $a_{1\sigma} = 0.2148 \pm 0.0822$, $a_{1v} = 0.0 \pm 0.07$, $b_{11} = b_{1\eta} = b_{1\sigma} = 0$, $b_{1v} = 0.012 \pm 0.0287$, $a_{21} = -0.919 \pm 0.107$, $a_{2\eta} = -1.399 \pm 0.348$, $a_{2\sigma} = 0.0 \pm 0.7$, $a_{2v} = -11.45 \pm 0.75$, $b_{21} = 0.0747 \pm 0.0503$, $b_{2\eta} = b_{2\sigma} = 0$, $b_{2v} = 4.83 \pm 1.94$;
for B-solution: $\underline{a_{11} = 0.0 \pm 0.003}$, $a_{1\eta} = -0.0913 \pm 0.0327$, $a_{1\sigma} = 0.1707 \pm 0.0899$, $a_{1v} = 0.0 \pm 0.07$, $b_{11} = b_{1\eta} = b_{1\sigma} = 0$, $b_{1v} = 0.006 \pm 0.029$, $a_{21} = -1.338 \pm 0.111$, $a_{2\eta} = -1.119 \pm 0.376$, $a_{2\sigma} = 0.0 \pm 0.8$, $a_{2v} = -12.13 \pm 0.77$, $b_{21} = 0.018 \pm 0.050$, $b_{2\eta} = b_{2\sigma} = 0$, $b_{2v} = 4.48 \pm 1.98$.

Further in Table, the obtained pole-clusters for resonances are shown on the $\sqrt{s}$-plane. The pole positions $\sqrt{s_r} = \mathrm{E}_r - i\Gamma_r/2$ are given in the MeV units.

Solution	Sheet		II	III		IV
A	$f_0(600)$	E_r	517.0±7.8	458.5±14.7		
		$\Gamma_r/2$	393.9±6.0	205.9±4.7		
	$f_0(980)$	E_r	1004.6±3.9	995.5±10.1		
		$\Gamma_r/2$	25.0±2.3	96.9±2.7		
	$f_0(1370)$	E_r		1351.5±32.5		1342.9±12.2
		$\Gamma_r/2$		369.0±45.7		221.6±30.7
	$f_0(1500)$	E_r		1498.7±5.8		1501.1±6.4
		$\Gamma_r/2$		56.7±5.6		56.6±6.0
	$f_0'(1500)$	E_r	1532.2±12.4	1489.1±16.2	1515.9±29.2	1519.3±18.7
		$\Gamma_r/2$	323.2±21.0	217.9±10.2	388.4±22.6	339.5±42.2
	$f_0(1710)$	E_r		1701.9±31.8		1717.0±34.9
		$\Gamma_r/2$		77.8±18.0		72.9±16.2
B	$f_0(600)$	E_r	550.6±9.0	664.5±12.1		
		$\Gamma_r/2$	502.1±7.2	188.2±2.6		
	$f_0(980)$	E_r	1003.2±3.0	995.4±7.3		
		$\Gamma_r/2$	28.9±2.0	96.7±2.7		
	$f_0(1370)$	E_r		1353.8±27.9		1336.7±14.1
		$\Gamma_r/2$		367.4±37.4		251.9±27.5
	$f_0(1500)$	E_r		1499.5±6.0		1500.3±6.3
		$\Gamma_r/2$		56.5±6.1		57.0±6.4
	$f_0'(1500)$	E_r	1528.4±12.5	1491.3±15.8	1510.8±29.1	1515.6±17.0
		$\Gamma_r/2$	328.0±20.2	217.9±8.0	388.3±16.3	340.3±34.9
	$f_0(1710)$	E_r		1703.1±31.5		1722.0±35.7
		$\Gamma_r/2$		81.7±19.9		92.3±20.3

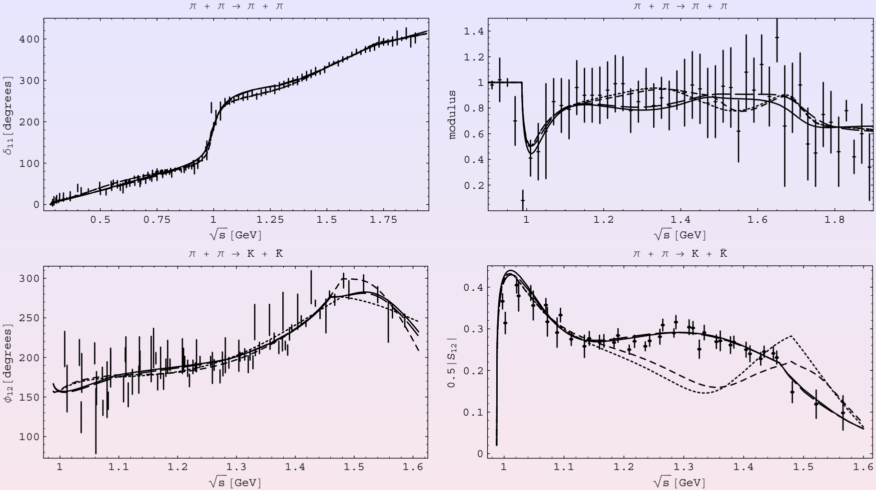


Figure: The S-wave phase shifts and modules of the $\pi\pi$-scattering and $\pi\pi \to K\overline{K}$ matrix elements. The dotted and short-dashed lines correspond to the analyses only of $\pi\pi$-scattering without and with the narrow $f_0(1500)$, respectively; in the lower row those curves give energy behaviour of the phase shift and module of the $\pi\pi \to K\overline{K}$ matrix element which are calculated using the resonance parameters from the analysis of only $\pi\pi$-scattering. The long-dashed and solid lines correspond to solutions A and B of the combined analysis of $\pi\pi \to \pi\pi, K\overline{K}$, respectively.

a_0^0 $[m_{\pi^+}^{-1}]$	Remarks	References
0.222 ± 0.008	Analysis only of $\pi\pi$ scattering	This paper
0.230 ± 0.004 0.282 ± 0.003	A-solution B-solution	This paper: combined analysis of processes $\pi\pi \to \pi\pi, K\overline{K}$
0.26 ± 0.05	Analysis of the $K \to \pi\pi e\nu$ using Roy's equation	L.Rosselet et al., PR D (1977)
0.24 ± 0.09	Analysis of $\pi^- p \to \pi^+\pi^- n$ using the effective range formula	A.A.Bel'kov et al., Pisma ZhETF (1979)
$0.2220 \pm 0.0128_{\rm stat}$ $\pm 0.0050_{\rm syst} \pm 0.0037_{\rm th}$	Experiment on K_{e4} decay	J.R.Batley et al., EPJ C (2010)
0.220 ± 0.005	ChPT + Roy's equations	G.Colangelo et al., PL B (2000); Phys.Rept. (2001)
0.220 ± 0.008	Dispersion relations and K_{e4} data	R.García-Martín et al., PR D (2011)
0.26	NJL model	M.K.Volkov, Sov.J.Part.Nucl. (1986)
0.28	Variant of the NJL model	A.N.Ivanov, N.Troitskaya, NC A (1995)

The obtained value for a_0^0 for the A-solution is in accordance with the ChPT predictions (non-linear realization of chiral symmetry), whereas the value for the B-solution agrees with predictions of models of NJL type (linear realization). If one considers the problem of precise determination of the $\pi\pi$ scattering length a_0^0 to be solved allowing for the results of the NA48/2 Collaboration (J.R.Batley et al., EPJ C**70** (2010) 635) and the DIRAC experiment at CERN (B.Adeva et al., PL B**704** (2011) 24), then the A-solution should be chosen.

In the combined analysis of data on coupled processes both above-indicated important flaws, related to the analysis of only $\pi\pi$-scattering, are cured.
Now the $\pi\pi$ background below the $K\overline{K}$ threshold is absent ($\underline{a_{11} = 0.0}$) because its contribution is practically completely accounted for by the left-hand branch-point at $s = 0$ included explicitly in the uniformizing variable. An arising pseudo-background at the $\eta\eta$ threshold ($a_{1\eta} < 0$) is also clear: *this is a direct indication to consider explicitly the $\eta\eta$-threshold branch-point.*
This was already done in our work Yu.S.Surovtsev, P.Bydžovský, V.E.Lyubovitskij, PR D**85** (2012) 036002. The 3-channel combined analysis of data on processes $\pi\pi \to \pi\pi, K\overline{K}, \eta\eta(\eta\eta')$ was carried out. As in the above 2-channel combined analysis, two solutions (A and B), mainly related with parameters of the $f_0(600)$, were found. Furthermore, it was shown that there are four scenarios of representation of resonances $f_0(1370)$, $f_0(1500)$ (as the superposition of two states, broad and narrow) and $f_0(1710)$ giving the similar description of the above processes and, however, the quite different parameters of some resonances. For the $f_0(600)$, $f_0(1370)$ and $f_0(1710)$, a spread of values is obtained for the masses and widths 605-735 and 567-686 MeV, 1326-1404 and 223-345 MeV, and 1751-1759 and 118-207 MeV, respectively. Thereat $f_0(600)$ and $f_0(980)$ are given by the pole clusters of the same types in all cases.
In order to diminish the number of possible scenarios, it is required to enlarge the analysis adding data on the relevant decays.

The combined 3-channel analysis of data on isoscalar S-wave processes $\pi\pi \to \pi\pi, K\overline{K}, \eta\eta$ and on decays $J/\psi \to \phi\pi\pi, \phi K\overline{K}$

Enlarging our analysis, to the data on processes $\pi\pi \to \pi\pi, K\overline{K}, \eta\eta$ we added also data on decays $J/\psi \to \phi\pi\pi, \phi K\overline{K}$ from Mark III (W.Lockman, Hadron'89, Proceedings, p.109), from DM2 (A.Falvard et al., PR D**38** (1988) 2706), and from BES (M.Ablikim et al., PL B**607** (2005) 243).

The 3-channel S-matrix is determined on the 8-sheeted Riemann surface. That function can be uniformized only on torus with the help of a simple mapping. This is unsatisfactory for our purpose. Therefore, we neglect the influence of the lowest ($\pi\pi$) threshold branch-point (however, unitarity on the $\pi\pi$ cut is taken into account). This approximation means the consideration of the nearest to the physical region semi-sheets of the Riemann surface of the S-matrix. In fact, we construct a 4-sheeted model of the initial 8-sheeted Riemann surface that is in accordance with our approach of a consistent account of the nearest singularities on all the relevant sheets.

In the corresponding uniformizing variable, we have neglected the $\pi\pi$-threshold branch-point and taken into account the $K\overline{K}$- and $\eta\eta$-threshold branch-points and the left-hand branch-point at $s = 0$:

$$w = \frac{\sqrt{(s-s_2)s_3} + \sqrt{(s-s_3)s_2}}{\sqrt{s(s_3-s_2)}} \qquad (s_2 = 4m_K^2 \text{ and } s_3 = 4m_\eta^2).$$

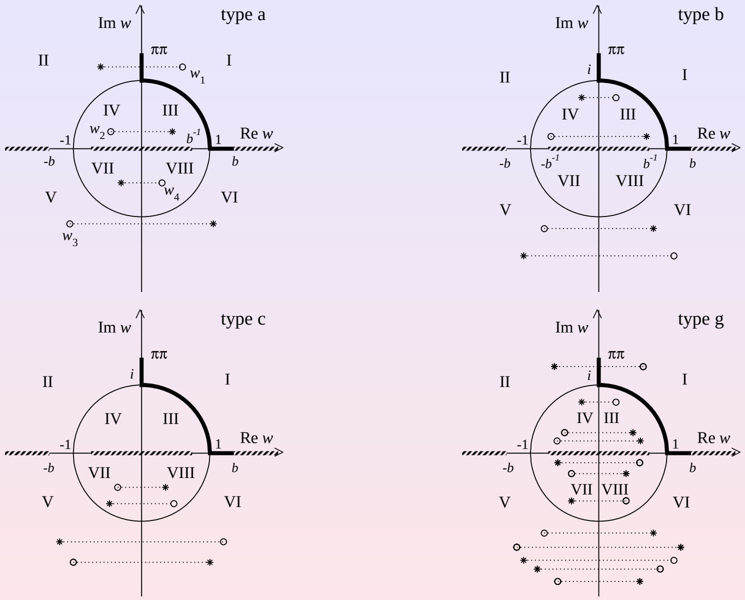


Figure: Uniformization w-plane for the 3-channel-$\pi\pi$-scattering amplitude: Representation of resonances of types (**a**), (**b**), (**c**) and (**g**).

On the w-plane, the Le Couteur-Newton relations are

$$S_{11} = \frac{d^*(-w^*)}{d(w)}, \qquad S_{22} = \frac{d(-w^{-1})}{d(w)}, \qquad S_{33} = \frac{d(w^{-1})}{d(w)},$$

$$S_{11}S_{22} - S_{12}^2 = \frac{d^*({w^*}^{-1})}{d(w)}, \qquad S_{11}S_{33} - S_{13}^2 = \frac{d^*(-{w^*}^{-1})}{d(w)}.$$

$$d = d_B d_{res}, \qquad d_{res}(w) = w^{-\frac{M}{2}} \prod_{r=1}^{M} (w + w_r^*)$$

M is the number of resonance zeros.

$$d_B = \exp[-i(a + \sum_{n=1}^{3} \frac{\sqrt{s - s_n}}{2m_n}(\alpha_n + i\beta_n))],$$

$$\alpha_n = a_{n1} + a_{n\sigma}\frac{s - s_\sigma}{s_\sigma}\theta(s - s_\sigma) + a_{nv}\frac{s - s_v}{s_v}\theta(s - s_v),$$

$$\beta_n = b_{n1} + b_{n\sigma}\frac{s - s_\sigma}{s_\sigma}\theta(s - s_\sigma) + b_{nv}\frac{s - s_v}{s_v}\theta(s - s_v).$$

s_σ is the $\sigma\sigma$ threshold; s_v is the combined threshold of the $\eta\eta'$, $\rho\rho$, $\omega\omega$ channels.

Formalism for calculating di-meson mass distributions of the decays $J/\psi \to \phi\pi\pi, \phi K\overline{K}$ can be found in Refs. (D.Morgan, M.R.Pennington, PR D**48** (1993) 1185; PR D**48** (1993) 5422; B.S.Zou, D.V.Bugg, PR D**50** (1994) 591).
There is assumed that pairs of pseudo-scalar mesons of final states have $I = J = 0$ and only they undergo strong interactions, whereas the ϕ meson acts as a spectator.
The amplitudes for $J/\psi \to \phi\pi\pi, \phi K\overline{K}$ decays are related with the scattering amplitudes T_{ij} $i,j = 1 - \pi\pi, 2 - K\overline{K}$ as follows

$$F(J/\psi \to \phi\pi\pi) = \sqrt{2/3}\,[c_1(s)T_{11} + c_2(s)T_{21}],$$

$$F(J/\psi \to \phi K\overline{K}) = \sqrt{1/2}\,[c_1(s)T_{12} + c_2(s)T_{22}]$$

where $c_i = \alpha_i/(s-\beta_i) + \gamma_{i0} + \gamma_{i1}s$ are functions of couplings of the J/ψ to channel i; α_i, β_i, γ_{i0} and γ_{i1} are free parameters. The first term in c_i is related to the on-shell manifestation of Adler zero.

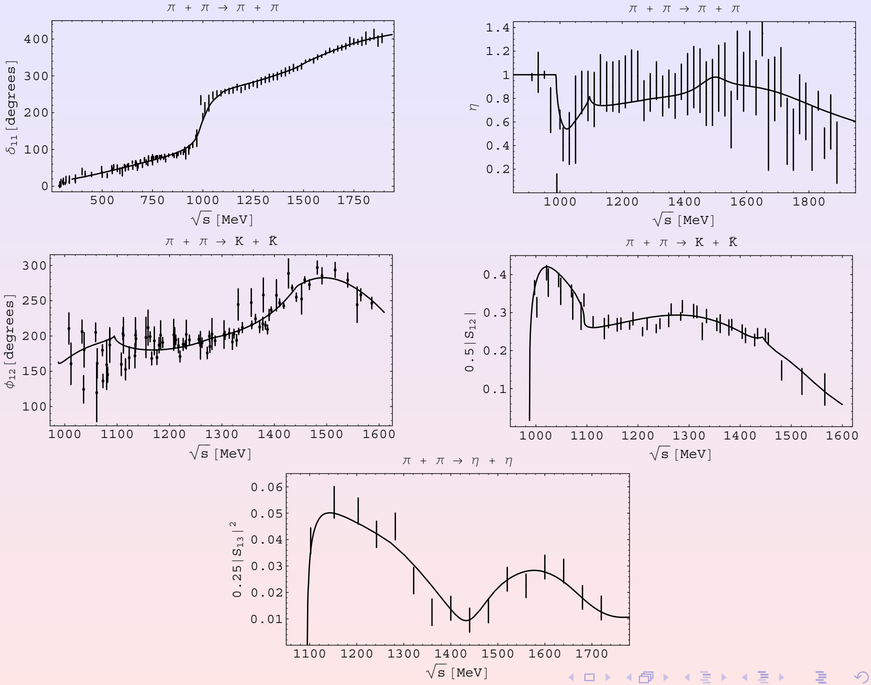
$$N|F|^2\sqrt{(s-s_i)(m_\psi^2 - (\sqrt{s}-m_\phi)^2)(m_\psi^2 - (\sqrt{s}+m_\phi)^2)}$$

gives the di-meson mass distributions. N (normalization to experiment) is 0.73 for Mark III and 0.28 for DM2. Parameters of the c_i-functions, obtained in the analysis, are $\alpha_1, \beta_1 \alpha_2, \beta_2 = 0.0306,\ 0.0646,\ 0.0222,\ 0.0701$ and $\gamma_{10}, \gamma_{11}, \gamma_{20}, \gamma_{21} = 1.6148, 1.3169, -1.0962, -1.64.$

In the analysis the $f_0(600)$ is described by the cluster of type (**a**); $f_0(1500)$, type (**c**) and $f_0'(1500)$, type (**g**); the $f_0(980)$ is represented only by the pole on sheet II and shifted pole on sheet III. However, the $f_0(1370)$ and $f_0(1710)$ can be described by clusters either of type (**b**) or (**c**). Analyzing only $\pi\pi \to \pi\pi, K\overline{K}, \eta\eta(\eta\eta')$ (Yu.S.Surovtsev, P.Bydžovský, V.E.Lyubovitskij, PR D**85** (2012) 036002), it is impossible to prefer surely any of four indicated possibilities; moreover, the data admit two possibilities for parameters of the $f_0(600)$ with mass, relatively near to the ρ-meson mass, and with the total widths about 600 and 950 MeV.

When adding to the combined analysis the data on decays $J/\psi \to \phi\pi\pi, \phi K\overline{K}$, one can give some preference for scenarios when the $f_0(1370)$ is described by the cluster of type (**b**), $f_0(1710)$ either of type (**b**) or (**c**). Further, for definiteness, we shall tell about the case when the $f_0(1710)$ is represented by the cluster of type (**c**).

It is interesting that *the di-pion mass distribution of the $J/\psi \to \phi\pi\pi$ decay of the BES data from the threshold to about 0.85 GeV prefers surely the solution with the more wide $f_0(600)$* – B-solution. Satisfactory combined description of all analyzed processes is obtained with the total χ^2/NDF $= 424.317/(389-55) \approx 1.26$; for the $\pi\pi$ scattering, χ^2/NDF ≈ 1.20; for $\pi\pi \to K\overline{K}$, χ^2/NDF ≈ 1.64; for $\pi\pi \to \eta\eta$, χ^2/ndp ≈ 0.82; for decays $J/\psi \to \phi\pi\pi, \phi K\overline{K}$, χ^2/NDF ≈ 1.55.



π + π → π + π
δ11 [degrees]
√s [MeV]
π + π → π + π
η
√s [MeV]
π + π → K + K̄
φ12 [degrees]
√s [MeV]
π + π → K + K̄
0.5 |S12|
√s [MeV]
π + π → η + η
0.25 |S13|^2
√s [MeV]

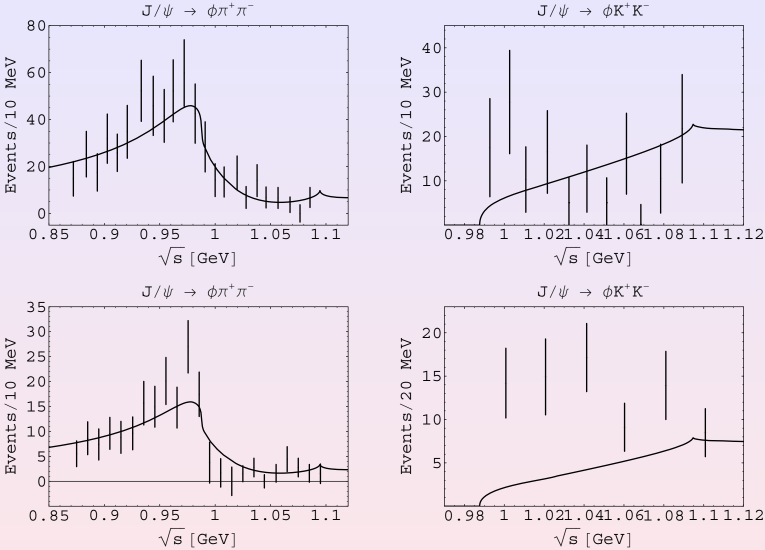


Figure: The $J/\psi \to \phi\pi\pi, \phi K\overline{K}$ decays. The upper panel shows the fit to data of Mark III, the lower to DM2.

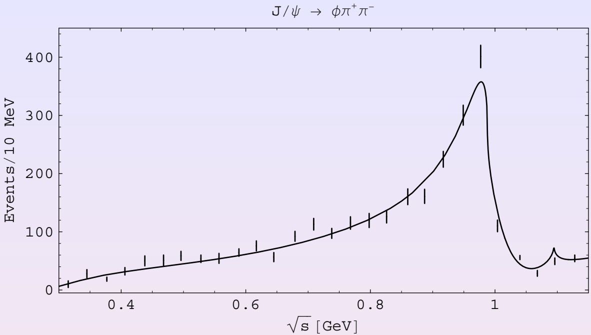


Figure: The $J/\psi \to \phi\pi\pi$ decay; the data of BES Collaboration.

The obtained background parameters are: $a_{11} = 0.0$, $a_{1\sigma} = 0.0198$, $a_{1v} = 0.0$, $b_{11} = b_{1\sigma} = 0.0$, $b_{1v} = 0.0336$, $a_{21} = -2.4808$, $a_{2\sigma} = -2.3021$, $a_{2v} = -6.62$, $b_{21} = b_{2\sigma} = 0.0$, $b_{2v} = 6.99$, $b_{31} = 0.6432$, $b_{3\sigma} = 0.489$, $b_{2v} = 0$; $s_\sigma = 1.6384\ \mathrm{GeV}^2$, $s_v = 2.0851\ \mathrm{GeV}^2$.

The obtained very simple description of the $\pi\pi$-scattering background confirms well our assumption: $d = d_B d_{res}$. Moreover, this shows that *the consideration of the left-hand branch-point at $s = 0$ in the uniformizing variable solves partly a problem (see, e.g., N.N.Achasov, G.N.Shestakov, PR D**49** (1994) 5779) that the wide-resonance parameters are strongly controlled by the non-resonant background.*

Table: The pole clusters for resonances on the $\sqrt{s}$-plane. $\sqrt{s_r} = \mathrm{E}_r - i\Gamma_r/2$ [MeV].

Sheet		$f_0(600)$	$f_0(980)$	$f_0(1370)$	$f_0(1500)$	$f_0'(1500)$	$f_0(1710)$
II	E_r	521.6 ± 12.4	1008.4 ± 3.1			1512.4 ± 4.9	
	$\Gamma_r/2$	467.3 ± 5.9	33.5 ± 1.5			287.2 ± 12.9	
III	E_r	552.5 ± 17.7	976.7 ± 5.8	1387.2 ± 24.4		1506.1 ± 9.0	
	$\Gamma_r/2$	467.3 ± 5.9	53.2 ± 2.6	167.2 ± 41.8		127.8 ± 10.6	
IV	E_r			1387.2±24.4		1512.4±4.9	
	$\Gamma_r/2$			178.2 ± 37.2		215.0 ± 17.6	
V	E_r			1387.2±24.4	1493.9 ± 3.1	1498.8 ± 7.2	1732.8 ± 43.2
	$\Gamma_r/2$			261.0 ± 73.7	72.8 ± 3.9	142.3 ± 6.0	114.8 ± 61.5
VI	E_r	573.4 ± 29.1		1387.2±24.4	1493.9 ± 5.6	1511.5 ± 4.3	1732.8±43.2
	$\Gamma_r/2$	467.3 ± 5.9		250.0 ± 83.1	58.4 ± 2.8	179.3 ± 4.0	111.2 ± 8.8
VII	E_r	542.5 ± 25.5			1493.9 ± 5.0	1500.4 ± 9.3	1732.8±43.2
	$\Gamma_r/2$	467.3 ± 5.9			47.8 ± 9.3	99.9 ± 18.0	55.2 ± 38.0
VIII	E_r				1493.9 ± 3.2	1512.4±4.9	1732.8±43.2
	$\Gamma_r/2$				62.2 ± 9.2	298.4 ± 14.5	58.8 ± 16.4

Generally, *wide multi-channel states are most adequately represented by pole clusters*, because the pole clusters give the main model-independent effect of resonances. The pole positions are rather stable characteristics for various models, whereas masses and widths are very model-dependent for wide resonances.

However, mass values are needed in some cases, e.g., in mass relations for multiplets. Therefore, we stress that such parameters of the wide multi-channel states, as *masses, widths and coupling constants with channels, should be calculated using the poles on sheets II, IV, VIII*, because only on these sheets the analytic continuations have the forms:

$$\propto 1/S_{11}^{\mathrm{I}}, \quad \propto 1/S_{22}^{\mathrm{I}}, \quad \propto 1/S_{33}^{\mathrm{I}},$$

respectively, i.e., the pole positions of resonances are at the same points of the complex-energy plane, as the resonance zeros on the physical sheet, and are not shifted due to the coupling of channels. E.g., if

$$T^{res} = \sqrt{s}\,\Gamma_{el}/(m_{res}^2 - s - i\sqrt{s}\,\Gamma_{tot}),$$

$$m_{res} = \sqrt{\mathrm{E}_r^2 + (\Gamma_r/2)^2} \quad \text{and} \quad \Gamma_{tot} = \Gamma_r,$$

where the pole position $\sqrt{s_r} = \mathrm{E}_r - i\Gamma_r/2$ must be taken on sheets II, IV, VIII, depending on the resonance classification.

	$f_0(600)$	$f_0(980)$	$f_0(1370)$	$f_0(1500)$	$f_0'(1500)$	$f_0(1710)$
m_{res}[MeV]	700.3±10.0	1009.0±3.1	1398.6±24.7	1495.2±3.2	1539.4±5.4	1733.8±43.2
Γ_{tot}[MeV]	934.6±11.8	67.0±3.0	356.4±74.4	124.4±18.4	574.4±25.8	117.6±32.8

- The Riemann-surface structure of the S-matrix of considered processes must be allowed for properly. For calculating masses, total widths and coupling constants of resonances with channels, one must use the poles on sheets II, IV and VIII, depending on the resonance type.
- When considering the $\pi\pi$ scattering and $\pi\pi \to K\overline{K}$ in the 2-channel approach, it is shown that the analysis of only $\pi\pi$ scattering data, gives an excellent description from the threshold to 1.89 GeV with the resonance parameters and $\pi\pi$-scattering length being the same as the ones indicated in the PDG tables as estimations. However, (1) the $\pi\pi \to K\overline{K}$ are not well described even qualitatively above 1.15 GeV when using the resonance parameters from the only $\pi\pi$ scattering analysis, and (2) in this case, the description of $\pi\pi$ background is unsatisfactory (pseudo-background arises). I.e., a combined analysis of $\pi\pi \to \pi\pi, K\overline{K}$ is needed, which also is carried out satisfactorily, curing flaws (1) and partly (2). The resonance parameters are inevitably changed. The remaining pseudo-background arising at the $\eta\eta$ threshold indicates clearly that it is necessary to consider explicitly also the $\eta\eta$-threshold branch-point in the 3-channel analysis.

- In the combined model-independent analysis of data on $\pi\pi \to \pi\pi, K\overline{K}, \eta\eta$ and on $J/\psi \to \phi\pi\pi, \phi K\overline{K}$ from Mark III, DM2 and BES Collaborations, an additional confirmation of the $f_0(600)$ with mass about 700 MeV and width 930 MeV is obtained. This mass value accords with prediction ($m_\sigma \approx m_\rho$) on the basis of mended symmetry by Weinberg (S.Weinberg, PRL **65** (1990) 1177) and with a refined analysis using the large-N_c consistency conditions between the unitarization and resonance saturation suggesting $m_\rho - m_\sigma = O(N_c^{-1})$ (J.Nieves, E.Ruiz Arriola, PR D**80** (2009) 045023).
- Indication for $f_0(980)$ is obtained to be a non-$q\bar{q}$ state, e.g., the bound $\eta\eta$ state, because this state lies slightly above the $K\overline{K}$ threshold and is described by the pole on sheet II and by the shifted pole on sheet III without the corresponding poles on sheets VI and VII.
- The $f_0(1370)$ and $f_0(1710)$ have the dominant $s\bar{s}$ component. Conclusion about the $f_0(1370)$ quite agrees with the one of work of Crystal Barrel Collaboration (C.Amsler et al., PL B**355** (1995) 425) where the $f_0(1370)$ is identified as $\eta\eta$ resonance in the $\pi^0\eta\eta$ final state of the $\bar{p}p$ annihilation. This explains also quite well why one did not find this state considering only the $\pi\pi$ scattering (W.Ochs, arXiv:1001.4486v1 [hep-ph]; P.Minkowski, W.Ochs, EPJ C**9** (1999) 283; arXiv: hep-ph/0209223; hep-ph/0209225). Conclusion about the $f_0(1710)$ is consistent with the experimental facts that this state is observed in $\gamma\gamma \to K_S K_S$ (S.Braccini, Frascati Phys. Series **XV** (1999) 53) and not observed in $\gamma\gamma \to \pi^+\pi^-$ (R.Barate et al., PL B**472** (2000) 189).

- In the 1500-MeV region, there are two states: the $f_0(1500)$ ($m_{res} \approx 1495$ MeV, $\Gamma_{tot} \approx 124$ MeV) and the $f_0'(1500)$ ($m_{res} \approx 1539$ MeV, $\Gamma_{tot} \approx 574$ MeV). The $f_0'(1500)$ is interpreted as a glueball taking into account its biggest width among enclosing states (V.V.Anisovich et al., NP Proc.Suppl. A**56** (1997) 270).
- We propose the following assignment of the scalar mesons to lower nonets, excluding the $f_0(980)$ as the non-$q\bar{q}$ state. The lowest nonet: the isovector $a_0(980)$, the isodoublet $K_0^*(800)$, and $f_0(600)$ and $f_0(1370)$ as mixtures of the 8th component of octet and the SU(3) singlet. The Gell-Mann–Okubo (GM-O) formula
$$3m_{f_8}^2 = 4m_{K_0^*}^2 - m_{a_0}^2$$
gives $m_{f_8} = 870$ MeV. In relation for masses of nonet
$$m_\sigma + m_{f_0(1370)} = 2m_{K_0^*(800)}$$
the left-hand side is by about 17% bigger than the right-hand one.
- For the next nonet we find: the isovector $a_0(1450)$, the isodoublet $K_0^*(1450)$, and two isoscalars $f_0(1500)$ and $f_0(1710)$. From the GM-O formula, $m_{f_8} \approx 1450$ MeV. In formula
$$m_{f_0(1500)} + m_{f_0(1710)} = 2m_{K_0^*(1450)}$$
the left-hand side is by about 11% bigger than the right-hand one.

- This assignment removes a number of questions, stood earlier, and does not put any new. The mass formulas indicate to non-simple mixing scheme. The breaking of 2nd mass relations tells us that the $\sigma - f_0(1370)$ and $f_0(1500) - f_0(1710)$ systems get additional contributions absent in the $K_0^*(900)$ and $K_0^*(1450)$, respectively. A search of the adequate mixing scheme is complicated by the fact that here there is also a remaining chiral symmetry, though, on the other hand, this permits one to predict correctly, *e.g.*, the σ-meson mass (S.Weinberg, PRL **65** (1990) 1177).

There is a number of properties of the scalar mesons, which do not allow one satisfactorily to make up the lowest nonet. The main of them is inaccordance of the approximately equal masses of the $f_0(980)$ and $a_0(980)$ and the found $s\bar{s}$ dominance in the wave function of the $f_0(980)$. If these states are in the same nonet, the $f_0(980)$ must be heavier than $a_0(980)$ for 250-300 MeV, because a difference of masses of s- and u-quarks is 120-150 MeV. We proposed our way to solve this problem.