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On manifestation of quark-hadron duality via the Adler D-function

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Outline

- ✓ Historical overview: the Adler function
- ✓ The Adler function: experiment and QCD model
- ✓ Motivation: *R*-related quantities
- ✓ Basic relations & *R-D* self-duality
- ✓ Example: hadronic contribution to the muon g-2
- ✓ Summary

To compare theoretical results and experimental data one often uses the concept of quark-hadron duality, which establishes a bridge between quarks and gluons, a language of theoreticians, and real measurements with hadrons performed by experimentalists.

The idea of quark-hadron duality was formulated by Poggio, Quinn, and Weinberg (1976) as follows: Inclusive hadronic cross sections, once they are appropriately averaged over an energy interval, must approximately coincide with the corresponding quantities derived from the quark-gluon picture

There are various areas of hadronic physics dealing with different manifestations of quark-hadron duality. In the talk we concentrate on physical quantities and functions which are defined through the Drell ratio, $R(s) = \sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$, and for which a corresponding interval of integration involves a low energy region (about 1 GeV and less) and therefore a standard perturbative QCD description can not be directly applied.

Clearly that it is fruitful to connect measured quantities with ``simplest'' theoretical objects. Some single-argument functions which are directly connected with experimentally measured quantities can play the role of these objects. The Adler D-function (Euclidean quantity) turned out to be a smooth function without traces of the resonance structure of the R(s) is a convenient object for comparing theoretical results with experimental data, and one can expect that it more precisely reflects the quark-hadron duality.

Short historical review

In 1974 S. Adler [Phys. Rev. D 10] observed that whereas R(s) (the normalized cross-section for the process e+e- annihilation into hadrons) is measured in the timelike region the natural place to compare experiment with scaling predictions of various theories in QCD is in the spacelike region and consequently he suggested to use the function T(−s) which is the first derivative of R(s).

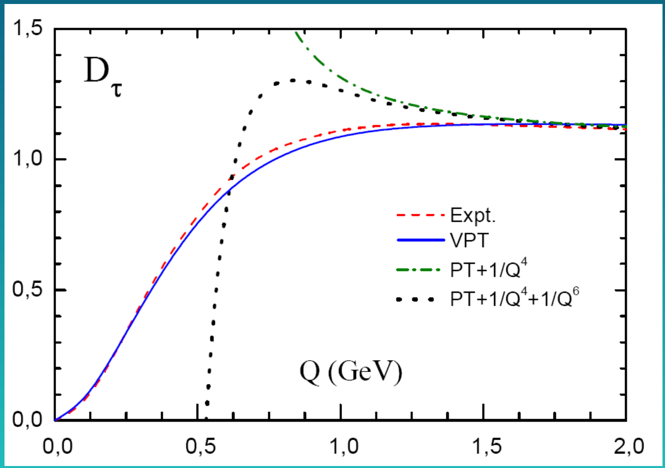
In 1976 A. De Rujula and H. Georgi [Phys. Rev. D 13] used a modified version of this idea, defining $D(s) = -sT(-s)$. ($-s=Q^2>0$ in the Euclidean region)

The function $D(Q^2)$ has been revived under the name of the ``Adler function".
The D-function was used in many papers, among which (more recent) are following:

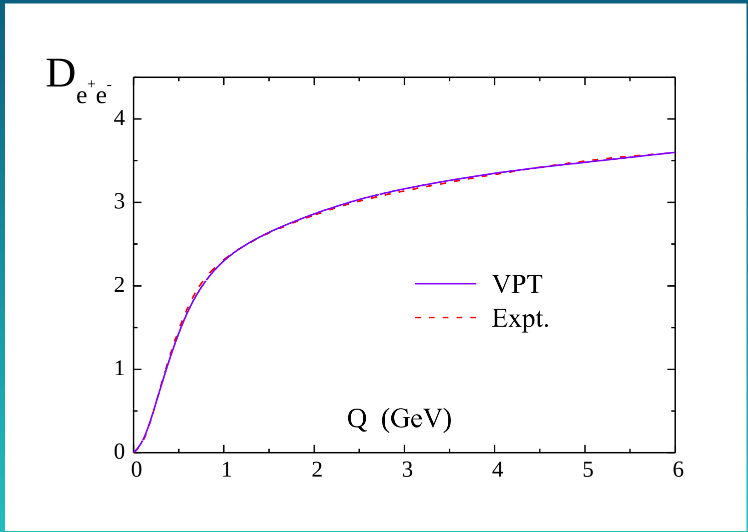
S. Peris, M. Perrottet, E. de Rafael, Matching long and short distances in large-N_c QCD, J. High Energy (1998),
S. Eidelman, F. Jegerlehner, A.I. Kataev, O. Veretin, Testing non-perturbative strong interaction effects via the Adler function, **Phys. Lett. B** (1999),
K. A. Milton, I. L. Solovtsov, O. P. Solovtsova, **Adler function for light quarks in analytic perturbation theory**, Phys. Rev. D (2001),
A.E. Dorokhov, Adler function and hadronic contribution to the muon g-2 in a nonlocal chiral quark model, Phys. Rev. D (2004),
A.L. Kataev, Is it possible to check urgently the 5-loop analytical results for the $e^+ e^-$ - annihilation Adler function? **Phys. Lett. B (2008),**
A.V. Nesterenko, On the low-energy behavior of the Adler function, Phys. Rev. D (2008),
F. Jegerlehner, The running fine structure constant via the Adler function, hep-ph:0807.4206 (2008),
P.A. Baikov, K.G. Chetyrkin, J. H. Kuhn, Hadronic Z and τ decays in order α_s^4, Phys. Rev. Lett. (2008),
B.A. Magradze, Testing the Concept of Quark-Hadron Duality with the ALEPH τ Decay Data., Few Body Syst. (2010),
T.Goecke, C.S.Fischer, R. Williams, Leading-order calculation of hadronic contributions to the muon g-2 using the Dyson-Schwinger approach, **Phys. Lett. B (2011),**
P.A. Baikov, K.G. Chetyrkin, J.H. Kuhn, J. Rittinger, Adler Function, Sum Rules and Crewther Relation of Order $O(\alpha_s^4)$: the Singlet Case. Phys. Lett. B (2012).

Behaviour of Adler D-function

- **A. N. Sissakian, I. L. Solovtsov, O. P. Solovtsova, A Nonperturbative *a* -Expansion Technique and the Adler *D* –function, *JETP Letts. (2001)***



Expt. and PT + OPE curves from paper by S. Peris, M. Perrottet, and E. de Rafael, JHEP 05, 011 (1998)



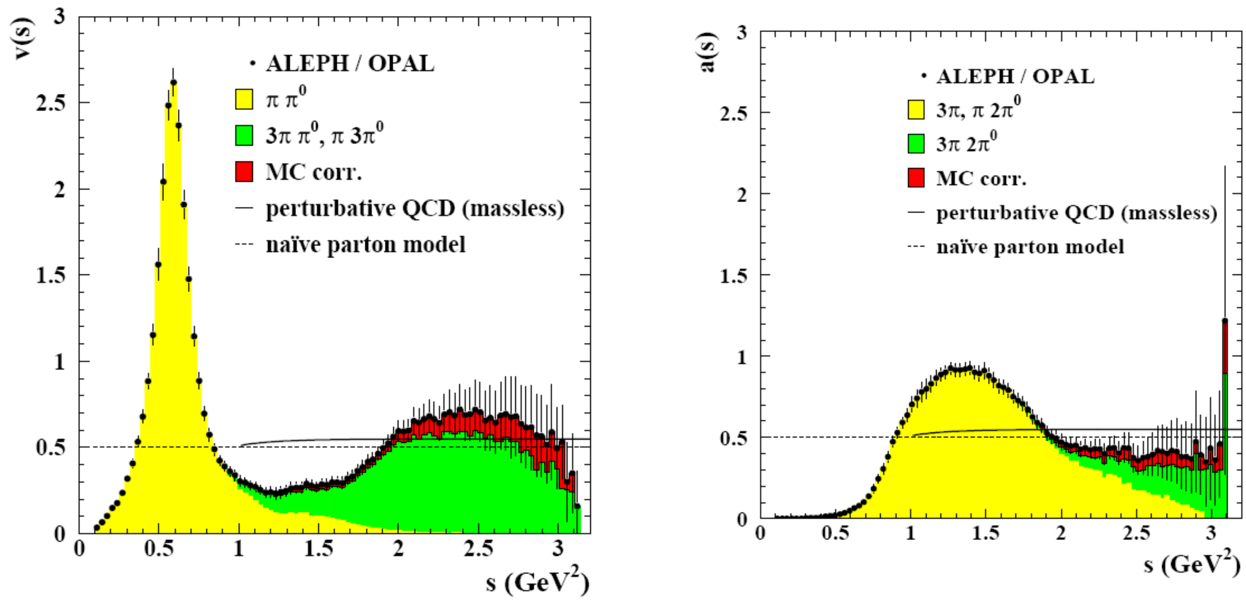
Data from the paper by S. Eidelman, F. Jegerlehner, A.I. Kataev, O. Veretin, Phys. Lett. B (1999)

The experimental *D*-function (dashed curve) turned out to be a smooth and monotone function without traces of the resonance structure.

The theoretical approach VPT *) which we used in this study (solid line) describes experimental curves rather well for the whole interval including the infrared region. Note that any finite order of the operator product expansion (OPE) fails to describe the infrared tail of the *D*-function (dotted curve) .

***) Variation Perturbation Theory (VPT) is the ghost-free modification of pQCD [A.N. Sissakian, I.L. Solovtsov, «*Variational expansions in quantum chromodynamics*», Phys. Part. Nucl. 30, 461-487 (1999)].**

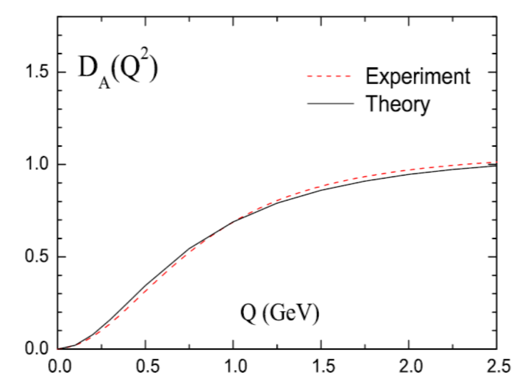
ALEPH/OPAL Spectral Functions



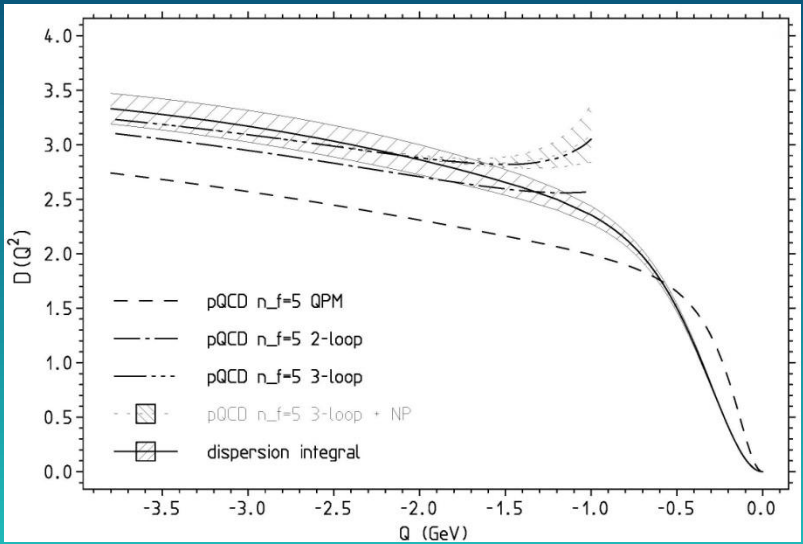
$$v(s)/a(s) = \frac{1}{2} R_{V/A}$$

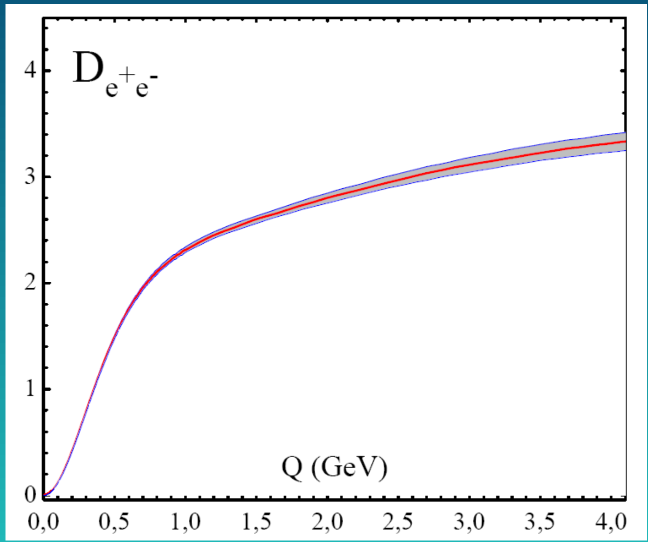
$$D_{V/A}^{\mathrm{exp/theor}}(Q^2) = Q^2 \int_0^\infty ds \, \frac{R_{V/A}^{\mathrm{exp/theor}}(s)}{(s+Q^2)^2} \, .$$

$$R_{V/A}(s) = R_{V/A}^{\mathrm{exp}}(s)\,\theta(s_0 - s) + R_{V/A}^{\mathrm{theor}}(s)\,\theta(s - s_0)\,,$$



The Adler function from $e^+e^- \to$ hadrons data





Figures from the paper S. Eidelman, F. Jegerlehner, A.I. Kataev, O. Veretin, Phys. Lett. B (1999).

Data from F. Jegerlehner (2008)

Experimental errors are small.

The Adler function provides a way of comparing theoretical predictions from QCD with timelike experimental e+e− → hadrons data.

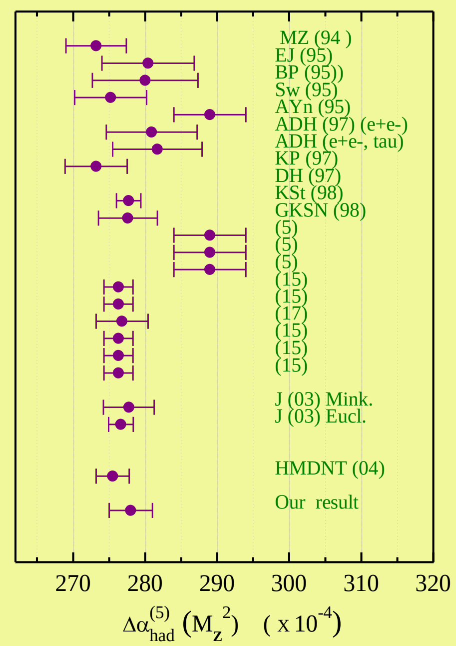
Some results

- The method has been proposed by **Igor Solovtsov** and has been reported at Baldin semina in 2006 (**see I.L. Solovtsov, Nonperturbative expantion method in QCD and R-related quantities, Proceedings of the XVIII International Baldin Seminar on High Energy Physics Problems Dubna, September 25-30, 2006, Editors: A. N. Sissakian V. V. Burov A. I. Malakhov, Vol. I, pp. 28-35.)**

Following quantities and functions have been investigated:

- R_τ^V – inclusive τ-decay characteristic in the vector channel;
- D – light Adler function;
- smeared R_Δ-function;
- a_μ^{had} – hadronic contribution to the anomalous magnetic moment of the muon;
- $\Delta\alpha_{\mathrm{had}}^{(5)}$ – hadronic contribution to the fine structure constant;

To this list we add a_e^{had} and a_τ^{had}



Hadronic contribution to the shift of the fine structure constant at the scale of the Z-bozon mass.

$$\Delta\alpha_{\mathrm{had}}^{(5)}(M_Z^2) = (279.9 \pm 4.0) \times 10^{-4}.$$

A common feature of all these quantities and functions is that they are defined through the function R(s) integrated with some other function.

By definition, all these quantities and functions include an infrared region as a part of the interval of integration and, therefore, they cannot be directly calculated within perturbative QCD.

About method

The method includes the non-perturbative approach based on the idea of variational perturbation theory which combines an optimization procedure of variational type with a regular method of calculating corrections. In the case of QCD the non-perturbative expansion parameter, a, obeys an equation whose solutions are always smaller than unity for any value of the original coupling constant. An important feature of this approach is the fact that for sufficiently small value of the running coupling the a -expansion reproduces the standard perturbative expansion, and, therefore, the perturbative high-energy physics is preserved. In moving to low energies, where ordinary perturbation theory breaks down, the parameter a remains small and we still stay within the region of applicability of the a-expansion method.

$$\lambda = \frac{g^2}{(4\pi)^2} = \frac{\alpha_s}{4\pi} = \frac{1}{C}\frac{a^2}{(1-a)^3}, \quad 0 \le a < 1$$

Summation of threshold singularities was performed.

Nonperturbative character of the light quark masses was taken into account.

Some numerical results

Hadronic contribution to the anomalous magnetic moment of the muon (in the leading order in electromagnetic coupling constant)

$$a_\mu^{\rm had} = \frac{1}{3}\left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty \frac{ds}{s} K(s)\, R(s)$$

Other results for $a_\mu \times 10^{-10}$:

$696.3 \pm 6.2_{\rm exp} \pm 3.6_{\rm rad}$	e^+e^--based [Davier]
$711.0 \pm 5.0_{\rm exp} \pm 0.8_{\rm rad} \pm 2.8_{SU(2)}$	τ-based [Davier]
$693.4 \pm 5.3_{\rm exp} \pm 3.5_{\rm rad}$	e^+e^--based [Hocker]
623.0 ± 40	Instanton liquid model [Dorokhov]
698 ± 13	APT-approach [Milton, Solovtsov, Solovtsova]

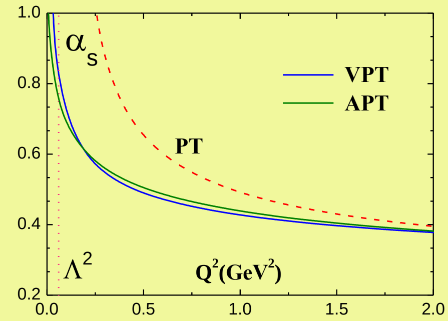
$$a_\mu^{\rm had} = (702 \pm 16) \times 10^{-10}.$$

VPT,Solovtsov, 2006

694.9 (3.7) – expt. (one of set of results)

Note

The method based on the analytic perturbation theory (APT) leads to very close result.



D.V. Shirkov and I.L.Solovtsov, «Analytic QCD running coupling with finite IR behavior and universal alpha_s(0) value», **Phys. Rev. Lett. 79 (1997) 1209.**

It was obtained a good agreement between our results and the experimental data down to the lowest energy scale for both Minkowskian (timelike) and Euclidean (spacelike) quantities.
There is a question: Wy?

Basic relations for investigation

The main object in a description of many physical processes is the correlator $\Pi(q^2)$ /or the corresponding Adler function

$$\Pi_{\mu\nu}(q^2) = i \int d^4x \, e^{iqx} \, \langle 0 \left| T V_\mu(x) V_\nu(0)^+ \right| 0 \rangle$$

$$\propto (q_\mu q_\nu - g_{\mu\nu} \, q^2) \, \Pi(q^2), \qquad V^\mu_{ij} = \bar{\psi}_j \gamma^\mu \psi_i$$

$$D(Q^2) \equiv -Q^2 \, \frac{d\,\Pi(-Q^2)}{d\,Q^2}, \qquad Q^2 = -q^2 > 0$$

[in Euclidian (spacelike) region]

The analytic properties of the D-function are contained within the relation

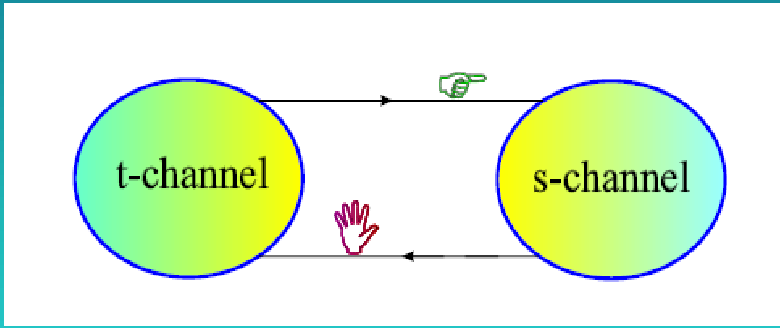
$$D(Q^2) = Q^2 \int_0^\infty \frac{ds}{(s+Q^2)^2} R(s),$$

where $R(s) = \mathrm{Im}\Pi(s)/\pi$ (timelike momentum transfer) has a resonance structure. D-function is an analytic function in the complex Q^2-plane with a cut along the negative real axis.

Relations between R and D function

$$D \propto 1 + d\,, \quad R \propto 1 + r$$

$$d(Q^2) = Q^2 \int_0^\infty \frac{ds}{(s+Q^2)^2}\, r(s)\,, \quad r(s) = -\frac{1}{2\pi i}\int_{s-i\epsilon}^{s+i\epsilon} \frac{dz}{z}\, d(-z)$$

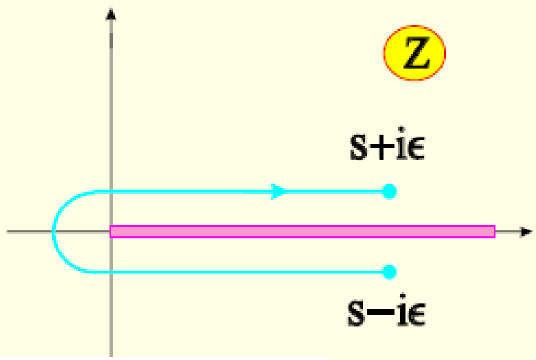


We concentrate on a study of cases when a process can be described in terms of Minkowskian or Euclidean variables equivalently.
The simplest case -> running coupling
Other case -> hadronic tau decays
One more case -> hadronic contribution to the muon anomalous magnetic moment (AMM)

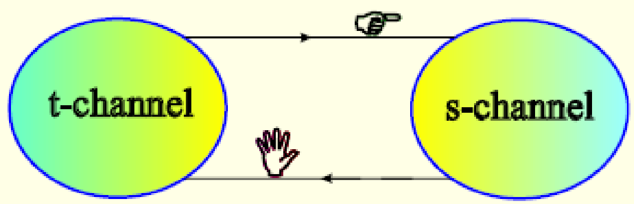
(!) If a calculation method maintains the correct analytic properties of the D-function, then both representations are equivalent.

Running coupling in the timelike region

$$\alpha_{\rm E}(Q^2) = Q^2 \int_0^\infty \frac{d\,s}{(s+Q^2)^2}\,\alpha_{\rm M}(s), \quad \alpha_{\rm M}(s) = -\frac{1}{2\pi {\rm i}} \int_{s-{\rm i}\,\epsilon}^{s+i\epsilon} \frac{d\,z}{z}\,\alpha_{\rm E}(-z).$$



Contour of integration.



LO PT coupling

$$a_{\rm pt}(z) = \frac{1}{\beta_0}\frac{1}{\ln(-z)}$$

$$a_{\rm pt}(z) \neq -\,z \int_0^\infty \frac{d\,s}{(s-z)^2}\,\tilde{a}_{\rm pt}(s)$$

It is very important that the method used maintains the correct analytic properties.
The VPT (as APT) preserves the correct analytic properties and leads to a selt-consistent definition of analytic continuation.

$$a_l^{\rm had} = \frac{1}{3}\left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty \frac{ds}{s} K_l\ (s)\, R_{e^+e^-}(s), \qquad (1)$$

$\alpha(0)^{-1} = 137.0335\,999\,59(38),\ l = \mu, \tau, e.$

If $s \le 4m_l^2$,

$$K(s \le 4m_l^2) = \frac{1}{2} - 4y_l - 4y_l\ (1 - 2y_l)\ln(4y_l) -$$

$$2\ (1 - 8\ y_l + 8\ y_l^2)\sqrt{y_l(1 - y_l)}\arccos\sqrt{y_l}, \qquad (2)$$

and then $s \ge 4m_l^2$

$$K(s \ge 4m_l^2) = \eta_l^2\left(1 - \frac{\eta_l^2}{2}\right) + (1 + \eta_l)^2\left(1 + \frac{1}{\eta_l^2}\right)\left[\ln(1 + \eta_l) - \eta_l + \frac{\eta_l^2}{2}\right] \qquad (3)$$

$$+ \frac{1 + \eta_l}{1 - \eta_l}\eta_l^2 \ln \eta_l,$$

$$y_l = \frac{s}{m_l^2}\,, \qquad \eta_l = \frac{1-v_l}{1+v_l}\,, \qquad v_l = \sqrt{1-\frac{4m_l^2}{s}}\,.$$

For $a_\mu^{\rm had}$

$$a_\mu^{\rm had} = \frac{1}{3}\left(\frac{\alpha}{\pi}\right)^2 \int\limits_0^\infty \frac{ds}{s}\, K(s)\, R_{e^+e^-}(s)\,, \tag{4}$$

where $K(s)$

$$K(s) = \int\limits_0^1 dx\, \frac{x^2(1-x)}{x^2+(1-x)s/m_\mu^2} \tag{5}$$

can be written as (3). Using relationship between functions $R(s)$ и $D(Q^2)$, one can write

$$a_\mu^{\rm had} = \frac{1}{3}\left(\frac{\alpha}{\pi}\right)^2 \frac{1}{2} \int\limits_0^1 \frac{dx}{x}\,(1-x)(2-x)\, D\left(\frac{x^2}{1-x}\, m_\mu^2\right). \tag{6}$$

(!) If a calculation method maintains the correct analytic properties

of the D-function, then both representations are equivalent.

Numerical result

$$a_\mu^{\rm had} = (690.9 \pm 4.4) \times 10^{-10} \quad \text{Davier'08}$$

$$a_\mu^{\rm had} = (702 \pm 16) \times 10^{-10}$$

$$a_e^{\rm had} = (1.67 \pm 2) \times 10^{-12} \quad \text{Passera'07}$$

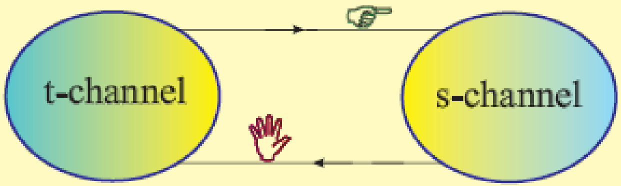
$$a_e^{\rm had} = 1.65 \times 10^{-12}$$

$$a_\tau^{\rm had} = (3.38 \pm 0.04) \times 10^{-6} \quad \text{Passera'07}$$

$$a_\tau^{\rm had} = 3.30 \times 10^{-6}$$

Minkowskian $\Longleftrightarrow$ Euclidean

$$Q_M = \int_0^\infty \frac{ds}{s} M(s) R(s),$$



$$Q_E = \int_0^\infty \frac{dt}{t} E(t) D(t),$$

$$D(t) = -t \frac{d\,\Pi(-t)}{d\,t} = t \int_0^\infty ds \frac{R(s)}{(s+t)^2}$$

$$R(s) = \frac{1}{2\pi \mathrm{i}} \left[\Pi(s + \mathrm{i}\,\epsilon) - \Pi(s - \mathrm{i}\,\epsilon)\right] = -\frac{1}{2\pi \mathrm{i}} \int_{s-\mathrm{i}\,\epsilon}^{s+\mathrm{i}\,\epsilon} \frac{d\,z}{z} D(-z).$$

What we can say about connection between kernels M(s) and E(t) in the case when

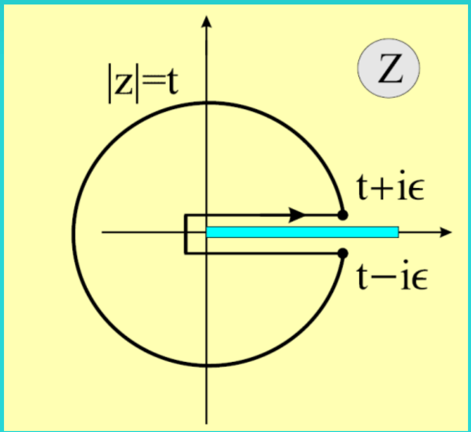
$$\boxed{Q_M = Q_E}$$

$$Q_E = \int_0^\infty \frac{dt}{t}\, E(t)\, t \int_0^\infty ds \frac{R(s)}{(s+t)^2} = \int_0^\infty \frac{ds}{s} \left[s \int_0^\infty dt \frac{E(t)}{(s+t)^2} \right] R(s).$$

Compare with

$$Q_M = \int_0^\infty \frac{ds}{s}\, M(s) R(s),$$

$$M(s) = s \int_0^\infty dt \frac{E(t)}{(s+t)^2}.$$



This relation is similar to D-function definition. M(s) should be an analytic function with a cut along the negative real axis.

$$E(t) = -\frac{1}{2\pi \mathrm{i}} \int_{t-\mathrm{i}\,\epsilon}^{t+\mathrm{i}\,\epsilon} \frac{dz}{z}\, M(-z).$$

$a_\mu^{\rm had}$ - example

$R - D$- self-duality presentations are

$$a^{\rm had} = \frac{1}{3}\left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty \frac{ds}{s}\, \mathrm{M(s)}\, R(s),$$

$$a^{\rm had} = \frac{1}{3}\left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty \frac{dt}{t}\, \mathrm{E(t)}\, D(t),$$

$$M(s) \equiv K(s) = \int_0^1 dx\, \frac{x^2(1-x)}{x^2 + (1-x)s/m^2}.$$

$$E(t) = -\frac{1}{2\pi \mathrm{i}} \int_{t-\mathrm{i}\,\epsilon}^{t+\mathrm{i}\,\epsilon} \frac{d\,z}{z} \int_0^1 dx \frac{x^2}{x^2/(1-x) - z/m^2}$$

$$= m^2 \int_0^1 dx\, x^2 \frac{1}{2\pi \mathrm{i}} \oint_{|z|=t} \frac{d\,z}{z(\tau - z)},$$

$$\tau = \tau(x) = m^2 \frac{x^2}{1-x}.$$

As a result we get

$$E(t) = \frac{1}{2}\left[\frac{\sqrt{1+4m^2/t}-1}{\sqrt{1+4m^2/t}+1}\right]^2$$

$$= \frac{1}{2}\,\frac{t+2m^2-\sqrt{t^2+4m^2}}{t+2m^2+\sqrt{t^2+4m^2}} = \frac{2m^4}{[t+2m^2+\sqrt{t^2+4m^2}]^2}.$$

$$a^{\text{had}} = \frac{1}{3}\left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty \frac{dt}{t}\,\mathrm{E(t)}\,D(t),$$

$$\mathrm{E(t)} = \frac{1}{2}\left[\frac{\sqrt{1+4m^2/t}-1}{\sqrt{1+4m^2/t}+1}\right]^2$$

Summary

We have analyzed various physical quantities and functions generated by *R(s)* based on the nonperturbative VPT-method **(Adler functions, hadronic contributions to anomalous magnetic moments of leptons and so on.).**

It was showed that VPT-method allows us to describe these quantities well down to low energy scale.

We investigate the reason of such good agreement and as a result we formulate a criterion which we name as the R-D self-duality.

Ковыляющий по прямой дороге опередит бегущего, который сбился с пути.

Ф. Бекон


Thanks for your
attention !