THE PHOTON CLUSTER EXCITATION MECHANISM OF THE e^-e^+ PLASMA CREATED FROM VACUUM IN A STRONG ELECTROMAGNETIC FIELD

S.A. Smolyansky

Saratov State University, Russia

Collaboration:

D.B. Blaschke (University of Wroclaw, Poland)

B. Kämpfer (Helmholtzzentrum Dresden-Rossendorf, Dresden, Germany)

S.M. Schmidt (Forschungszentrum Jülich GmbH, Jülich, Germany)

V.V. Dmitriev, A.V. Prozorkevich, A.V. Tarakanov
(Saratov State University, Russia)

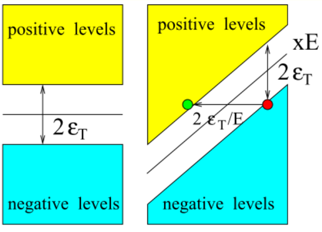
XXI Baldin ISHEPP, September 10-15, 2012, Dubna

Content

- Problem: field pulse shape and residual density
- Schwinger effect:
  - tunnel mechanism
  - multiphoton mechanism
- Kinetic description of $e^- e^+$ pair (EPP) production
  - kinetic equation (KE)
  - quasiparticle (QP) excitation and residual EPP
- Results:
  - photon cluster mechanism
  - stratum structure of EPP spectrum
  - influence of the pulse shape
- Conclusion

Schwinger effect: tunnel mechanism

Pair creation as barrier penetration in a strong constant field



- To materialize a virtual e^+e^- pair in a constant electric field E the separation d must be sufficiently large

$$eEd = 2mc^2$$

- Probability for separation d as quantum fluctuation

$$P \propto \exp\left(-\frac{d}{\lambda_c}\right) = \exp\left(-\frac{2E_c}{E}\right)$$

- Emission sufficient for observation when $E \sim E_c$

Schwinger result
(rate for pair production)

$$\frac{dN}{d^3x dt} = \frac{(eE)^2}{4\pi^3} \sum_{n=1}^{\infty} \frac{1}{n^2} \exp\left(-n\pi\frac{E_c}{E}\right),$$

$$E_c = \frac{m^2c^3}{e\hbar} \simeq 1.3 \times 10^{16} V/cm$$

J. Schwinger, On Gauge Invariance and Vacuum Polarization, Phys.Rev. 82 (1951) 664

Kinetic description of EPP production

Kinetic equation for $\mathbf{A}(t) = (0, 0, A(t))$ is a non-perturbative consequence of QED (S. Schmidt et al, Int. J. Mod. Phys. 1998, E7, 709)

$$\dot{f}(\mathbf{p}, t) = \frac{1}{2}\lambda(\mathbf{p}, t) \int\limits_{t_0}^{t} dt' \lambda(\mathbf{p}, t')[1 \pm 2f(\mathbf{p}, t')] \cos\theta(\mathbf{p}; t, t'),$$

$$\theta(\mathbf{p}; t, t') = 2 \int\limits_{t'}^{t} \omega(\mathbf{p}, \tau) d\tau,$$

$E \gg \nu^2$

$$\lambda(\mathbf{p}, t) = eE(t)\varepsilon_\perp / \omega^2(\mathbf{p}, t),$$

$$\omega(\mathbf{p}, t) = \sqrt{\varepsilon_\perp^2(\mathbf{p}) + (p_\parallel - eA(t))^2},$$

$$E(t) = -\dot{A}(t),$$

$$\varepsilon_\perp = (m^2 + p_\perp^2)^{1/2}.$$

Low density approximation

S. Schmidt et al, Phys. Rev. D59 (1999) 094005

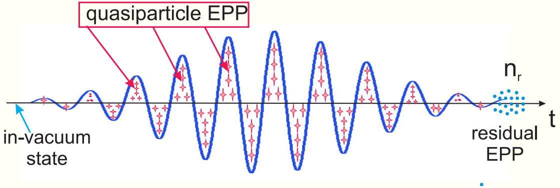
$E \ll E_c \rightarrow f \ll 1$

Distribution function of the **quasiparticle EPP**

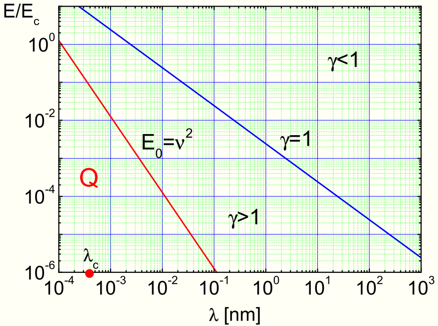
$$f_{low}(\mathbf{p},t) = \frac{1}{4}\left| \int_{-\infty}^{t} dt' \lambda(\mathbf{p},t') \mathrm{e}^{i\theta(t,t')} \right|^2 .$$

Distribution function of the **residual EPP** $(t \rightarrow +\infty)$

$$f_{out}(\mathbf{p}) = \lim_{t\rightarrow\infty} f_{low}(\mathbf{p},t) = \frac{1}{4}\left| \int_{-\infty}^{\infty} dt' \lambda(\mathbf{p},t') \mathrm{e}^{i\theta(t,t')} \right|^2 .$$



Landscape



Adiabacity parameter

$$\gamma = 2\pi \frac{E_c}{E_0}\frac{\lambda_c}{\lambda}$$

$\gamma > 1 \sim$ multiphoton mechanism,

$\gamma < 1 \sim$ tunnel mechanism

$E_0 = \nu^2$ is the boundary between the quasiclassical (right) and quantum (left) domains of the external field

Model limitations

1. Low density approximation, $f \ll 1$
2. Multiphoton domain, $\gamma > 1$
3. Laser field is quasiclassical, $E_0 \gg \nu^2$

Models of laser pulse:

- nonadiabatic switching on(off)
  $E(t) = E_0 \cos \nu t, \quad 0 \leq t \leq NT, \quad \nu = 2\pi/T$
- adiabatic (Gaussian) pulse
  $E(t) = E_0 \exp\left(-t^2/2\tau\right) \cos \nu t, \quad \nu\tau = \sigma$

EPP creation

- at $t \gtrsim \tau$ – transient processes, $f(\mathbf{p}, t)$
- at $t \to \infty$ – residual EPP, $f_{out}(\mathbf{p}) = \lim_{t\to\infty} f(\mathbf{p}, t)$

Leading approximation

$$\omega(\mathbf{p},t) \to \omega_0(\mathbf{p}) = \sqrt{\mathbf{p}^2 + m^2};$$
$$\lambda(\mathbf{p},t) \to \lambda_0(\mathbf{p})E(t), \qquad \lambda_0(\mathbf{p}) = e\varepsilon_\perp/\omega_0^2(\mathbf{p});$$
$$\theta(t) = 2\omega_0 \cdot t.$$

It means, that contribution of the high frequency harmonics of the external field is omitted. It leads to the simple result

$$f_{out}(\mathbf{p}) = \frac{1}{4}\lambda_0^2 \Big| E(\omega = 2\omega_0) \Big|^2,$$

where $E(\omega)$ is the Fourier transform of the field strength $E(t)$.
The threshold $\nu = 2\omega_0$ corresponds to the one photon mechanism of EPP creation.

Multiphoton processes

The working hypothesis: account of the multiphoton processes should lead to a decrease in the threshold frequency

Transformation of the high frequency phase ($A(t) = A_0 \cos \nu t$)

$$\omega(\mathbf{p}, t) = \sum_{n=1} \Omega_n \cos n\nu t,$$

$$\theta(t) = 2\Omega_0 t + \sum_{n=1} a_n \sin n\nu t, \qquad a_n = 2\Omega_n/\nu n$$

Ω_0 is the renormalized frequency (E.Brezin and C. Itzykson, 1970),

$$\exp\left(ia \sin\phi\right) = \sum_{-\infty}^{\infty} J_k(a) \mathrm{e}^{ik\phi},$$

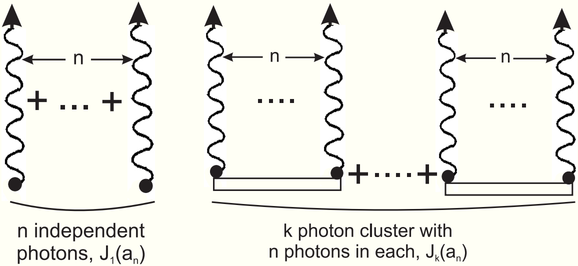
$J_k(a)$ are the Bessel functions

Cluster decomposition

The typical integral

$$\sum_k \int_{-\infty}^{\infty} E(t) \prod_n \exp\left\{ it\Big(2\Omega_0 + \nu\, n\, k\Big)\right\} J_k(a_n)\ dt$$

n is the harmonic number $=$ the photon number, k is the order of the photon cluster



D. B. Blaschke, B. Kaempfer, A. D. Panferov, A. V. Prozorkevich, S. A. Smolyansky, arXiv:1205.3154v1 [physics.plasm-ph]

The working formula

Distribution function of the residual EPP in approximation of the limited number of the photons (N_{ph}):

$$f_{out}(\mathbf{p}; N_c, N_{ph}) = \frac{\pi^2 \lambda_0^2}{8\nu \mathcal{N}_{ph}} \sum_{k=-N_c}^{N_c} \prod_{n=1}^{N_{ph}} J_k^2(a_n) \delta[2\Omega_0 - \nu(k\mathcal{N}_{ph} + 1)] \quad (*)$$

N_{ph} is the maximal number of the photons in a cluster,

$\mathcal{N}_{ph} = N_{ph}(N_{ph} + 1)/2$,

N_c is is the maximal number of the photon clusters

The renormalized frequency (approximation)

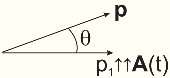
$$\Omega_0 = \sqrt{\varepsilon_\perp^2 + (p_\parallel - eA_0)^2}$$

Spectrum

The roots of the δ-function in Eq. (*)

$$p_{(1,2)} = \frac{1}{\gamma}\cos\theta \pm \left\{[\nu(\mathcal{N}_{ph}+1)/2m]^2 - \gamma^{-2}\sin^2\theta - 1\right\}^{1/2},$$

γ is the adiabacity parameter.



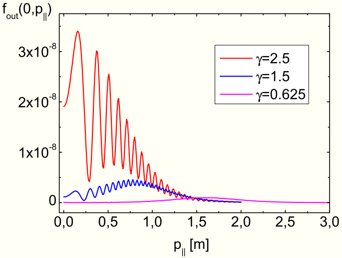
Features:

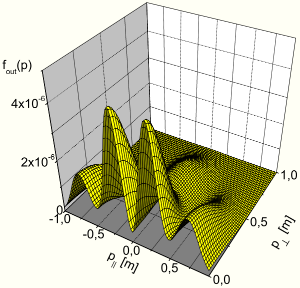
- absence of the symmetry $p_1 \to -p_1$
- cylindrical symmetry
- the angle dependence is defined by γ
- quasi-discrete (stratum) structure with quasi-equidistant distribution
- existence of the threshold (isotropic approximation $\gamma \gg 1$)

$$\nu(\mathcal{N}_{ph} \pm 1) \geq 2m \qquad (**)$$

For the given frequency ν the threshold is reached at participation of some minimal threshold photon number $\mathcal{N}_{ph}^{th}$, defined from (**)

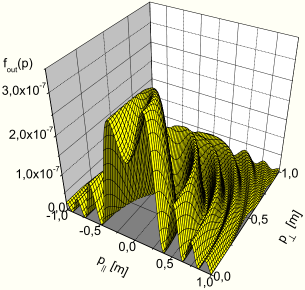
Stratum structure of spectrum

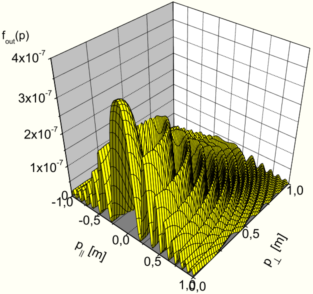




Left: Sections $p_{\perp} = 0$ of $f_{\text{out}}(\mathbf{p})$ for the Gaussian pulse for different γ.
Right: The shape of $f_{\text{out}}(\mathbf{p})$ for a Gaussian pulse with $\gamma = 10$, $\sigma = 5$ and $E_0 = 0.1\ E_c$

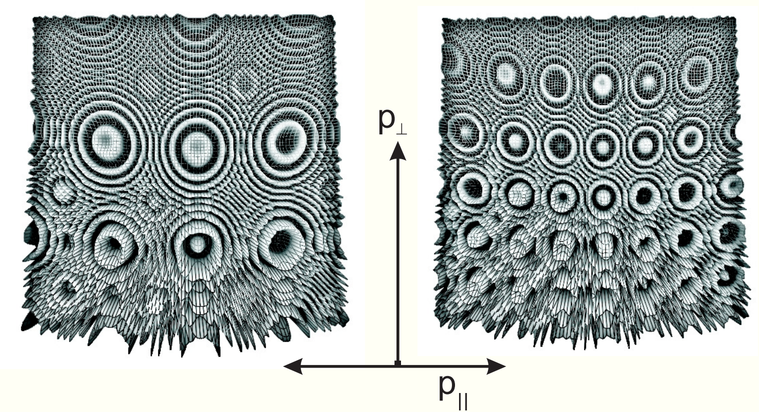
Stratum structure of spectrum





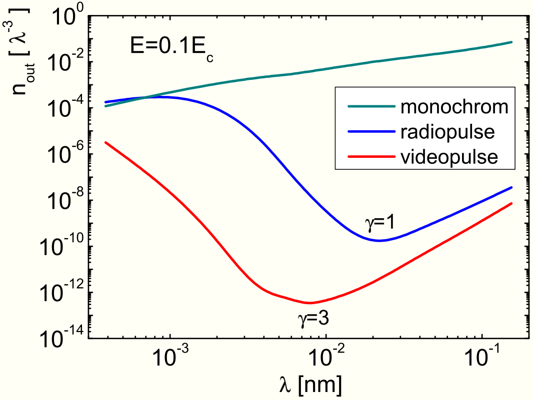
Complication of $f_{\rm out}(\mathbf{p})$ in the monochromatic field with increasing N at $E_0 = 0.01\ E_c$ and $\gamma = 24$. Left panel: N=1; right panel: N=2

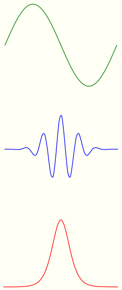
Cellular structure of spectrum



The complex cellular structure of the shape of $f_{\rm out}(\mathbf{p})$ at large N in a survey top view. Left panel: left N=50; right panel: N=100.

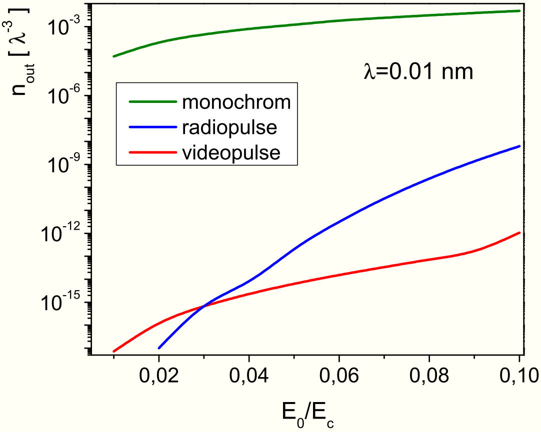
Comparison of different pulse shapes

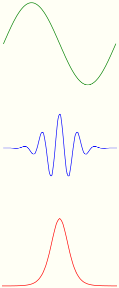




λ dependence of n_{out} for different pulse forms

Comparison of different pulse shapes





Field dependence of n_{out} for different pulse forms

Conclusion

The following results had obtained for the distribution function $f_{\rm out}(\mathbf{p})$ and the number density $n_{\rm out}$ of residual EPP on the basis of exact kinetic equation for the subcritical fields $E_0 \ll E_c$ in the multiphoton domain $\gamma \gg 1$:

- complicated character of $f_{\rm out}(\mathbf{p})$:
  - stratum structure in the case of the Gaussian type pulse and
  - cellular structure in the case of the periodical field
- strong dependence of the field and frequency bechaviour of the particle number density $n_{\rm out}$ from the way of switching on (off) an electromagnetic pulse (adiabatical or nonadiabatical)

The most actual task

Next problem: the search of some satisfactory estimations for $f_{\mathrm{out}}(\mathbf{p})$ and n_{out} for the typical model of laser pulse for the following investigation of the secondary effects of the type the annihilation photons generation

D. B. Blaschke, V. V. Dmitriev, G. Röpke, S. A. Smolyansky, Phys. Rev. **D84**, 085028 (2011)

THANK FOR YOUR ATTENTION!