

LONGITUDINAL PROFILES AND CORRELATIONS OF ELECTROMAGNETIC CASCADES PRODUCED BY 100-3500 MeV GAMMA QUANTA IN DENSE AMORPHOUS MEDIA

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The XXI International Baldin Seminar on High Energy Physics Problems
"Relativistic Nuclear Physics and Quantum Chromodynamics"
September 10 to 15, 2012 in Dubna, Russia



OUTLINE OF THE TALK

- **MOTIVATION**
- **ABSTRACT**
- **SHORT HISTORY**
- **RESULTS OF MODELING**
- **SUMMARY AND CONCLUSION**



Motivation

- Longitudinal profiles of electromagnetic cascades (EMC) produced in heavy amorphous media by high enough energy gamma quanta and electrons (or positrons) are the basic characteristics of the phenomenon both from cognitive and application viewpoints.
- The fluctuations determine the energy resolution and accuracy of flight direction of particles initiating EMC.
- A knowledge of correlation enables to measure the energy of EMC in conditions of limited geometry of detectors.
- The information about correlation and fluctuations in EMC is needed for electromagnetic calorimeters under construction as PANDA (GSI), as well for radiation shielding construction and radiation material physics.



Abstract

- We study the average longitudinal and transverse **profiles** of electromagnetic cascades (EC) created in most popular dense amorphous media (liquid xenon, PWO, CdWO₄, GaAs, NaI, Pb, lead glass and BGO) by gamma quanta of energy $E_\gamma=100\div 3500$ MeV at three different cut-off energies (0.6, 1.2 and 3.0 MeV). The work has been performed using the EGS4 & GEANT4 modeling codes.
- Analyzed is the correlation of energy release in longitudinal cascade development in order to estimate the possibility to reconstruct its primary energy when a part of it has been registered.
- The results are compared with available experiment. The ultimate objective of this investigation is to obtain concise information about average profiles, fluctuations and correlation in EC **suitable for practical purposes**.



Basic steps in the investigation of electromagnetic cascades:

1. Rossi B. Phys. Zs., 1932, vol.33, p.304 - discovery of the phenomenon.
2. Rossi B. High-Energy Particles. Prentice-Hall, New York, 1952 - one-dimensional theory of EMC.
3. Longo E., Sestili J. Nucl. Instr. Meth., 1975, vol.128, p.283 – computer model of EMC (*neither EGS4, nor GEANT*).
4. De Angelis A. Nucl. Instr. Meth. A., 1988, vol.271, p.455 - computer model of EMC (*neither EGS4, nor GEANT*).
5. Słowiński B. Phys. Part. Nucl. 25 (2), March-April 1994 - overview of experimental and theoretical description of EMC.
6. **Modern description of EMC is needed urgently** (*with EGS4 and GEANT confronted with experiment*).



What is the EMC in short?

$$\gamma \rightarrow e^+ + e^-$$

$$e^+ \rightarrow e^+ + \gamma$$

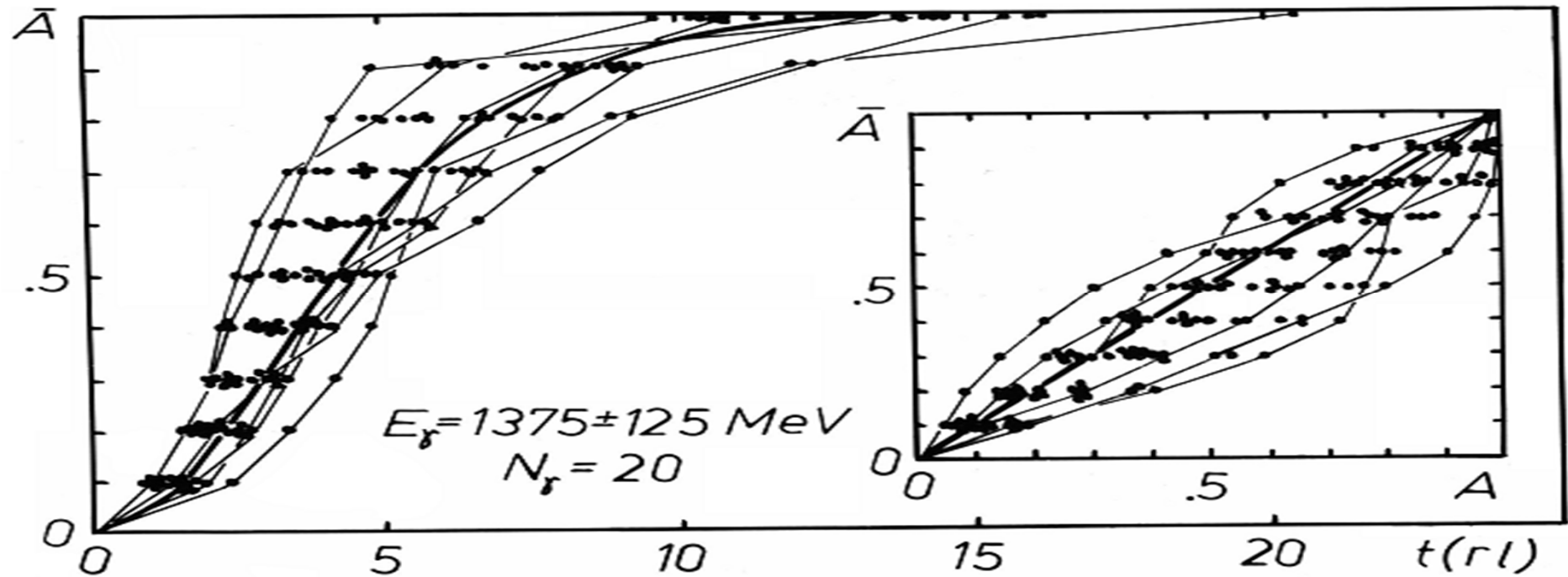
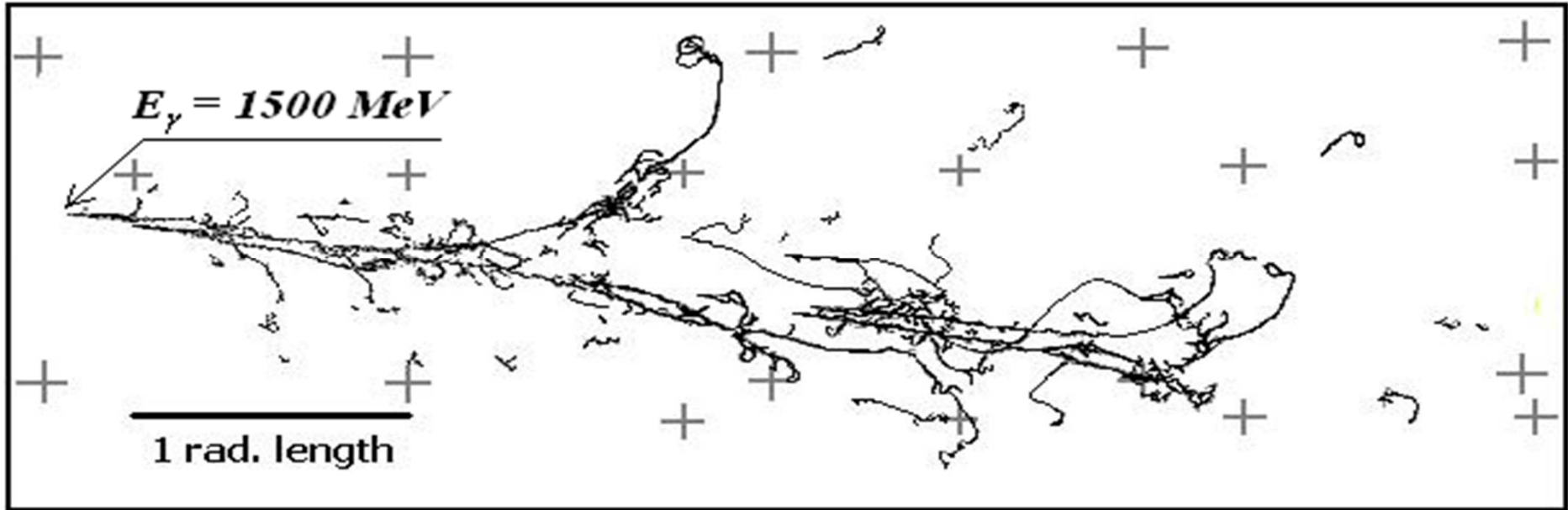
$$e^- \rightarrow e^- + \gamma$$

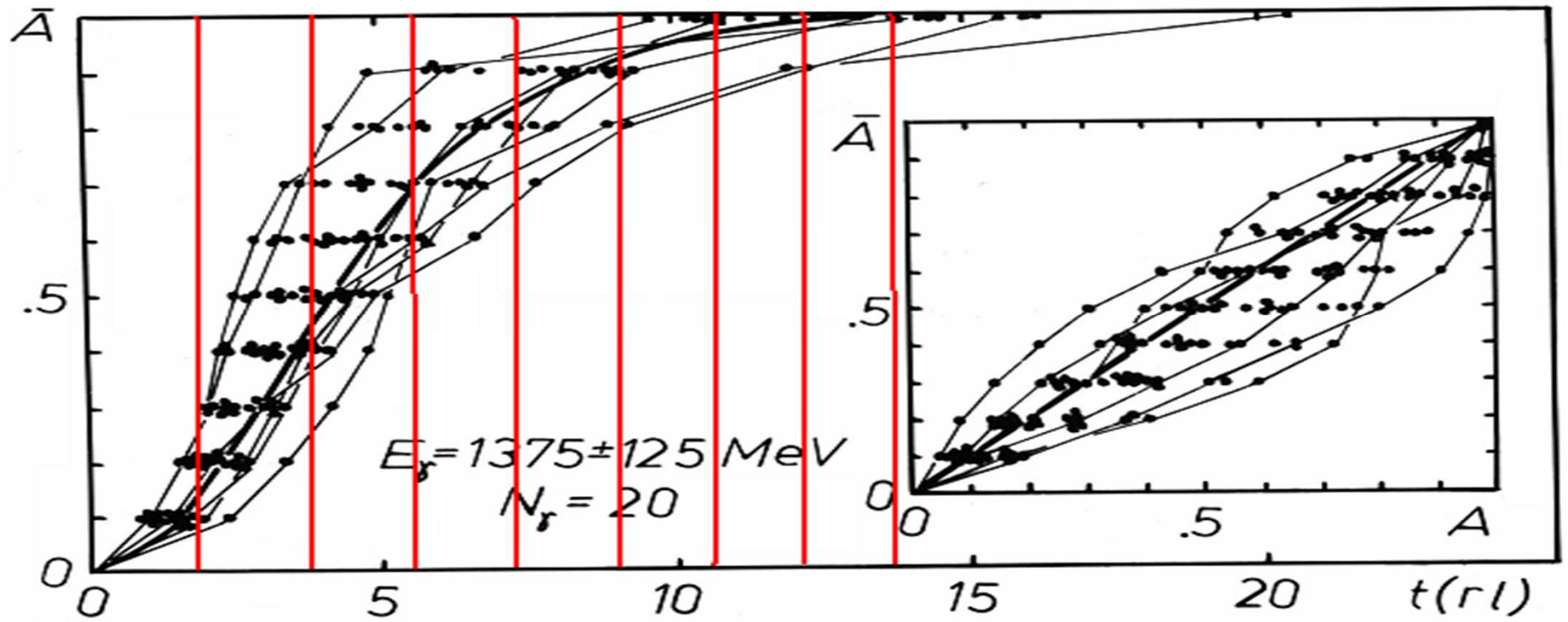
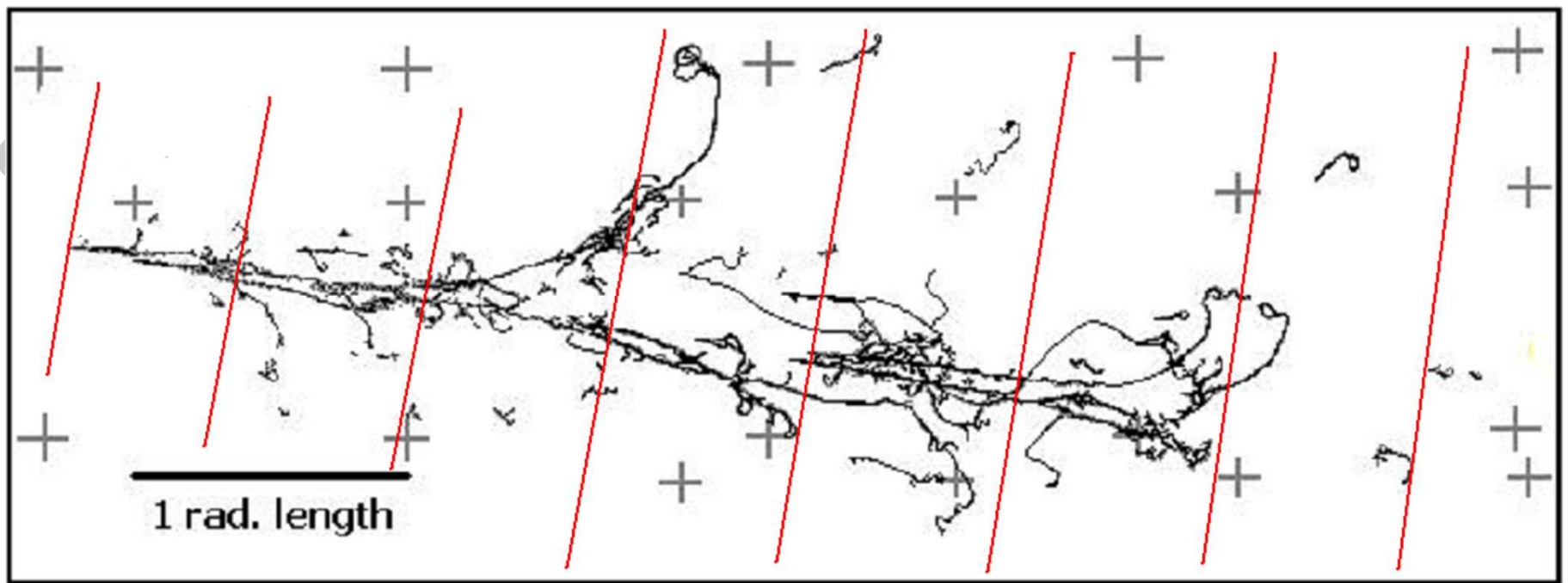
...

Leading elementary processes:

- Pair creation
- Bremsstrahlung (radiation emission)
- Ionization
- Multiple Coulomb scattering

A picture of EMC from 26 liter Xenon Bubble Chamber (LHE JINR)







Average description of EMC profiles

Profiles

- **longitudinal:** $(-dE / dt) = A \cdot t^\alpha \cdot \exp(-bt)$
- **lateral (or radial):** $(-dE / dr) = B \cdot \exp(-\mu(t) \cdot r)$
- **three-dimensional EMC picture:**

$$(-d^2 E / dt \cdot dr) = C \cdot t^\alpha \cdot \exp\{-[bt + \mu(t) \cdot r]\}$$



and fluctuation

(as an estimation of energy resolution when a cascade totally develops inside a target material)

$$\frac{\sigma}{E} \propto \frac{\sqrt{t}}{\sqrt{E}},$$

t is the step of sampling in units of r.l.

(C.Grupen. Particle detectors. Cambridge University Press. 1996)

$$\frac{\sigma_E}{E} = \left(\frac{a^2}{E^2} + \frac{b^2}{E} + c^2 \right)^{1/2}$$

E is in GeV (for $PbWO_4 - PWO$; ALICE experiment)



Modeling of EMC

Programs: EGS4 and GEANT4

GQ energies: $E_\gamma = 210, 555, 875, 1625, 2375, 3125$ MeV

Cut-off energies: $E_{c.o.} = 0.6, 1.2, 2.0, 3.0$ MeV

Materials:

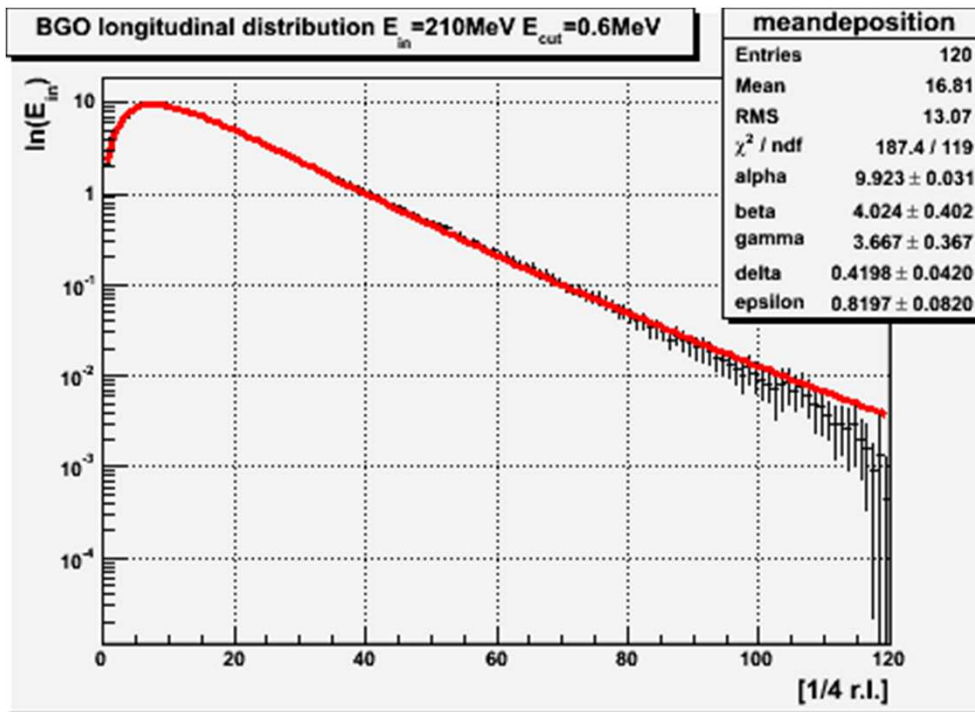
- liquid xenon
- PWO
- CdWO_4
- GaAs
- NaI
- Pb
- lead glass
- BGO

For every set of this parameters (E_γ , $E_{c.o.}$ and material) we modeled 20000 events (history).

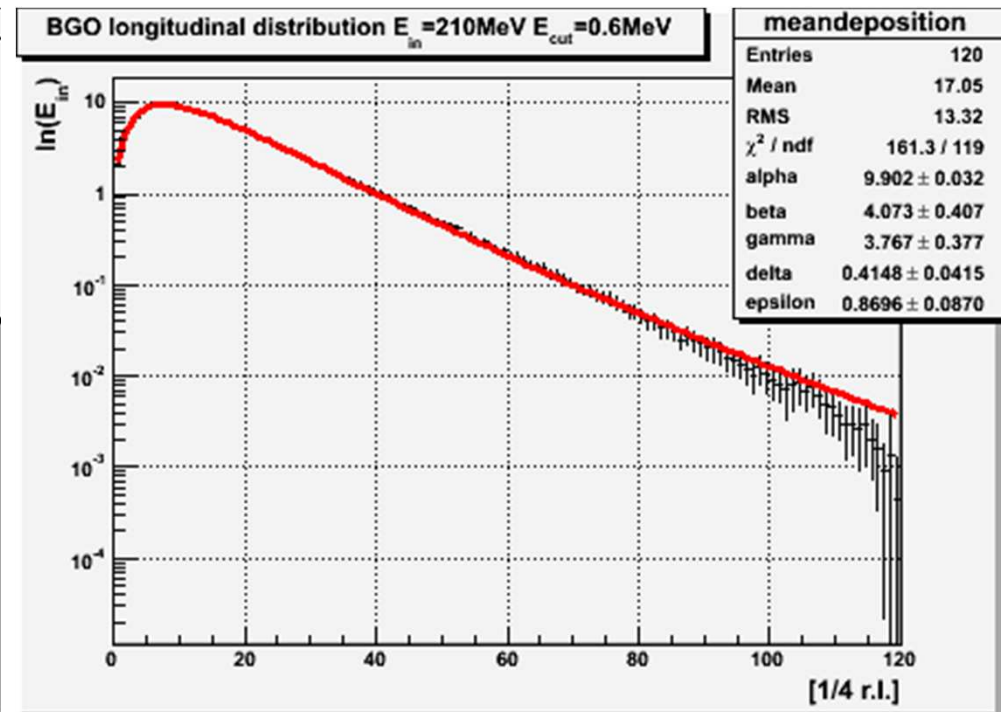
LONGITUDINAL PROFILES

An example for illustration:

Average longitudinal profile of EMC produced in BGO by gammas of energy 210 MeV ($E_{co}=0.6$ MeV).



EGS4



GEANT4



FITTING LONGITUDINAL PROFILES

to the approximating function:

$$(-dE / dt) = \alpha_t (t - \varepsilon_t)^{\beta_t} \exp(-\gamma_t t^{\delta_t})$$

$\alpha_t, \beta_t, \delta_t, \gamma_t, \varepsilon_t$ are the fit parameters depending on cut-off energy $E_{c.o.}$, E_γ and material properties.

$$(-dE / dt) = \alpha_t t^{\beta_t} \exp(-\gamma_t t)$$

Dependence of parameters:

$\beta_t, \delta_t, \gamma_t$ and ε_t

on

$E_\gamma, E_{c.o.}, Z/A.$

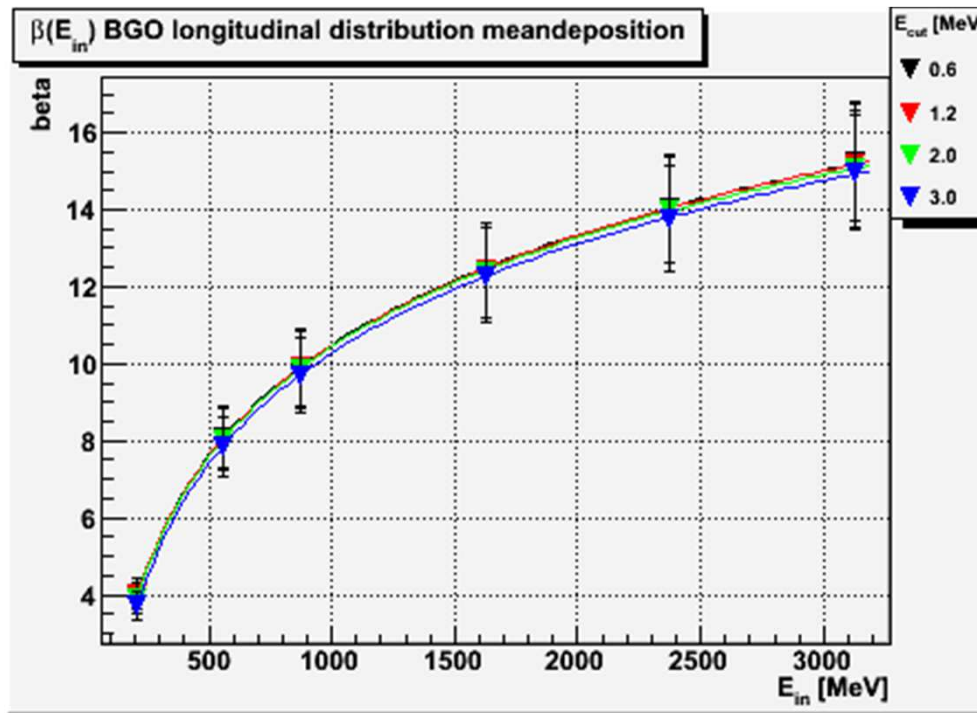
Parameter

 β_t

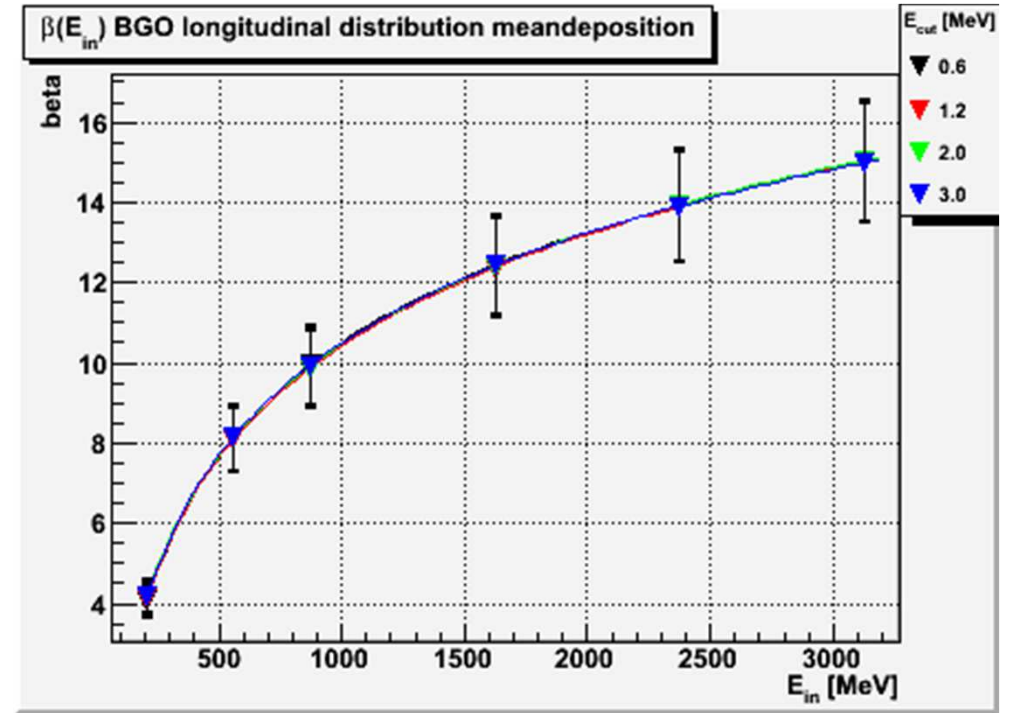


BGO

$$\beta_t(E_\gamma) = a \cdot E_\gamma^b + c$$



EGS4

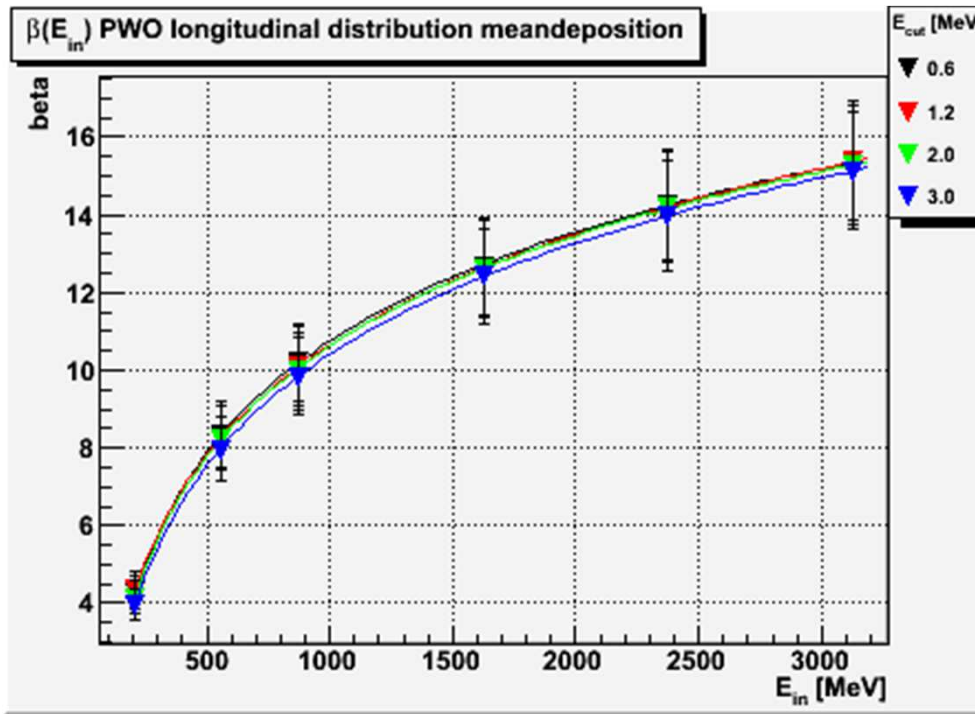


GEANT4

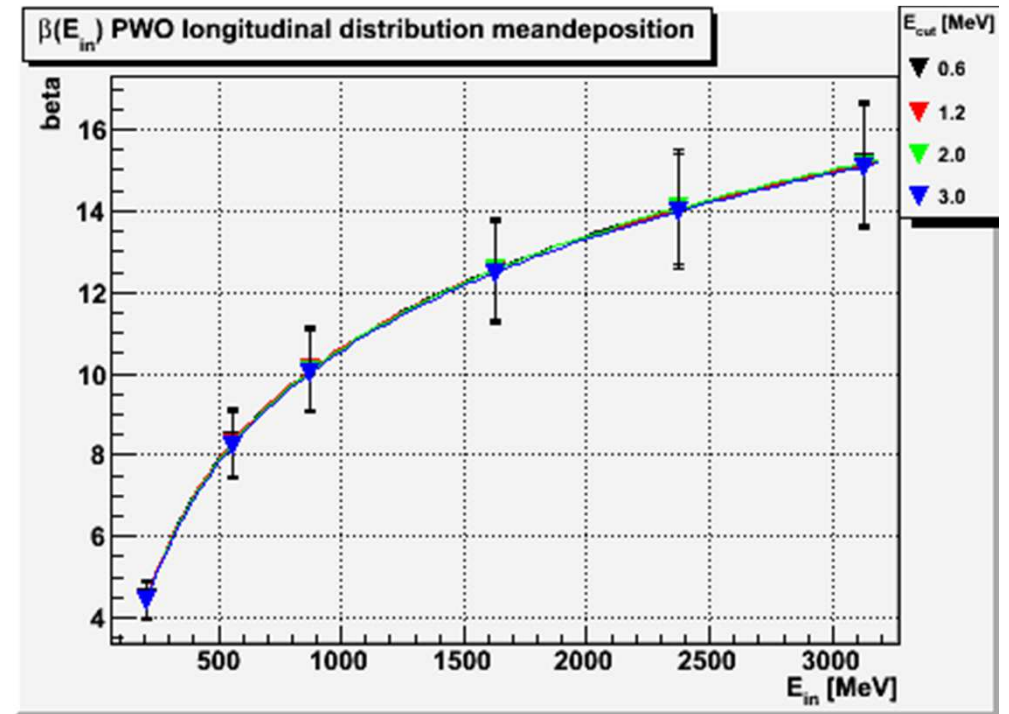


PWO

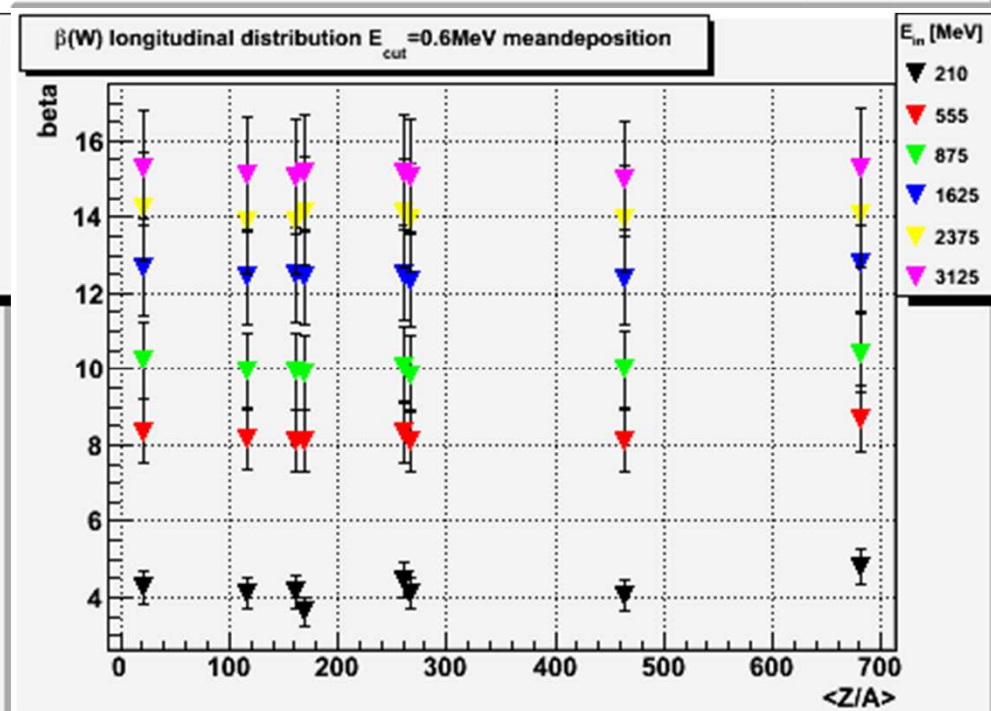
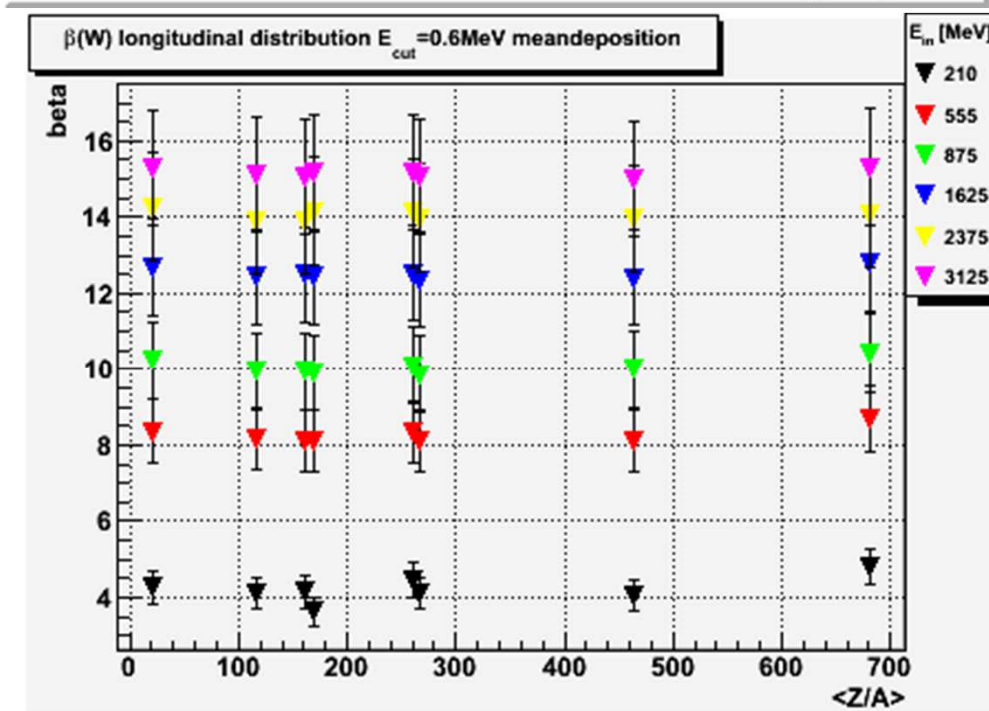
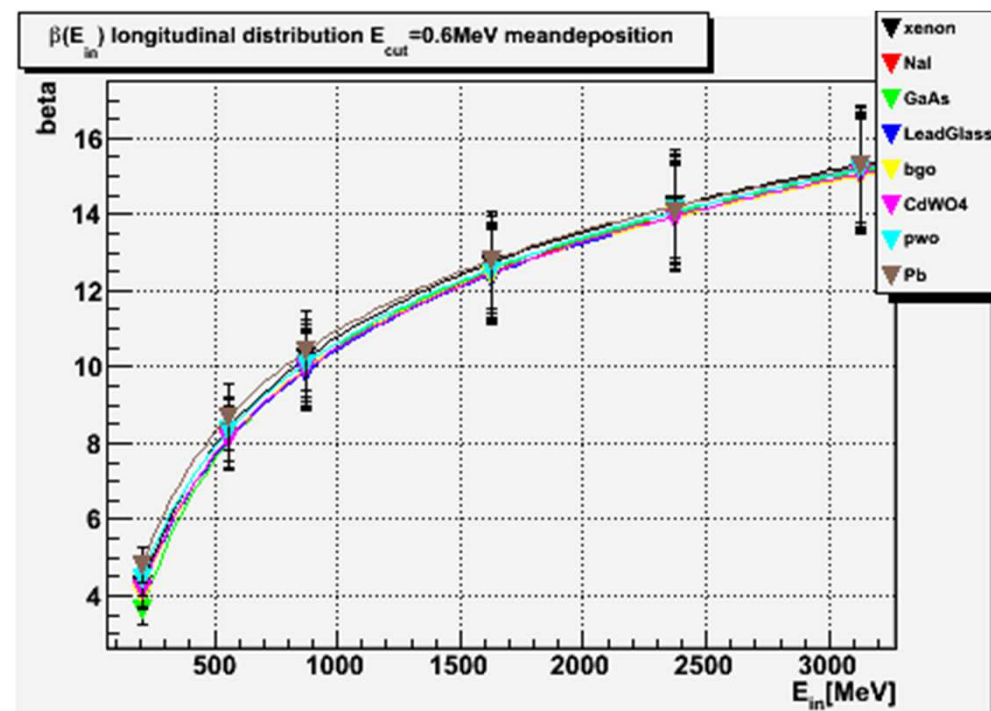
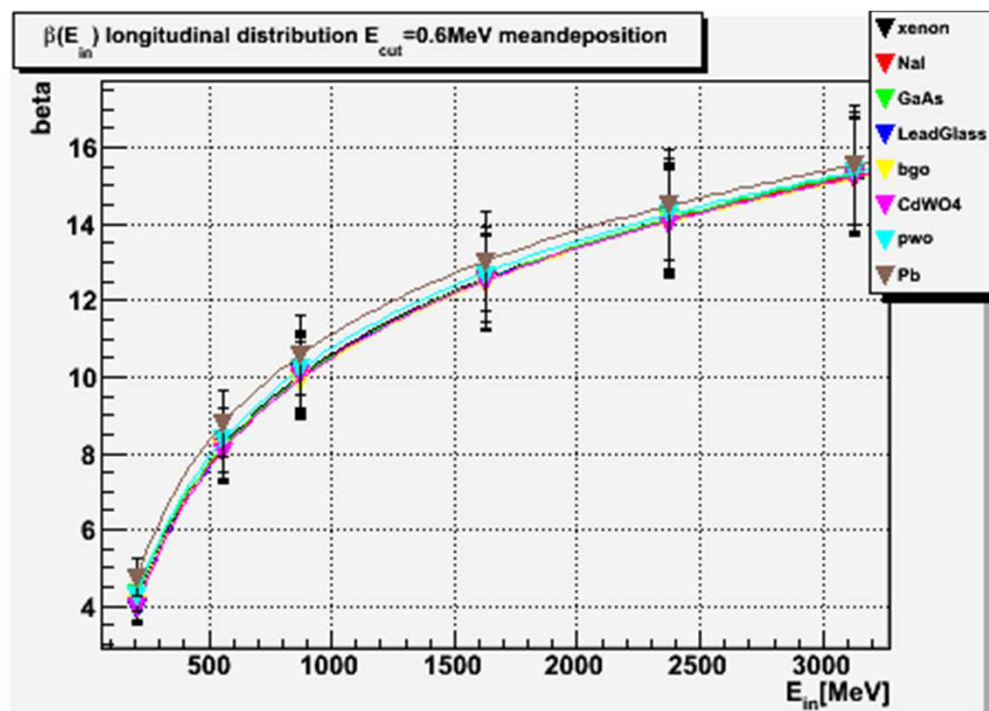
$$\beta_t(E_\gamma) = a \cdot E_\gamma^b + c$$



EGS4



GEANT4



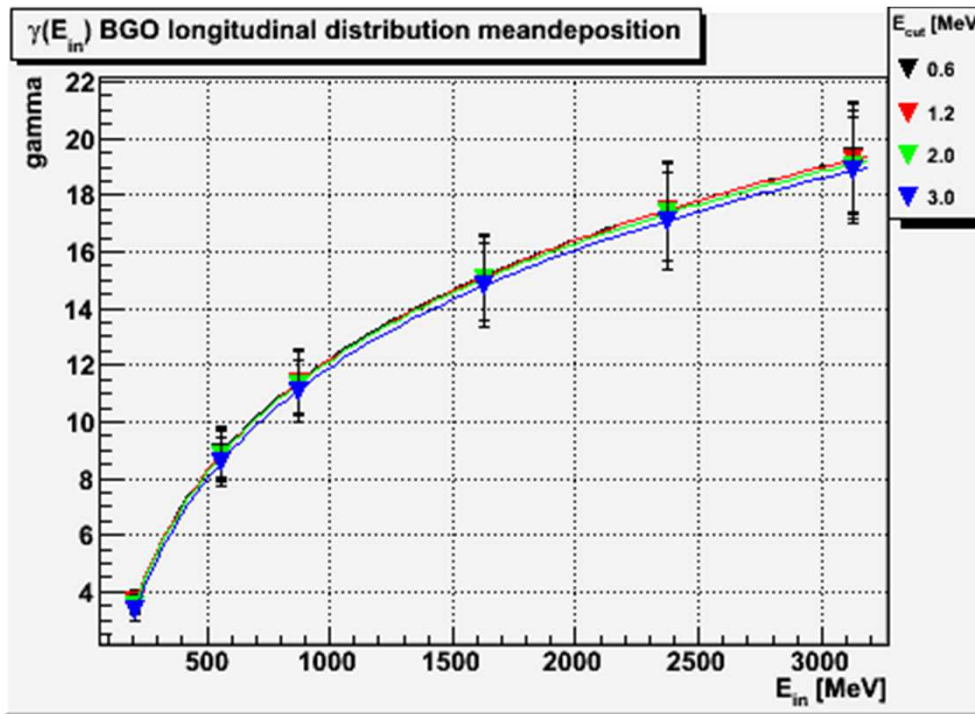
Parameter

γ_t

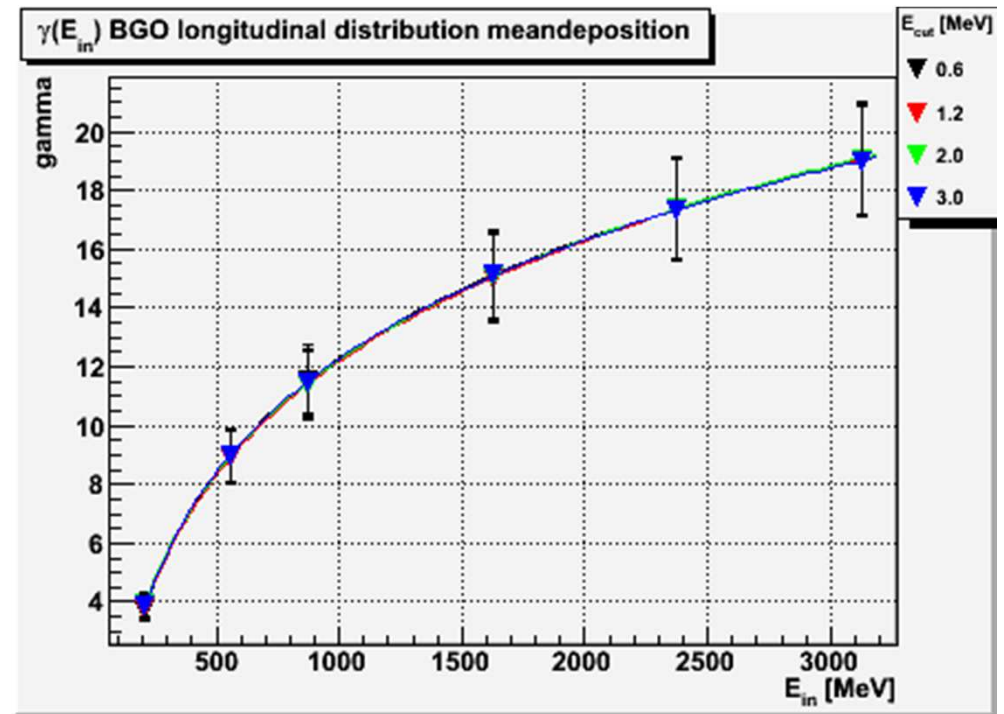


BGO

$$\gamma_t(E_\gamma) = a \cdot E_\gamma^b + c$$



EGS4

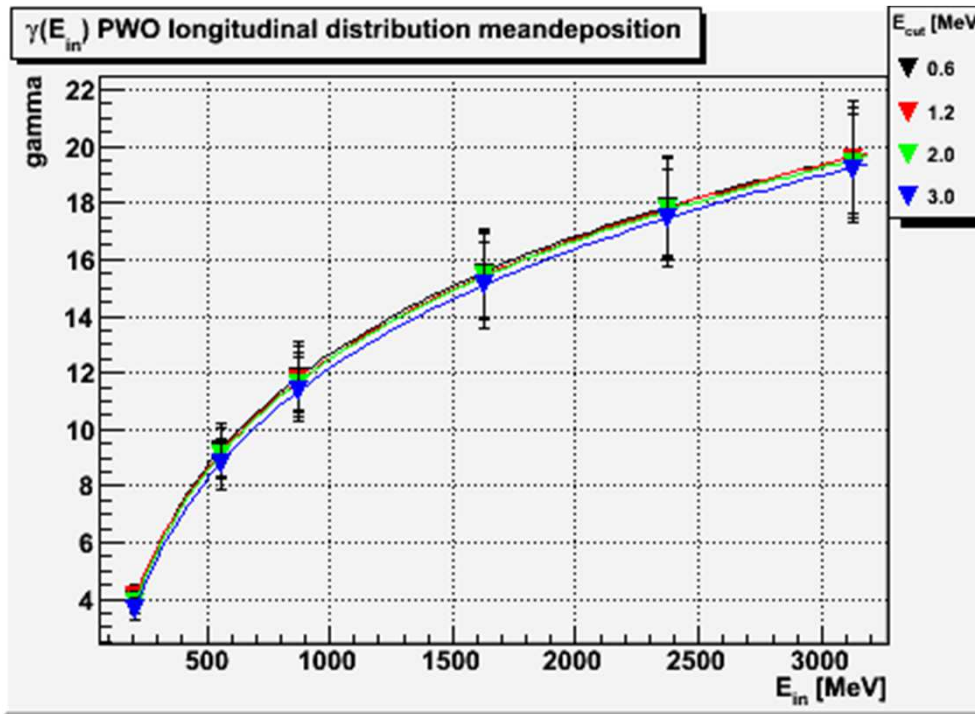


GEANT4

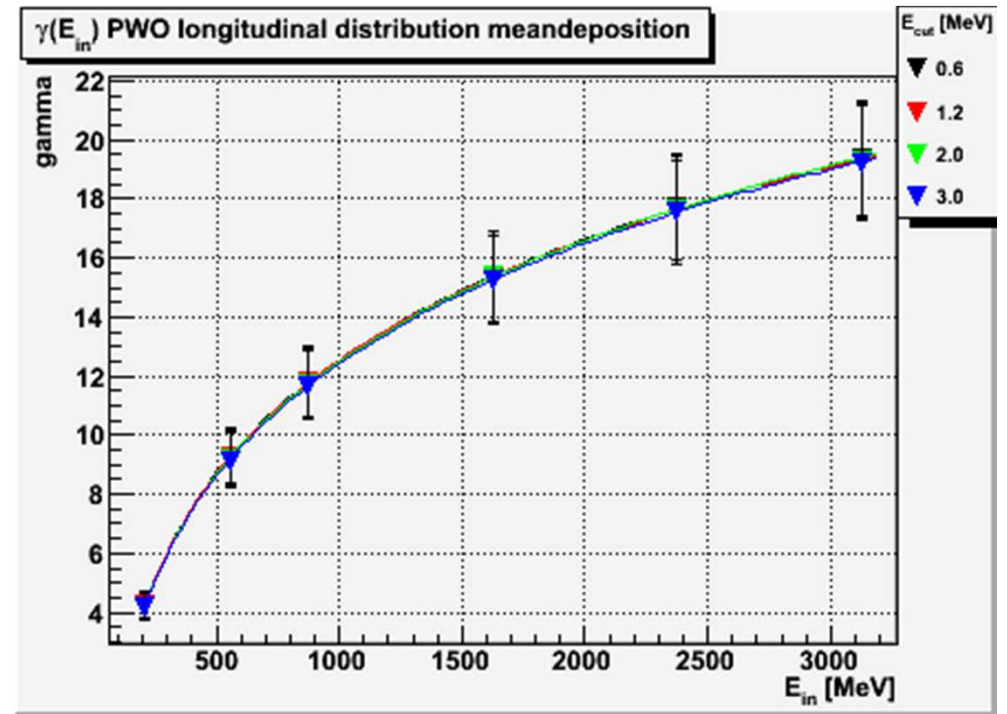


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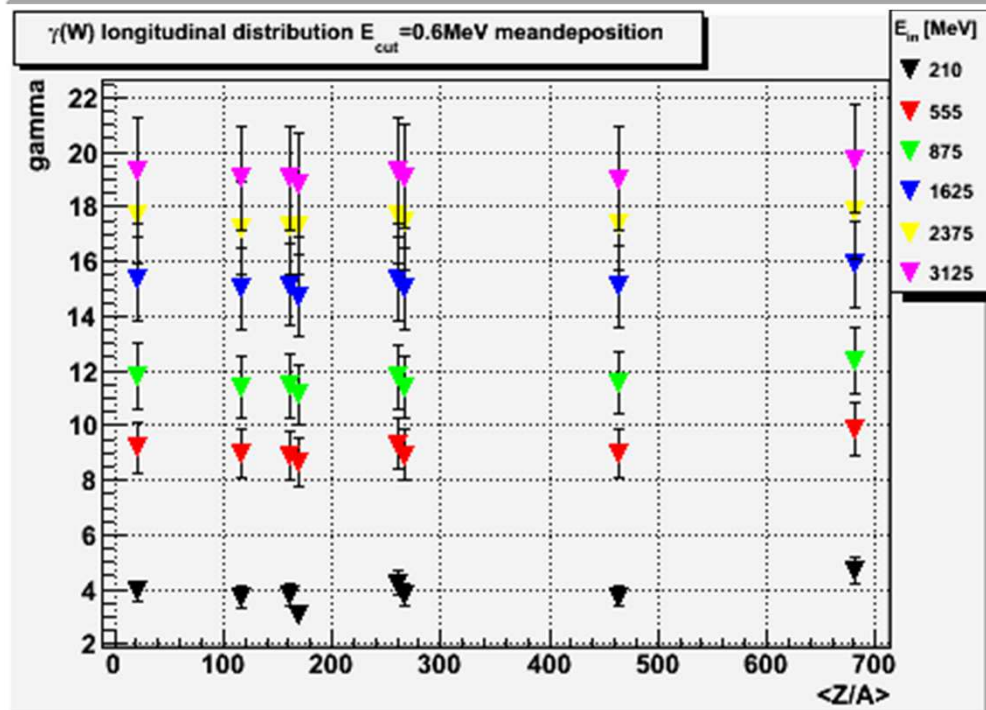
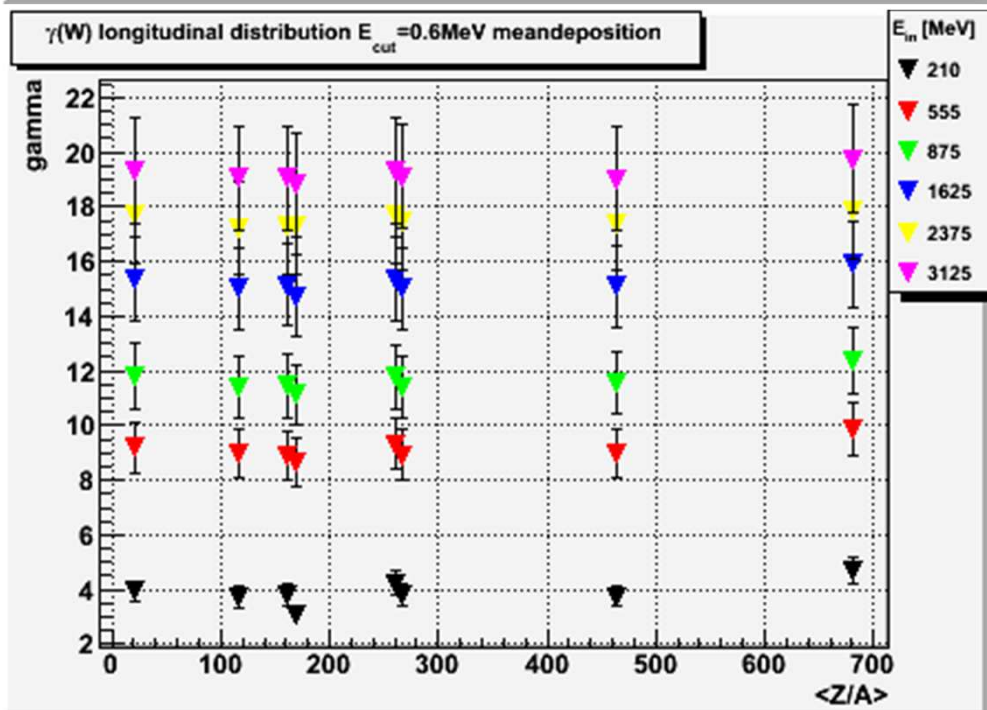
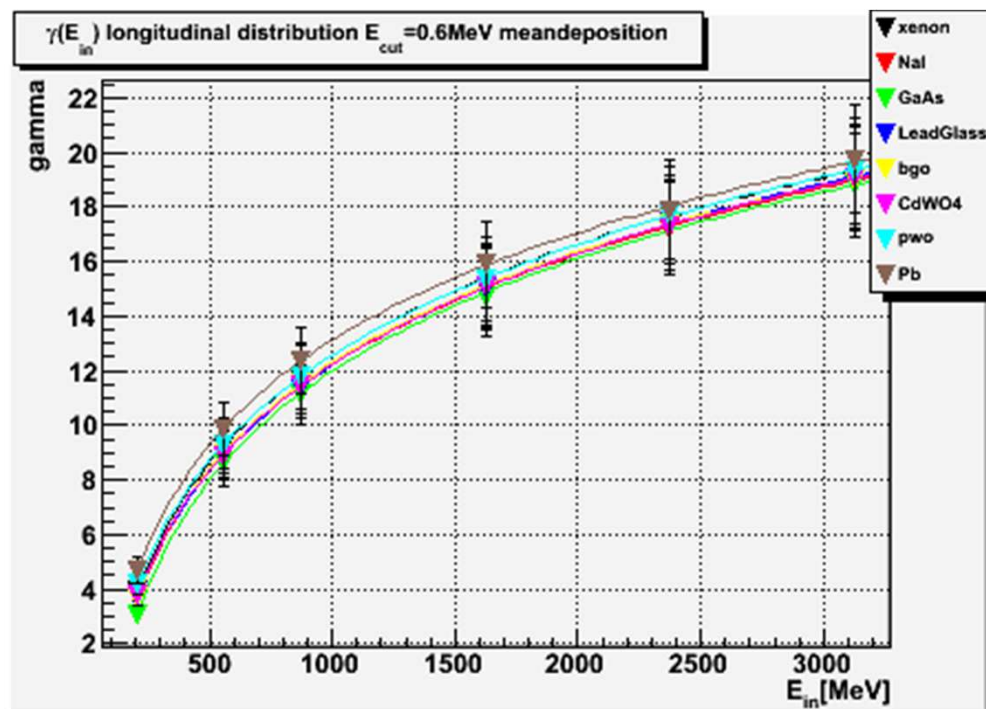
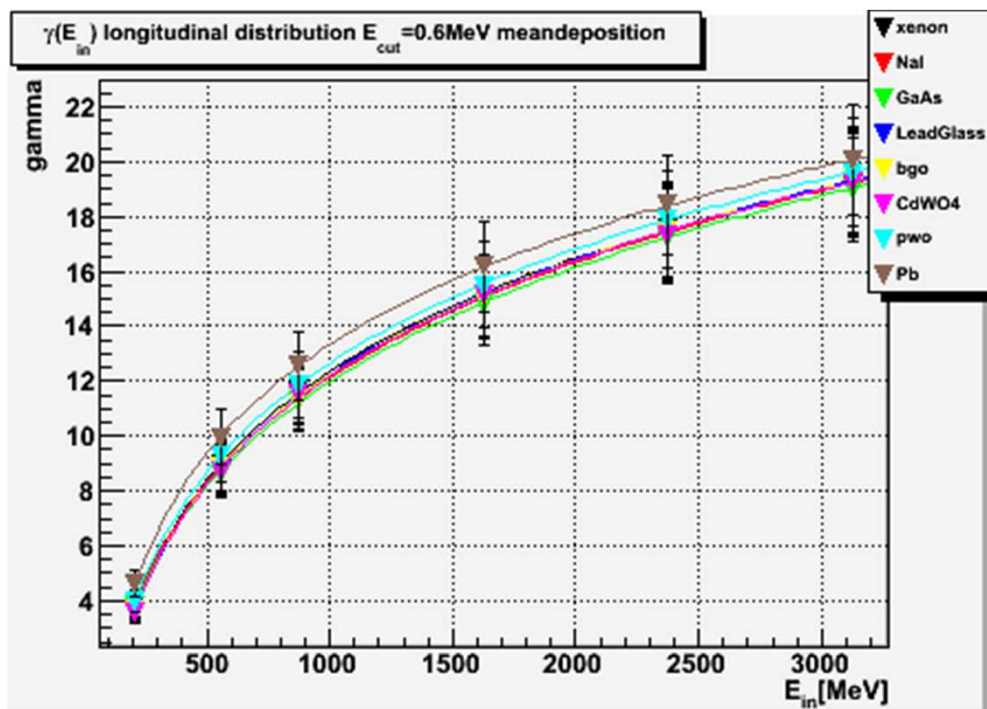
$$\gamma_t(E_\gamma) = a \cdot E_\gamma^b + c$$



EGS4



GEANT4



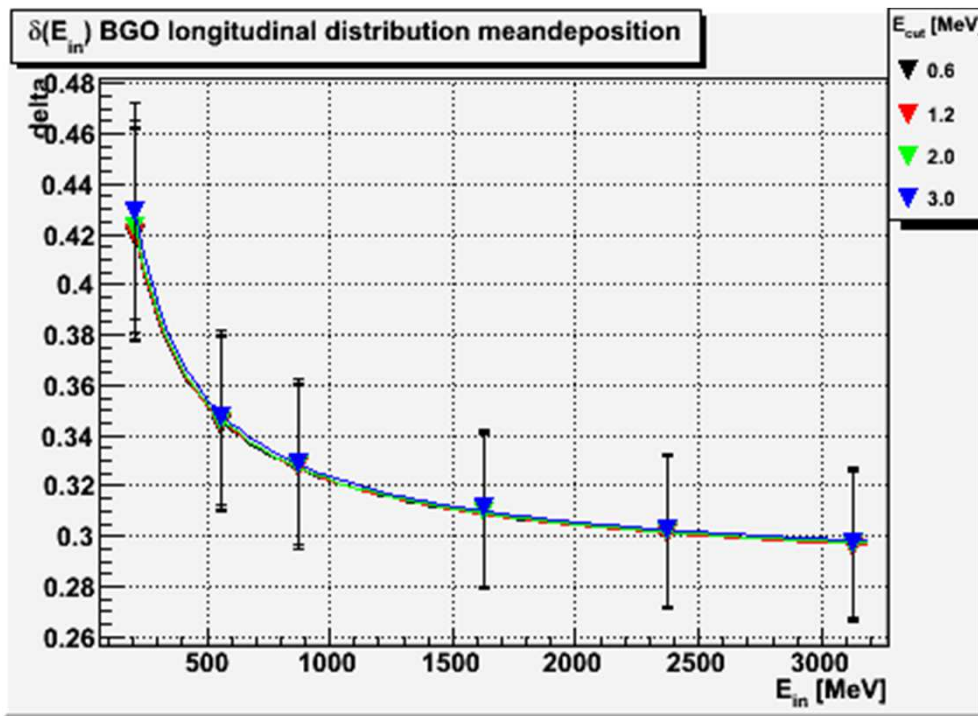
Parameter

δ_t

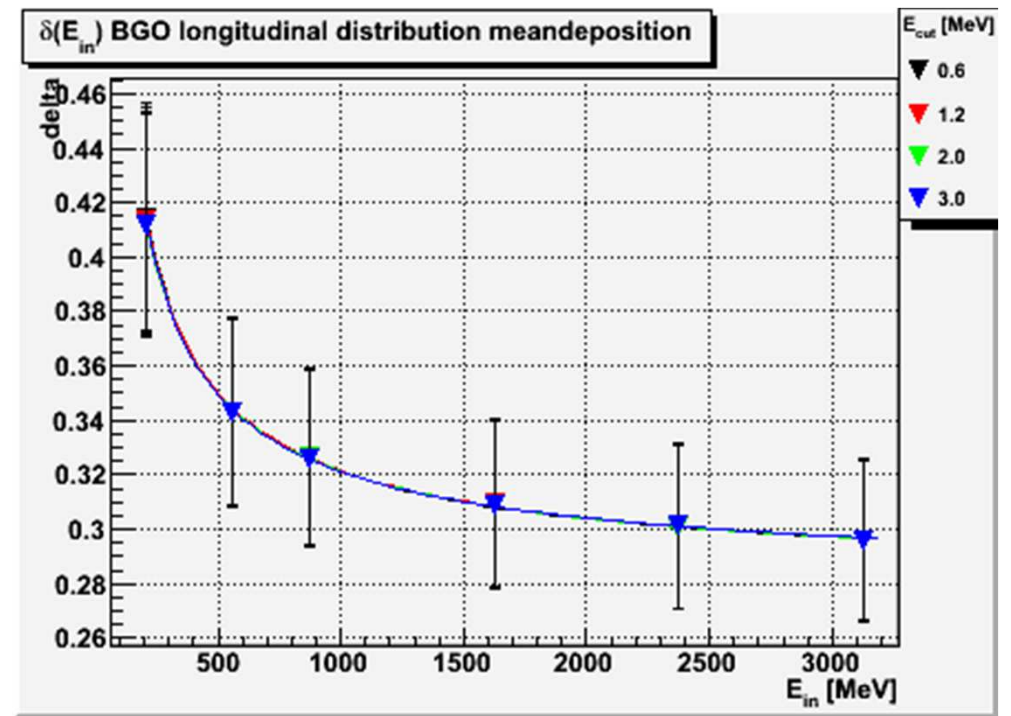


BGO

$$\delta_t(E_\gamma) = a \cdot E_\gamma^b + c$$



EGS4

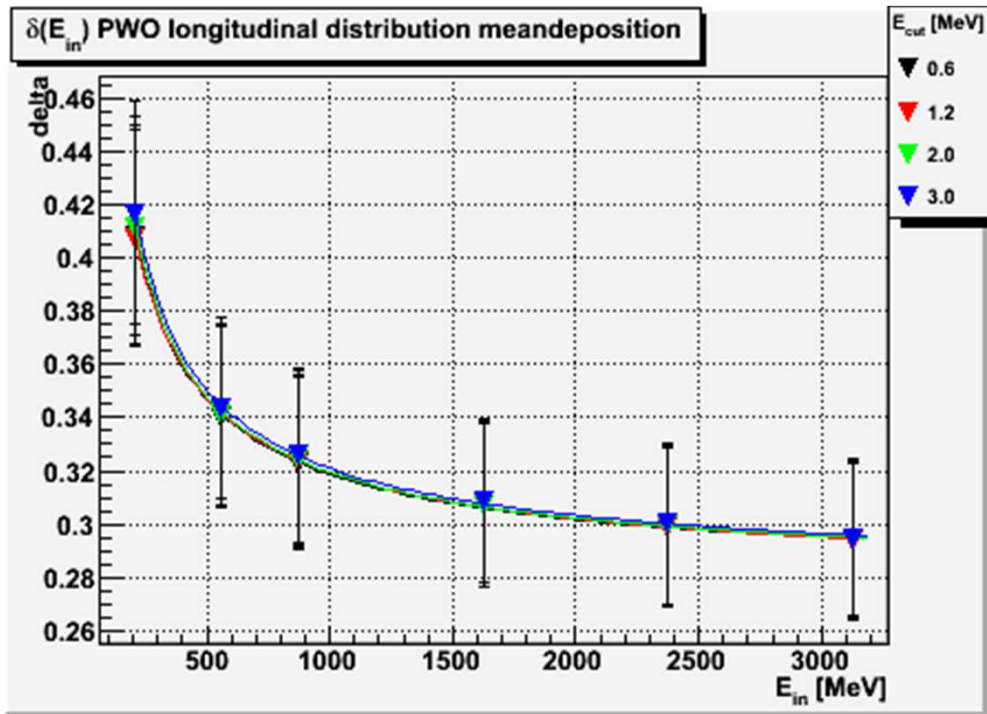


GEANT4

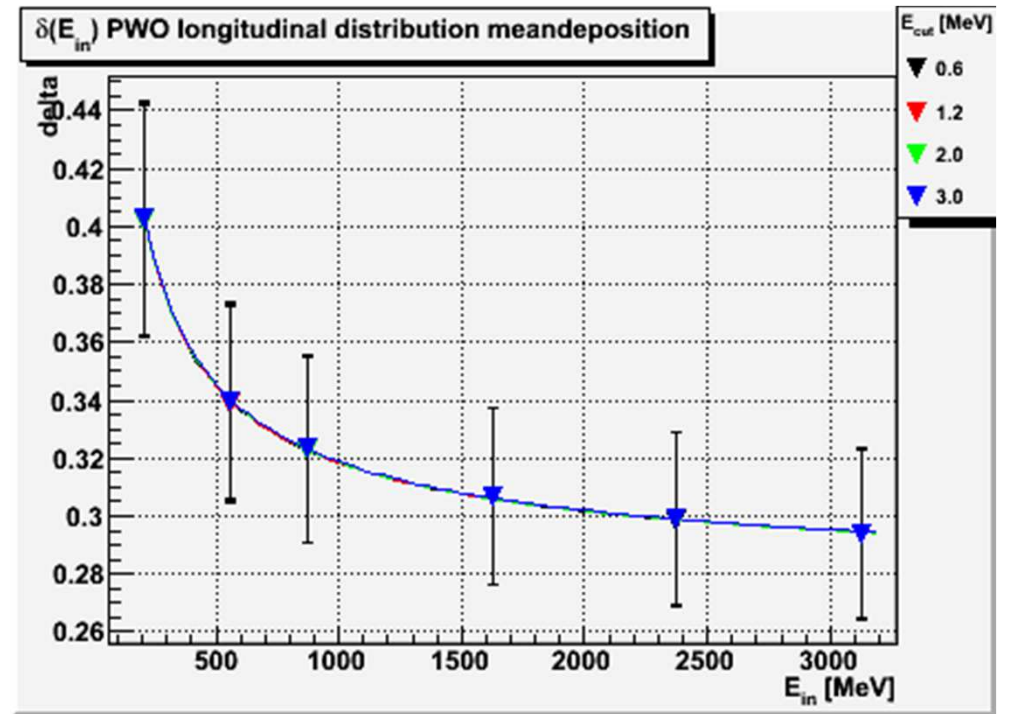


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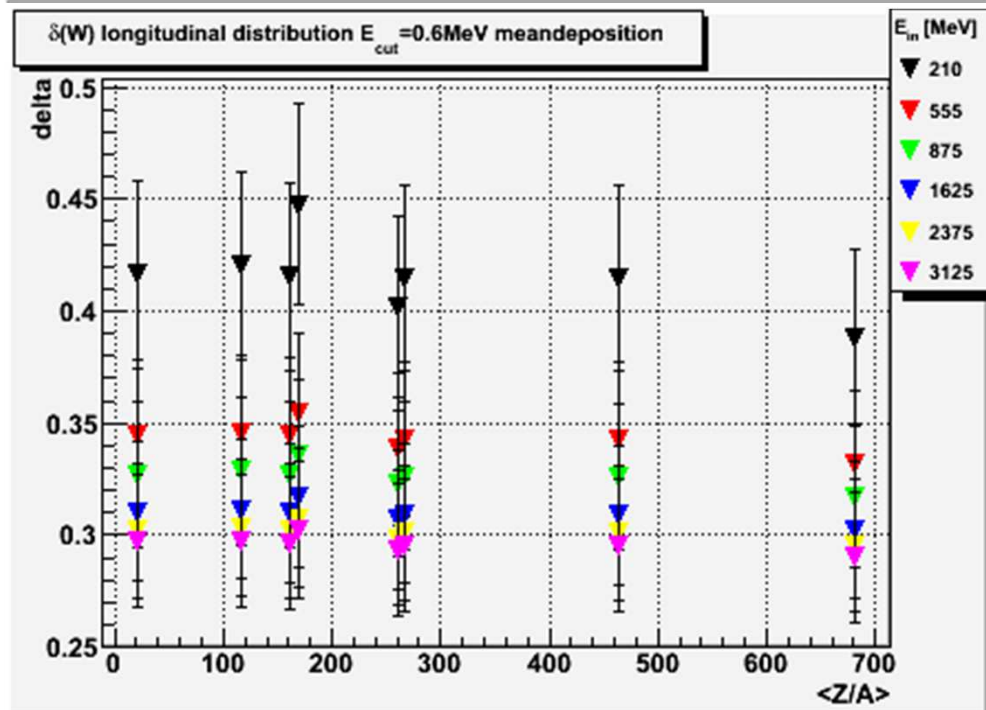
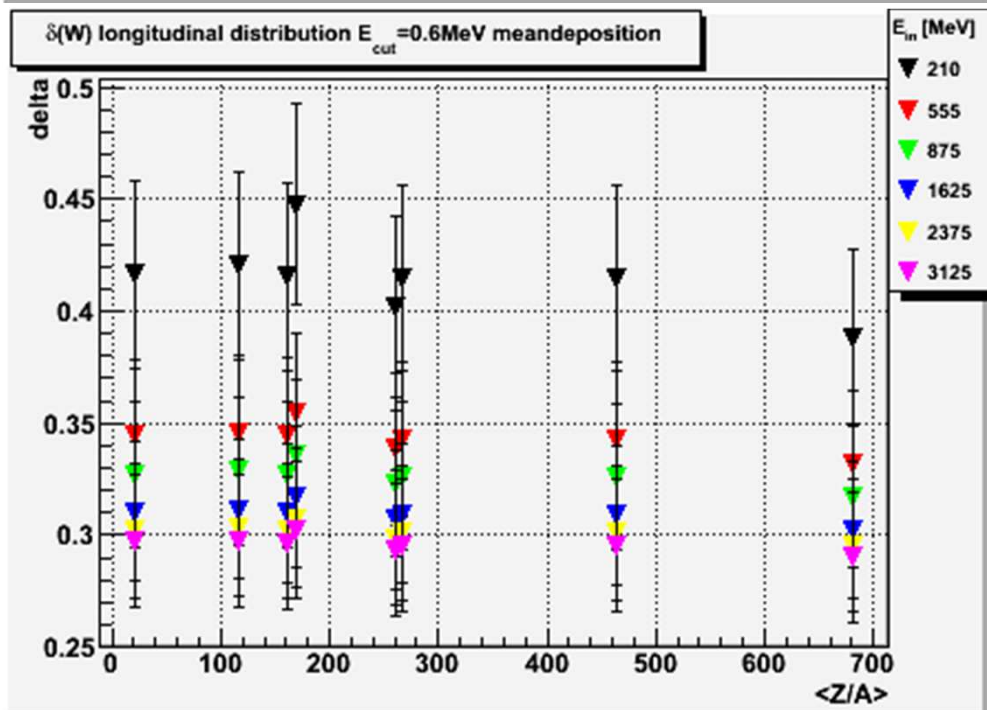
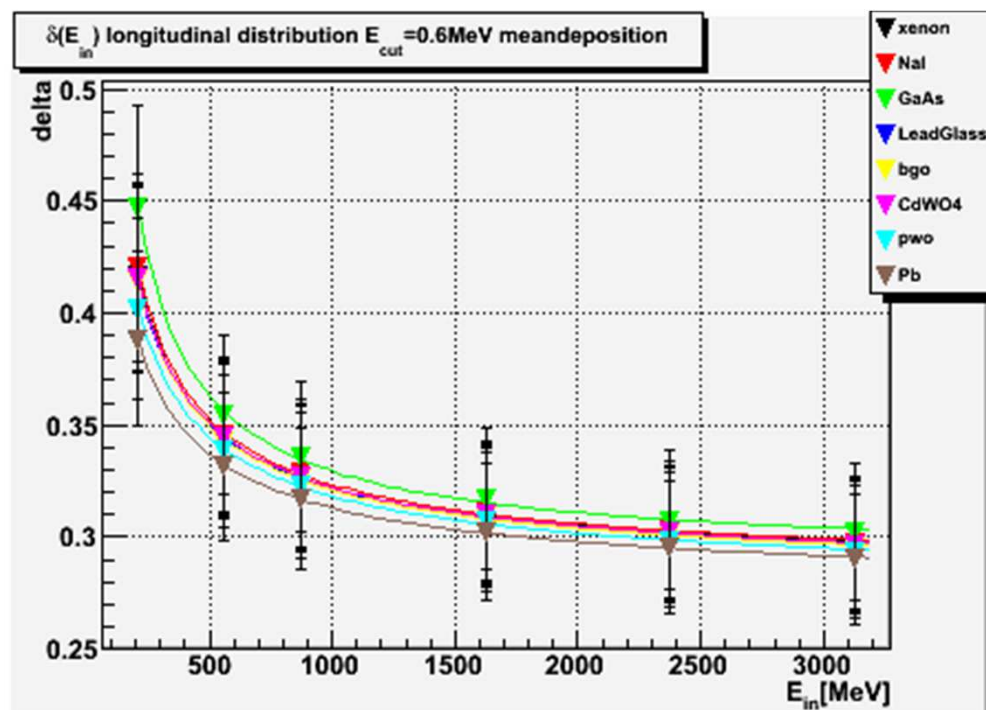
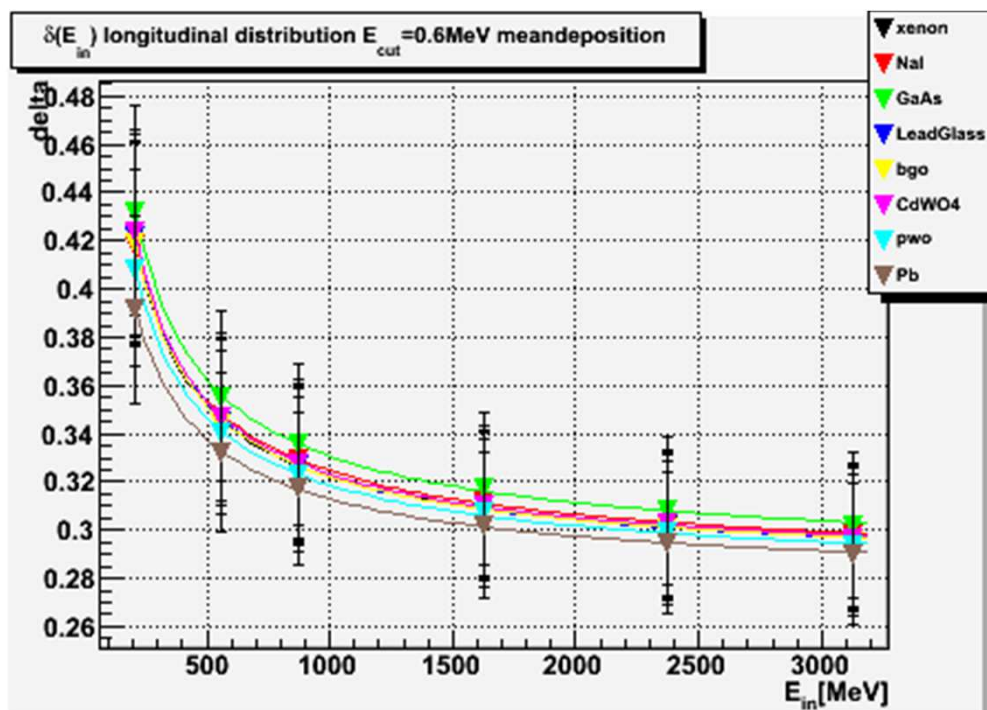
$$\delta_t(E_\gamma) = a \cdot E_\gamma^b + c$$



EGS4



GEANT4



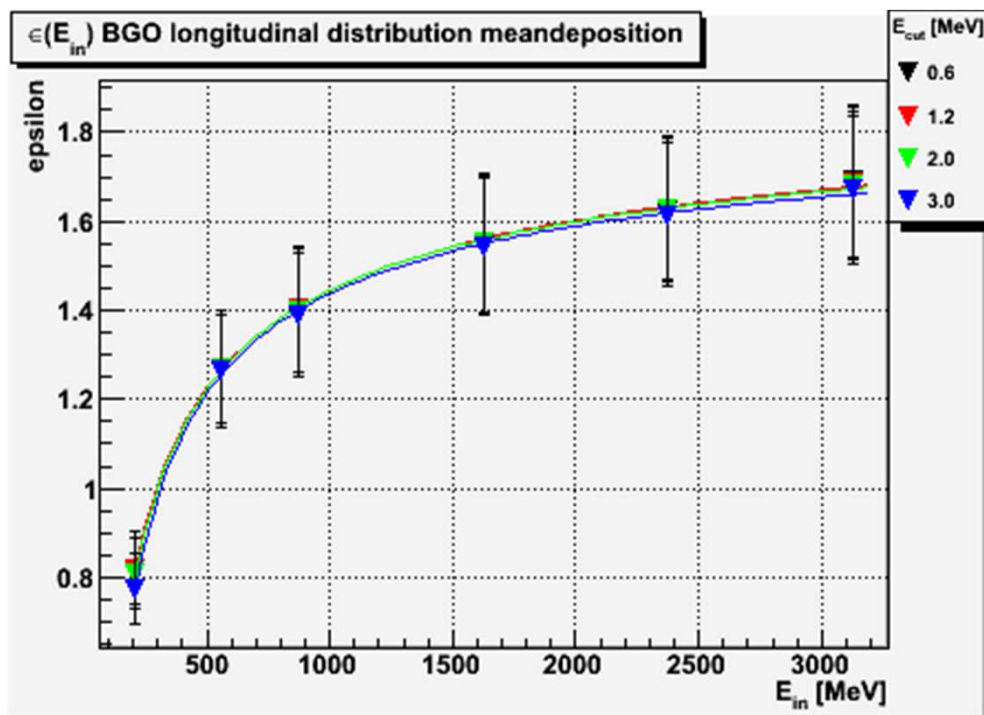
Parameter

ε_t

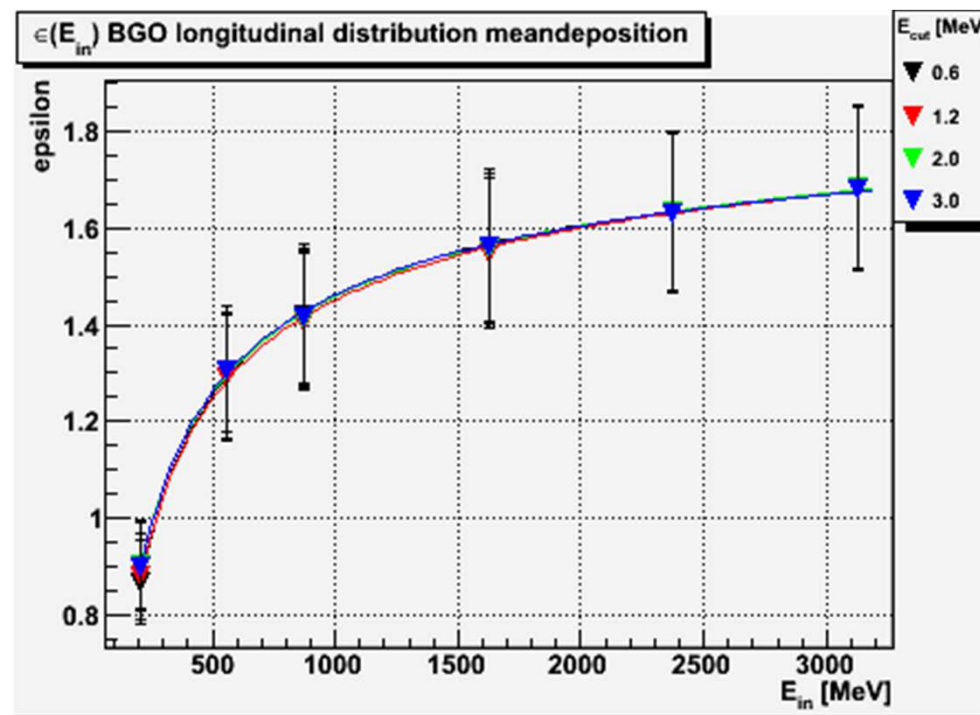


BGO

$$\varepsilon_t(E_\gamma) = a \cdot E_\gamma^b + c$$



EGS4

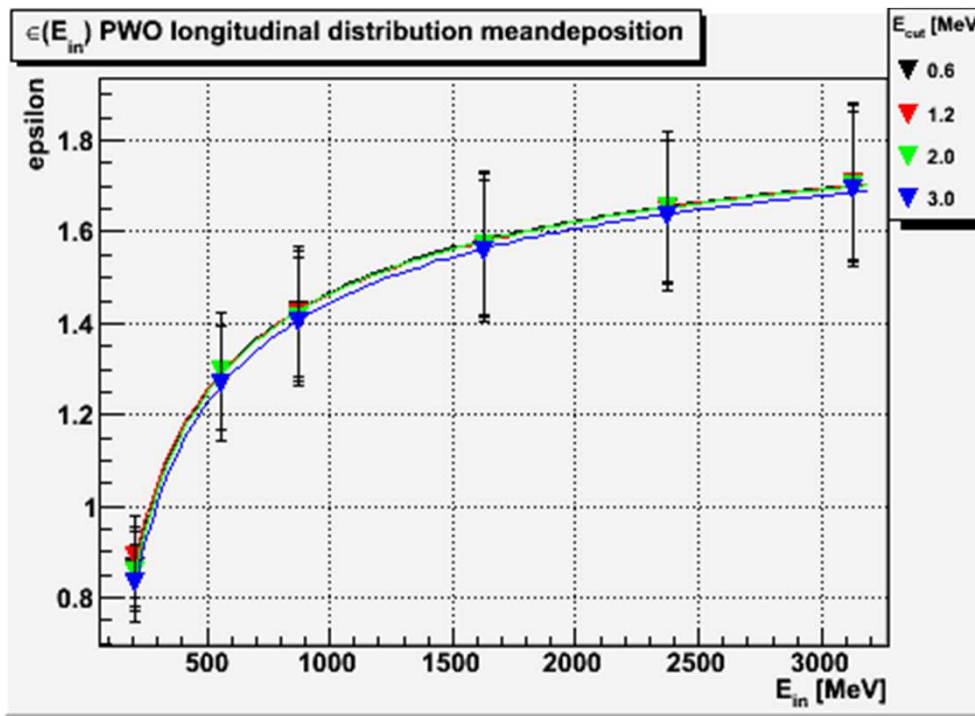


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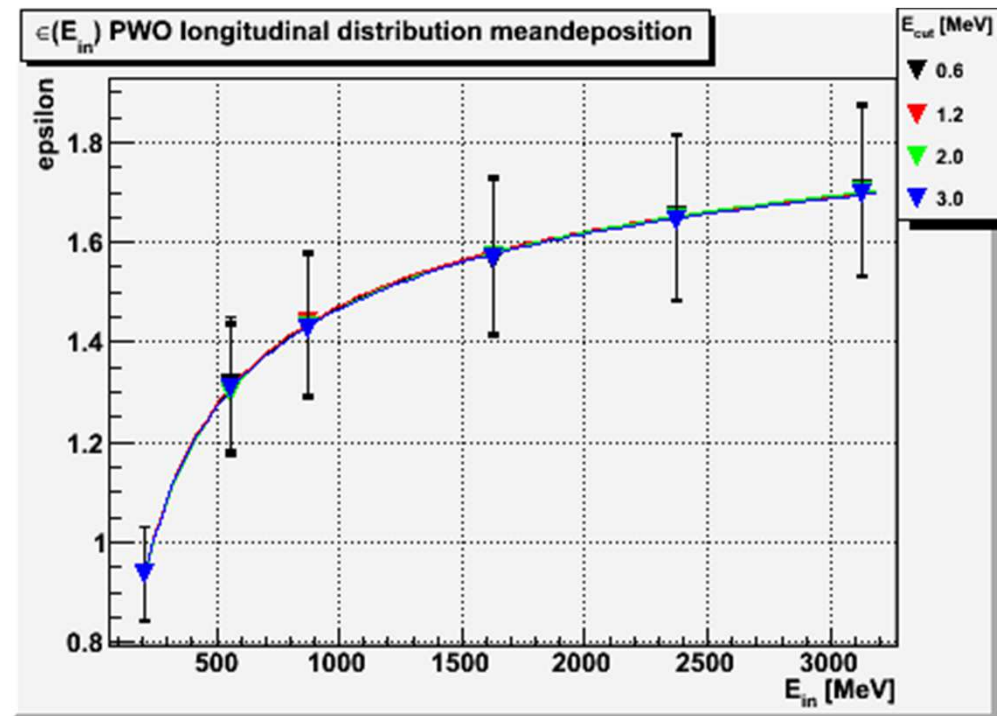


PWO

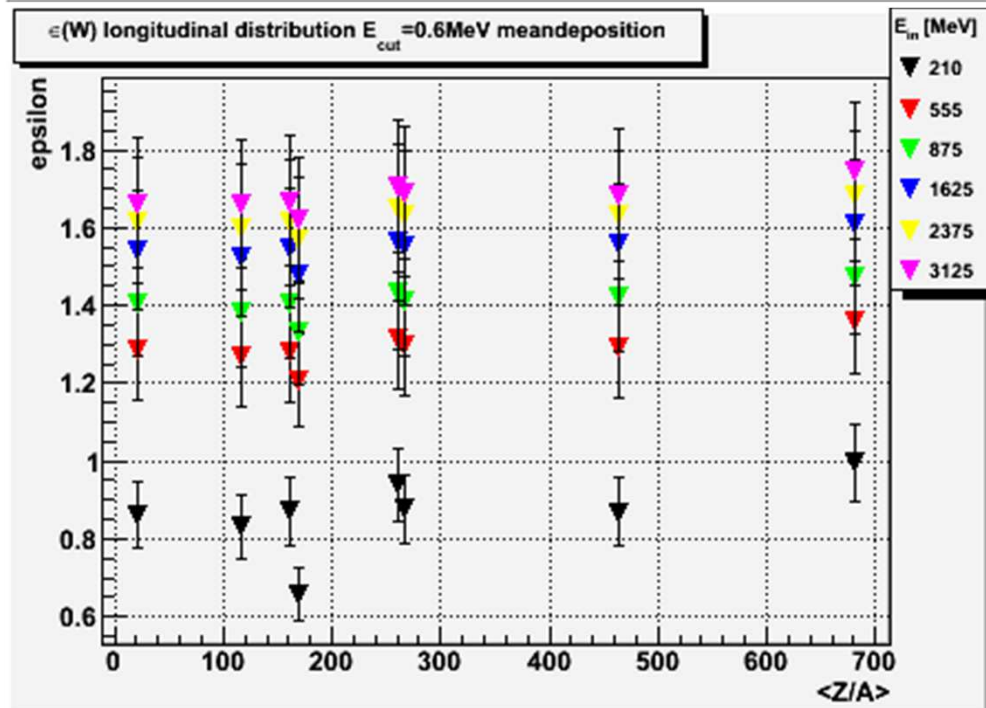
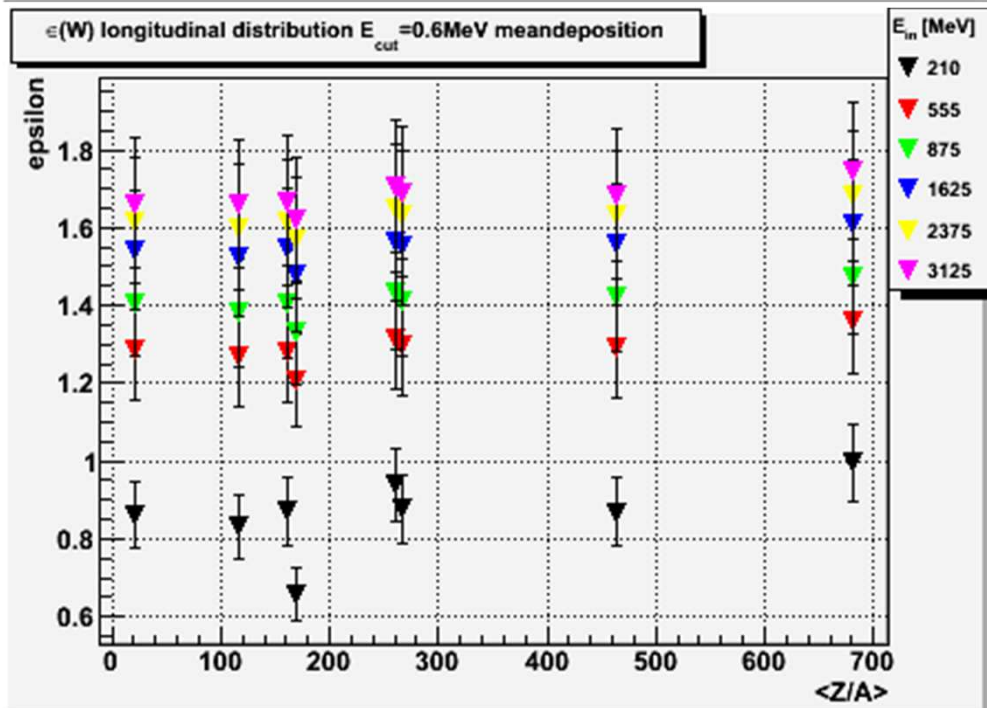
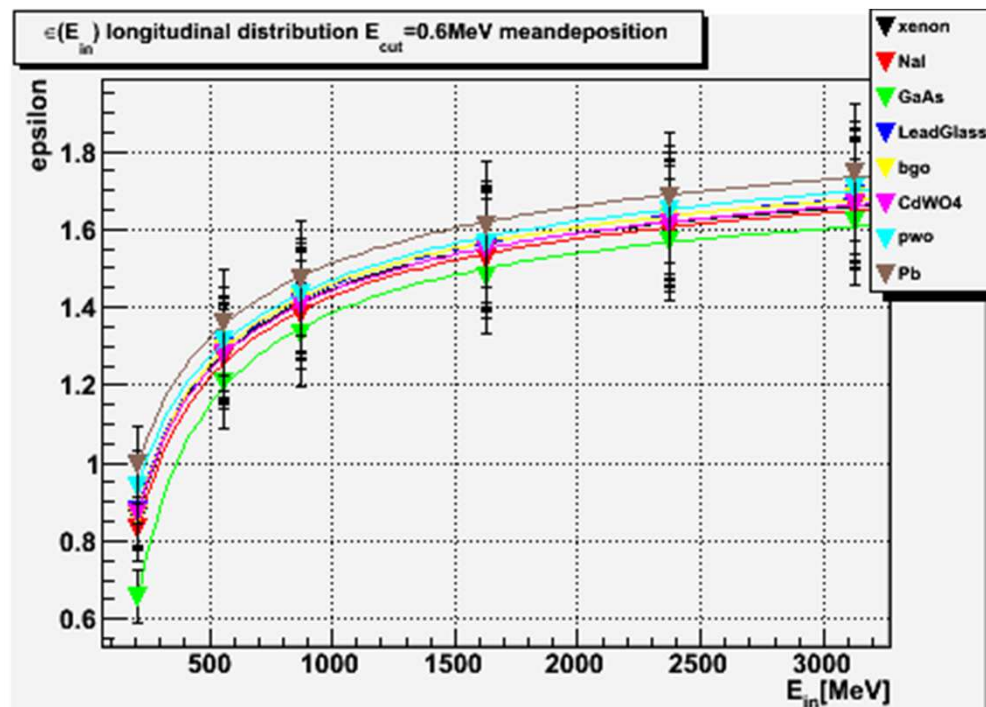
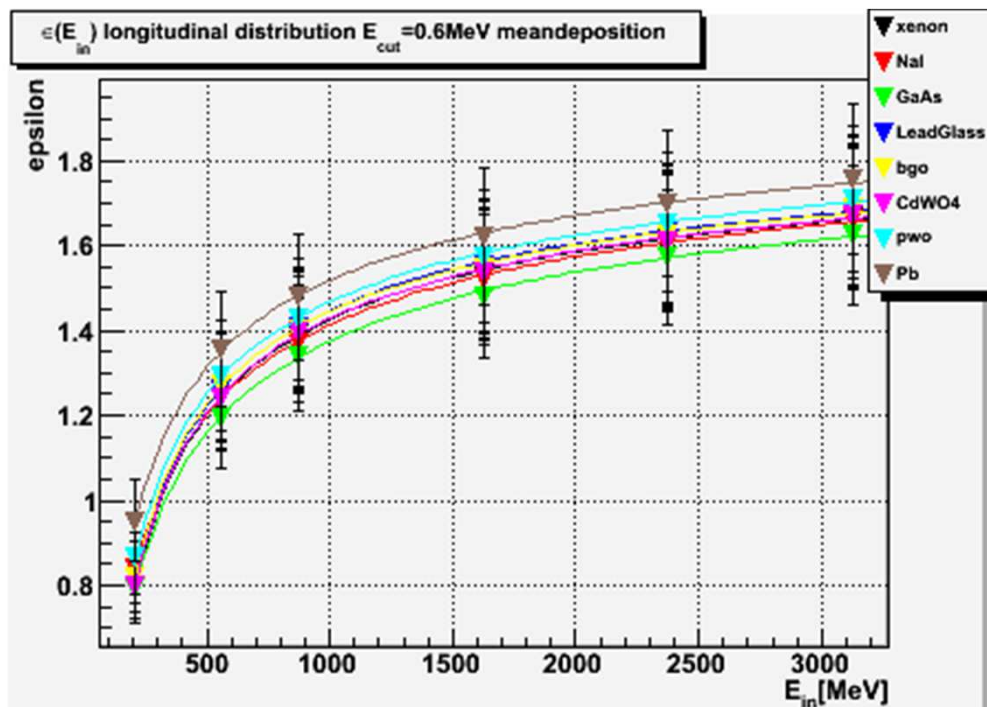
$$\varepsilon_t(E_\gamma) = a \cdot E_\gamma^b + c$$



EGS4

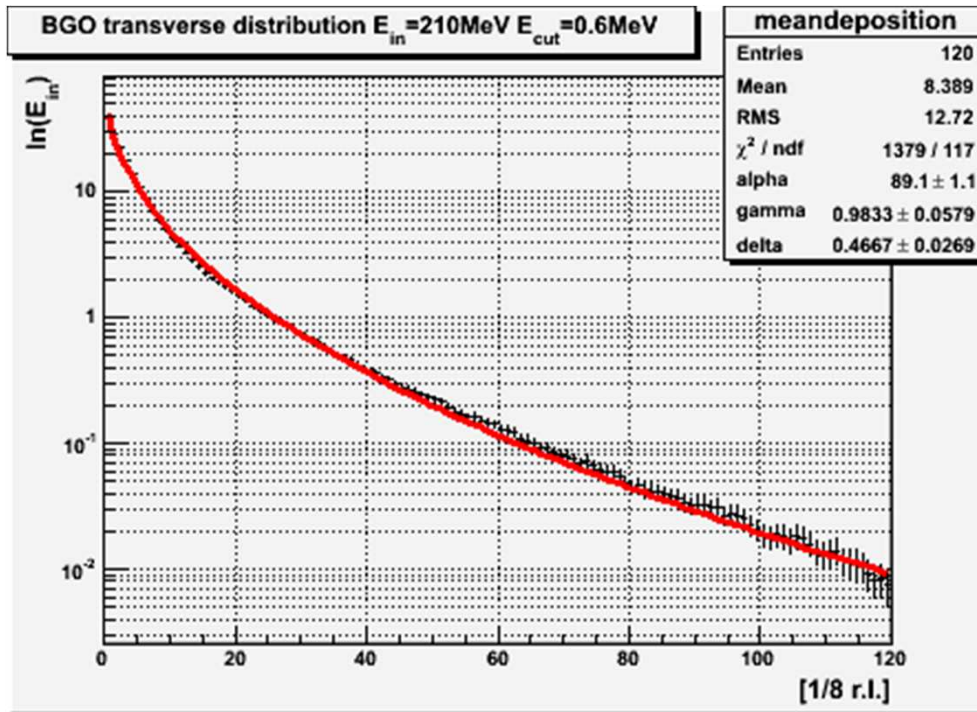


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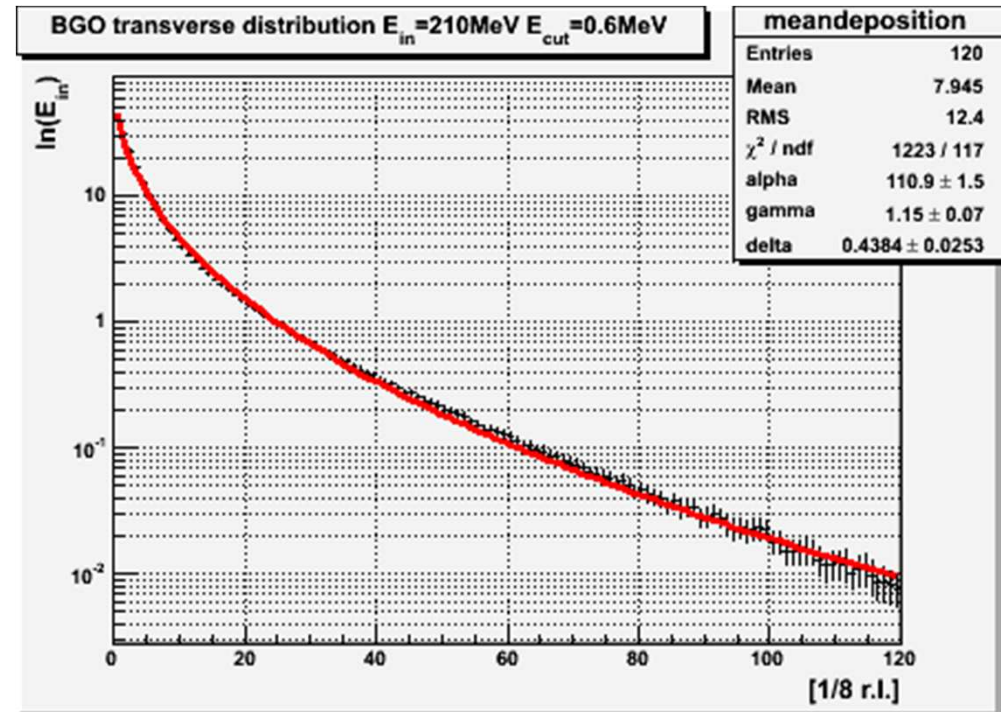


TRANSVERSE PROFILES

Average transverse profile of EMC produced in BGO by gammas of energy 210 MeV ($E_{co}=0.6$ MeV).



EGS4



GEANT4



APPROXIMATING FUNCTION

$$(-dE / dr) = \alpha_r \exp(-\gamma_r r^{\delta_r})$$

α_r , δ_r , γ_r are the fit parameters depending on cut-off energy $E_{c.o.}$, E_γ and material properties Z/A .

Dependence of parameters:

γ_r and δ_r

on

E_γ , $E_{c.o.}$, Z/A .

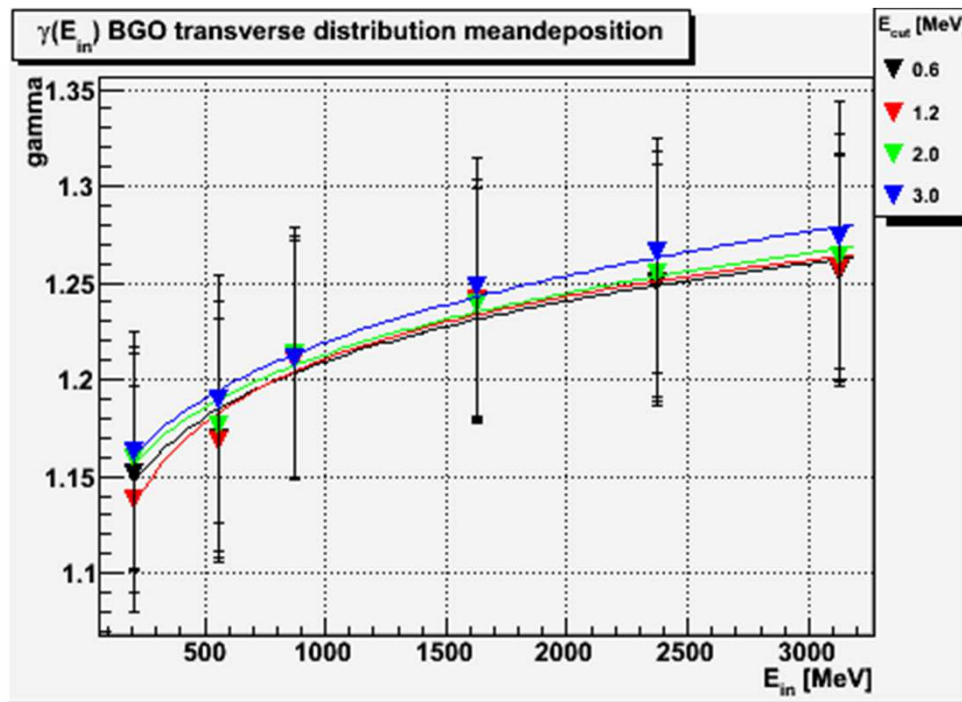
Parameter

γ_r

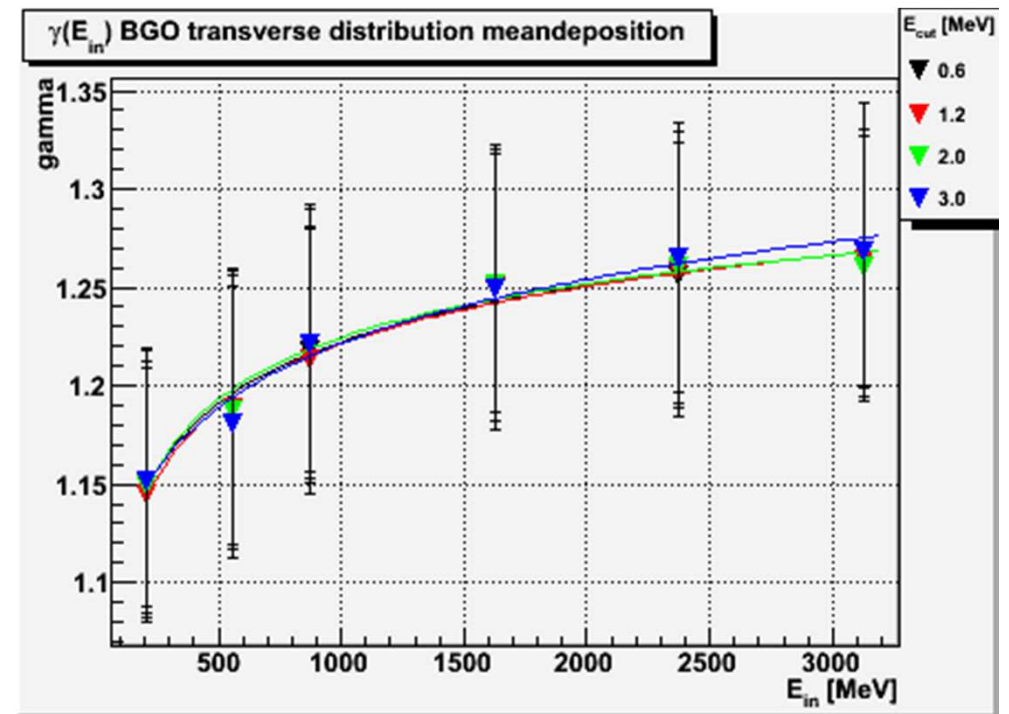


BGO

$$\gamma_r(E_\gamma) = a \cdot E_\gamma^b + c$$



EGS4

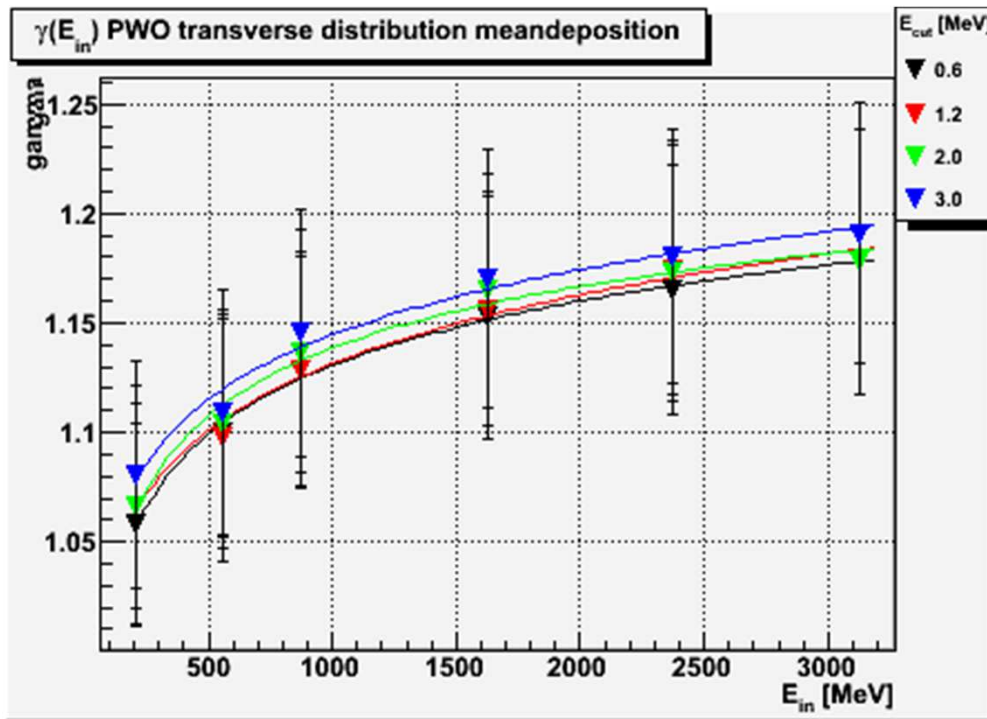


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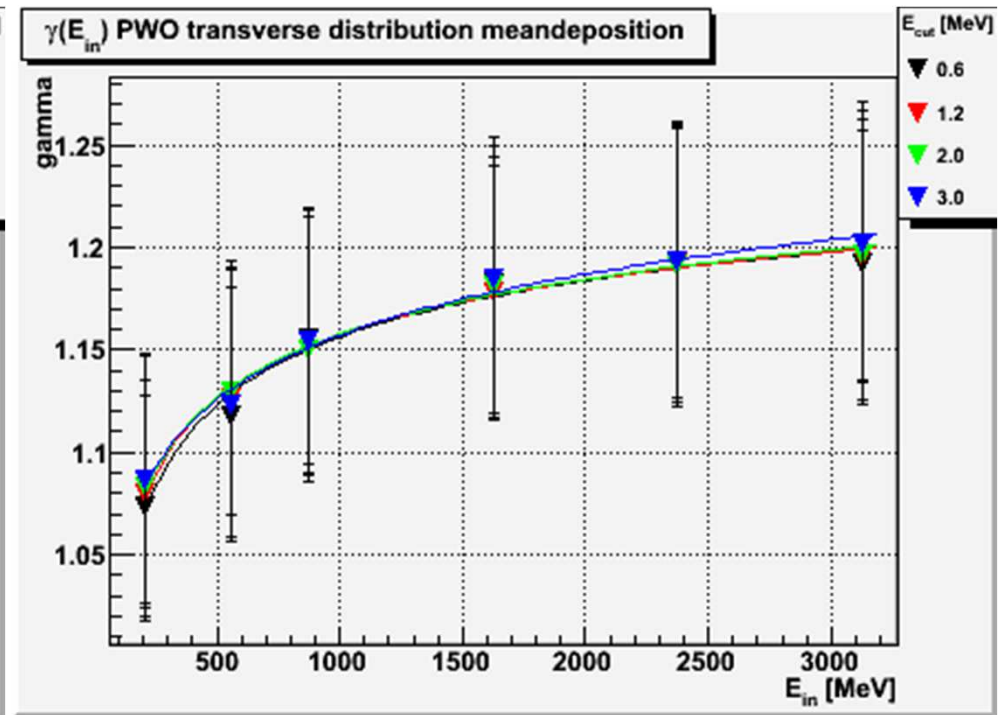


PWO

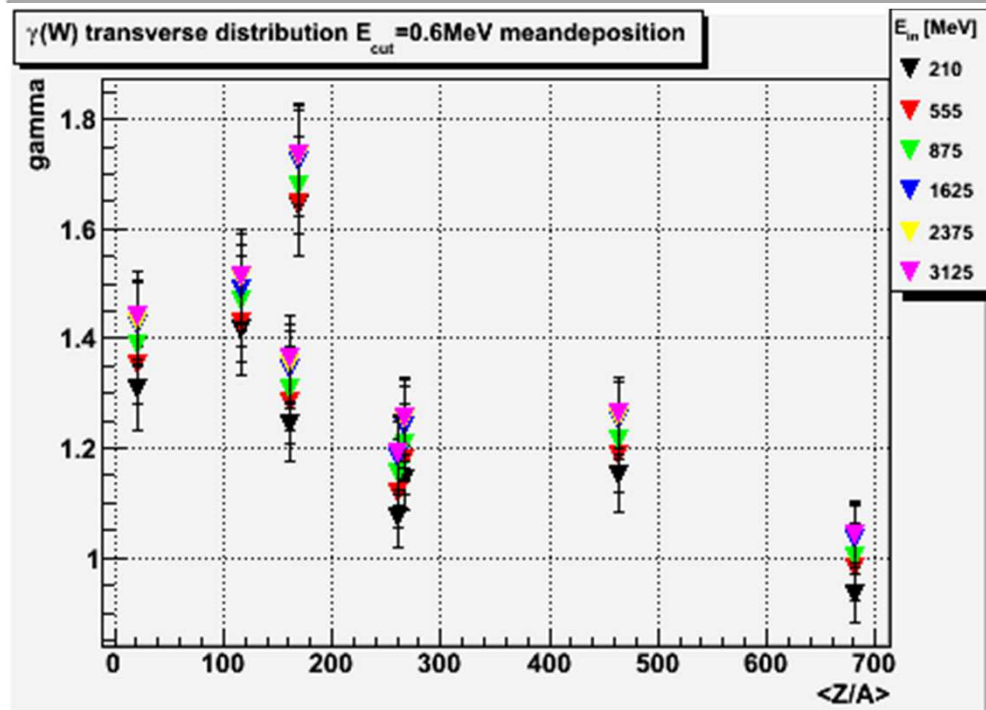
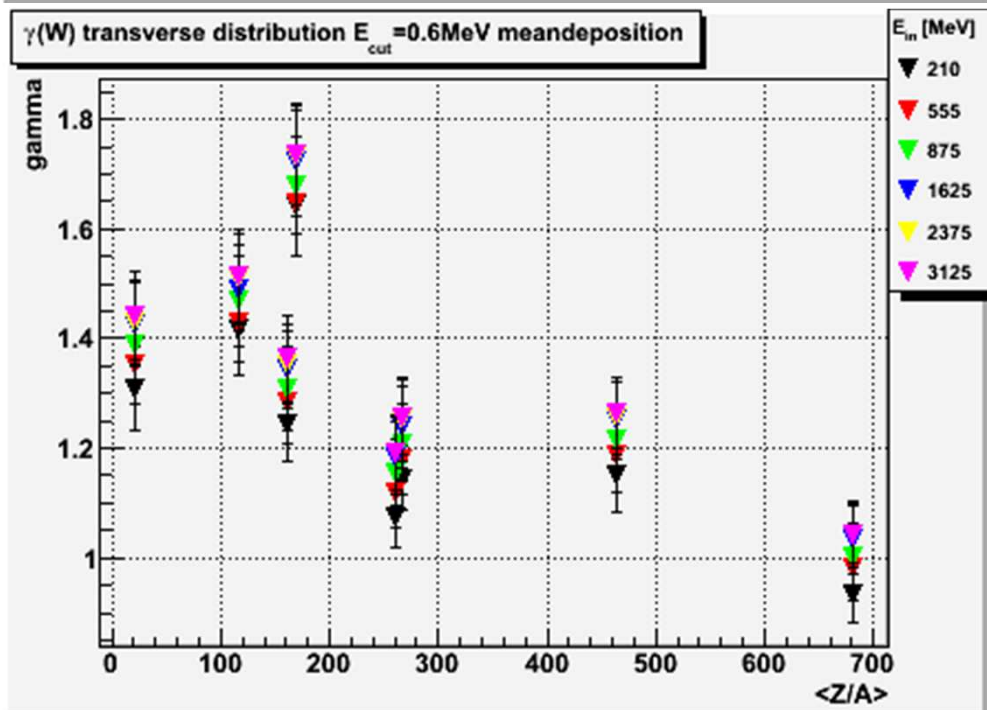
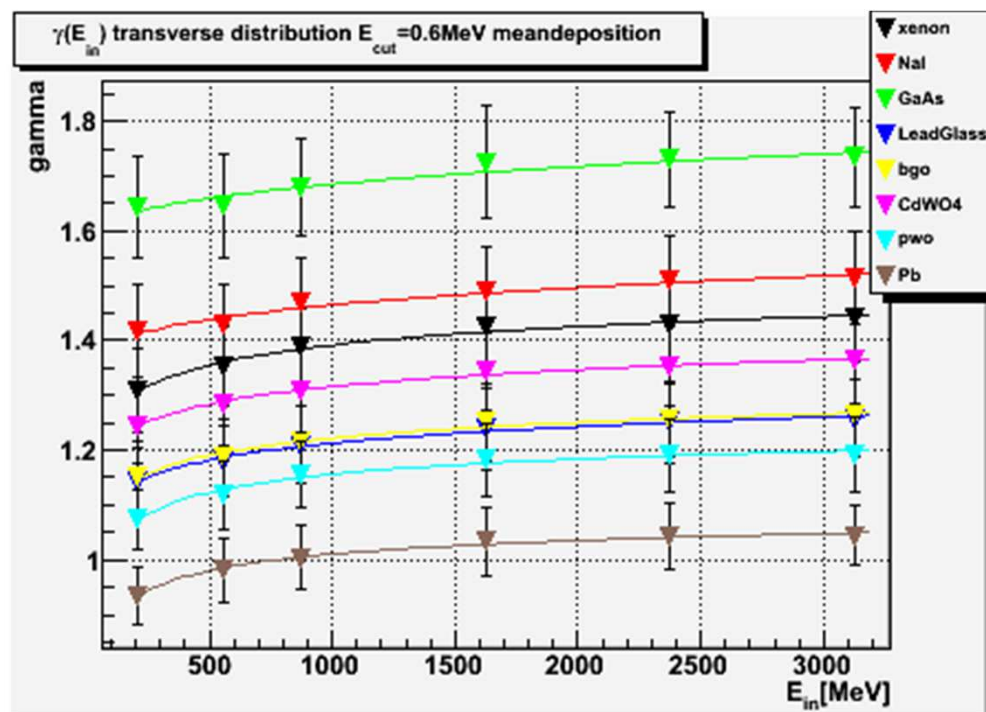
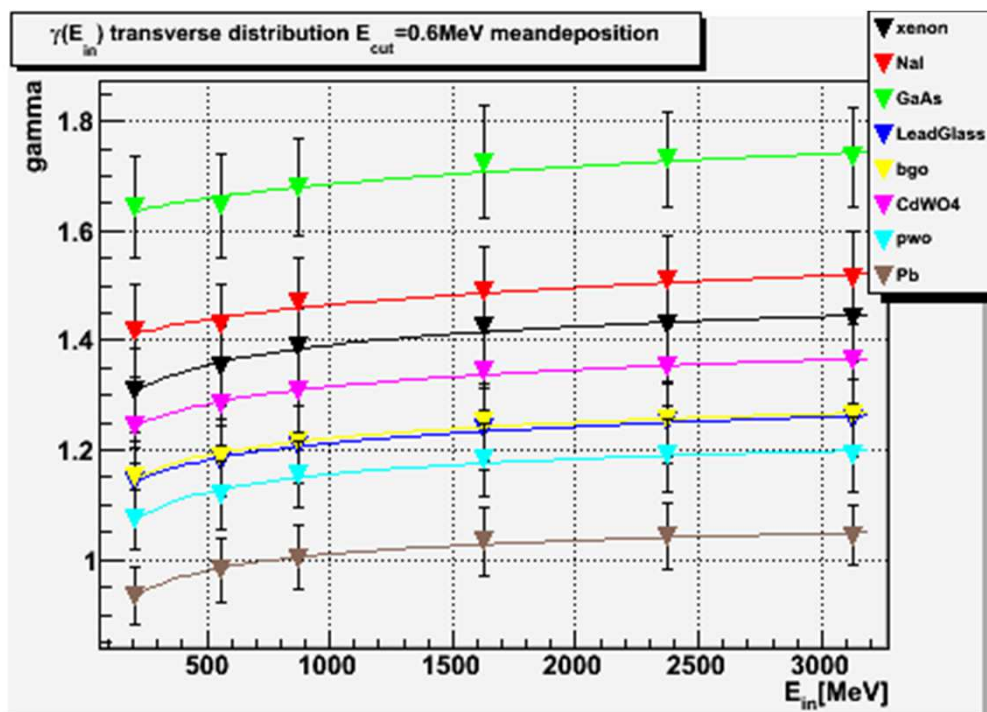
$$\gamma_r(E_\gamma) = a \cdot E_\gamma^b + c$$



EGS4



GEANT4



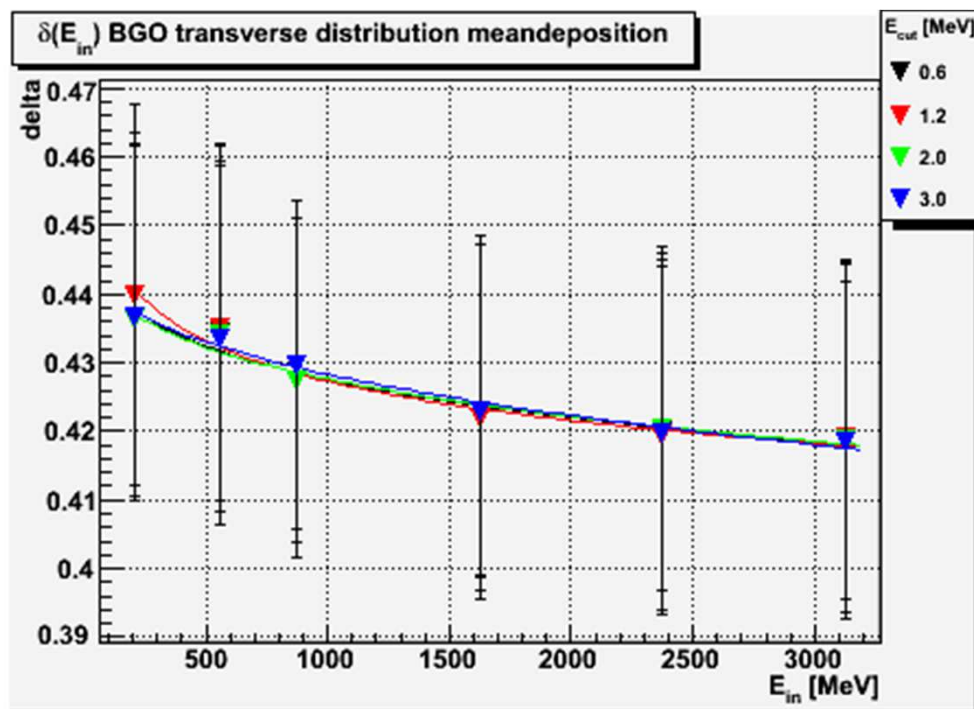
Parameter

δ_t

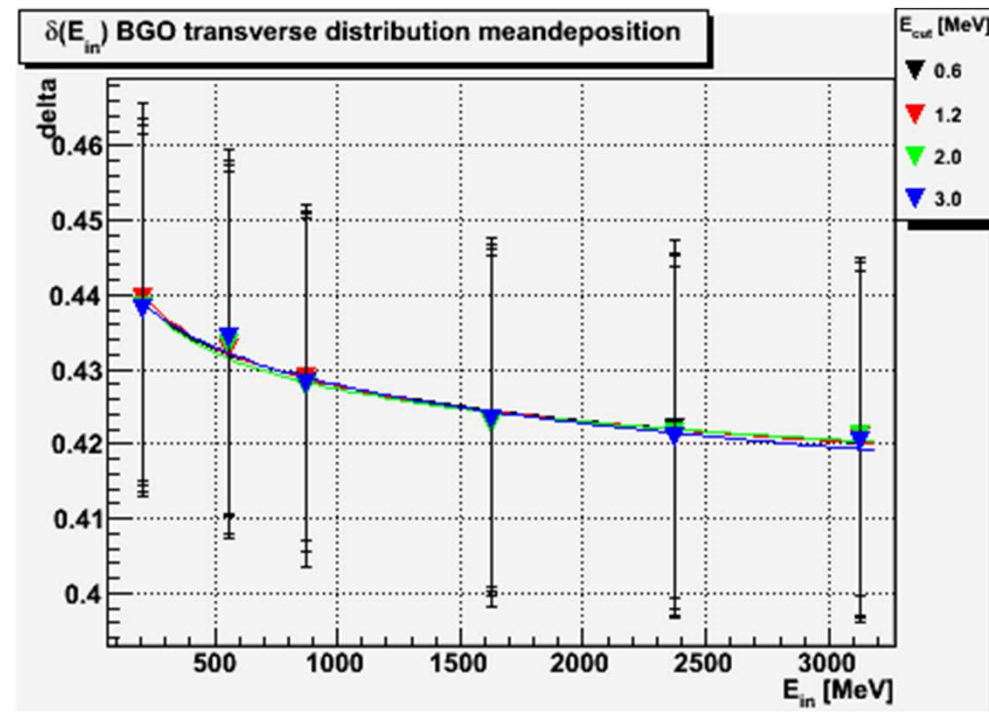


BGO

$$\delta_r(E_\gamma) = a \cdot E_\gamma^b + c$$



EGS4

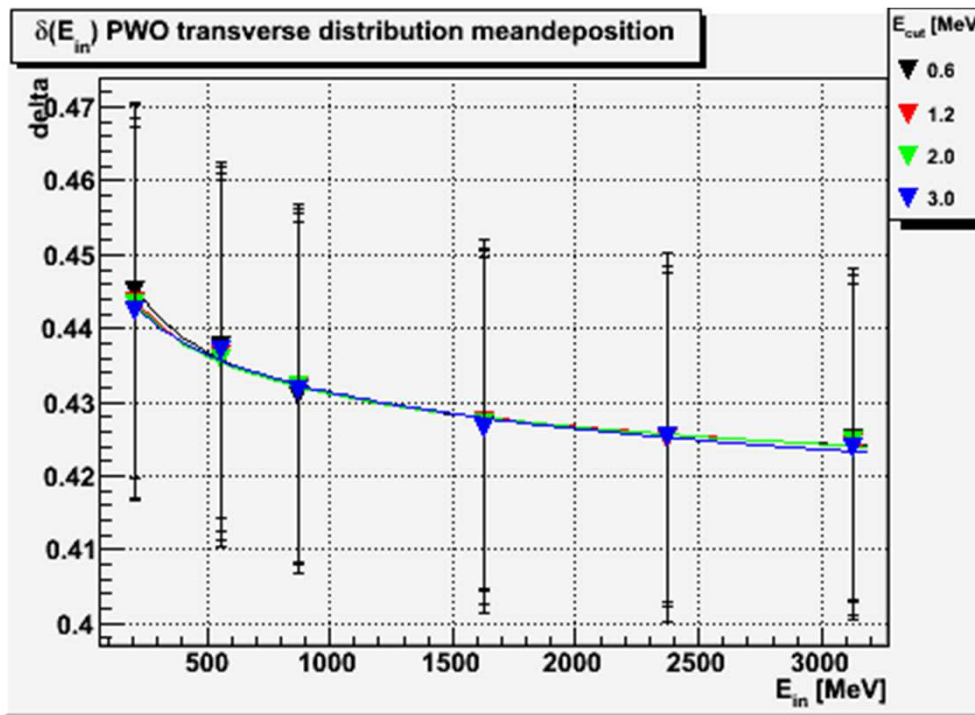


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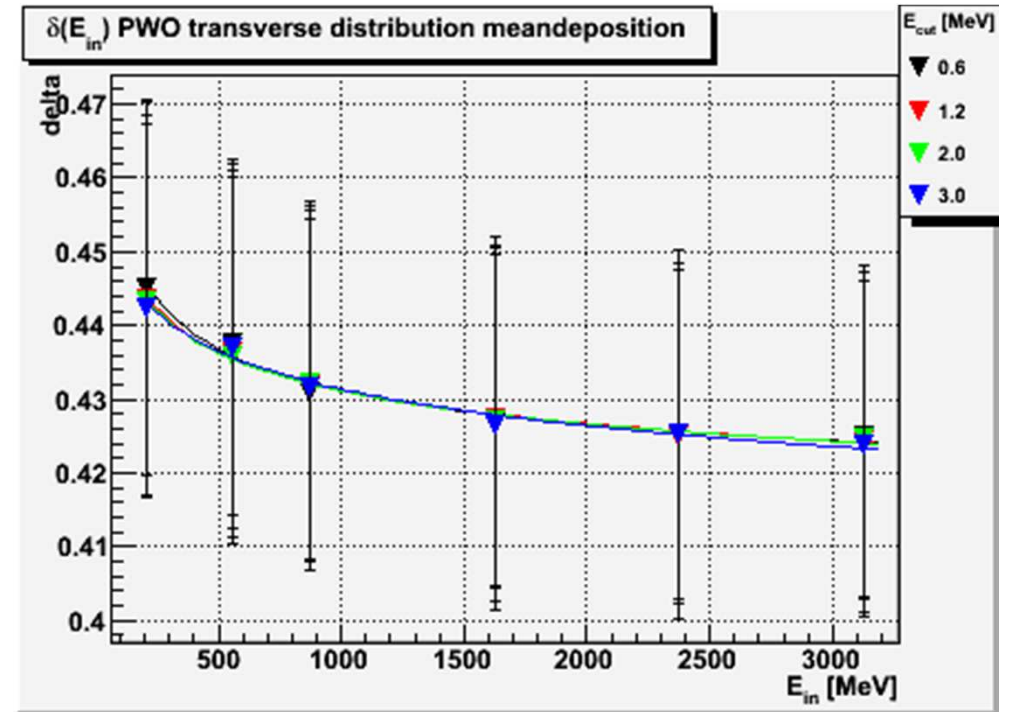


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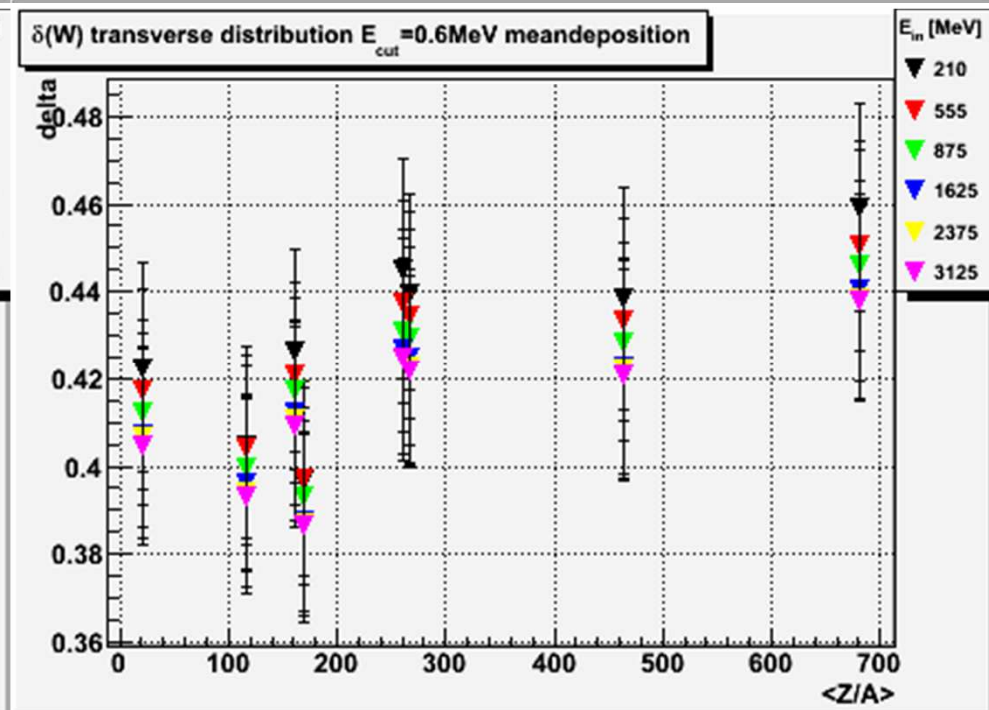
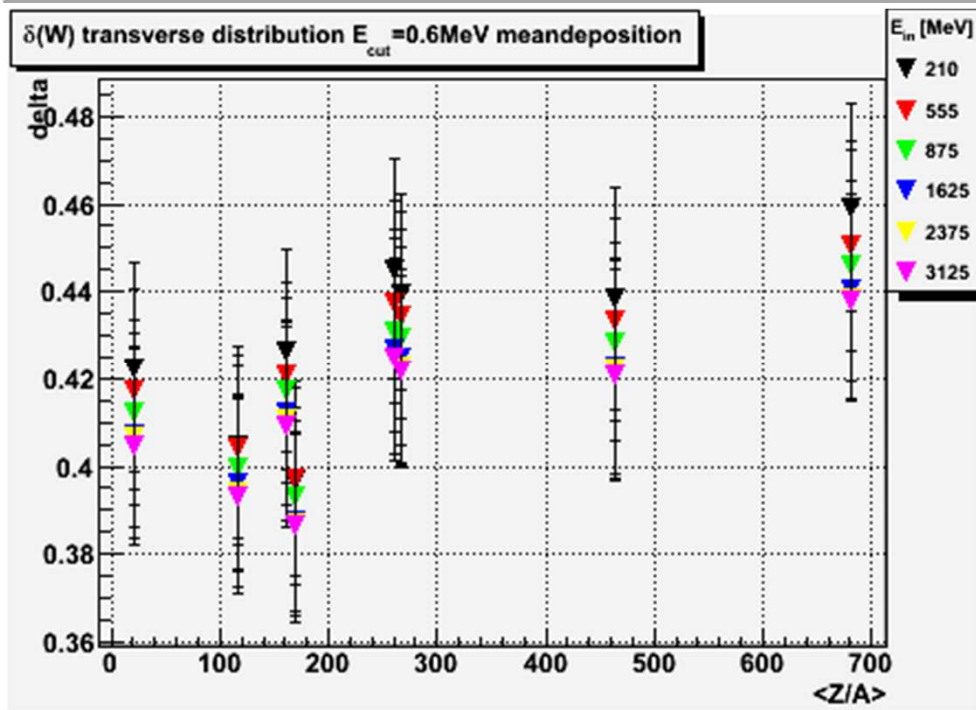
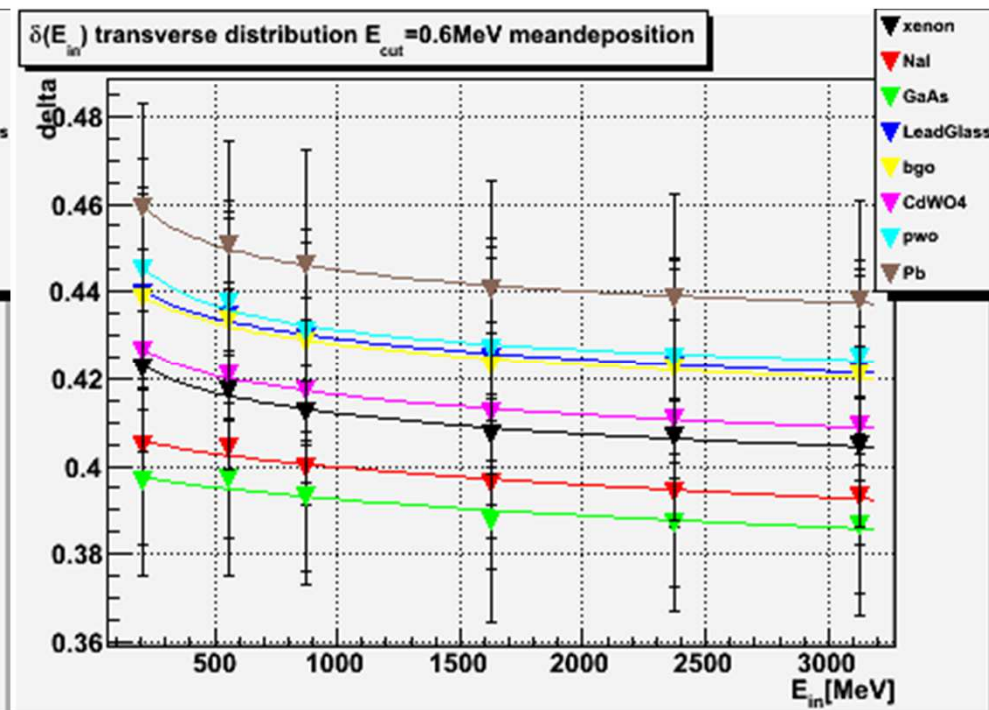
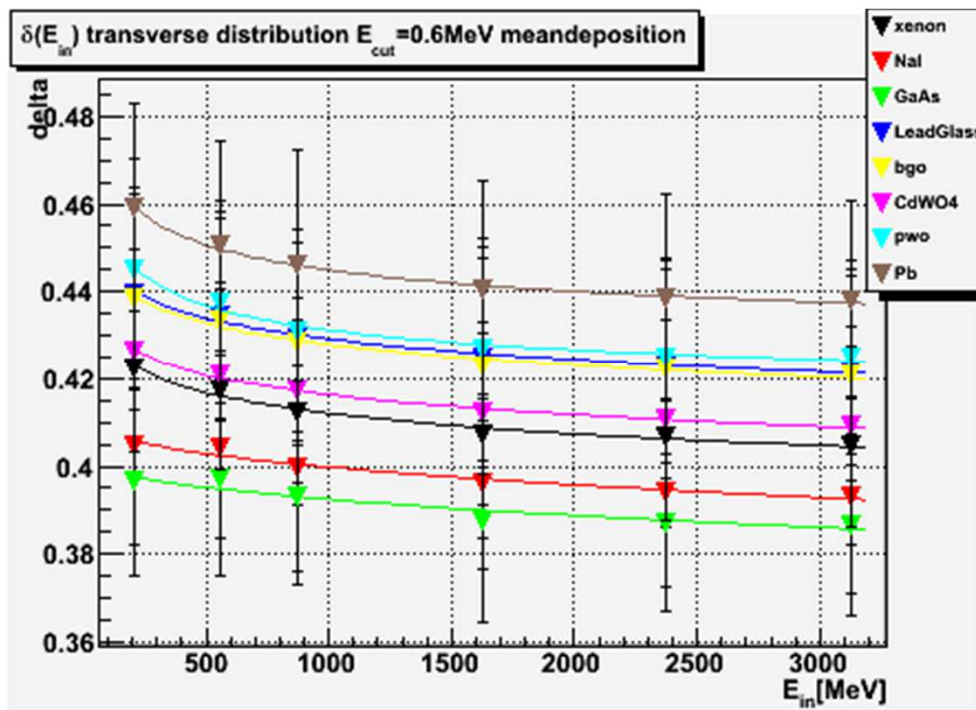
$$\delta_r(E_\gamma) = a \cdot E_\gamma^b + c$$



EGS4



GEANT4





FLUCTUATIONS

**Distributions of the depth t at
which a fraction A of the total EMC
energy is deposited**



Fitting function for longitudinal
and transverse fluctuation of
EMC:

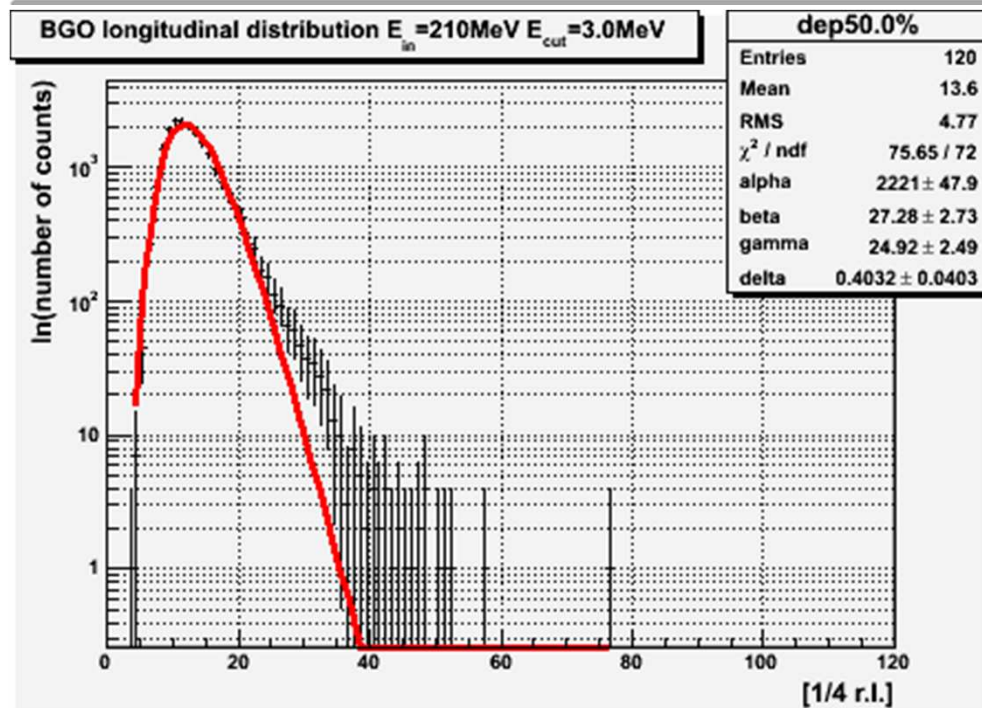
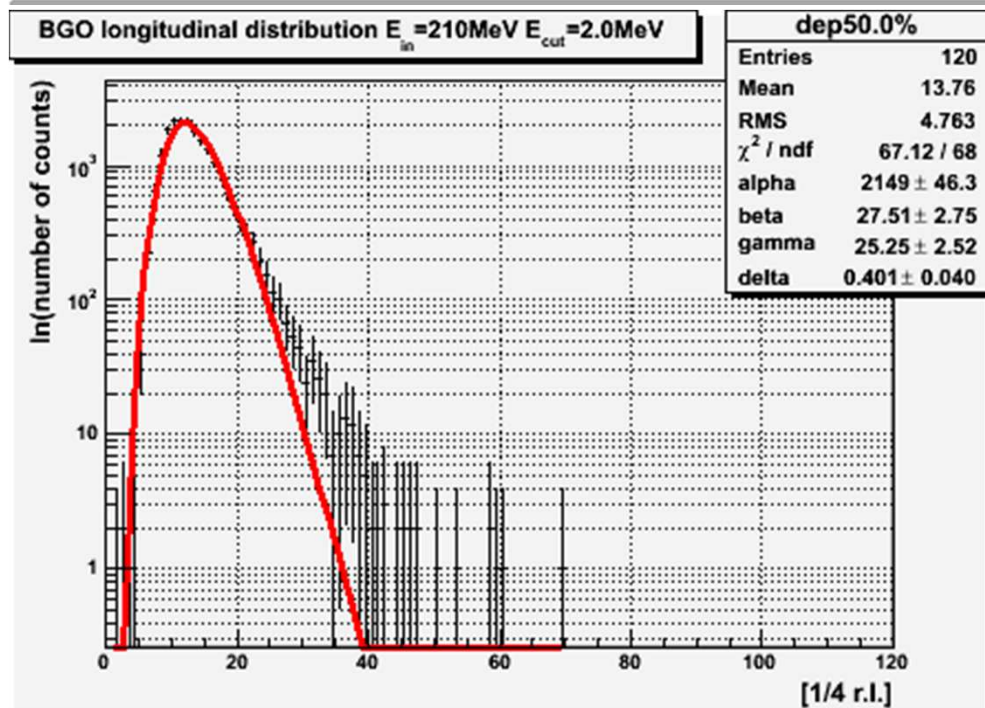
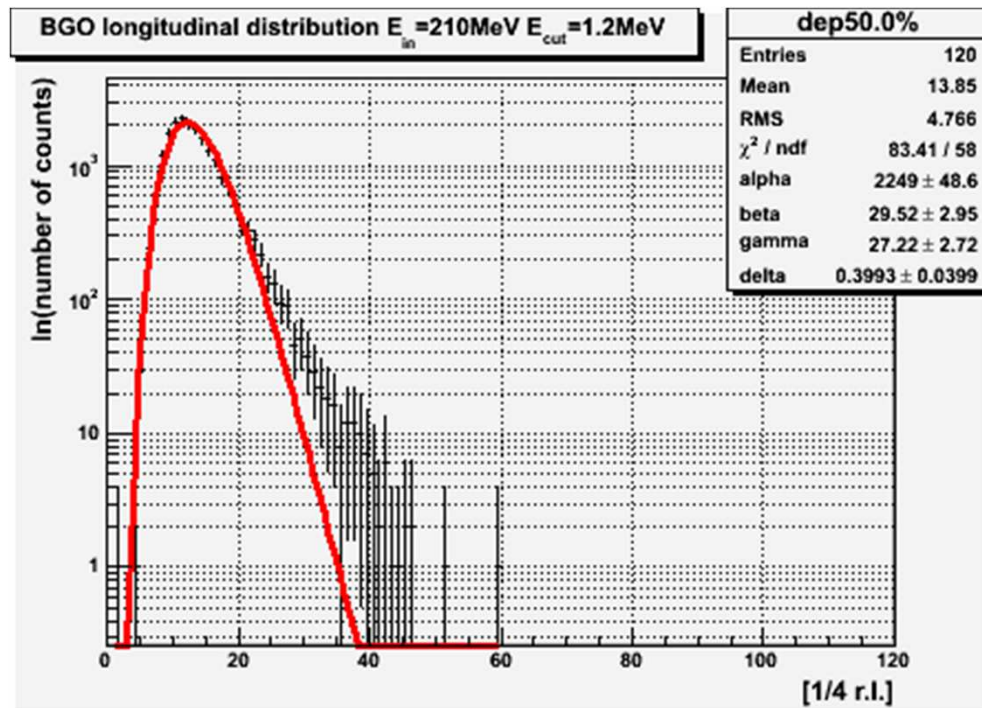
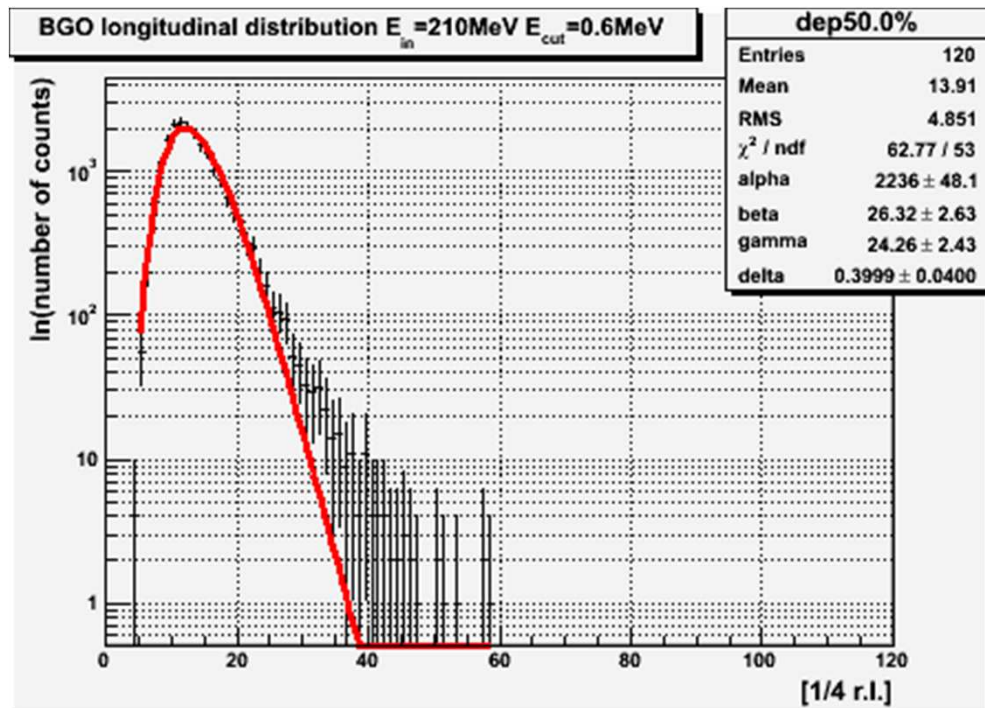
$$P(t_A) = \alpha t_A^\beta \exp(-\gamma t_A^\delta)$$

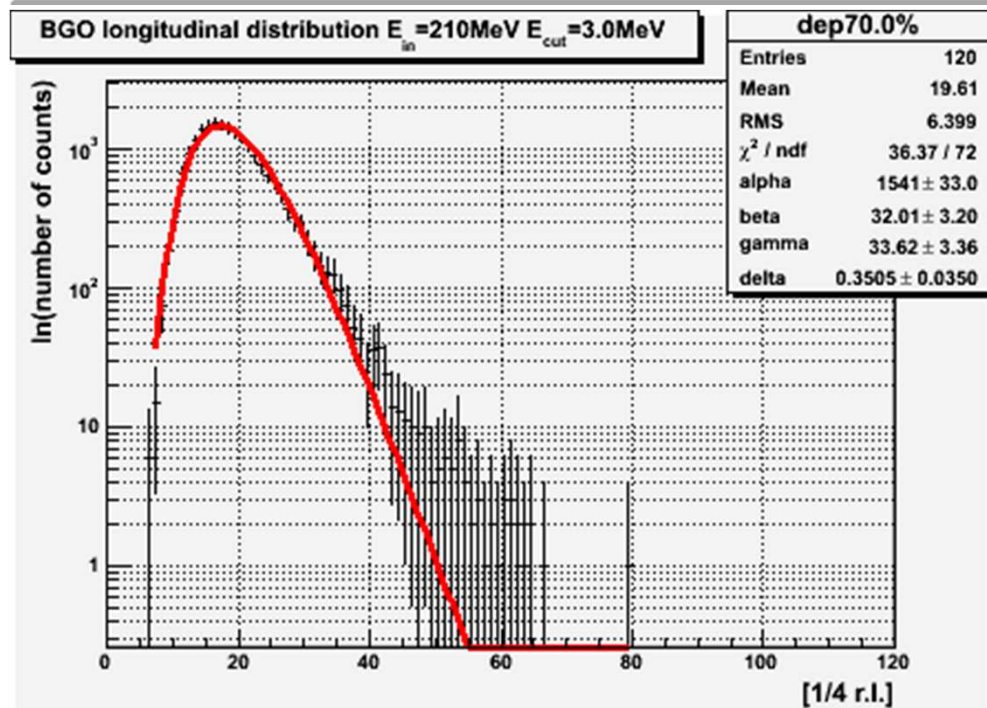
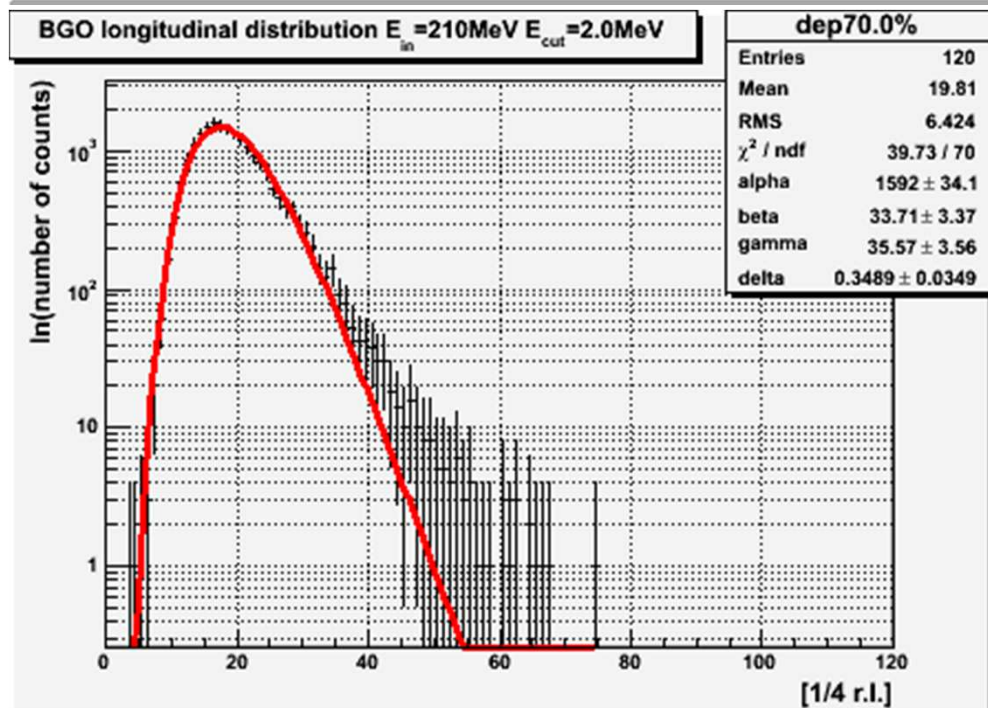
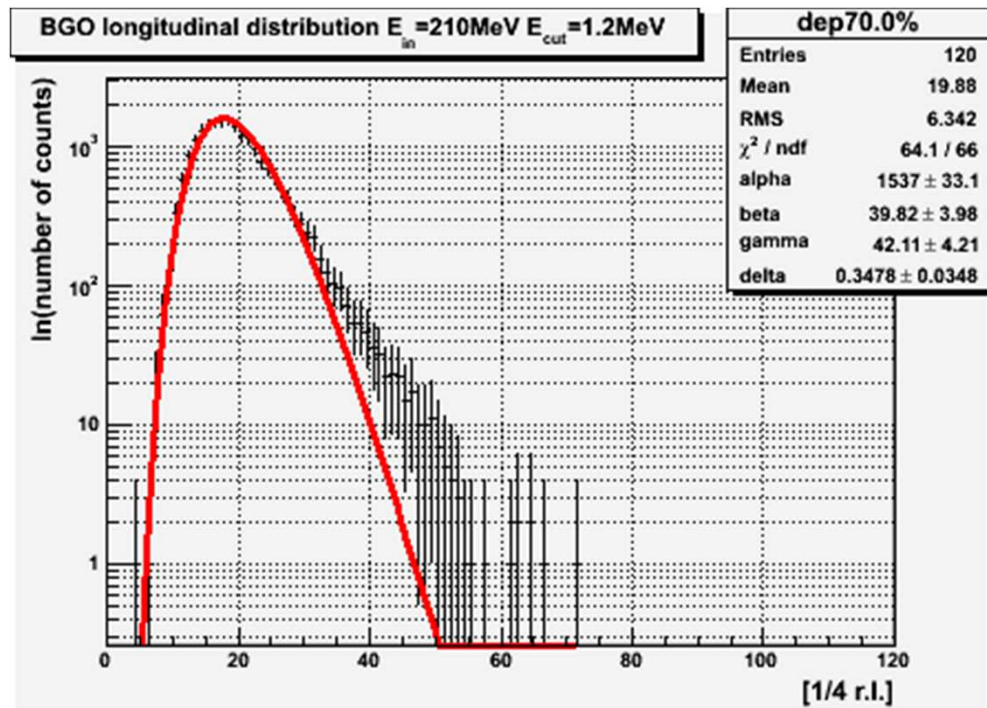
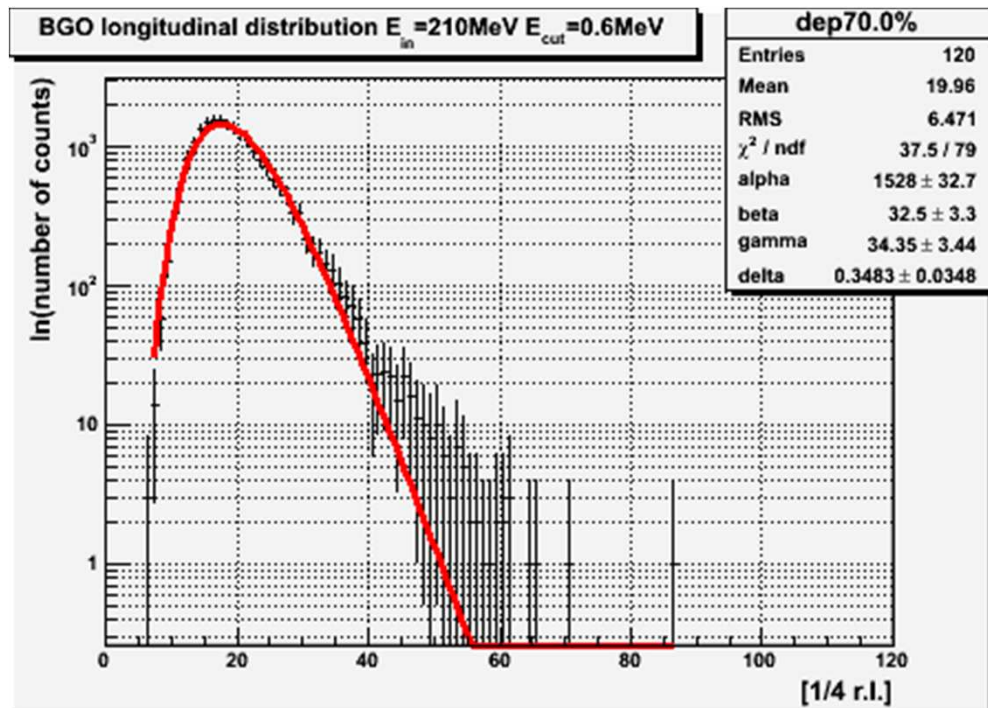
LONGITUDINAL

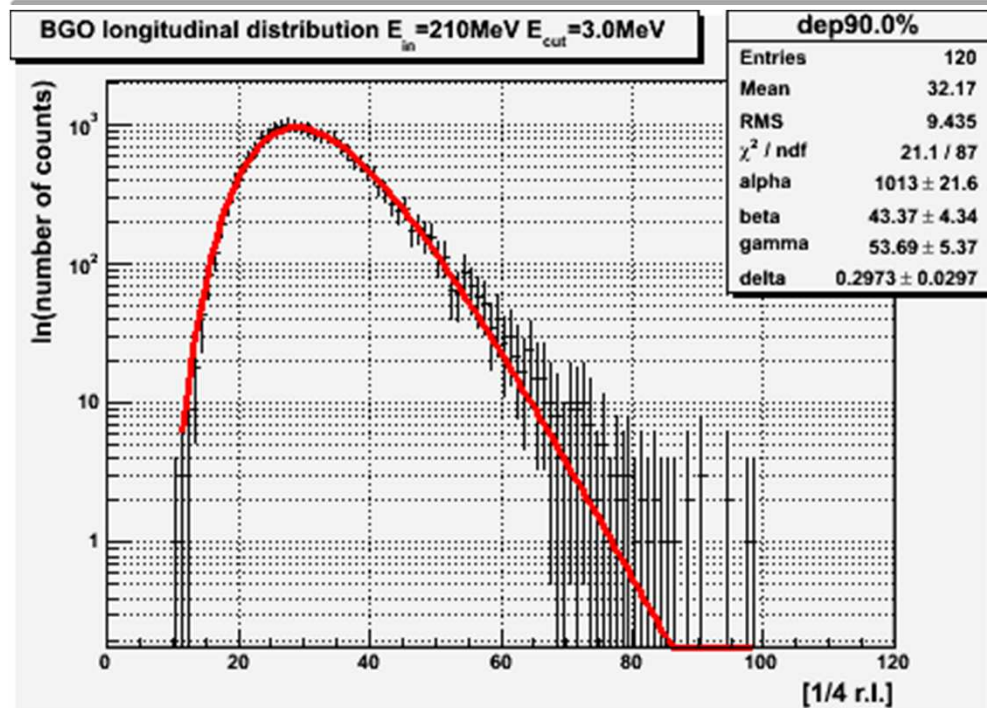
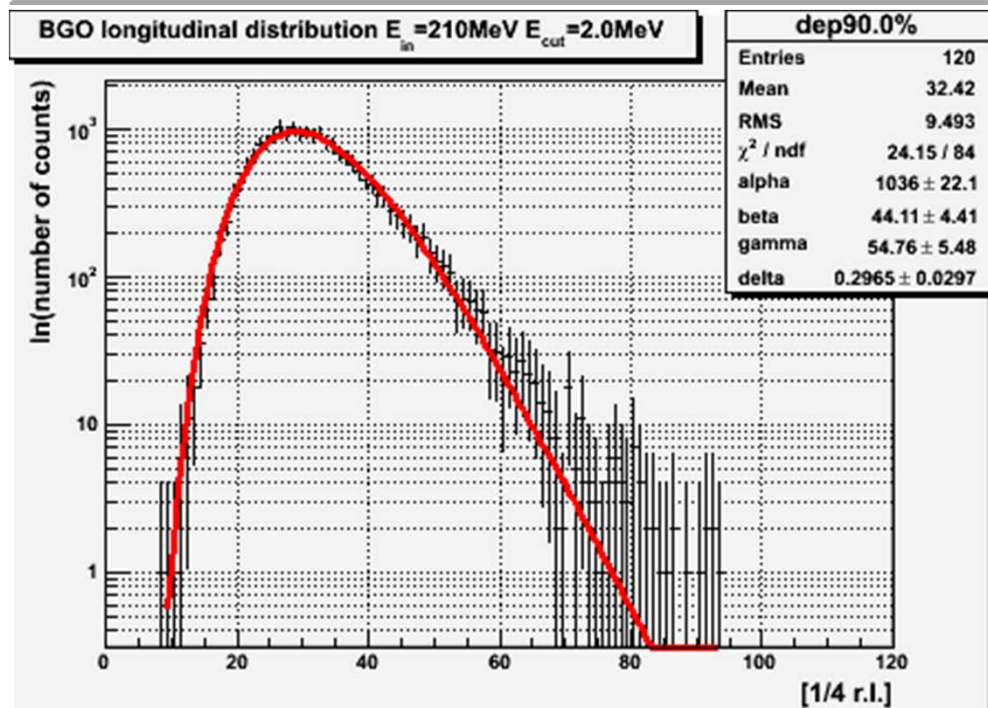
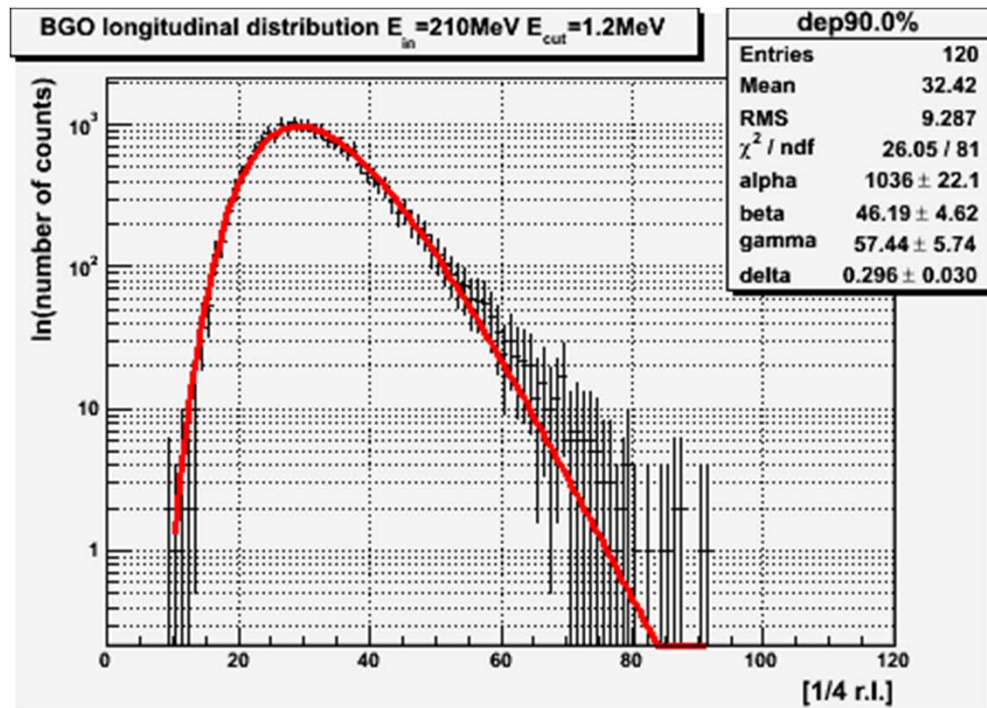
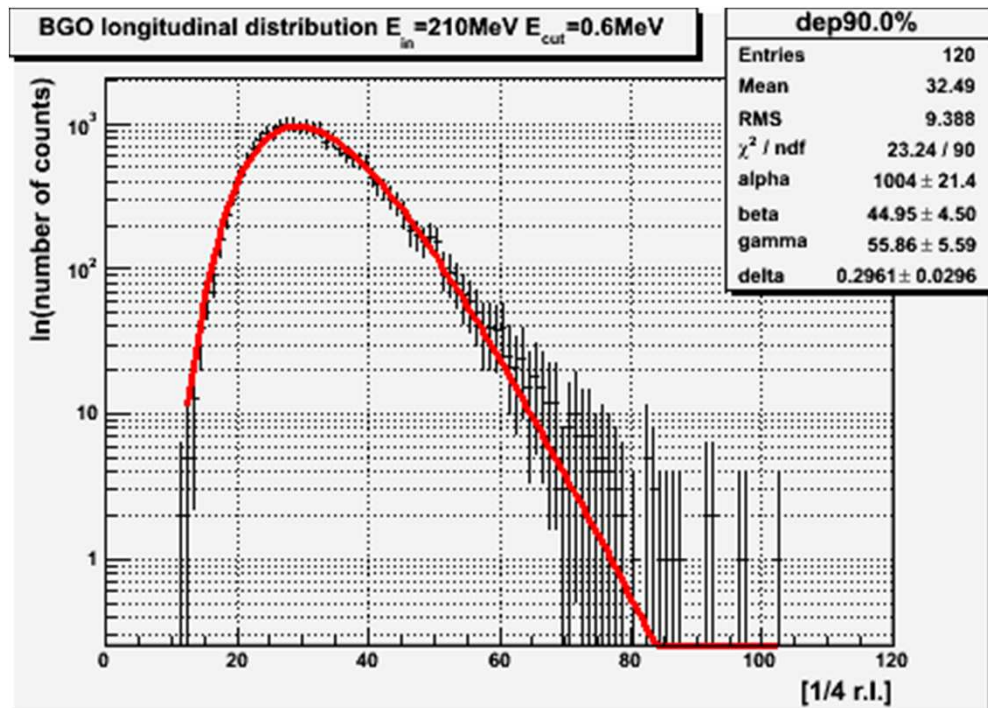


Illustration for $E_\gamma = 210$ MeV in BGO

Distributions of the shower depth t_A at which a fixed part A of average cascade energy is released when the cascade is initiated by gamma quanta of energy $E_\gamma = 1625$ MeV and detected with the cut-off energies $E_{\text{c.o.}} = 0.6, 1.2, 2.0$ and 3.0 MeV (histograms).



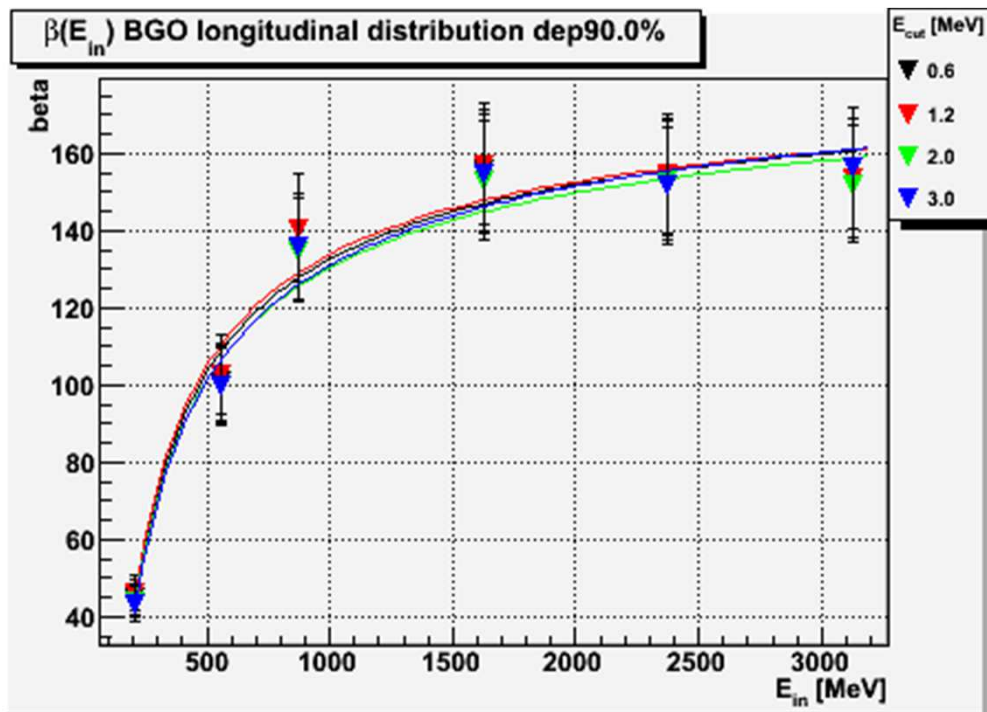
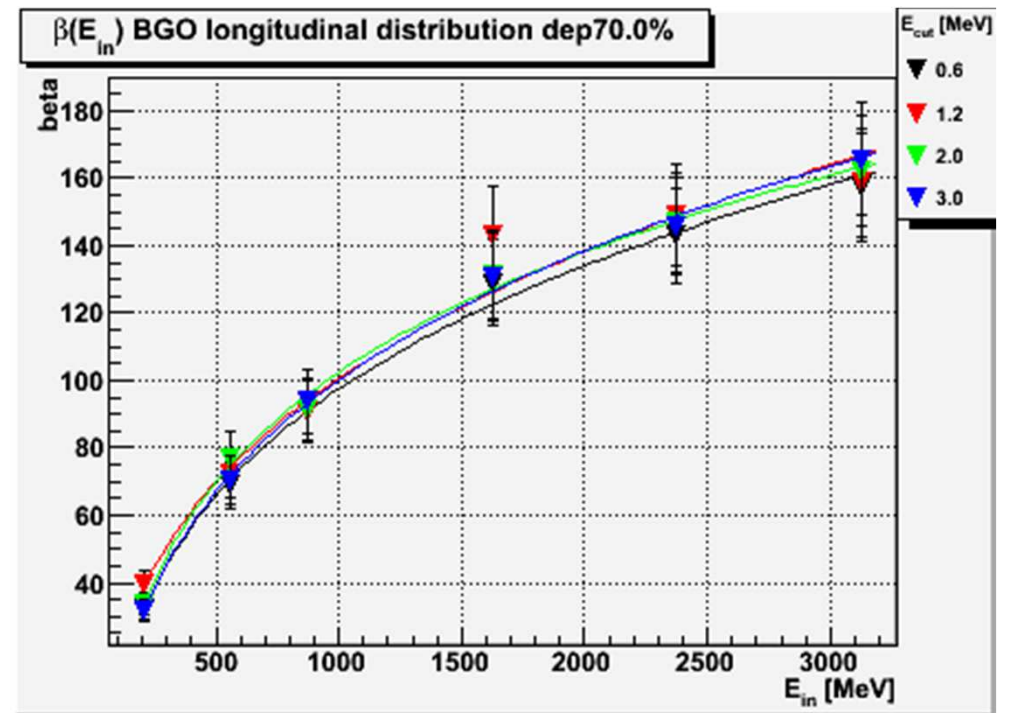
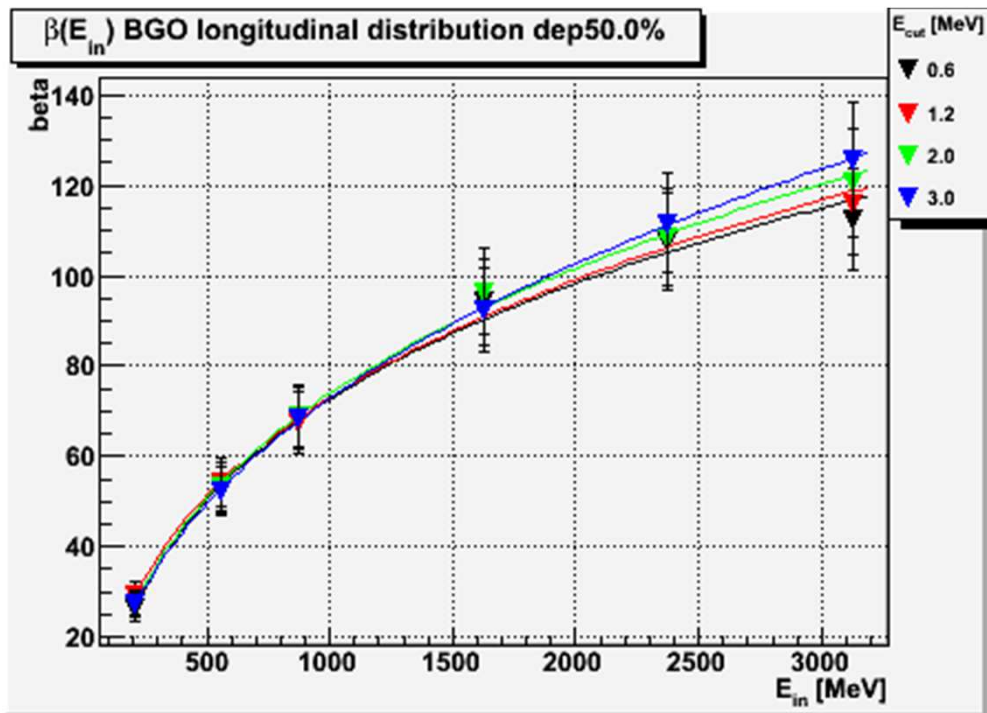




Dependence of fit parameters

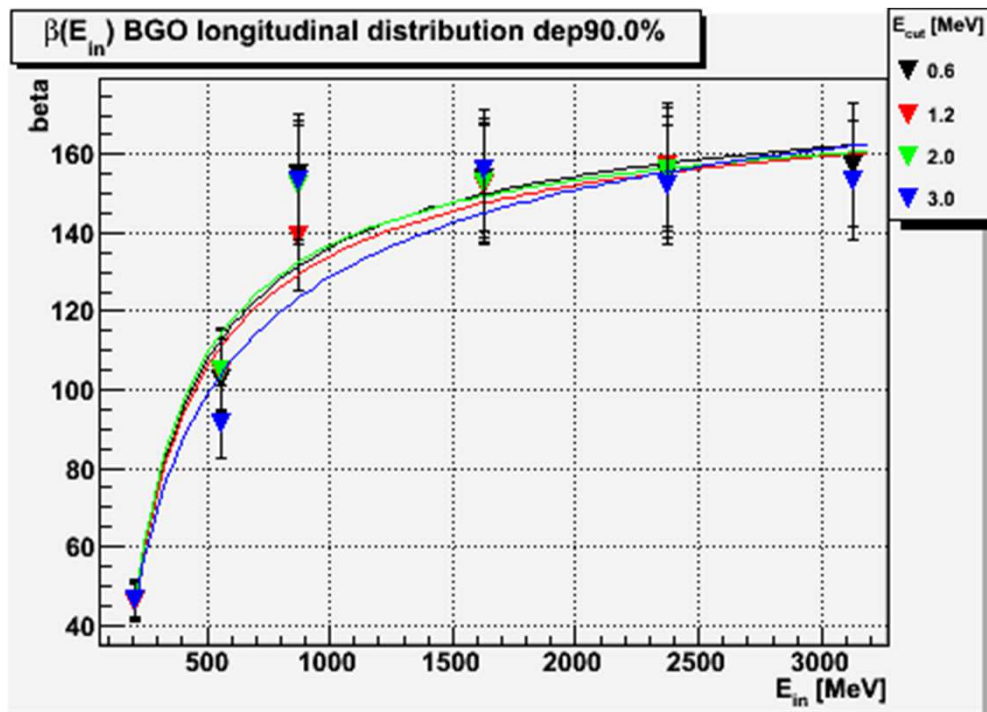
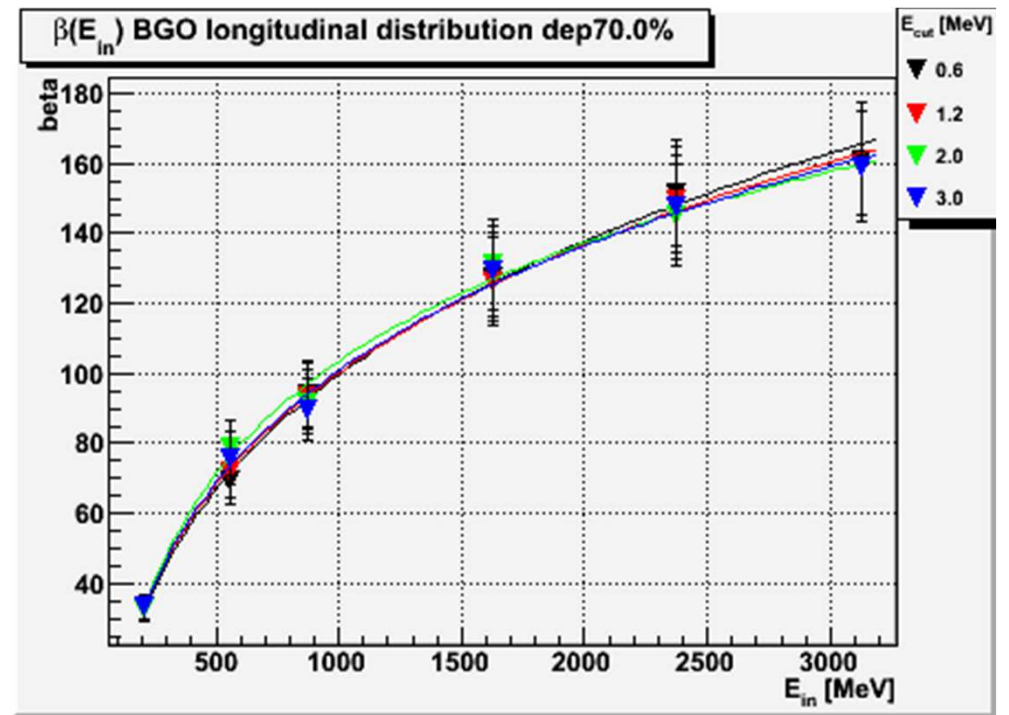
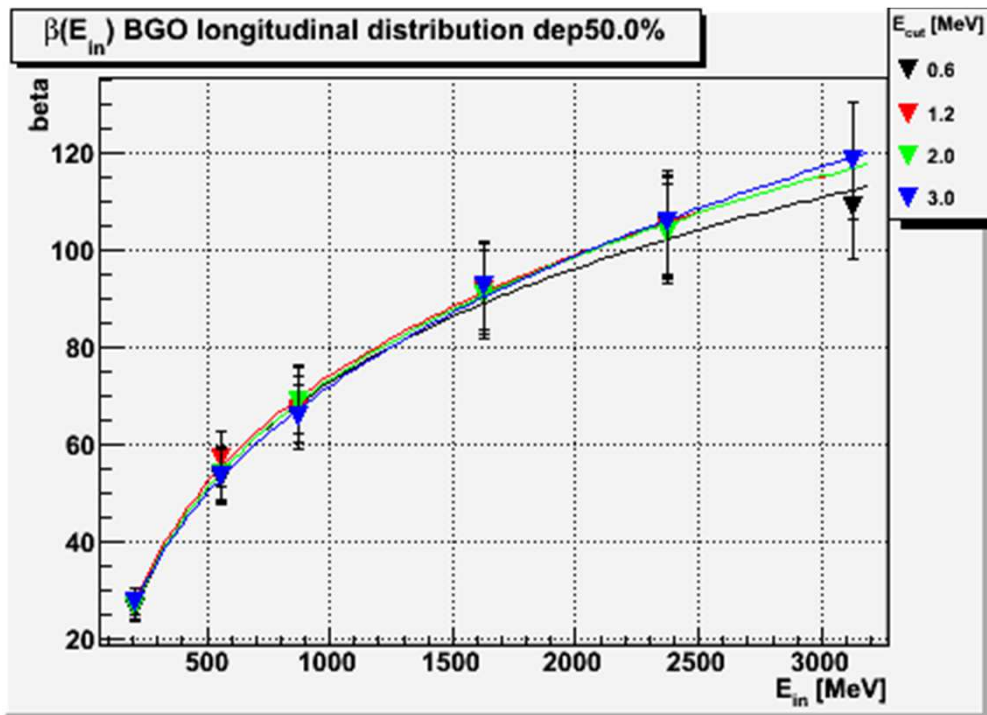
β, γ, δ

on E_γ , $E_{c.o.}$ and Z/A



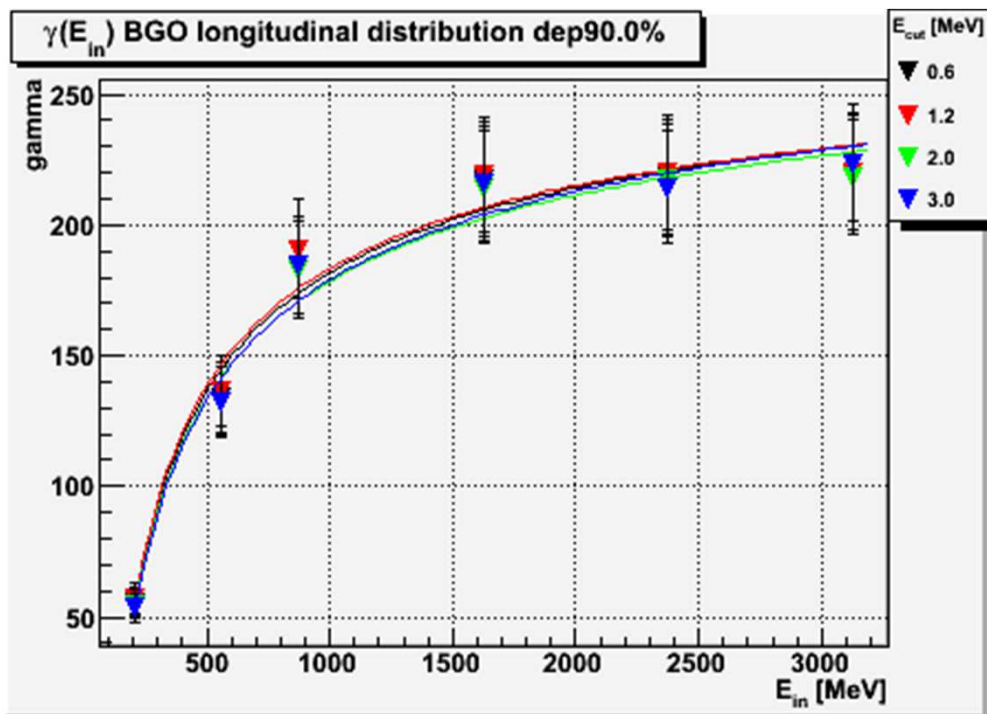
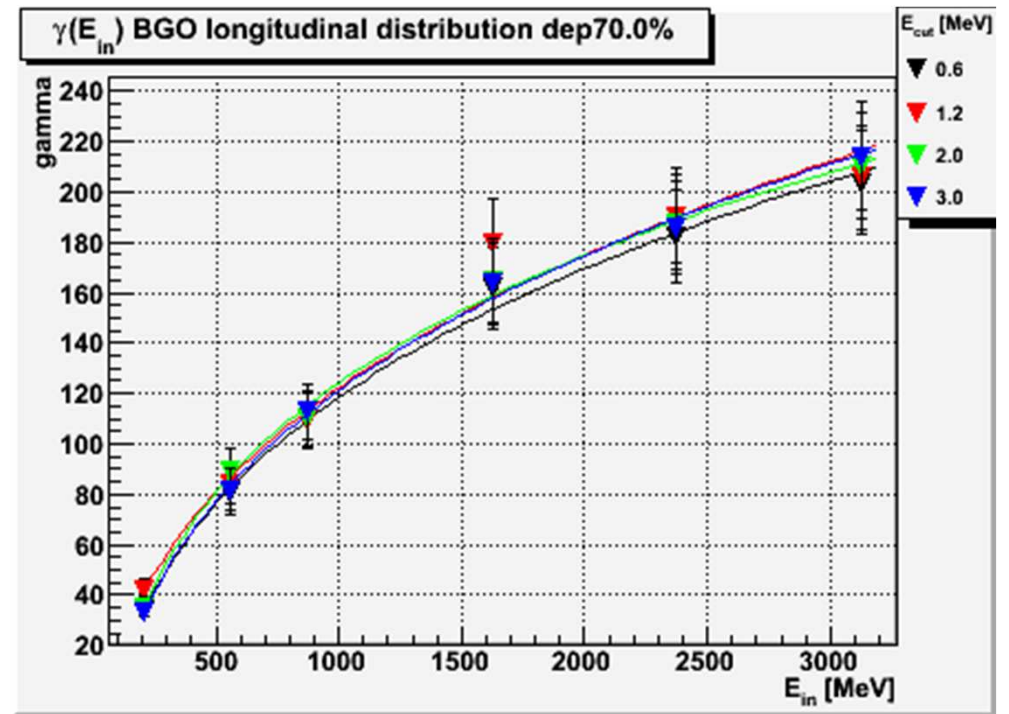
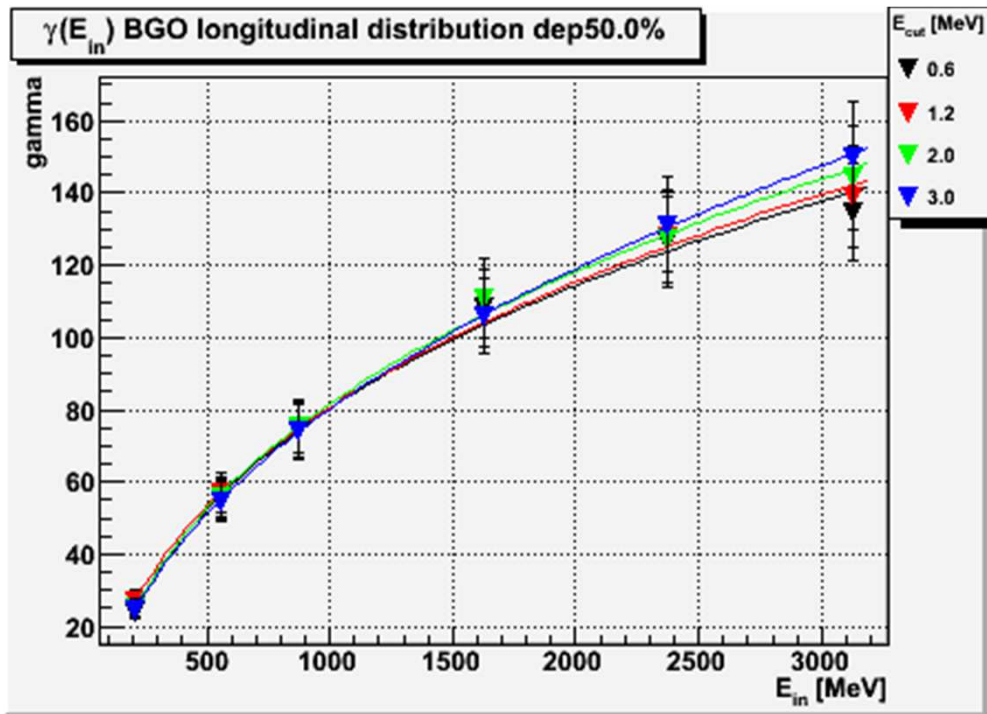
$$\beta(E_{\gamma}) = a \cdot E_{\gamma}^b + c$$

EGS4



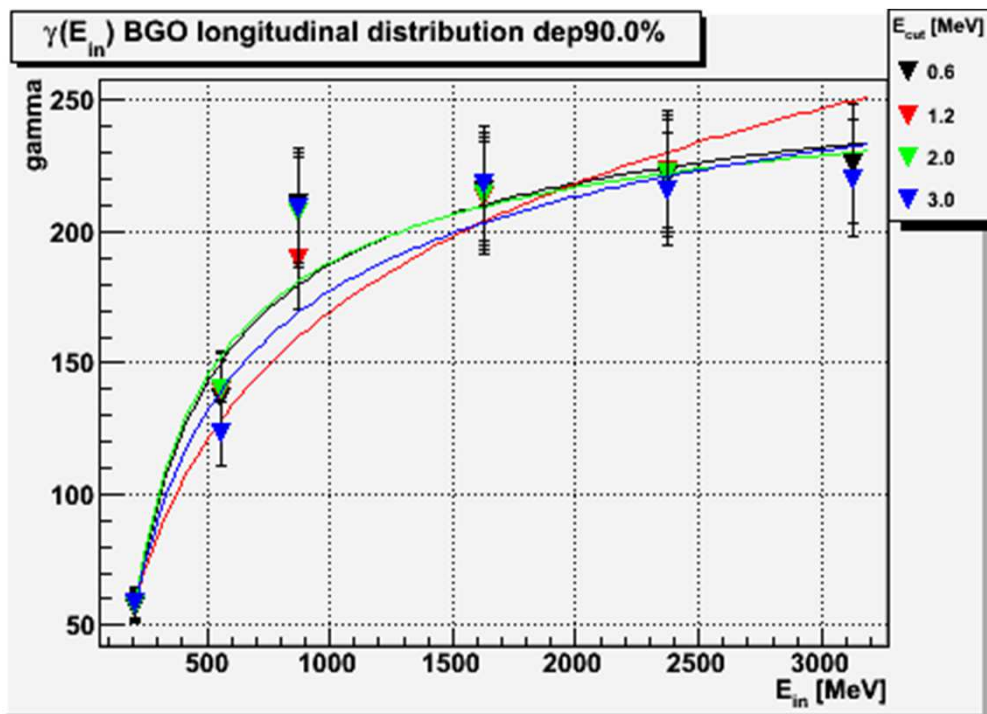
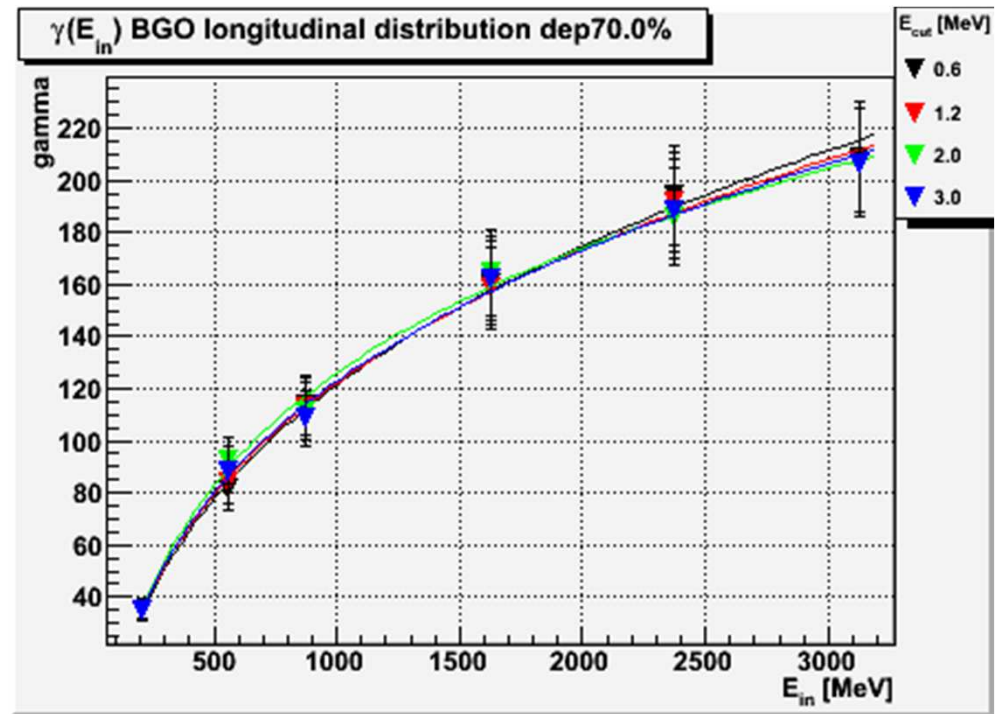
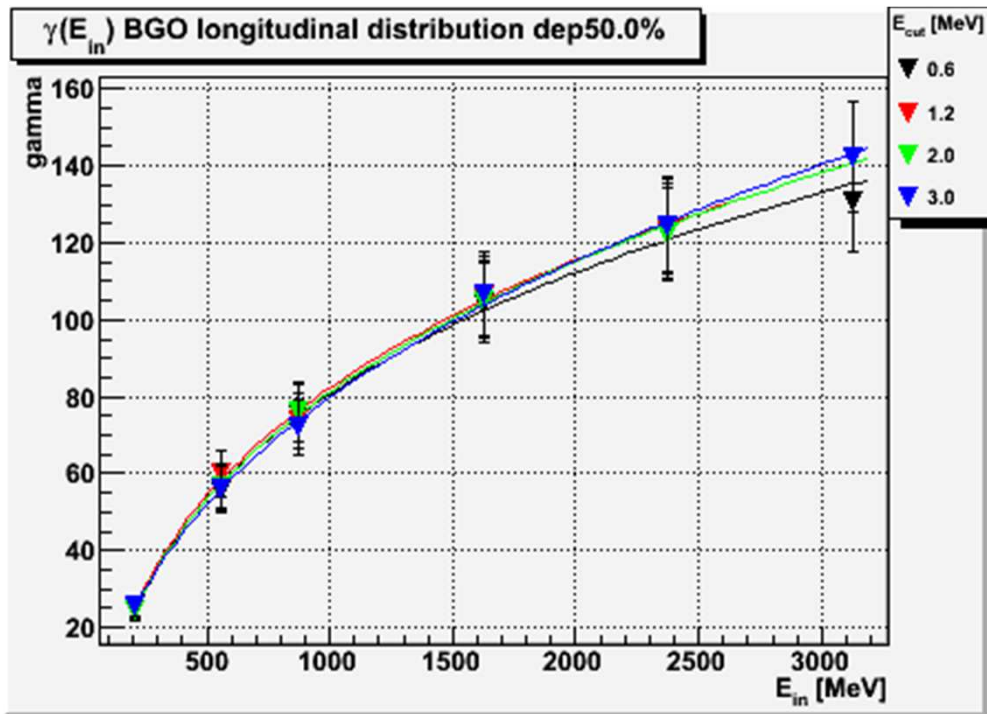
$$\beta(E_{\gamma}) = a \cdot E_{\gamma}^b + c$$

GEANT4



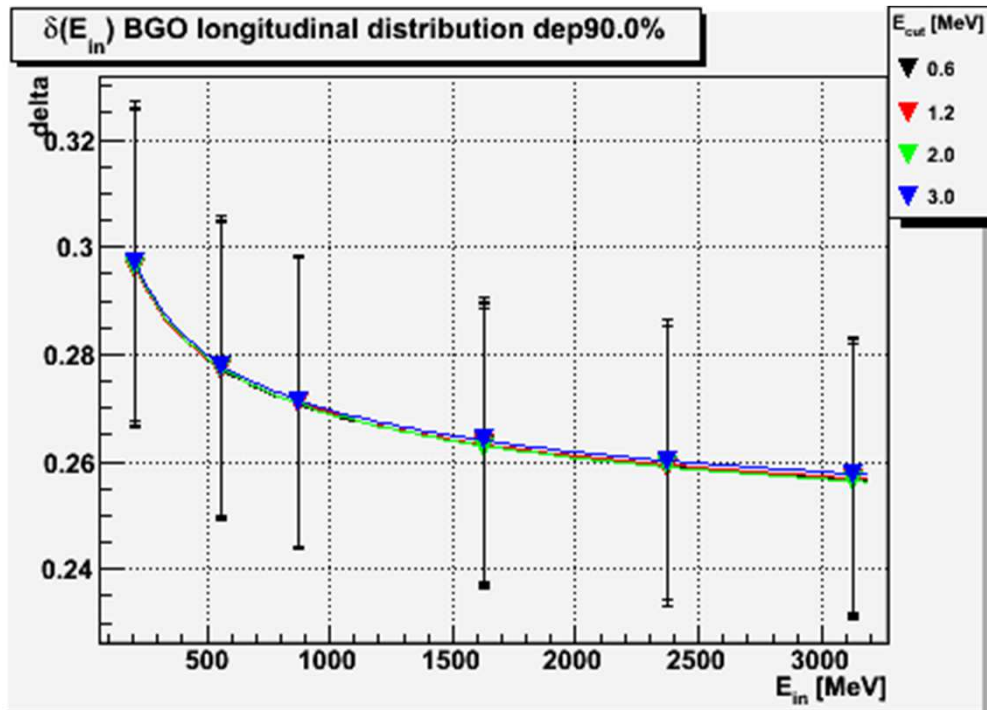
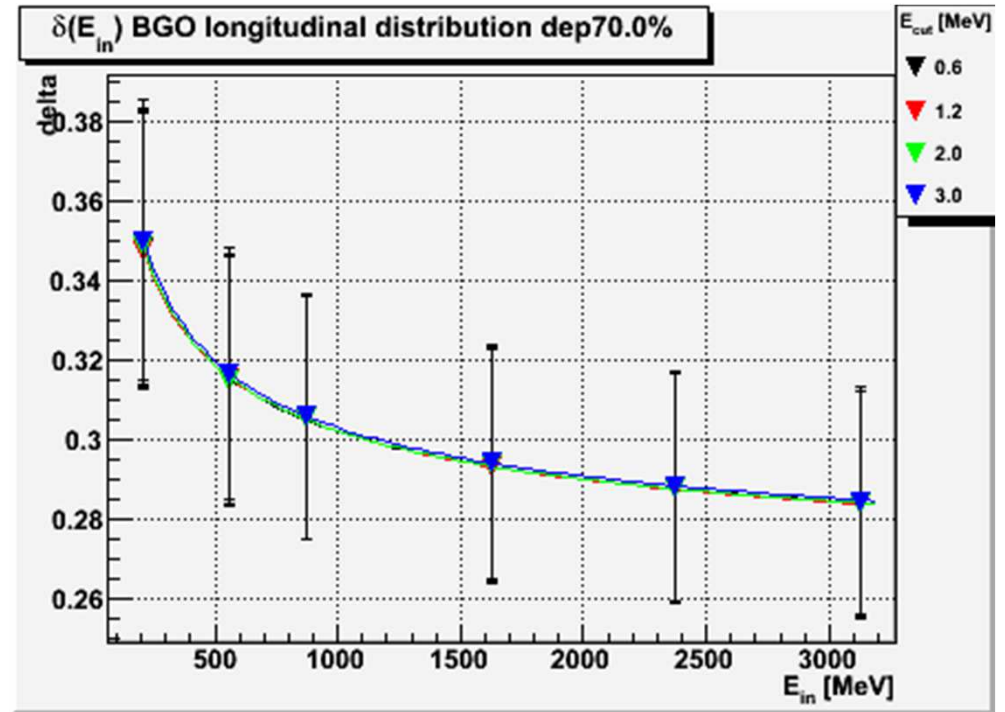
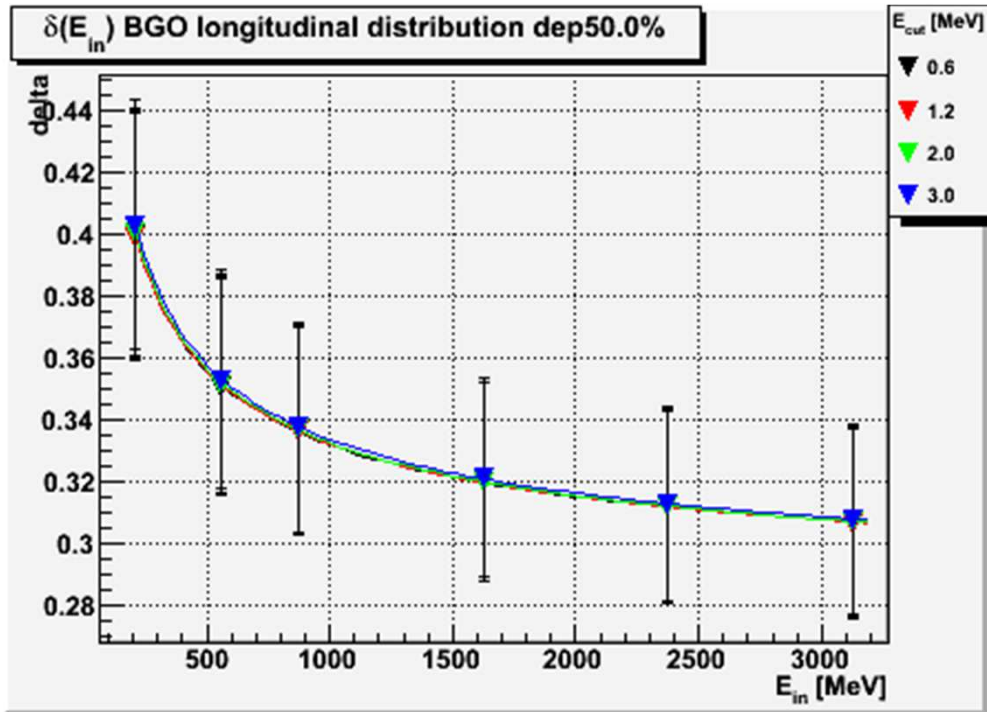
$$\gamma(E_{\gamma}) = a \cdot E_{\gamma}^b + c$$

EGS4



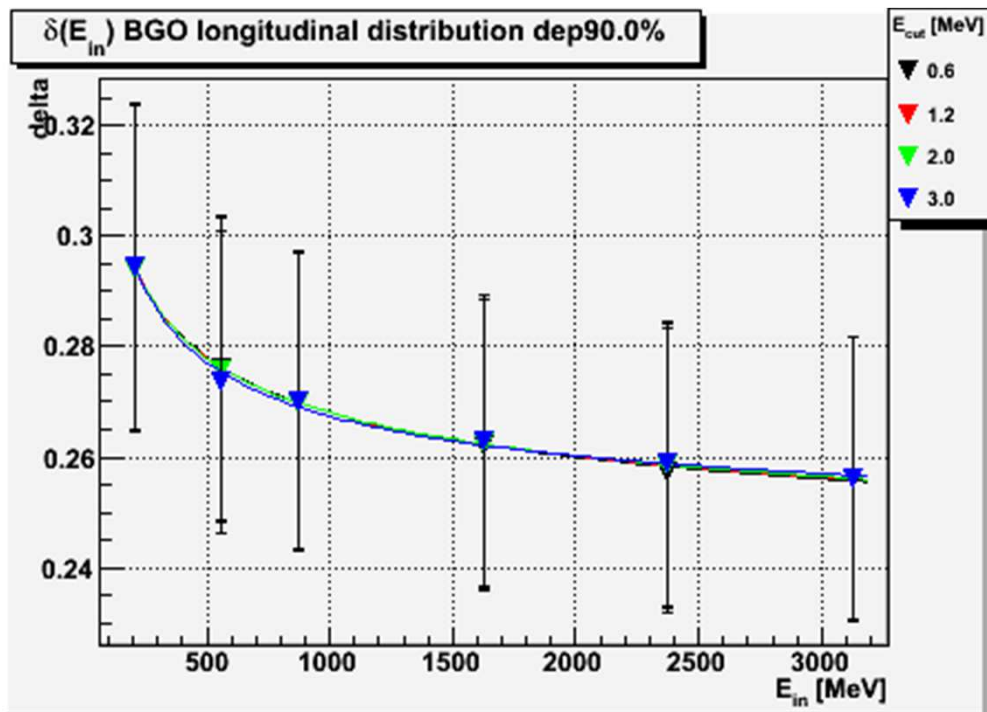
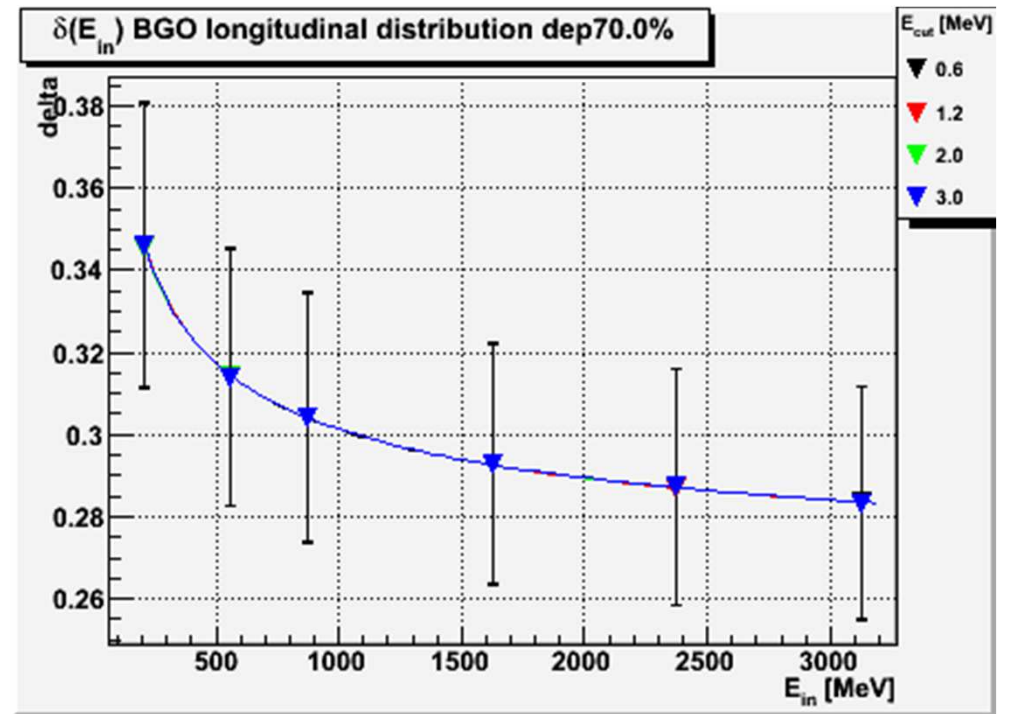
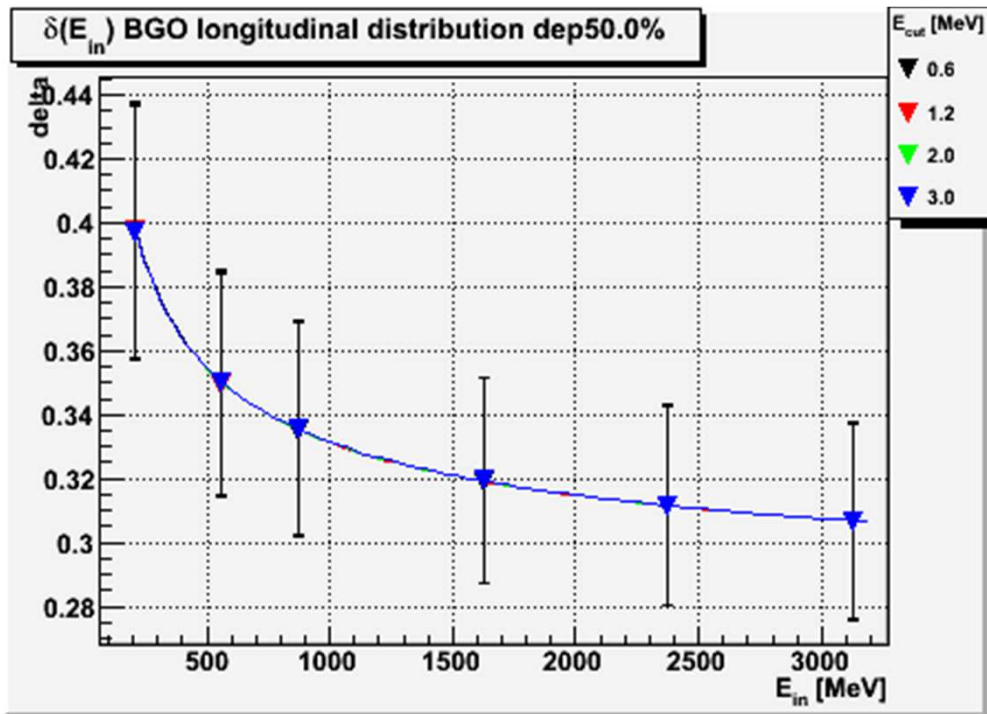
$$\gamma(E_{\gamma}) = a \cdot E_{\gamma}^b + c$$

GEANT4



$$\delta(E_{\gamma}) = a \cdot E_{\gamma}^b + c$$

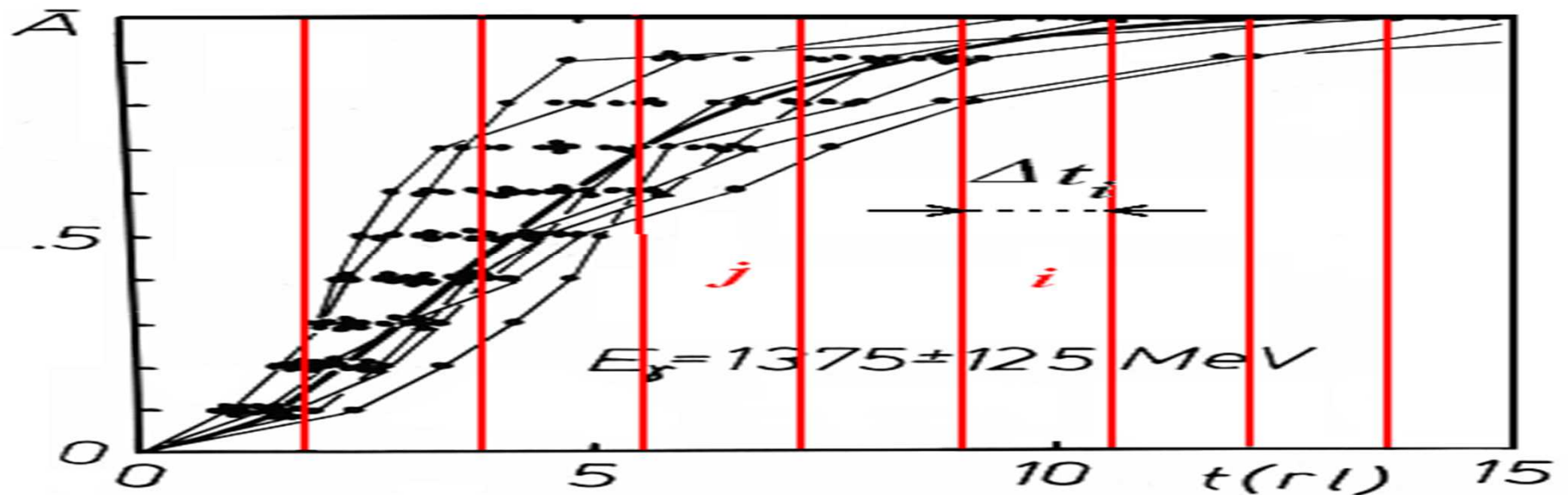
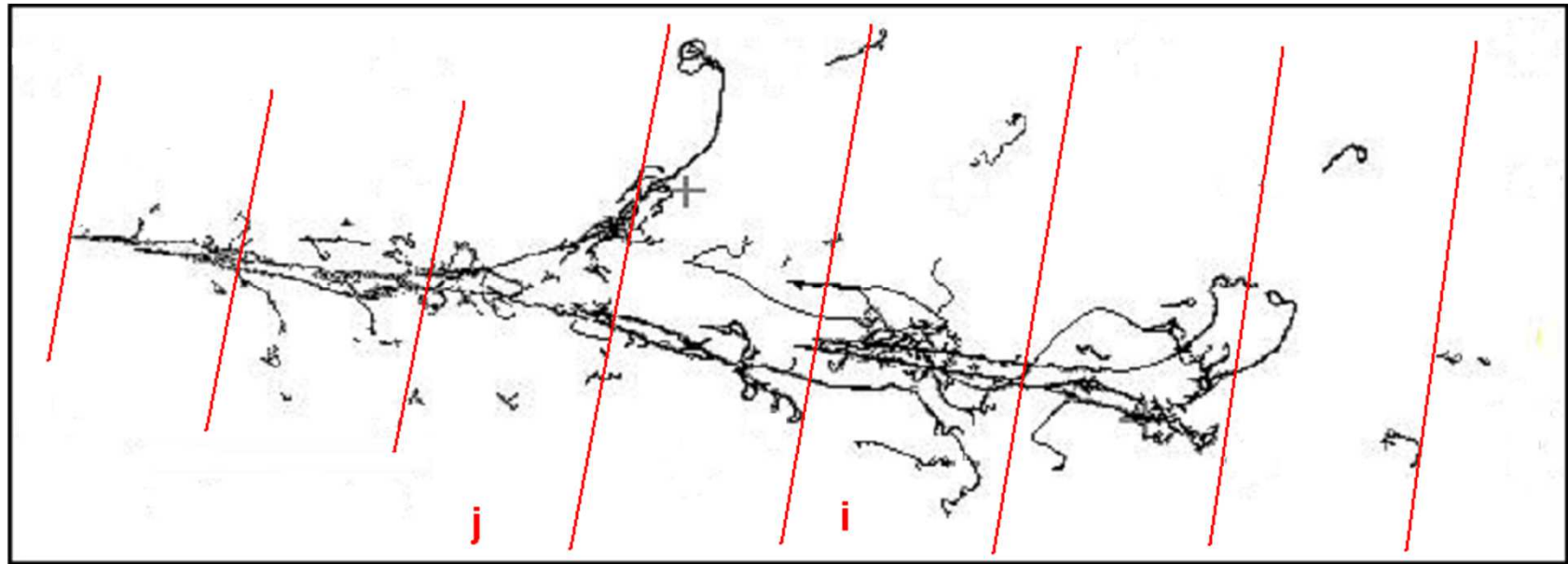
EGS4



$$\delta(E_{\gamma}) = a \cdot E_{\gamma}^b + c$$

GEANT4

LONGITUDINAL CORRELATION





Correlation coefficient

$$r_{ij} = \text{cov}(X_i, X_j) / [\sigma(X_i)\sigma(X_j)]$$

$$X_i = \Delta \bar{A} / \Delta t_i$$

is the slope of the cumulative longitudinal profile at the depth t_i at which on the average $i/10$ of the total shower energy is absorbed



$$E_\gamma = 210 \text{ MeV}, \text{XeC}_2\text{H}_4, E_{\text{cut}} = 1 \text{ MeV}$$

i/j	3	4	5	6	7	8	9	10
2	$0,29 \pm 0,03$	$0,10 \pm 0,03$	$0,04 \pm 0,03$	$0,02 \pm 0,03$	$0,01 \pm 0,03$	$0,00 \pm 0,03$	$-0,02 \pm 0,03$	$0,01 \pm 0,03$
3	<u>$0,32 \pm 0,03$</u>	$0,13 \pm 0,03$	$0,06 \pm 0,03$	$0,02 \pm 0,03$	$-0,01 \pm 0,03$	$-0,02 \pm 0,03$	$-0,01 \pm 0,03$	
4	<u>$0,35 \pm 0,03$</u>	$0,10 \pm 0,03$	$0,02 \pm 0,03$	$-0,02 \pm 0,03$	$-0,02 \pm 0,03$	$-0,02 \pm 0,03$		
5	<u>$0,33 \pm 0,03$</u>	$0,07 \pm 0,03$	$-0,02 \pm 0,03$	$-0,04 \pm 0,03$	$-0,02 \pm 0,03$			
6	$0,29 \pm 0,03$	$0,02 \pm 0,03$	$-0,06 \pm 0,03$	$-0,03 \pm 0,03$				
7	$0,20 \pm 0,03$	$-0,07 \pm 0,03$	$-0,05 \pm 0,03$					
8	$0,06 \pm 0,03$	$-0,06 \pm 0,03$						
9	$-0,08 \pm 0,03$							



$$E_\gamma = 555 \text{ MeV}, \text{XeC}_2\text{H}_4, E_{cut} = 1 \text{ MeV}$$

$i \backslash j$	3	4	5	6	7	8	9	10
2	$0,38 \pm 0,03$	$0,16 \pm 0,03$	$0,08 \pm 0,03$	$0,04 \pm 0,03$	$0,00 \pm 0,03$	$-0,01 \pm 0,03$	$-0,01 \pm 0,03$	$-0,02 \pm 0,03$
3	<u>$0,42 \pm 0,03$</u>	$0,19 \pm 0,03$	$0,08 \pm 0,03$	$0,03 \pm 0,03$	$0,00 \pm 0,03$	$-0,02 \pm 0,03$	$-0,02 \pm 0,03$	
4	<u>$0,41 \pm 0,03$</u>	$0,15 \pm 0,03$	$0,05 \pm 0,03$	$0,00 \pm 0,03$	$-0,02 \pm 0,03$	$-0,02 \pm 0,03$		
5	$0,39 \pm 0,03$	$0,12 \pm 0,03$	$0,00 \pm 0,03$	$-0,02 \pm 0,03$	$-0,01 \pm 0,03$			
6	$0,32 \pm 0,03$	$0,03 \pm 0,03$	$-0,03 \pm 0,03$	$-0,04 \pm 0,03$				
7	$0,23 \pm 0,03$	$-0,02 \pm 0,03$	$-0,05 \pm 0,03$					
8	$0,11 \pm 0,03$	$-0,05 \pm 0,03$						
9	$-0,04 \pm 0,03$							

● ● ● | $E_\gamma = 1125 \text{ MeV}, \text{XeC}_2\text{H}_4, E_{\text{cut}} = 1 \text{ MeV}$

i/j	3	4	5	6	7	8	9	10
2	$0,45 \pm 0,03$	$0,22 \pm 0,03$	$0,12 \pm 0,03$	$0,07 \pm 0,03$	$0,04 \pm 0,03$	$0,02 \pm 0,03$	$0,00 \pm 0,03$	$-0,03 \pm 0,03$
3	<u>$0,48 \pm 0,03$</u>	$0,26 \pm 0,03$	$0,14 \pm 0,03$	$0,05 \pm 0,03$	$0,02 \pm 0,03$	$-0,01 \pm 0,03$	$-0,03 \pm 0,03$	
4	<u>$0,49 \pm 0,03$</u>	$0,23 \pm 0,03$	$0,11 \pm 0,03$	$0,03 \pm 0,03$	$-0,02 \pm 0,03$	$-0,04 \pm 0,03$		
5	$0,45 \pm 0,03$	$0,18 \pm 0,03$	$0,06 \pm 0,03$	$-0,02 \pm 0,03$	$-0,05 \pm 0,03$			
6	$0,39 \pm 0,03$	$0,12 \pm 0,03$	$0,00 \pm 0,03$	$-0,06 \pm 0,03$				
7	$0,30 \pm 0,03$	$0,00 \pm 0,03$	$-0,06 \pm 0,03$					
8	$0,12 \pm 0,03$	$-0,05 \pm 0,03$						
9	$-0,05 \pm 0,03$							



$$E_{\gamma} = 2625 \text{ MeV}, \text{XeC}_2\text{H}_4, E_{cut} = 1 \text{ MeV}$$

i/j	3	4	5	6	7	8	9	10
2	$0,53 \pm 0,02$	$0,32 \pm 0,03$	$0,22 \pm 0,03$	$0,12 \pm 0,03$	$0,07 \pm 0,03$	$0,04 \pm 0,03$	$0,01 \pm 0,03$	$-0,10 \pm 0,03$
3	<u>$0,57 \pm 0,02$</u>	$0,35 \pm 0,03$	$0,21 \pm 0,03$	$0,11 \pm 0,03$	$0,04 \pm 0,03$	$0,02 \pm 0,03$	$-0,11 \pm 0,03$	
4	<u>$0,56 \pm 0,02$</u>	$0,31 \pm 0,03$	$0,17 \pm 0,03$	$0,08 \pm 0,03$	$0,02 \pm 0,03$	$-0,09 \pm 0,03$		
5	$0,52 \pm 0,02$	$0,28 \pm 0,03$	$0,12 \pm 0,03$	$0,02 \pm 0,03$	$-0,11 \pm 0,03$			
6	$0,46 \pm 0,03$	$0,18 \pm 0,03$	$0,04 \pm 0,03$	$-0,09 \pm 0,03$				
7	$0,37 \pm 0,03$	$0,08 \pm 0,03$	$-0,09 \pm 0,03$					
8	$0,22 \pm 0,03$	$-0,09 \pm 0,03$						
9	$-0,09 \pm 0,03$							



$$E_\gamma = 3375 \text{ MeV}, \text{ XeC}_2\text{H}_4, E_{cut} = 1 \text{ MeV}$$

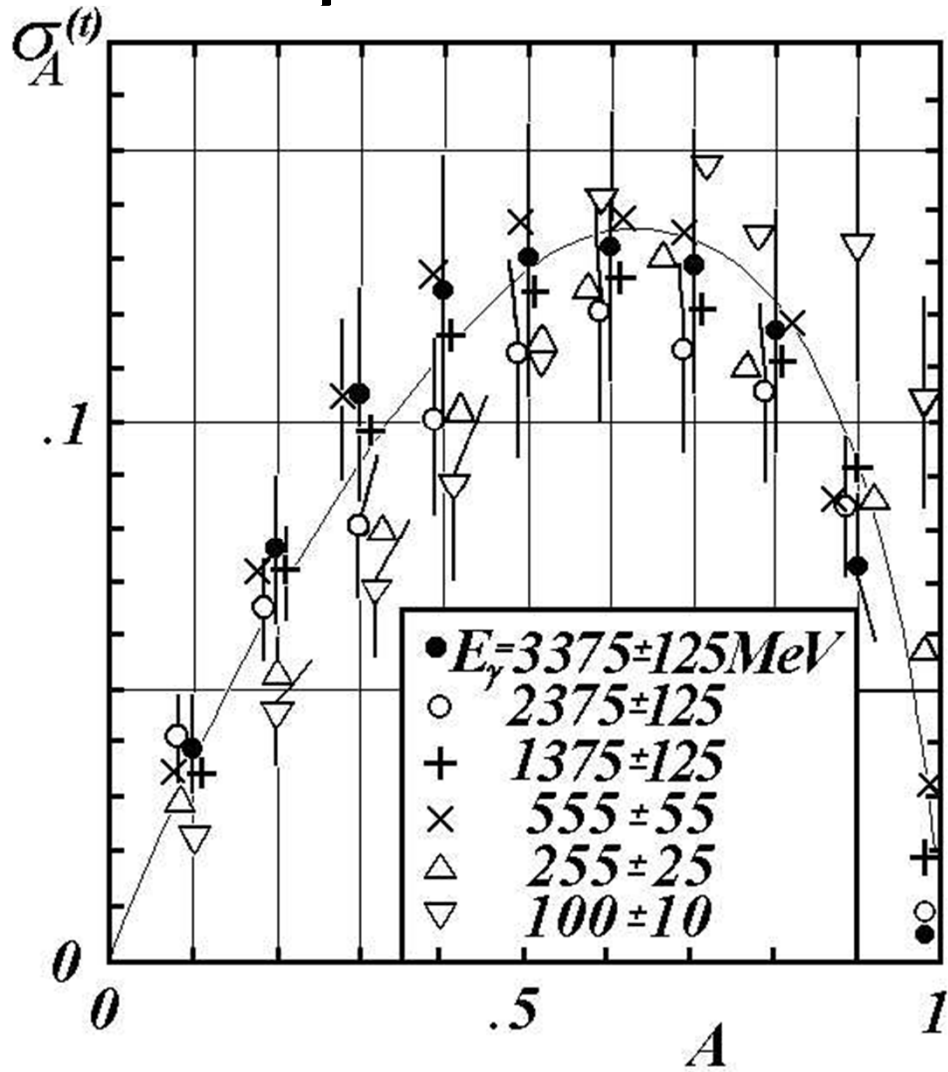
i/j	3	4	5	6	7	8	9	10
2	$0,55 \pm 0,02$	$0,35 \pm 0,03$	$0,23 \pm 0,03$	$0,16 \pm 0,03$	$0,07 \pm 0,03$	$0,02 \pm 0,03$	$0,00 \pm 0,03$	$-0,11 \pm 0,03$
3	<u>$0,59 \pm 0,02$</u>	$0,37 \pm 0,03$	$0,25 \pm 0,03$	$0,13 \pm 0,03$	$0,06 \pm 0,03$	$0,02 \pm 0,03$	$-0,13 \pm 0,03$	
4	<u>$0,59 \pm 0,02$</u>	$0,36 \pm 0,03$	$0,20 \pm 0,03$	$0,08 \pm 0,03$	$0,04 \pm 0,03$	$-0,13 \pm 0,03$		
5	<u>$0,56 \pm 0,02$</u>	$0,29 \pm 0,03$	$0,12 \pm 0,03$	$0,05 \pm 0,03$	$-0,13 \pm 0,03$			
6	$0,48 \pm 0,03$	$0,20 \pm 0,03$	$0,07 \pm 0,03$	$-0,13 \pm 0,03$				
7	$0,38 \pm 0,03$	$0,10 \pm 0,03$	$-0,13 \pm 0,03$					
8	$0,24 \pm 0,03$	$-0,11 \pm 0,03$						
9	$-0,10 \pm 0,03$							



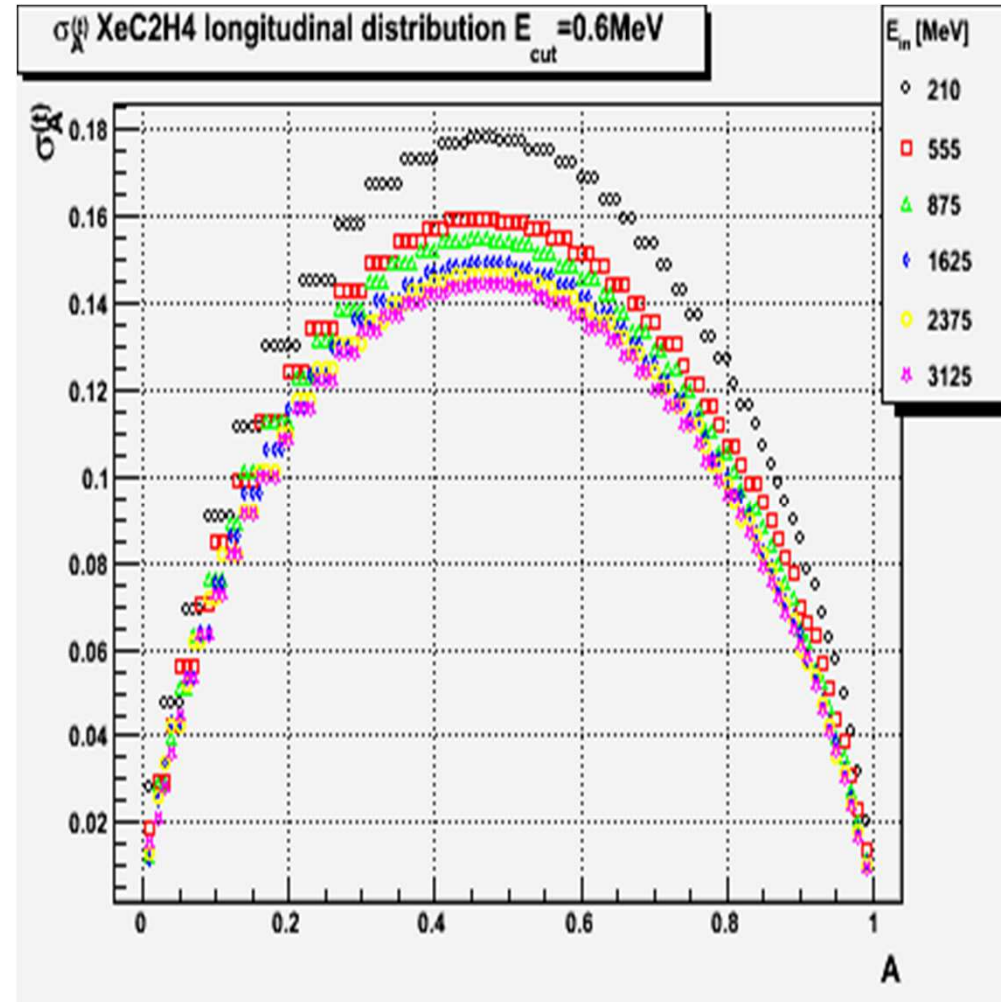
COMPARISON EGS4 & GEANT4 WITH EXPERIMENT

(material: liquid xenon)

Longitudinal fluctuation [EGS4]

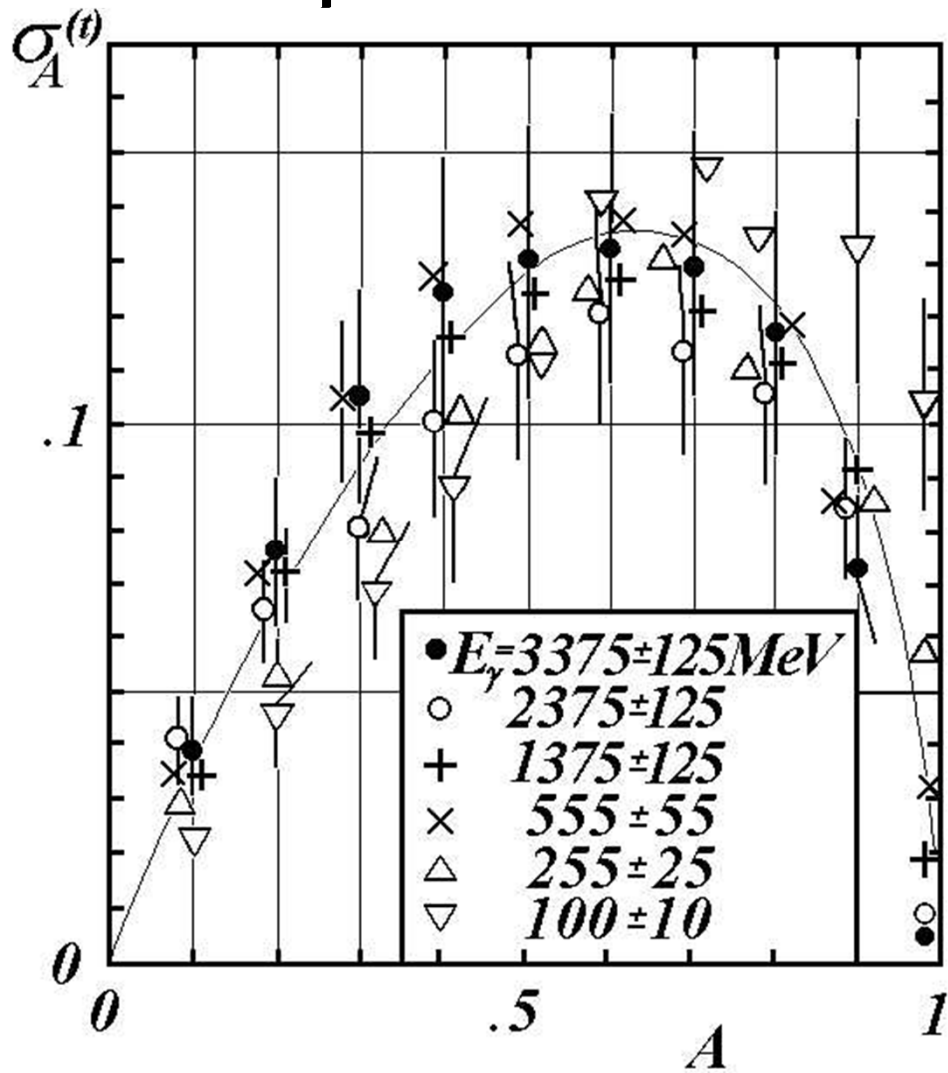


Experiment

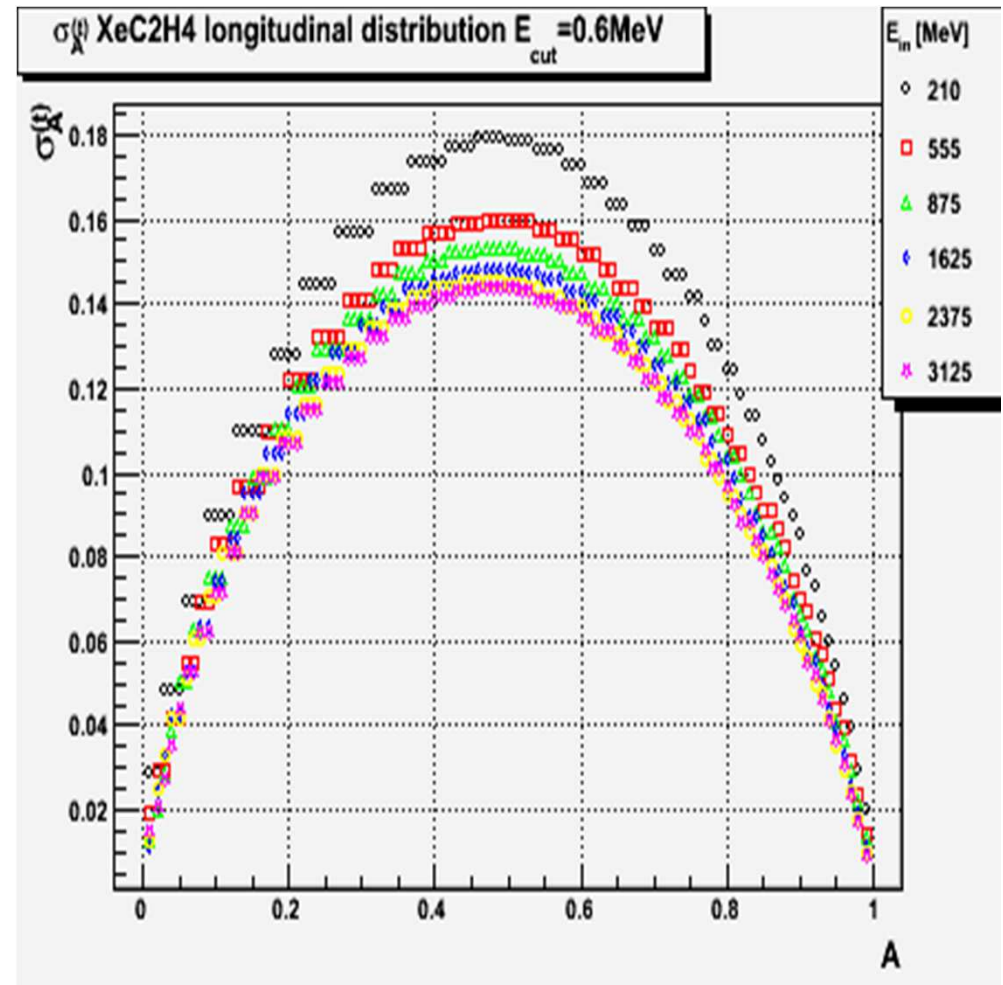


Simulation

Longitudinal fluctuation [GEANT4]

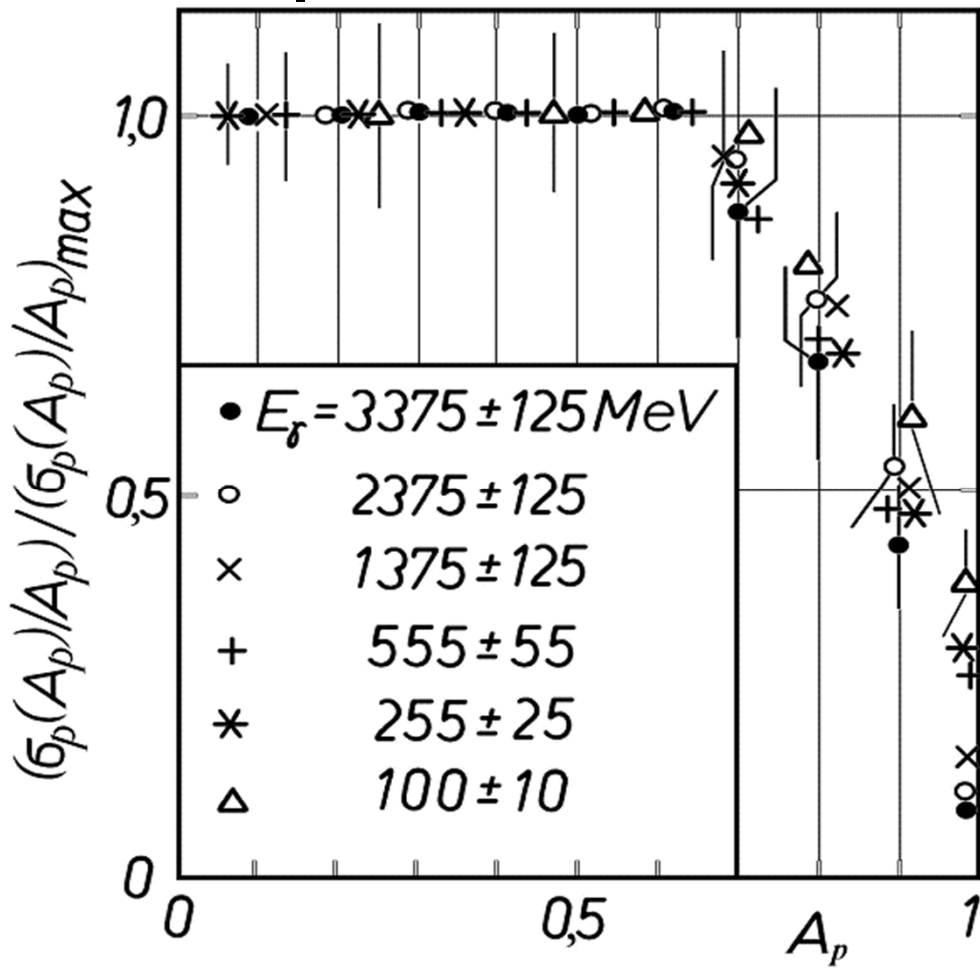


Experiment

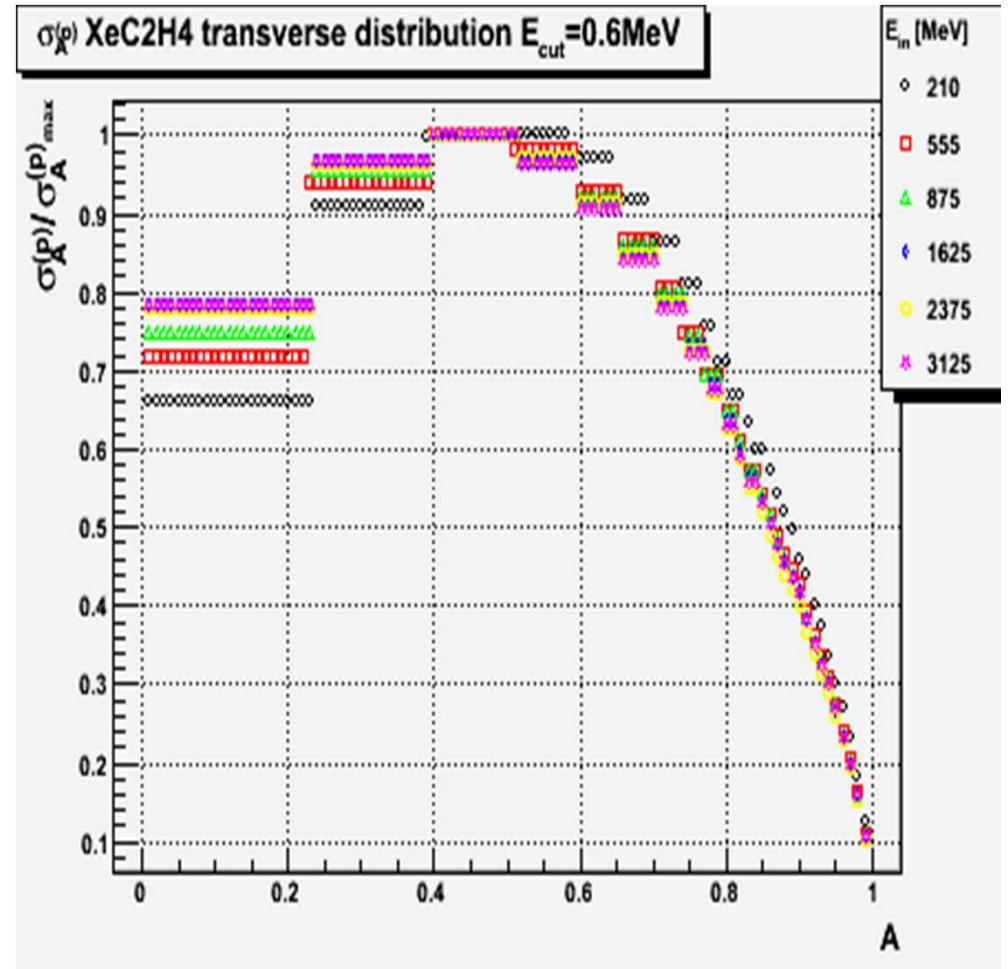


Simulation

Transverse fluctuation [EGS4]

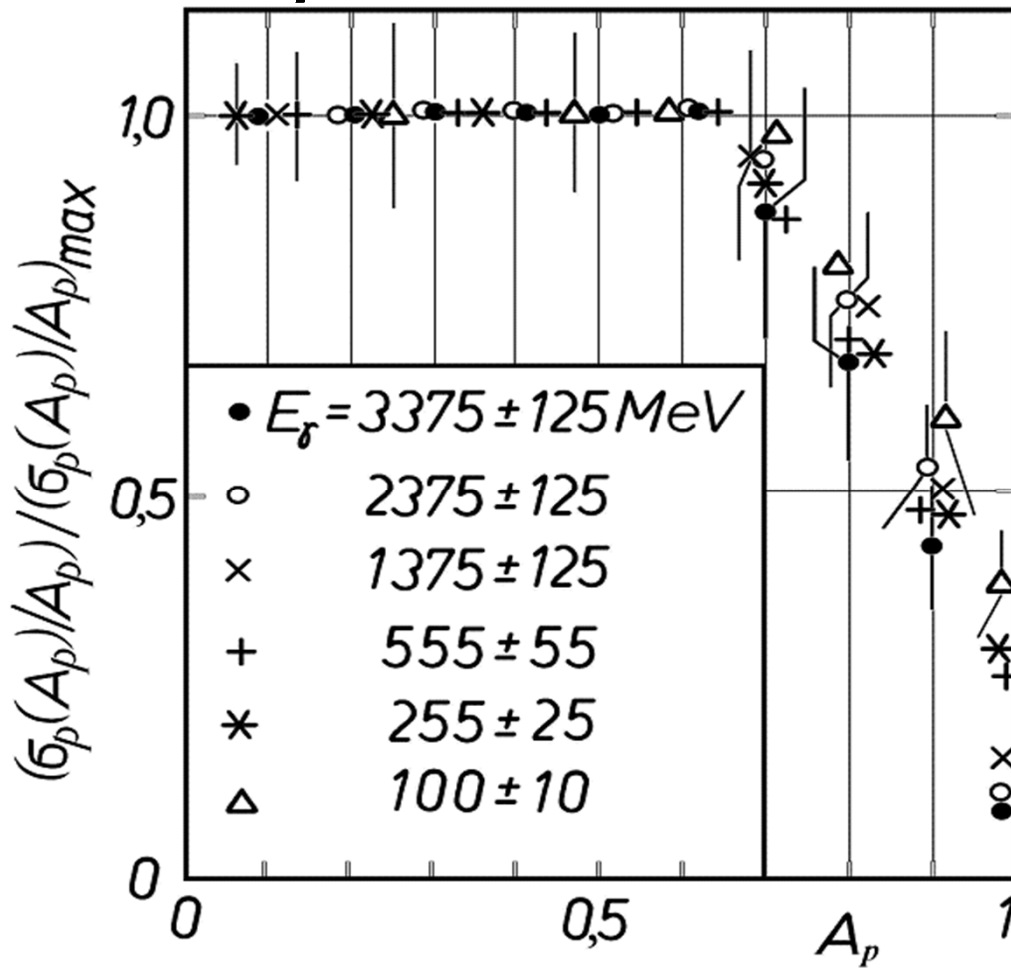


Experiment

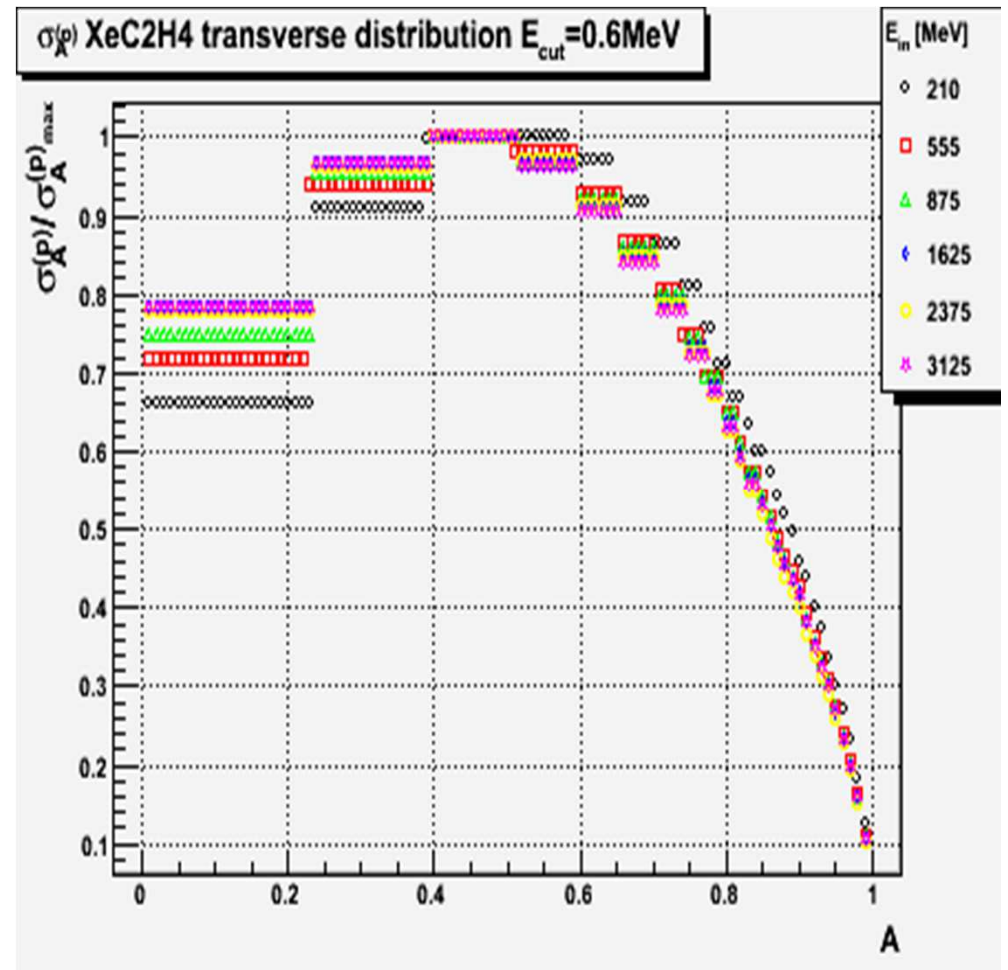


Simulation

Transverse fluctuation [GEANT4]



Experiment

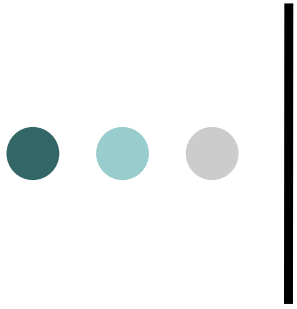


Simulation



SUMMARY AND CONCLUSION

- The comprehensive analysis of the longitudinal and transverse profiles of EMC initiated in 8 various amorphous materials by gamma quanta of energy $E_\gamma = 210, 555, 875, 1625, 2375$ and 3125 MeV has been performed by using EGS4 and GEANT4 code at four values of cut-off energy $E_{c.o.} = 0.6, 1.2, 2.0$ and 3.0 MeV.
- All the obtained approximating formulas in the form of simple functions **reveal a quite acceptable scaling description** of the electromagnetic cascade process initiated by gamma quanta of transitional energy interval $100\div 3500$ MeV in the most often used dense materials. **They can be applied both for hard gamma detection and radiation shielding construction.**



- In the range $E_\gamma = 100\text{-}3375$ MeV of energies of gamma quanta producing cascades in liquid xenon mainly short-range correlation of the longitudinal energy release in the vicinity of the cascade maximum (i.e., for $i = 3\text{-}7$) takes place.