A NEW APPROACH TO THE THEORY OF ELECTROMAGNETIC INTERACTIONS WITH BOUND SYSTEMS: CALCULATIONS OF DEUTERON MAGNETIC AND QUADRUPOLE MOMENTS

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To the memory of M. I. Shirokov
Excellent Scientist and
Modest Person

Outline

1 Some recollections

The present work goes on our recent studies of nucleon-nucleon (N-N) scattering below pion production threshold within a field-theoretical approach based upon method of unitary clothing transformations (UCTs). Now we present the first application of UCT method to description of deuteron and its electromagnetic properties.

1. Shebeko A., Shirokov M.: Prog. Part. Nucl. Phys. **44** (2000) 75; {SheShi00}

2. Shebeko, A.V., Shirokov, M.I.: Phys. Part. Nucl. **32** (2001) 31; {SheShi01}
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4. Dubovyk I., Shebeko O.: Few Body Syst. **48** (2010) 109; {DuShe10}
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5. Shebeko A.V., Frolov P.A.: Few Body Syst. **52** (2012) 125; {SheFro12}
arXiv:1107.5877 [hep-th]

Electromagnetic (EM) Static Moments of Bound Systems

Relativistic calculations: eventful history within:
light-front dynamics {Chung et al. PR C**39** (1989), Lev et al. PRL**83** (1999)}, the Bethe-Salpeter formalism {Honzawa, Ishida PR C**45** (1992), Bondarenko et al. Prog. Part. Nucl. Phys.**48** (2002)}, inclusion of meson exchange and boost contributions of leading order {Tamura et al. NP A**536** (1992), Arenhövel et al. PR C **61**(2000)} and Refs. therein.
Note also a possible way for constructing generators of the Poincaré group (Π) in quantum field theory (QFT) {Shebeko, Frolov FBS **52** (2012)}

Starting from operator

$$\vec{\mu} = \frac{1}{2} \int d\vec{z}\, \vec{z} \times \vec{J}(\vec{z}) \tag{1}$$

of magnetic dipole moment for a system with current density $\vec{J}(\vec{x})$ (reminescent of Biot-Savart formula from magnetostatics) one can show after {Sachs62} that its matrix elements between narrow wave packets are expressed through limit

$$\lim_{\vec{q}\to 0} \left[-\frac{i}{2} curl_{\vec{q}} \langle \frac{\vec{q}}{2} | \vec{J}(0) | -\frac{\vec{q}}{2} \rangle \right], \tag{2}$$

where matrix elements $\langle \frac{\vec{q}}{2} | \vec{J}(0) | -\frac{\vec{q}}{2} \rangle$ (total angular moment J, its projection M_J and other quantum numbers, if any, are implied) determine the corresponding current in Breit frame. Magnetic dipole moment of system, being defined as z-component of vector (1) for stretched configuration, for deuteron

$$\mu_d = \lim_{\vec{q}\to 0} \left[-\frac{i}{2} curl_{\vec{q}} \langle \frac{\vec{q}}{2}; 1 | \vec{J}(0) | -\frac{\vec{q}}{2}; 1 \rangle \right]^z . \tag{3}$$

In parallel, considering interaction energy of system with charge density $\rho(\vec{x}) = J^0(\vec{x})$ in static external electric field and expanding it in Cartesian electric moments one encounters quadrupole moment tensor

$$Q_{ij} = \int d\vec{x}\,[3x_i x_j - \delta_{ij}\vec{x}^2]\rho(\vec{x}) \;\; (i,j = 1(x), 2(y), 3(z)). \qquad (4)$$

Then repeating the same trick with wave packets one gets matrix elements

$$\langle JM'_J|Q_{ij}|JM_J\rangle = -\lim_{\vec{q}\to 0}\left[\left\{3\frac{\partial^2}{\partial q_i \partial q_j} - \delta_{ij}\frac{\partial^2}{\partial q_l^2}\right\}\langle\frac{\vec{q}}{2}|\rho(0)| - \frac{\vec{q}}{2}\rangle\right]. \qquad (5)$$

to introduce electric quadrupole moment $Q = \langle JJ|Q_{33}|JJ\rangle$ Again, not necessarily for deuteron. In this context, let us a little deviation.

Two Forms of Electron-Deuteron Scattering Amplitude

To ensure gauge-independent (GI) treatment for inevitably approximate calculations of amplitudes of one-photon radiative processes we prefer to employ a generalization {Levchuk,Shebeko Phys.At.Nucl.**52** (1993),*ibid.* **62** (1999)}, {Levchuk, Canton, Shebeko EPJ A **21** (2004)} of the Siegert theorem (cf., {Friar, Fallieros PR C **34** (1986)}, in which elastic *e-d* scattering amplitude can be represented in explicitly GI form:

$$T(ed \to e'd') = \left[\omega\vec{\varepsilon}(e',e) - \vec{q}\varepsilon_0(e',e)\right]\vec{D}(\vec{q}) + \left[\vec{q}\times\vec{\varepsilon}(e',e)\right]\vec{M}(\vec{q}), \quad (6)$$

with generalized electric dipole moment

$$\vec{D}(\vec{q}) = -i\omega^{-1}\int_0^1 \frac{d\lambda}{\lambda}\nabla_{\vec{q}}\left\{\left[\sqrt{\lambda^2\vec{q}^2 + m_d^2} - m_d\right]\langle\lambda\vec{q};M'|\rho(0)|\vec{0};M\rangle\right\} \quad (7)$$

and generalized magnetic dipole moment

$$\vec{M}(\vec{q}) = -i\int_0^1 d\lambda\nabla_{\vec{q}}\times\langle\lambda\vec{q};M'|\vec{J}(0)|\vec{0};M\rangle, \quad (8)$$

where we have introduced notation $\varepsilon_\mu(e,e') = \bar{u}_{e'}(k')\gamma_\mu u_e(k)$ omitting electron polarization labels. As usually, the Dirac spinor $u_e(k)$ ($u_{e'}(k')$) describes the incident (outgoing) electron with the 4-momentum k (k') and $q_\mu = (\omega,\vec{q}) = k_\mu - k'_\mu$ ($q_\mu^2 = \omega^2 - \vec{q}^2 < 0$) the 4-momentum transfer.

These equations have been derived using property

$$J^{\mu}(\vec{x}) = e^{-i\vec{P}\vec{x}} J^{\mu}(0) e^{i\vec{P}\vec{x}} \tag{9}$$

and Foldy representation for $\vec{a}\exp(i\vec{b}\vec{c})$ with arbitrary vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ (Foldy L.L.: Phys. Rev. **92** (1953) 178).

One should note that the parametrization of deuteron current in terms of three (no more) covariant FFs becomes possible once current, first, meets gauge-independence (GI) condition

$$q_\mu \langle \vec{q}; M' | J^\mu(0) | \vec{0}; M \rangle = 0 \tag{10}$$

and, second, has property

$$U(\Lambda) J^\mu(0) U^\dagger(\Lambda) = J^\nu(0) \Lambda_\nu^{\ \mu} \tag{11}$$

to be 4-vector. By definition, correspondence $\Lambda \Rightarrow U(\Lambda)\ \forall \Lambda$ that belong to the Lorentz group realizes its irreducible representation in Hilbert space $\mathcal{H}$.
Evidently, eq. (10) does not follow merely from continuity equation (CE)

$$[P_\mu, J^\mu(0)] = 0, \tag{12}$$

this e^1-order consequence of gauge-invariance principle after Fock and Weyl {Kazes et al. Ann. Phys. **142** (1982)} Here is spring of some problem in practical calculations.
Thus, one has to handle matrix elements of Nöther operator $J^\mu(x) = (J^0(x) \equiv \rho(x), \vec{J}(x))$ at space-time point $x = (t, \vec{x}) = 0$, sandwiched between initial $|\vec{0}; M\rangle$ in rest frame and final $|\vec{P} = \vec{q}; M'\rangle$ deuteron states, and we will show finding of them within the so-called clothed particle representation (CPR) that has been constructed via the method of unitary clothing transformations (henceforth, UCT method) {[1] Shebeko, Shirokov Prog. Part. Nucl. Phys. **44**(2000);[2] Phys. Part. Nucl. **32**(2001)},{[3] Korda, Canton, Shebeko Ann. Phys. **322** (2007) }

Hamiltonian and Boost Generator for Meson-Nucleon System in CPR

Following {Greenberg, Schweber Nuovo Cim. (1958)}, CPR for primary Hamiltonian $H \equiv H(\alpha) = H_F(\alpha) + H_I(\alpha)$ and other operators (for instance, in case of interacting meson and nucleon fields) being expressed through the set α of bare-particles creation/an operators is constructed via unitary clothing transformation (UCT) $W(\alpha) = W(\alpha_c) = \exp[R(\alpha_c)]$ ($R^\dagger = -R$) that does transition $\alpha = W(\alpha_c)\alpha_c W^\dagger(\alpha_c)$ to new set α_c of clothed-particles creation/annihilation operators. Its generator R is determined in such a way to remove from Hamiltonian with help of similarity transformation

$$H(\alpha) = W(\alpha_c)H(\alpha_c)W^\dagger(\alpha_c) \qquad (13)$$

the so-called bad terms that prevent the bare vacuum and the bare one-particle state to be the H eigenvectors (details in [1]). It results in representation $H = K_F(\alpha_c) + K_I(\alpha_c) = K$, where free part $K_F(\alpha_c) = H_F(\alpha_c)$ while operator $K_I(\alpha_c)$ contains interactions between clothed particles (mesons, nucleons) and destroys physical vacuum Ω (the lowest-energy H eigenstate) and the clothed one-particle states.

In case with conventional scalar (s), pseudoscalar (ps)and vector (v) meson-nucleon couplings determined by formulae (3)-(5) in {[4] Dubovik, Shebeko FBS **48**(2010)} we encounter separate four-operator contributions of class [2.2]

$$K_I^{(2)}(\alpha_c) = K(NN \to NN) + K(\bar{N}\bar{N} \to \bar{N}\bar{N}) + K(N\bar{N} \to N\bar{N})$$
$$+ K(bN \to bN) + K(b\bar{N} \to b\bar{N}) + K(bb' \to N\bar{N}) + K(N\bar{N} \to bb'), \qquad (14)$$

responsible for different $2 \rightleftarrows 2$ processes. In particular, N-N interaction operator can be written as $K(NN \to NN) = \sum_b K_b(NN \to NN) \equiv K_{NN}$,

$$K_b(NN \to NN) = \int \sum_\mu d\vec{p}_1' d\vec{p}_2' d\vec{p}_1 d\vec{p}_2\, V_b(1', 2'; 1, 2) \times b_c^\dagger(1') b_c^\dagger(2') b_c(1) b_c(2) \sim b_c^\dagger b_c^\dagger b_c b_c, \quad (15)$$

where symbol $\sum_\mu$ denotes summation over nucleon spin projections, $1 = \{\vec{p}_1, \mu_1\}$, etc.
Analytic expressions for c-number matrices V_b are given by eqs. (19)-(22) in [4].
In CPR boost operator $\vec{B}(\alpha_c) \equiv W(\alpha_c)\vec{N}(\alpha_c)W^\dagger(\alpha_c) = \vec{B}(\alpha_c) = \vec{B}_F(\alpha_c) + \vec{B}_I(\alpha_c)$ consists of free $\vec{B}_F$ and interaction $\vec{B}_I$ parts. Here $\vec{N} = \vec{N}_F + \vec{N}_I$ is entire boost operator for interacting fields in instant form of relativistic dynamics employed. The interaction part $\vec{B}_I$ repeats operator structure of K_I. Relevant expressions can be found in {Shebeko, Frolov FBS **52** (2012)}

Here in subspace $\mathcal{H}_{2N}$ (spanned onto the basis $b_c^\dagger b_c^\dagger|\Omega\rangle$) we have

$$\vec{B}_F = \sum\!\!\!\!\!\!\int d\vec{p}\,' d\vec{p} \vec{B}(\vec{p}\,'\mu', \vec{p}\mu) b_c^\dagger(\vec{p}\,'\mu') b_c(\vec{p}\mu),$$

with

$$\vec{B}(\vec{p}\,'\mu', \vec{p}\mu) = i\frac{m}{4}\frac{E_{\vec{p}\,'} + E_{\vec{p}}}{\sqrt{E_{\vec{p}\,'}E_{\vec{p}}}} u^\dagger(\vec{p}\,'\mu') u(\vec{p}\mu) \left[\frac{\partial}{\partial\vec{p}\,'} - \frac{\partial}{\partial\vec{p}}\right] \delta(\vec{p}\,' - \vec{p}).$$

It also can be divided into two parts

$$\vec{B}_F = \vec{B}_F^{orb} + \vec{B}_F^{spin}$$

with

$$\vec{B}_F^{orb} = \frac{i}{2} \sum\!\!\!\!\!\!\int d\vec{p}\, E_{\vec{p}} \left\{ \frac{\partial b_c^\dagger(\vec{p}\mu)}{\partial\vec{p}} b_c(\vec{p}\mu) - b_c^\dagger(\vec{p}\mu)\frac{\partial b_c(\vec{p}\mu)}{\partial\vec{p}} \right\}$$

and

$$\vec{B}_F^{spin} = -\frac{1}{2} \sum\!\!\!\!\!\!\int d\vec{p}\, \vec{p} \times \frac{\chi^\dagger(\mu)\vec{\sigma}\chi(\mu)}{E_{\vec{p}} + m} b_c^\dagger(\vec{p}\mu) b_c(\vec{p}\mu),$$

3 Deuteron Equation in CPR

Deuteron state $|\Psi_d(\vec{P})\rangle \in \mathcal{H}_{2N}$ satisfies eigenvalue equation

$$[H_F(\alpha) + H_I(\alpha)]|\Psi_d(\vec{P})\rangle = E_d|\Psi_d(\vec{P})\rangle$$

or in CPR

$$[K_F(\alpha_c) + K_I(\alpha_c)]|\Psi_d(\vec{P})\rangle = E_d|\Psi_d(\vec{P})\rangle,$$

with $E_d = \sqrt{m_d^2 + \vec{P}^2}$, where $\vec{P}$ is total deuteron momentum, $m_d = m_p + m_n - \varepsilon_d$ is deuteron mass and ε_d represents binding energy of deuteron.
Using approximation with $K_I(\alpha_c) = K_{NN}$ we arrive to a simpler eigenvalue problem

$$[K_0 + K_{NN}]\,|\vec{P}; M\rangle = E_d|\vec{P}; M\rangle$$

in the subspace $\mathcal{H}_{2N}$ spanned onto the basis $b_c^\dagger b_c^\dagger|\Omega\rangle$ with $K_{NN} \sim b_c^\dagger b_c^\dagger b_c b_c$ and $K_0 \sim b_c^\dagger b_c$. Here M denotes deuteron spin projection on quantization axis.
Solution of this equation can be represented as

$$|\vec{P}; M\rangle = \int d\vec{p}_1 d\vec{p}_2 D_M(\vec{P}; \vec{p}_1\mu_1, \vec{p}_2\mu_2) b_c^\dagger(\vec{p}_1\mu_1) b_c^\dagger(\vec{p}_2\mu_2)|\Omega\rangle,$$

with c-number coefficients $D_M(\vec{P}; \vec{p}_1\mu_1, \vec{p}_2\mu_2) = \delta(\vec{P} - \vec{p}_1 - \vec{p}_2)\psi_M(\vec{p}_1\mu_1, \vec{p}_2\mu_2)$ that have property

$$\psi_M(1,2) = -\psi_M(2,1).$$

In deuteron rest frame we have

$$|\psi_M\rangle = [m_d - K_0]^{-1} K_{NN} |\psi_M\rangle$$

where

$$|\psi_M\rangle \equiv |\vec{P}=0; M\rangle = \int d\vec{p}\psi_M(\vec{p}\mu_1, -\vec{p}\mu_2) b_c^\dagger(\vec{p}\mu_1) b_c^\dagger(-\vec{p}\mu_2)|\Omega\rangle.$$

Using basis vectors $|p(lS)JM_J, TM_T\rangle$ from {DuShe10}, vector $|\psi_M\rangle$ can be written as

$$|\psi_{M,TM_T}\rangle = \frac{1}{\sqrt{2}} \sum \int_0^\infty p^2 dp\, |p(lS)1M, TM_T\rangle \psi_{lST}(p),$$

since deuteron has the invariant spin equal to $J = 1$. The permissible values of quantum numbers l, S and T are restricted to property

$$\mathcal{P}_{ferm} |\psi_{M,TM_T}\rangle = |\psi_{M,TM_T}\rangle,$$

with space inversion operator determined in {DuShe10}. In fact, there are only two combinations of T, S and l, namely, $T = 0$, $S = 1$ and $l = 0, 2$

$$|\psi_{M,00}\rangle \equiv |\psi_M\rangle = \frac{1}{\sqrt{2}} \sum_{l=0,2} \int_0^\infty p^2 dp\, |p(l1)1M\rangle \psi_l(p),$$

$$\psi_M(\vec{p}\mu_1\tau_1, -\vec{p}\mu_2\tau_2) = \frac{1}{\sqrt{2}} \sum \psi_l(p) Y_{lm_l}(\hat{\vec{p}})(lm_l 1M_S|1M) \left(\tfrac{1}{2}\mu_1 \tfrac{1}{2}\mu_2|SM_S\right) \left(\tfrac{1}{2}\tau_1 \tfrac{1}{2}\tau_2|00\right).$$

At this point, we accept normalization condition

$$\langle \psi_{M'} | \psi_M \rangle = \delta_{M'M}$$

which is equivalent to

$$2 \sum \int d\vec{p} \psi^*_{M'}(\vec{p}\mu_1, -\vec{p}\mu_2) \psi_M(\vec{p}\mu_1, -\vec{p}\mu_2) = \delta_{M'M},$$

that implies

$$\int_0^\infty p^2 dp \left[\psi_0^2(p) + \psi_2^2(p) \right] = 1.$$

More precisely, these relations should be formulated for wave packets $\int d\vec{P} a(\vec{P}) |\vec{P}; M\rangle$. Each of them is a superposition of eigenvectors of total momentum operator with eigenvalues $\vec{P}$ close to $\vec{P} = 0$ by letting packet width goes to zero at end of calculations. Finally, we get set of homogeneous integral equations for "radial" components $\psi_l(p)(l = 0, 2)$

$$\psi_l(p) = \frac{1}{m_d - 2E_{\vec{p}}} \sum_{l'} \int_0^\infty k^2 dk V_{l\,l'}^{J=S=1,T=0}(p,k) \psi_{l'}(k).$$

In a moving frame corresponding eigenvector that belongs to value $E_d = \sqrt{\vec{P}^2 + m_d^2}$ can be determined either by solving directly eigenvector equation or using relation

$$|\vec{P}; M\rangle = \exp[-i\vec{\beta}\vec{B}(\alpha_c)]|\psi_M\rangle$$

Perhaps, one should note that the required

$$\hat{P}^\mu|\vec{P}; M\rangle = P^\mu|\vec{P}; M\rangle$$

follows from the property of the energy-momentum operator $\hat{P}^\mu = (H, \hat{P}^1, \hat{P}^2, \hat{P}^3)$ to be the four-vector, viz.,

$$e^{-i\vec{\beta}\vec{B}}\hat{P}^\mu e^{i\vec{\beta}\vec{B}} = \hat{P}^\nu L_\nu^{\ \mu}(\vec{\beta}).$$

In these formulae parameters $(\beta^1, \beta^2, \beta^3) = \vec{\beta}$ of the Lorentz transformation $m_d(1, 0, 0, 0) \Rightarrow (P^0, P^1, P^2, P^3) = P$ are related to velocity $\vec{v} = \vec{P}/m_d$ of moving frame

$$\vec{\beta} = \beta\vec{n}, \quad \vec{n} = \vec{v}/v, \quad \tanh\beta = v.$$

4 Current Density Operator in the CPR

In our approach we consider expansion in R-commutators {SheShi00}

$$J^{\mu}(0) = WJ_c^{\mu}(0)W^{\dagger} = J_c^{\mu}(0) + [R, J_c^{\mu}(0)] + \frac{1}{2}[R, [R, J_c^{\mu}(0)]] + ...,$$

where $J_c^{\mu}(0)$ is initial current in which bare operators $\{\alpha\}$ are replaced by clothed ones $\{\alpha_c\}$. This decomposition involves one-body, two-body and more complicated interaction currents.
In its turn, this operator being between clothed two-nucleon states

$$\eta_c J^{\mu}(0)\eta_c = J^{\mu}_{one-body} + J^{\mu}_{two-body},$$

where operator

$$J^{\mu}_{one-body} = \int d\vec{p}\,' d\vec{p} F^{\mu}_{p,n}(\vec{p}\,', \vec{p}) b_c^{\dagger}(\vec{p}\,') b_c(\vec{p})$$

with coefficients

$$F^{\mu}_{p,n}(\vec{p}\,', \vec{p}) = e\frac{m}{\sqrt{E_{\vec{p}\,'}E_{\vec{p}}}}\bar{u}(\vec{p}\,')\left\{F_1^{p,n}[(p'-p)^2]\gamma^{\mu} + i\sigma^{\mu\nu}(p'-p)_{\nu}F_2^{p,n}[(p'-p)^2]\right\}u(\vec{p})$$

which determines interaction of virtual photon with physical (clothed) proton (neutron).

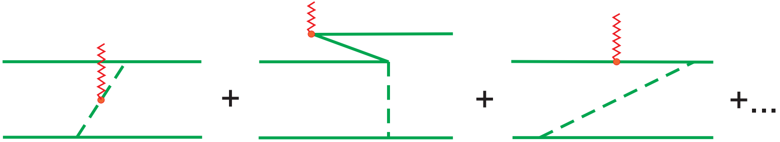
Such a form follows from representation, in which primary Nöther current operator, being sandwiched between physical (clothed) states $|\Psi_N\rangle = b_c^\dagger|\Omega\rangle$, yields usual on-mass-shell expression

$$\langle\Psi_{p,n}(\vec{p}\,')|J^\mu(0)|\Psi_{p,n}(\vec{p})\rangle = F^\mu_{p,n}(\vec{p}\,',\vec{p})$$

in terms of the Dirac and Pauli nucleon FFs.
Of course, the RIA results should be corrected including more complex mechanisms of e-d scattering, that are contained in

$$J^\mu_{two-body} = \int d\vec{p}_1' d\vec{p}_2' d\vec{p}_1 d\vec{p}_2 F^\mu_{MEC}(\vec{p}_1', \vec{p}_2'; \vec{p}_1, \vec{p}_2) b_c^\dagger(\vec{p}_1') b_c^\dagger(\vec{p}_1') b_c(\vec{p}_1) b_c(\vec{p}_2). \tag{16}$$

Analytic expressions for coefficients F^μ_{MEC} stem from the R-commutators which, first, belong to the class $[2.2]$ and, second, depend on even numbers of mesons involved.



5 Some Details of Calculations. Numerical Results and Discussion

First of all, one should note that

$$\mu_d = M^z(\vec{q}=0) = \frac{1}{2m_d}\langle\vec{0};1|[\vec{B}\times\vec{J}(0)]^z|\vec{0};1\rangle \tag{17}$$

and we will show our calculations in approximation, where $\vec{B} \Rightarrow \vec{B}_F$ and $\vec{J}(0) \Rightarrow \vec{J}_{one-body}$ with usual normalizations $F^{\nu}_{\tau}(\vec{p}'\mu',\vec{p}\mu) = e\frac{p^{\nu}}{E_{\vec{p}}}F^{\tau}_{1}(0)\delta_{\mu'\mu}$, $F^{p}_{1}(0)=1$, $F^{n}_{1}(0)=0$, $2mF^{p}_{2}(0) = \mu_p - 1 = 1.793$, $2mF^{n}_{2}(0) = \mu_n = -1.913$.

Now after modest effort we get decomposition

$$\mu_d = \mu_d^{\rm NR} + \mu_d^{\rm RC},$$

into "nonrelativistic" contribution

$$\mu_d^{\rm NR} = \left\{\mu_n + \mu_p - \frac{3}{2}[\mu_n+\mu_p-\frac{1}{2}]P_D\right\}\left[\frac{e}{2m}\right].$$

and relativistic correction

$$\mu_d^{\rm RC} = \frac{1}{3\sqrt{2}}\int\limits_0^\infty p^2dp(E_{\vec{p}}-m)\left[\frac{\mu_n+\mu_p}{E_{\vec{p}}} + \frac{1-\mu_n-\mu_p}{m}\right]$$
$$\times\left\{\sqrt{2}\left[\psi_2^2(p)-\psi_0^2(p)\right]+\psi_0(p)\psi_2(p)\right\}\left[\frac{e}{2m}\right].$$

Our calculation with u and w components of deuteron state, depicted in Figs. gives

All we need to do our derivations is
i) to calculate vacuum expectation $\langle\Omega|b_c b_c\, b_c^\dagger b_c\, b_c^\dagger b_c\, b_c^\dagger b_c^\dagger|\Omega\rangle$

$$\langle\vec{0};M'|B^j J^k(0)|\vec{0};M\rangle = 2\sum\!\!\!\!\!\!\int d\vec{p}d\vec{q}\,\{\psi^*_{M'}(\vec{p}\mu'_1,-\vec{p}\mu_2)B^j(\vec{p}\mu'_1,\vec{q}\mu'_2) -\psi^*_{M'}(\vec{q}\mu'_1,-\vec{q}\mu'_2)B^j(\vec{q}\mu'_1,\vec{p}\mu_2)\}\,F^k_\tau(\vec{q}\mu'_2,\vec{p}\mu_1)\psi_M(\vec{p}\mu_1,-\vec{p}\mu_2), \quad (18)$$

ii) to perform integration in parts and necessary differentiations in r.h.s. (**??**) coming $\mu_d = \mu_d^{NR} + \mu_d^{RC}$ with

$$\mu_d^{\mathrm{NR}} = \frac{2m}{m_d}\sum\!\!\!\!\!\!\int d\vec{p}\,\psi_{l'}(p)\psi_l(p)\langle Y^{l'1}_{11}(\hat{\vec{p}})|[\mu_n+\mu_p]J^z - [\mu_n+\mu_p-\frac{1}{2}]L^z|Y^{l1}_{11}(\hat{\vec{p}})\rangle, \quad (19)$$

$$\mu_d^{\rm RC} = \frac{m}{m_d} \sum\!\!\!\!\!\!\int d\vec{p}\, \psi_{l'}(p)\psi_l(p)(E_{\vec{p}} - m)\left[\frac{\mu_n + \mu_p}{E_{\vec{p}}} + \frac{1 - \mu_n - \mu_p}{m}\right]$$
$$\langle Y_{11}^{l'1}(\hat{\vec{p}})|\hat{p}^z(\hat{\vec{p}}\vec{S}) - S^z|Y_{11}^{l1}(\hat{\vec{p}})\rangle. \quad (20)$$

$$\vec{L} = -i\,\vec{p} \times \frac{\partial}{\partial \vec{p}},\ \vec{S} = \frac{1}{2}[\vec{\sigma}(1) + \vec{\sigma}(2)],\ \vec{J} = \vec{L} + \vec{S}$$

iii) to utilize properties of spin-angular functions $|Y_{1M}^{l1}(\hat{\vec{p}})\rangle$ deriving final result.

In parallel, we prefer to handle

$$Q_d = -\frac{1}{m_d^2}\left\{\langle\vec{0};1|B_z^2 J^0(0)|\vec{0};1\rangle - \langle\vec{0};0|B_z^2 J^0(0)|\vec{0};0\rangle\right\} \quad (21)$$

to get in a similar way,

$$Q_d = Q_d^{NR} + Q_d^{RC}. \quad (22)$$

As in case of magnetic moment we separate a "nonrelativistic" contribution

$$Q_d^{NR} = \frac{1}{20}\int dq\, q^2 \left\{2\sqrt{2}\frac{d\psi_0(q)}{dq}\left(\frac{d\psi_2(q)}{dq} + 3\frac{\psi_2(q)}{q}\right) - \left(\frac{d\psi_2(q)}{dq}\right)^2 - 6\frac{\psi_2^2(q)}{q^2}\right\} \quad (23)$$

and relativistic correction

$$Q_d^{RC} = \frac{1}{m_d^2}\sum_{i=1}^{4} I^{(i)}, \quad (24)$$

$$I^{(1)} = -\frac{1}{5}\int dq\, q^4 \left\{2\sqrt{2}\frac{d\psi_0(q)}{dq}\left(\frac{d\psi_2(q)}{dq} + 3\frac{\psi_2(q)}{q}\right) - \left(\frac{d\psi_2(q)}{dq}\right)^2 - 6\frac{\psi_2^2(q)}{q^2}\right\}, \quad (25)$$

$$I^{(2)} = -\frac{3\sqrt{2}}{5}\int dq\,\frac{q^3 E_q}{E_q+m}\left[\mu_n+\mu_p+\frac{E_q}{m}(\mu_n+\mu_p-1)\right] \times\left\{\psi_0(q)\frac{d\psi_2(q)}{dq}-\psi_2(q)\frac{d\psi_0(q)}{dq}+3\frac{\psi_0(q)\psi_2(q)}{q}+\sqrt{2}\frac{\psi_2^2(q)}{q}\right\} \quad (26)$$

$$I^{(3)} = \frac{1}{3}\int dq\,\frac{q^4}{(E_q+m)^2}\left[\mu_n+\mu_p-\frac{1}{2}+\frac{E_q}{m}(\mu_n+\mu_p-1)\right] \times\left\{4\psi_0^2(q)+\frac{14\sqrt{2}}{5}\psi_0(q)\psi_2(q)+\frac{37}{35}\psi_2^2(q)\right\}, \quad (27)$$

$$I^{(4)} = \frac{4}{5}\int dq\,\frac{q^4}{E_q^2}\left\{2\sqrt{2}\psi_0(q)\psi_2(q)-\psi_2^2(q)\right\}\left[2E_q^2\left(\frac{dG_E^p(t)}{dt}+\frac{dG_E^n(t)}{dt}\right) -\frac{q^2-10E_q m-4m^2}{4(E_q+m)^2}+(\mu_p+\mu_n)\Gamma_1(q)-(\mu_p+\mu_n-1)\Gamma_2(q)\right], \quad (28)$$

$$\Gamma_1(q)=\frac{q^2-2E_q m}{(E_q+m)^2},\quad \Gamma_2(q)=\frac{q^4+3q^2m^2+4mE_q q^2+8E_q m^3+4m^4}{2m^2(E_q+m)^2}. \quad (29)$$

In the last integral $\frac{dG_E^\tau}{dt}|_{t=0}=\frac{1}{6}<r^2>_\tau^{ch}$ $(t=(p'-p)^2)$, where $<r^2>_\tau^{ch}$ is charge r.m.s. radius of proton or neutron.

Relying upon the experience [**] we find

Table: The best-fit parameters for the two models. The row *Potential B* (*UCT*) taken from Table A.1 in [*] (obtained by least squares fitting to OBEP values in Table 1 of Ref. [**] including deuteron binding energy and low-energy parameters). All masses are in MeV, and $n_b = 1$ except for $n_\rho = n_\omega = 2$.

Model	Meson	π	η	ρ	ω	δ	$\sigma,\ T = 0\,(T = 1)$
Potential B	$g^2/4\pi\ [f/g]$	14.4	3	0.9 [6.1]	24.5	2.488	18.3773 (8.9437)
	Λ	1700	1500	1850	1850	2000	2000 (1900)
	m	138.03	548.8	769	782.6	938	720 (550)
UCT	$g^2/4\pi\ [f/g]$	13.395	5.0	1.2 [6.1]	17.349	5.0	22.015 (5.514)
	Λ	2500	1219	1593	2494	2169	1200 (2500)
	m	138.03	548.8	769	782.6	938	720 (550)

***** Machleidt, R.: Adv. Nucl. Phys. **19** (1989) 189
****** Dubovyk I., Shebeko O.: Few Body Syst. **48** (2010) 109

Table: Low-energy parameters of the nucleon-nucleon scattering. Experimental values are from Table 4.2 of Ref. [*].

Parameter	Bonn B	UCT	Experiment
a_s (fm)	-23.71	-23.57	-23.748±0.010
r_s (fm)	2.71	2.65	2.75±0.05
a_t (fm)	5.426	5.44	5.419±0.007
r_t (fm)	1.761	1.79	1.754±0.008

Table: Deuteron properties.

Parameter	Bonn B	UCT		Experiment
		NR	RC	
ε_d (MeV)	2.223	2.224		2.224575
P_D (%)	4.99	4.89		
μ_d $(e/2m_p)$	0.8516	0.8521	$-8.981\cdot 10^{-4}$	0.857406∓000001
Q_d (fm^2)	0.2783	0.2972	$3.115\cdot 10^{-3}$	0.2860∓0.0015

***** Machleidt, R.: Adv. Nucl. Phys. **19** (1989) 189
cf. Chung P.L., Keister B.D., Coester F.: Phys. Rev. C**39** (1989) 1544

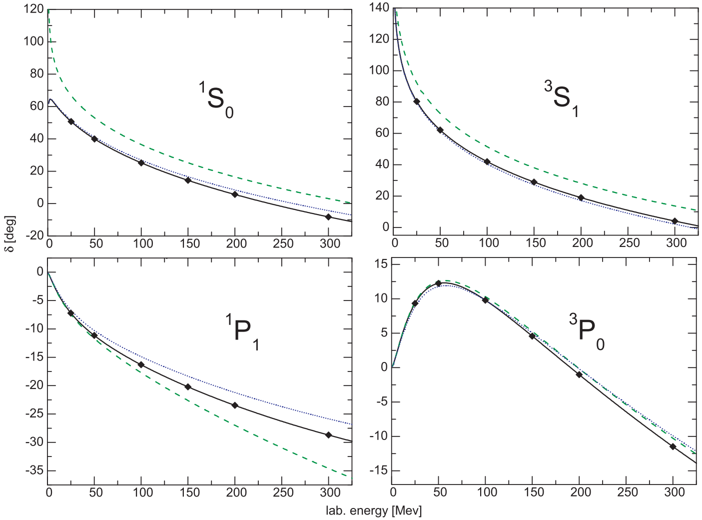


Figure: Neutron-proton phase parameters plotted versus nucleon kinetic energy in lab. system. Solid curves calculated for Potential B. Dashed (dotted) - for UCT potential with *Potential B* (*UCT*) parameters from Table 1. The rhombs show original OBEP results.

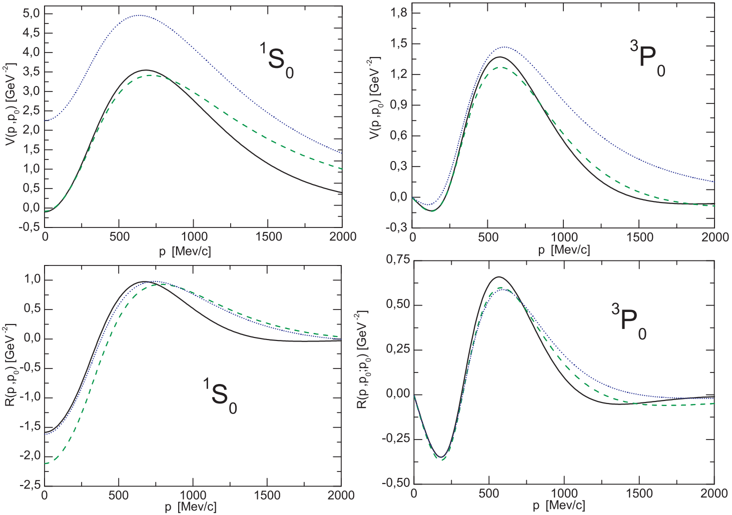


Figure: Half-off-shell R-matrices and off-shell potentials at laboratory energy equal to 150 MeV ($p0 = 265$ MeV). Solid curves calculated for Potential B. Dashed (dotted) - for UCT potential with *Potential B* (*UCT*) parameters from Table 1.

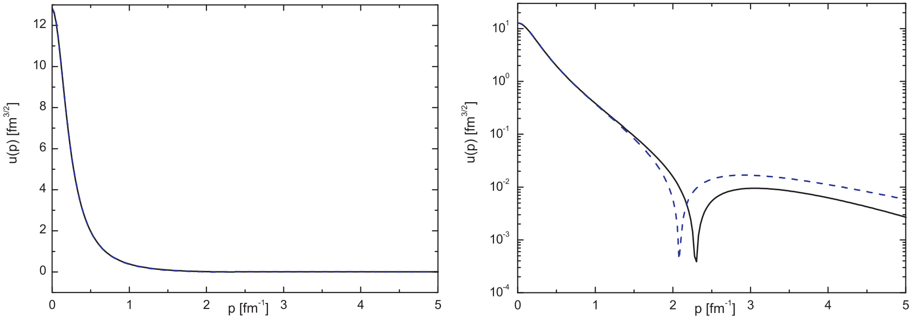


Figure: Deuteron wave function $\psi_0^d(p) = u(p)$. Solid (dashed) curves for *Bonn B* (*UCT*) potential.

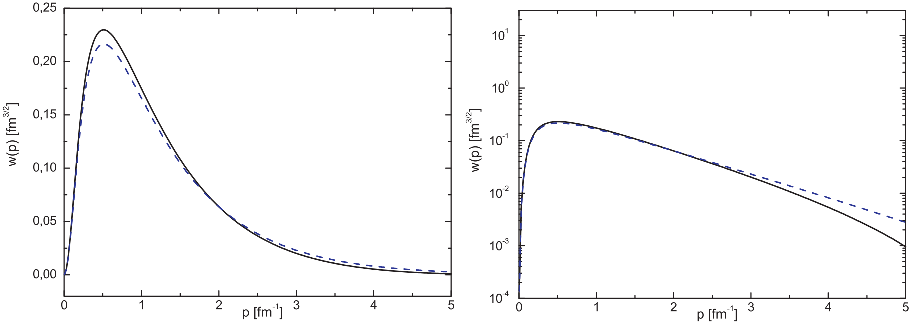


Figure: Deuteron wave function $\psi_2^d(p) = w(p)$. Solid (dashed) curves for *Bonn B* (*UCT*) potential.

Summary

For this exposition we have seen a reasonable treatment of the low-energy N-N scattering and deuteron properties. In the course of our current work we are trying to understand to what extent deuteron electromagnetic form factors and structure functions that determine deuteron photo- and electrodisintegration are sensitive to off-shell effects inherent in the quasipotentials and currents obtained within the UCT method.

In our opinion, exposed approach has promising prospects, e.g., in the theory of decaying states (after evident refinements), certainly in quantum electrodynamics and, we believe, in quantum chromodynamics. Such endeavors are under way.