بسم الله الرحمن الرحيم

لا يعزب عنه مثقال ذرة في السماوات ولا في الأرض

ولا أصغر من ذلك ولا أكبر إلا في كتاب مبين

Scattering of K^+ Meson from ^{40}Ca Nucleus at Intermediate Energies

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Aim of the work

Study of keon-nucleus interactions where the experimental data of the total elastic cross section and elastic differential cross section for keon-nucleus interactions are found to be higher than the theoretical calculations.

Guidelines

we construct realistic optical potential for keon-nucleon and keon-nucleus interactions based on one bosone exchange model.

We test our constructive potential by calculating the total elastic cross section for $(K^{+}-{}^{40}Ca)$

To understated the interactions mechanism of the kaon-nucleon we consider the inner structure of the kaon and nucleon and their interactions

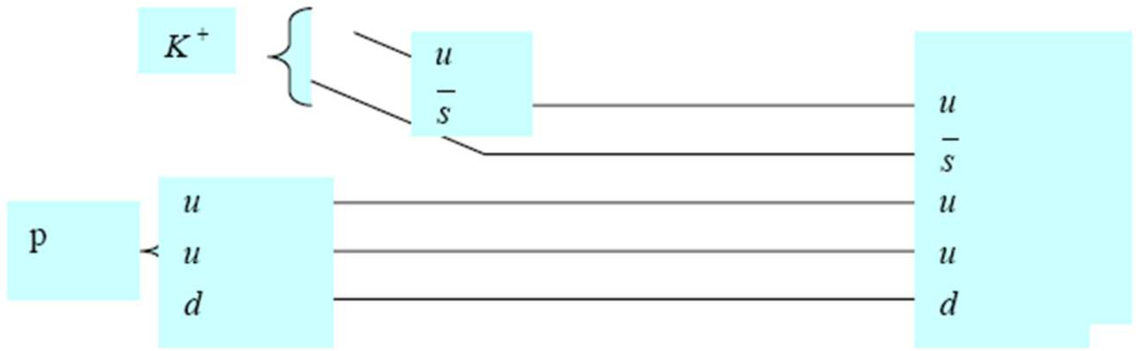


Fig. (1-A): Quark diagrams for ($K^+ - P$)

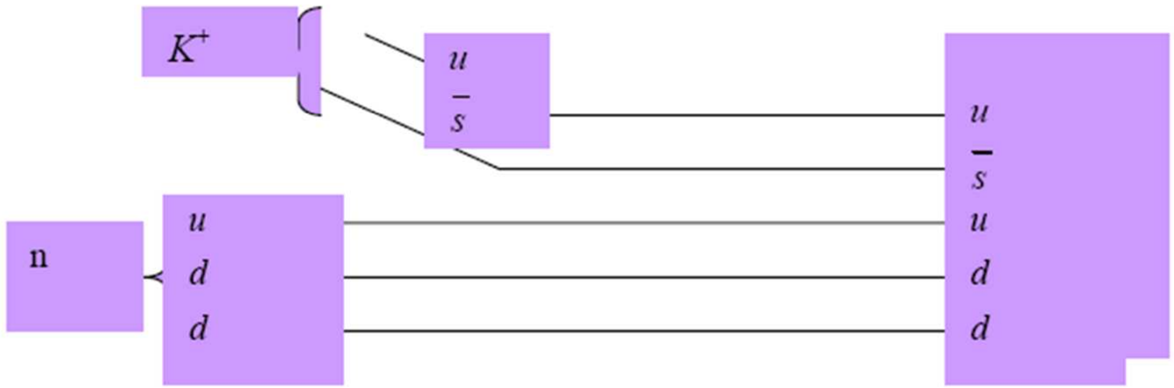


Fig. (1-B): Quark diagrams for ($K^+ - n$)

This means that the quark-antiquark annihilation has not been suppressed, so k+ has long mean free path i.e. smaller cross section.

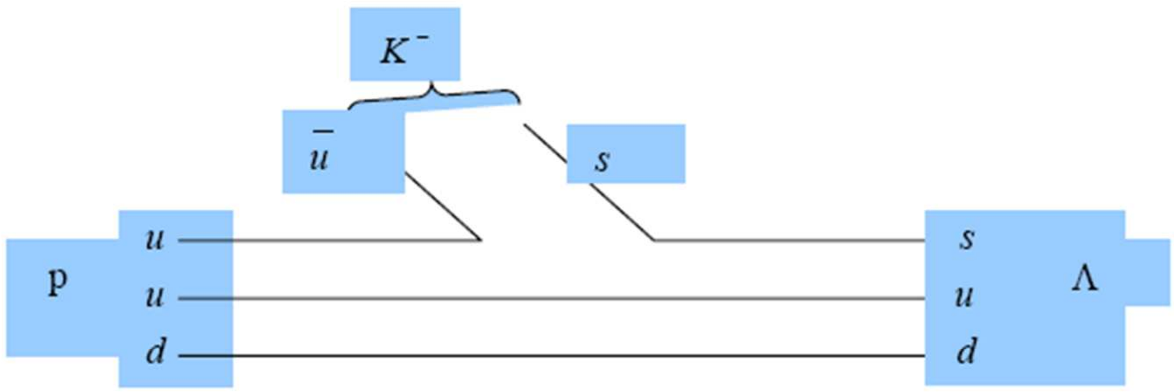


Fig. (1-C): Quark diagrams for ($K^- - P$)

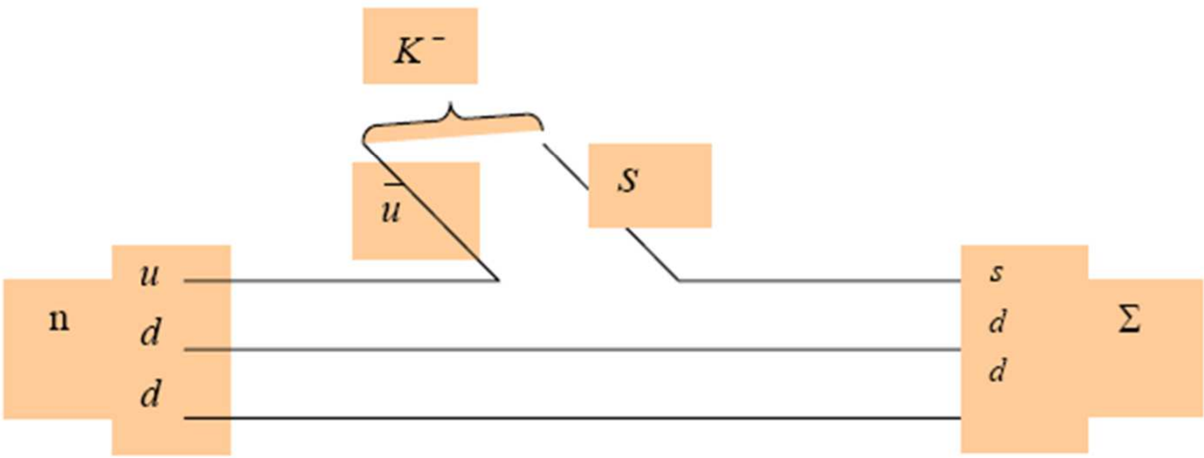
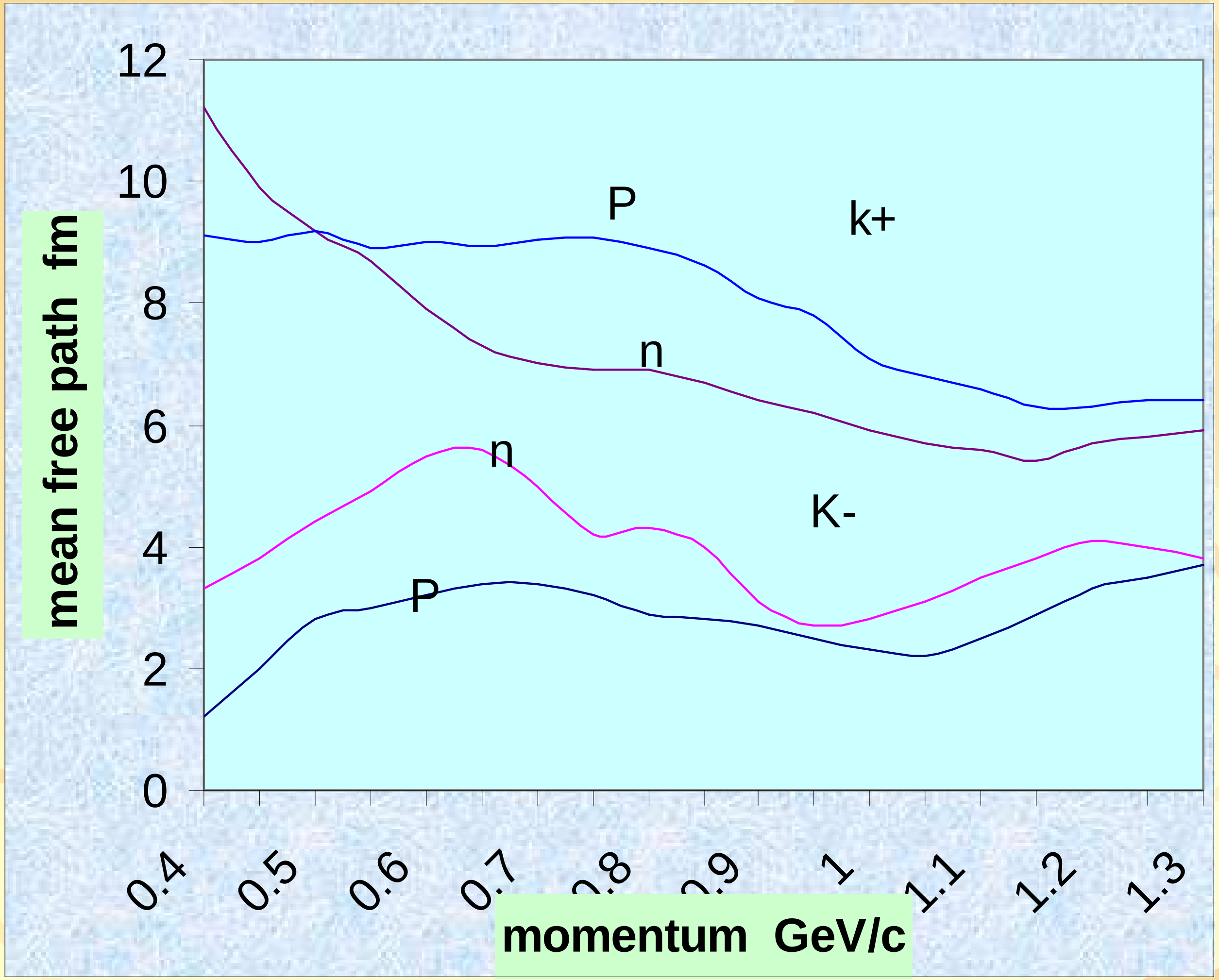


Fig. (1-D): Quark diagrams for ($K^- - n$)

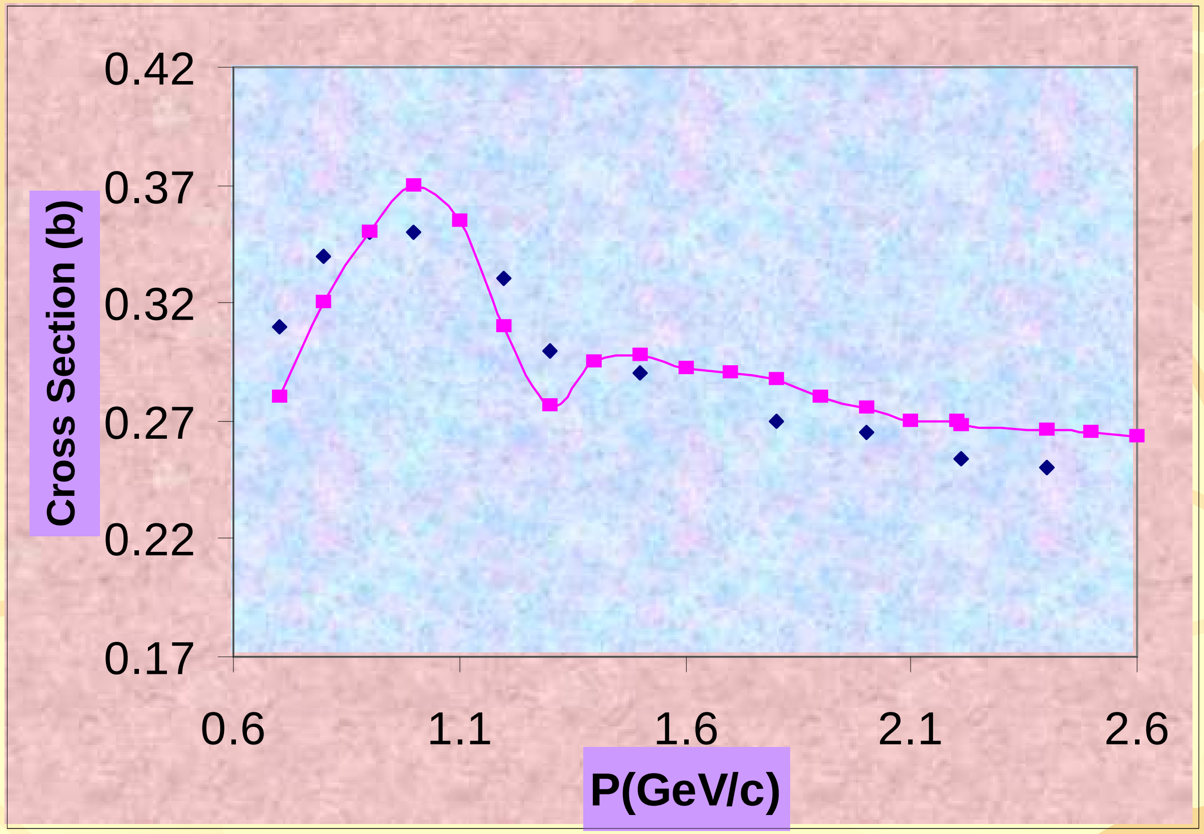
This means that the quark-antiquark annihilation has been suppressed, so k- has short mean free path i.e. larger cross section.

This figure shows that the longer mean free path of K+ and shorter for K- with proton and neutron in nuclear matter .



<u>Fig.(2)</u>: Mean free path versus. and kaons momentum with proton and neutron in nuclear matter .

This figure shows that the agreement of the theoretical calculations of total cross section for K^- C interaction with experimental data



<u>Fig. (3)</u>: Comparison between experimental cross sections (K^- C) **interaction (diamond points) and the corresponding calculated one (square points), in the framework of the Glauber model, vs. meson momentum.**

Mathematical Formulations.

The One Boson Exchange Kaon-Nucleon Potential Form.

The one-boson- exchange $(K^+ - N)$ interaction potential is composed of four parts

$$V_{(K^+, N)}(r) = V_\sigma(r) + V_\rho(r) + V_\omega(r) + V_{\sigma_0}(r)$$

Their structure are as follows ,

with $V_\sigma(r) = -\gamma_1^0 \gamma_2^0 J_\sigma(r)$ $\quad V_\rho(r) = \gamma_1^0 \gamma_2^0 \gamma_1^\mu \gamma_2^\mu J_\rho(r)$

$$V_\omega(r) = \gamma_1^0 \gamma_2^0 \gamma_1^\mu \gamma_2^\mu J_\omega(r) \qquad V_{\sigma_0}(r) = \gamma_1^0 \gamma_2^0 J_{\sigma_0}(r)$$

$$\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad \vec{\gamma} = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix} \qquad \gamma_1^\mu \gamma_2^\mu = \gamma_1^0 \gamma_2^0 - \vec{\gamma}_1 \vec{\gamma}_2$$

Coordinates and Momenta for Kaon Nucleon in Relative and Center of Mass System

Relative and center of mass Position vectors $\vec{r} = \sqrt{2}\,\frac{m_1\vec{r}_1 - m_2\vec{r}_2}{m_1+m_2}$ $\quad \vec{R} = \sqrt{2}\,\frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1+m_2}$

Relative and center of mass momenta $\vec{P}_r = \frac{m_2\vec{p}_1 - m_1\vec{p}_2}{m_1+m_2}$ $\quad \vec{P}_R = \vec{p}_1 + \vec{p}_2$

Normalization of the Wave Functions

The relativistic wave function in terms of the large and small components is

$$\left\langle f_\gamma(\vec{r})\right| = \left\langle \varphi_\gamma(\vec{r})\right| + \left\langle \chi_\gamma(\vec{r})\right|$$

where $\varphi(\vec{r})$ and $\chi(\vec{r})$ are complete sets in the Dirac coordinate space

$$\chi_\gamma(\vec{r})\,;\ \frac{\vec{\sigma}_2.\vec{P}_2}{2m_2c}\varphi_\gamma(\vec{r})$$

where $\vec{\sigma}_2$, m_2 and $\vec{p}_2$ are Pauli spin matrix, the nucleon mass and its momentum,

in Dirac space the normalization condition is

$$\left\langle f_\gamma(\vec{r})\middle| f_\gamma(\vec{r})\right\rangle = \left\langle \varphi_\gamma(\vec{r})\middle|\varphi_\gamma(\vec{r})\right\rangle + \left\langle \chi_\gamma(\vec{r})\middle|\chi_\gamma(\vec{r})\right\rangle = 1$$

$$\left|\varphi_n(\vec{r})\right\rangle = \sqrt{1+\frac{p^2}{4m_2^2c^2}}\left|\varphi_{un}(\vec{r})\right\rangle$$

Expansion of the Kaon and Nucleon Wave Functions

The kaon wave function can be expanded in terms of its angular and isotopic spin functions as

$$\varphi_\alpha(\vec{r}) = \sum_{m_{l_\alpha}} (l_\alpha 0 m_{l_\alpha} 0 | l_\alpha m_{l_\alpha}) \varphi_{n_\alpha l_\alpha m_{l_\alpha}}(\vec{r}) \hat{P}_{T_\alpha}$$

The bracket $(l_\alpha 0 m_{l_\alpha} 0 | l_\alpha m_{l_\alpha})$ is the Clebsch-Gordon coefficient

Also nucleon wave function can be expanded in terms of the angular, spin and its isotopic spin wave functions as

$$\varphi_\gamma(\vec{r}) = \sum_{m_{l_\gamma} m_{s_\gamma}} (l_\gamma \tfrac{1}{2} m_{l_\gamma} m_{s_\gamma} | J_\gamma m_{J_\gamma}) \varphi_{n_\gamma l_\gamma m_{l_\gamma}}(\vec{r}) \chi^{1/2}_{m_{s_\gamma}} \hat{P}_{T_\gamma}$$

so, the bra of (K^+, N) System wave function as

$$\left\langle \varphi_\alpha(\vec{r}_1)\varphi_\gamma(\vec{r}_2) \right| = \sum_{m_{l_\alpha} m_{l_\gamma} m_{s_\gamma}} (l_\alpha 0 m_{l_\alpha} 0 | J_\alpha m_{J_\alpha}) (l_\gamma \tfrac{1}{2} m_{l_\gamma} m_{s_\gamma} | J_\gamma m_{J_\gamma}) \left\langle \varphi_{n_\alpha l_\alpha m_{l_\alpha}}(\vec{r}_1) \varphi_{n_\gamma l_\gamma m_{l_\gamma}}(\vec{r}_2) \chi^{1/2}_{m_{s_\gamma}} \hat{P}_{T_\alpha} \hat{P}_{T_\gamma} \right|$$

The two angular wave functions are coupled by

$$\left\langle \varphi_{n_\alpha l_\alpha m_{l_\alpha}}(\vec{r}_1) \varphi_{n_\gamma l_\gamma m_{l_\gamma}}(\vec{r}_2) \right| = \sum_{\lambda m_\lambda} (l_\alpha l_\gamma m_{l_\alpha} m_{l_\gamma} | \lambda m_\lambda) \left\langle \varphi_{n_\alpha l_\alpha n_\gamma l_\gamma \lambda m_\lambda}(\vec{r}_1, \vec{r}_2) \right|$$

The bra of collective wave function is transformed to relative and C.M. system as

$$\left\langle \varphi_{n_\alpha l_\alpha n_\gamma l_\gamma \lambda m_\lambda}(\vec{r}_1,\vec{r}_2)\right| = \sum_{nlNL} (n_\alpha l_\alpha n_\gamma l_\gamma \lambda \mid NLnl\,\lambda) \left\langle \varphi_{NL\,nl\,\lambda m_\lambda}(\vec{R},\vec{r})\right|$$

$(n_\alpha l_\alpha n_\gamma l_\gamma \lambda \mid NLnl\,\lambda)$ is the GTMS transformation bracket

with $\lambda = l_\alpha + l_\gamma$ $\qquad m_\lambda = m_{l\alpha} + m_{l\gamma}$

$n_\alpha + n_\gamma = N + n$ and $l_\alpha + l_\gamma = \lambda = L + l$

The wave function of the relative and center of mass coordinate can be splited into two components as

$$\left\langle \varphi_{NL\,nl\,\lambda m_\lambda}(\vec{R},\vec{r})\right| = \sum_{Mm} (LlMm|\lambda m_\lambda) \left\langle \varphi_{NLM}(\vec{R})\right|\left\langle \varphi_{nlm}(\vec{r})\right|$$

Also the coupled wave functions of spin and isotropic spin are

$$\left\langle \hat{P}_{T_\alpha} \chi^{s_\gamma}_{m_{s_\gamma}} \hat{P}_{T_\gamma}\right| = \sum_{s\, m_{s_\gamma} T M_T} \left(0\,\tfrac{1}{2}\,0\,m_{s_\gamma}\middle| s\,m_s\right)\left(\tfrac{1}{2}\,\tfrac{1}{2}\,T_\alpha T_\gamma\middle| T\,m_T\right)\left\langle \chi^{s}_{m_s} \hat{P}_{T_\alpha T_\gamma}\right|$$

Kaon-Nucleon and Kaon-Nucleus Potentials

The interaction potential for the $(K^{+}-N)$ system takes the form

$$V_{(K^{+},N)}(r,R)=-J_{\sigma}(r)+\frac{1}{8m_2^2c^2}\{P_r^2J_{\sigma}(r)-\frac{2m_2}{m_1+m_2}\vec{P}_r\vec{P}_R J_{\sigma}(r)+(\frac{m_2}{m_1+m_2})^2P_R^2J_{\sigma}(r)+J_{\sigma}(r)P_r^2-\frac{2m_2}{m_1+m_2}J_{\sigma}(r)\vec{P}_r\vec{P}_R+$$

$$(\frac{m_2}{m_1+m_2})^2J_{\sigma}(r)P_R^2\}+\frac{2}{8m_2^2c^2}\{(-\vec{\sigma}_2.\vec{P}_r)+(\frac{m_2}{m_1+m_2})(\vec{\sigma}_2.\vec{P}_R)\}J_{\sigma}(r)\{(-\vec{\sigma}_2.\vec{P}_r)+(\frac{m_2}{m_1+m_2})(\vec{\sigma}_2.\vec{P}_R)\}+$$

$$J_{\rho}(r)-\frac{1}{8m_2^2c^2}\{P_r^2J_{\rho}(r)-\frac{2m_2}{m_1+m_2}\vec{P}_r\vec{P}_R J_{\rho}(r)+(\frac{m_2}{m_1+m_2})^2P_R^2J_{\rho}(r)+J_{\rho}(r)P_r^2-\frac{2m_2}{m_1+m_2}J_{\rho}(r)\vec{P}_r\vec{P}_R+$$

$$(\frac{m_2}{m_1+m_2})^2J_{\rho}(r)P_R^2\}+\frac{2}{8m_2^2c^2}\{(-\vec{\sigma}_2.\vec{P}_r)+(\frac{m_2}{m_1+m_2})(\vec{\sigma}_2.\vec{P}_R)\}J_{\rho}(r)\{(-\vec{\sigma}_2.\vec{P}_r)+(\frac{m_2}{m_1+m_2})(\vec{\sigma}_2.\vec{P}_R)\}++J_{\omega}(r)-$$

$$\frac{1}{8m_2^2c^2}\{P_r^2J_{\omega}(r)-\frac{2m_2}{m_1+m_2}\vec{P}_r\vec{P}_R J_{\omega}(r)+(\frac{m_2}{m_1+m_2})^2P_R^2J_{\omega}(r)+J_{\omega}(r)P_r^2-\frac{2m_2}{m_1+m_2}J_{\omega}(r)\vec{P}_r\vec{P}_R+(\frac{m_2}{m_1+m_2})^2J_{\omega}(r)P_R^2\}+$$

$$\frac{2}{8m_2^2c^2}\{(-\vec{\sigma}_2.\vec{P}_r)+(\frac{m_2}{m_1+m_2})(\vec{\sigma}_2.\vec{P}_R)\}J_{\omega}(r)\{(-\vec{\sigma}_2.\vec{P}_r)+(\frac{m_2}{m_1+m_2})(\vec{\sigma}_2.\vec{P}_R)\}+J_{\sigma_0}(r)-\frac{1}{8m_2^2c^2}\{P_r^2J_{\sigma_0}(r)-$$

$$\frac{2m_2}{m_1+m_2}\vec{P}_r\vec{P}_R J_{\sigma_0}(r)+(\frac{m_2}{m_1+m_2})^2P_R^2J_{\sigma_0}(r)+J_{\sigma_0}(r)P_r^2-\frac{2m_2}{m_1+m_2}J_{\sigma_0}(r)\vec{P}_r\vec{P}_R+(\frac{m_2}{m_1+m_2})^2J_{\sigma_0}(r)P_R^2\}-$$

$$\frac{2}{8m_2^2c^2}\{(-\vec{\sigma}_2.\vec{P}_r)+(\frac{m_2}{m_1+m_2})(\vec{\sigma}_2.\vec{P}_R)\}J_{\sigma_0}(r)\{(-\vec{\sigma}_2.\vec{P}_r)+(\frac{m_2}{m_1+m_2})(\vec{\sigma}_2.\vec{P}_R)\}$$

Then the expectation value of the $(K^+ - N)$ elastic interaction potential is given by

$$\left\langle f_\alpha f_\gamma \middle| V_{(K^+,N)}(r) \middle| f_\beta f_\delta \right\rangle = \sum_{\lambda m_\lambda} \sum_{n l N L} \sum_{m M} \sum_{m_{l_\alpha} m_{l_\gamma}} \sum_{m_{s_\alpha} m_{s_\gamma}} \sum_{T m_T} \sum_{S m_s} \sum_{j m_j}$$

$$(L\, l\, M\, m|\lambda m_\lambda)^2 \left(s_\alpha s_\gamma m_{s_\alpha} m_{s_\gamma} \middle| S\, m_s\right)^2 \left(\tfrac{1}{2}\tfrac{1}{2} T_\alpha\, T_\gamma \middle| T\, m_T\right)^2 \left(n_\alpha l_\alpha n_\gamma l_\gamma \lambda \middle| NnLl\,\lambda\right)^2$$

$$\left(l_\alpha s_\alpha m_{l_\alpha} m_{s_\alpha} \middle| j_\alpha m_{j_\alpha}\right)^2 \left(l_\gamma s_\gamma m_{l_\gamma} m_{s_\gamma} \middle| j_\gamma m_{j_\gamma}\right)^2 \left(l_\alpha l_\gamma m_{l_\alpha} m_{l_\gamma} \middle| \lambda m_\lambda\right)^2 \left(l\ s\ m_l\ m_s \middle| j\ m_j\right)^2$$

$$\{-\langle R_{nl}(r)|J_\sigma(r)|R_{nl}(r)\rangle + \langle R_{nl}(r)|J_\rho(r)|R_{nl}(r)\rangle + \langle R_{nl}(r)|J_\omega(r)|R_{nl}(r)\rangle +$$

$$\langle R_{nl}(r)|J_{\sigma_0}(r)|R_{nl}(r)\rangle + Q_1[\langle R_{nl}(r)|J_\sigma(r)|R_{nl}(r)\rangle + \langle R_{nl}(r)|J_{\sigma_0}(r)|R_{nl}(r)\rangle] +$$

$$Q_2[\langle R_{nl}(r)|J_\sigma(r)|R_{nl}(r)\rangle + \langle R_{nl}(r)|J_\rho(r)|R_{nl}(r)\rangle + \langle R_{nl}(r)|J_\omega(r)|R_{nl}(r)\rangle -$$

$$\langle R_{nl}(r)|J_{\sigma_0}(r)|R_{nl}(r)\rangle] - Q_3[\langle R_{nl}(r)|J_\sigma(r)|R_{nl}(r)\rangle - \langle R_{nl}(r)|J_{\sigma_0}(r)|R_{nl}(r)\rangle] +$$

$$Q_4[\langle R_{nl}(r)|\frac{1}{r}\frac{dJ_\sigma(r)}{r}|R_{nl}(r)\rangle + \langle R_{nl}(r)|\frac{1}{r}\frac{dJ_\rho(r)}{r}|R_{nl}(r)\rangle + \langle R_{nl}(r)|\frac{1}{r}\frac{dJ_\omega(r)}{r}|R_{nl}(r)\rangle -$$

$$\langle R_{nl}(r)|\frac{1}{r}\frac{dJ_{\sigma_0}(r)}{r}|R_{nl}(r)\rangle] + \quad Q_6[-\langle R_{nl}(r)|\frac{dJ_\sigma(r)}{r}\frac{d}{dr}|R_{nl}(r)\rangle -$$

$$\langle R_{nl}(r)|\frac{dJ_\rho(r)}{r}\frac{d}{dr}|R_{nl}(r)\rangle - \langle R_{nl}(r)|\frac{dJ_\omega(r)}{r}\frac{d}{dr}|R_{nl}(r)\rangle + \langle R_{nl}(r)|\frac{dJ_{\sigma_0}(r)}{r}\frac{d}{dr}|R_{nl}(r)\rangle]\}$$

$$R_{nl}(r) = \left[\frac{2(n!)}{\Gamma(n+l+3/2)}\right]^{1/2} \left(\frac{1}{b}\right)^{3/2} \left(\frac{r}{b}\right)^{l} \exp\left(-\frac{1}{2}\left(\frac{r}{b}\right)^{2}\right) L_n^{l+1/2}\left(\frac{r}{b}\right)^{2}$$

Is the normalized Laugurre Polynomial

$$P_r^{\,2} = m_1 m_2\left[(2n+l+3/2)\hbar\omega + \frac{1}{2}\frac{m_1 m_2}{m_1+m_2}\omega^2 r^2\right] \qquad b_r = \sqrt{\frac{\hbar(m_1+m_2)}{m_1 m_2 \omega}}$$

$$P_R^{\,2} = 2(m_1+m_2)\left[(2N+L+3/2)\hbar\omega + \frac{1}{2}(m_1+m_2)\omega^2 R^2\right] \qquad b_R = \sqrt{\frac{\hbar}{(m_1+m_2)\omega}}$$

$$\vec{S}_2 = \frac{\hbar}{2}\vec{\sigma}_2, \quad \vec{P}_r = -i\hbar\nabla_r, \quad \vec{P}_R = -i\hbar\nabla_R, \quad \vec{P}_r.\vec{P}_R = -\hbar^2 \frac{d}{dr}\frac{d}{dR}$$

$$Q_1 = \frac{m_1(2n+l+3/2)\hbar\omega}{m_2 c^2 (m_1+m_2)}, \qquad Q_2 = \frac{3(2N+L+3/2)\hbar\omega}{4c^2(m_1+m_2)}, \qquad Q_3 = \frac{\omega^2}{2c^2}\left(\frac{m_1}{m_1+m_2}\right)^2$$

$$Q_4 = \frac{\hbar^2}{4m_2^2 c^2}(\vec{s}.\vec{l}), \quad Q_5 = \frac{\hbar^2}{4m_2^2 c^2(m_1+m_2)}, \quad Q_6 = \frac{\hbar^2}{4m_2^2 c^2}, \qquad Q_7 = \frac{3\omega^2}{8c^2}$$

the impulse approximation can be applied to obtain the interaction potential between the kaon and the nucleus

$$V_{(K^+ - A)}(r) = \sum_{N=1}^{A} V_{(K^+ - N)}(r)$$

where, *A* is the mass number of nucleus

Relation Between our Potential and Phase Shift

$$tan(\delta_l(k)) = \frac{2\mu}{h^2}(1+i\beta)\Big[-k\{\int_r r\, j_l(kr)V_a(r)r\, j_l(kr)dr +$$

$$Qh^2\int_r r\, j_l(kr)\frac{d^2V_a(r)}{dr^2}r\, j_l(kr)dr + Qh^2\int_r r\, j_l(kr)V_a(r)\frac{d^2}{dr^2}r\, j_l(kr)dr -$$

$$2Qh^2\int_r r\, j_l(kr)V_c(r)\frac{d^2}{dr^2}r\, j_l(kr)dr - 2Qh^2\int_r r\, j_l(kr)\frac{dV_c(r)}{dr}\frac{d}{dr}r\, j_l(kr)dr +$$

$$2Qh^2\int_r j_l(kr)\frac{dV_c(r)}{dr}[J(J+1)-l(l+1)-S(S+1)]r\, j_l(kr)dr\}\Big]$$

The Elastic total Cross Section.

the differential scattering cross section

$$\frac{d\sigma}{d\Omega}(k,\theta) = |f(k,\theta)|^2 = \frac{1}{k^2}\sum_{l=0}^{\infty}\sum_{l'=0}^{\infty}(2l+1)(2l'+1)\exp\{i(\delta_l(k)-\delta_{l'}(k)\}\times$$

$$sin(\delta_l(k))\; sin(\delta_{l'}(k))P_l(\cos\theta)P_{l'}(\cos\theta)$$

By integration the differential cross section over all scatting angle we obtain the total elastic cross section as

$$\sigma_{tot}(k) = \frac{2\pi}{k^2}\sum_{l=0}^{\infty}(2l+1)Sin^2\delta_l(k)$$

Mesons Function and Mesons Parameters

We have used static meson function for numerical calculation of the potential

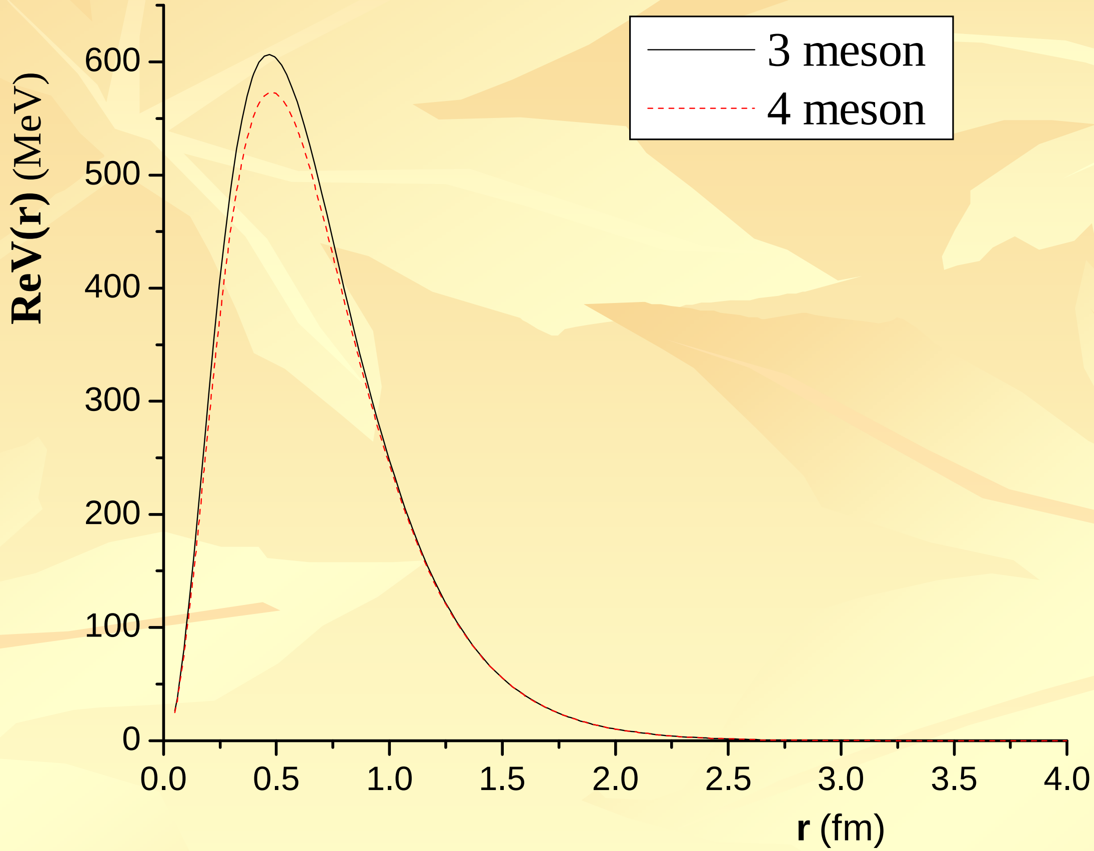
The associated Generalized Yukawa function (GY) is given by

$$J_i(r) = g_i^2 \hbar c \left[\frac{\exp(-u_i r)}{r} - \frac{\exp(-\Lambda_i r)}{r} \left(1 + \frac{\Lambda_i^2 - u_i^2}{2\Lambda_i} r\right) \right]$$

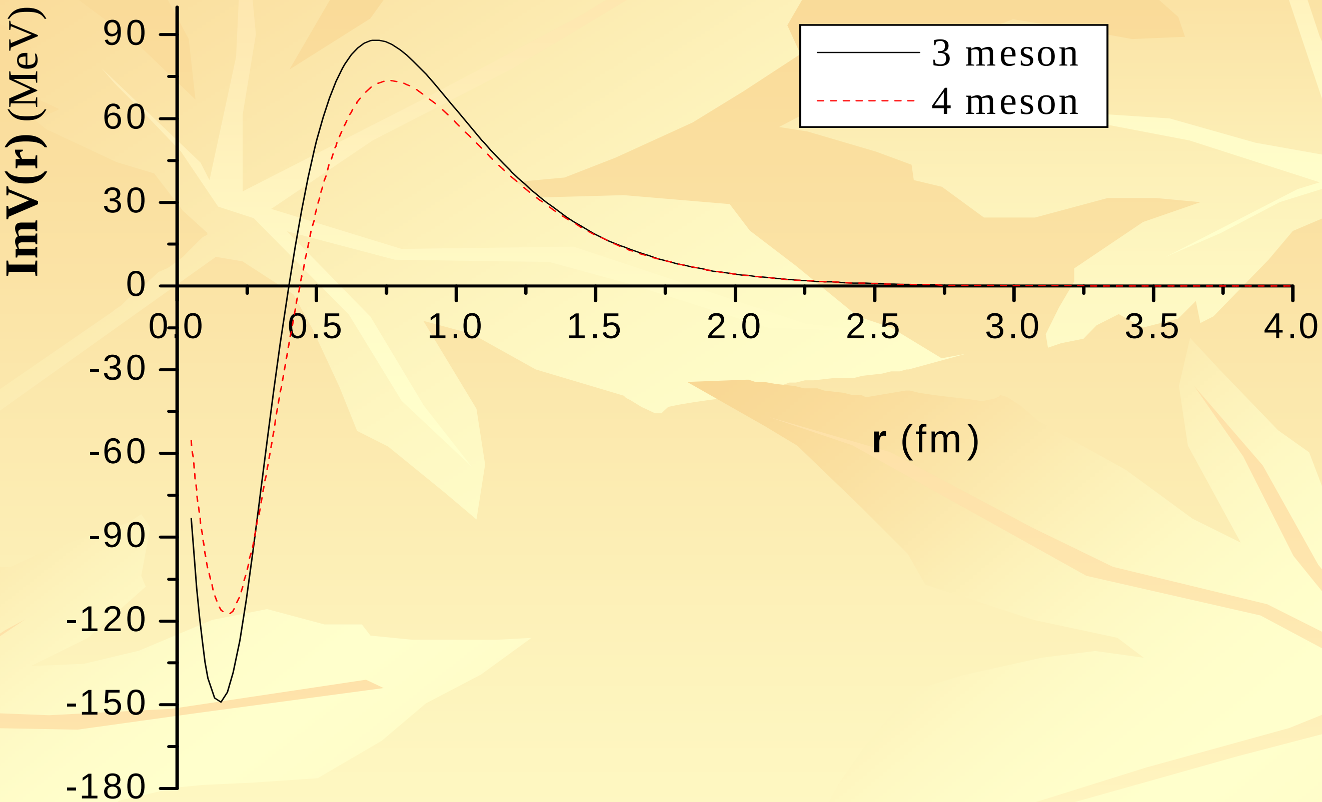
RESULTS AND DISCUSSION

The Elastic Interaction Potential

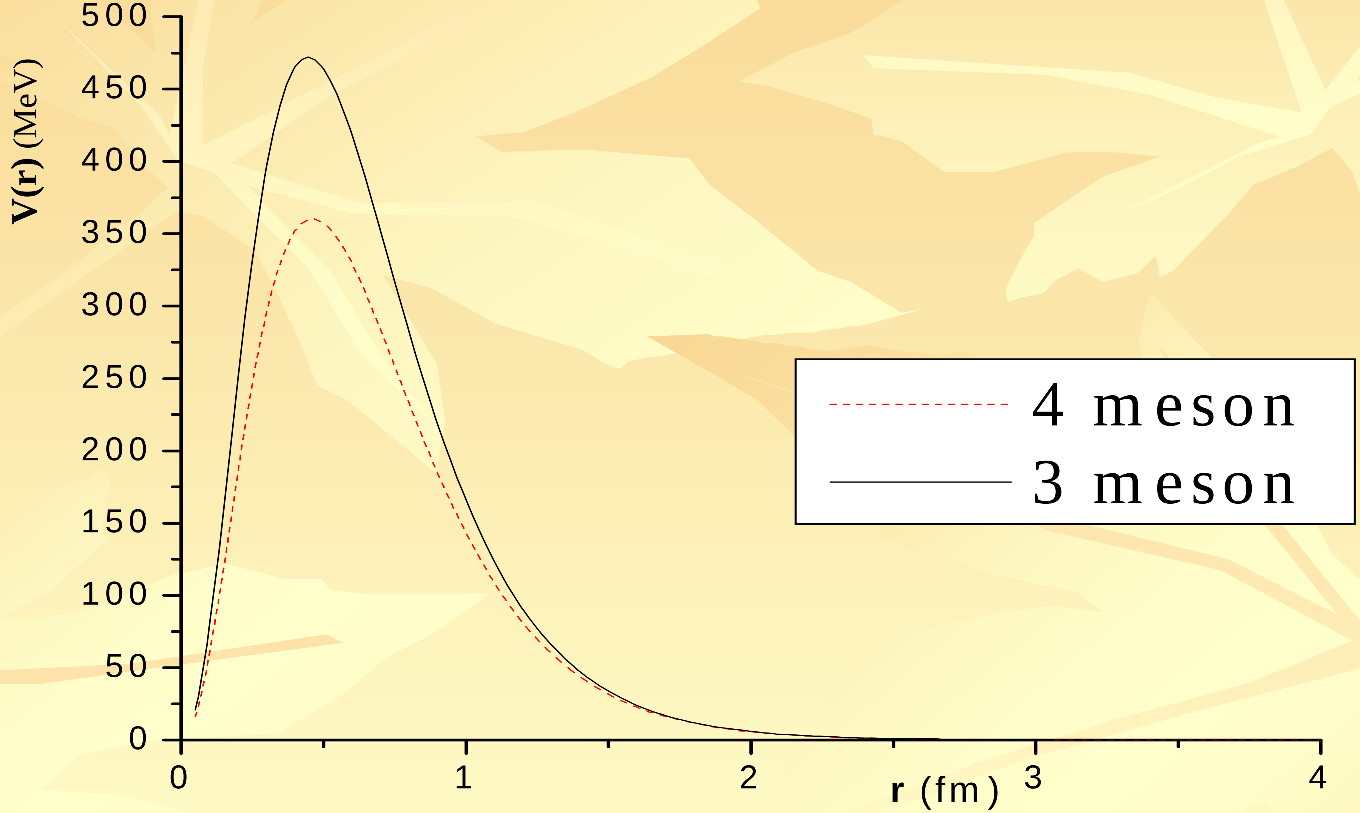
We have calculated, the real and imaginary potential forms for the $(K^{+} - {}^{40}Ca)$ interaction



<u>Fig. (I-A)</u> :The real potential using (GY) meson exchange function, at Kaon momentum (656 MeV/c) and mesons Parameters [16]



<u>Fig. (I-B)</u>: Imaginary potential. using (GY) meson exchange function, at Kaon momentum (656 MeV/c) and mesons Parameters [16]



<u>Fig. (II-A)</u> :The real potential using (GY) meson exchange function, at Kaon momentum (656 MeV/c) and mesons Parameters [17]

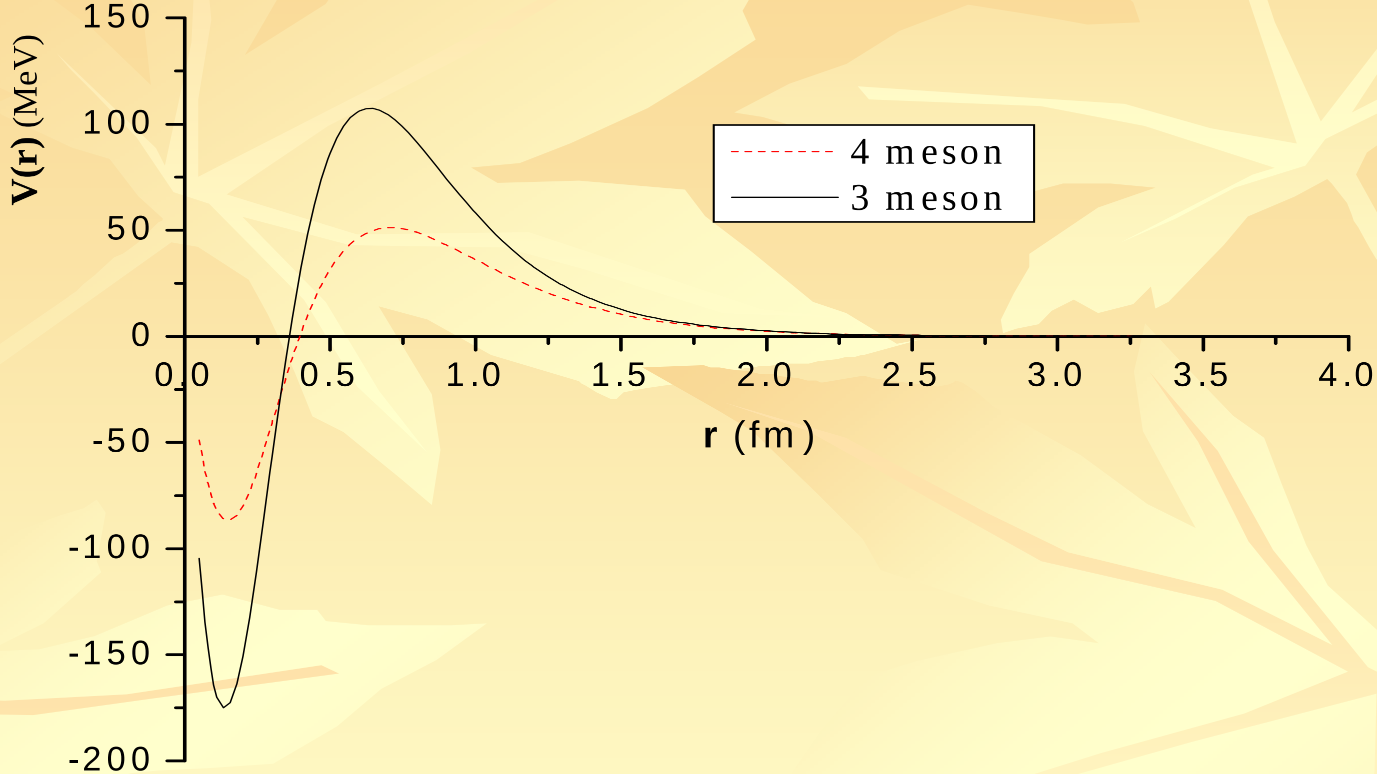


Fig. (I-B): Imaginary potential. using (GY) meson exchange function, at Kaon momentum (656 MeV/c) and mesons Parameters [17]

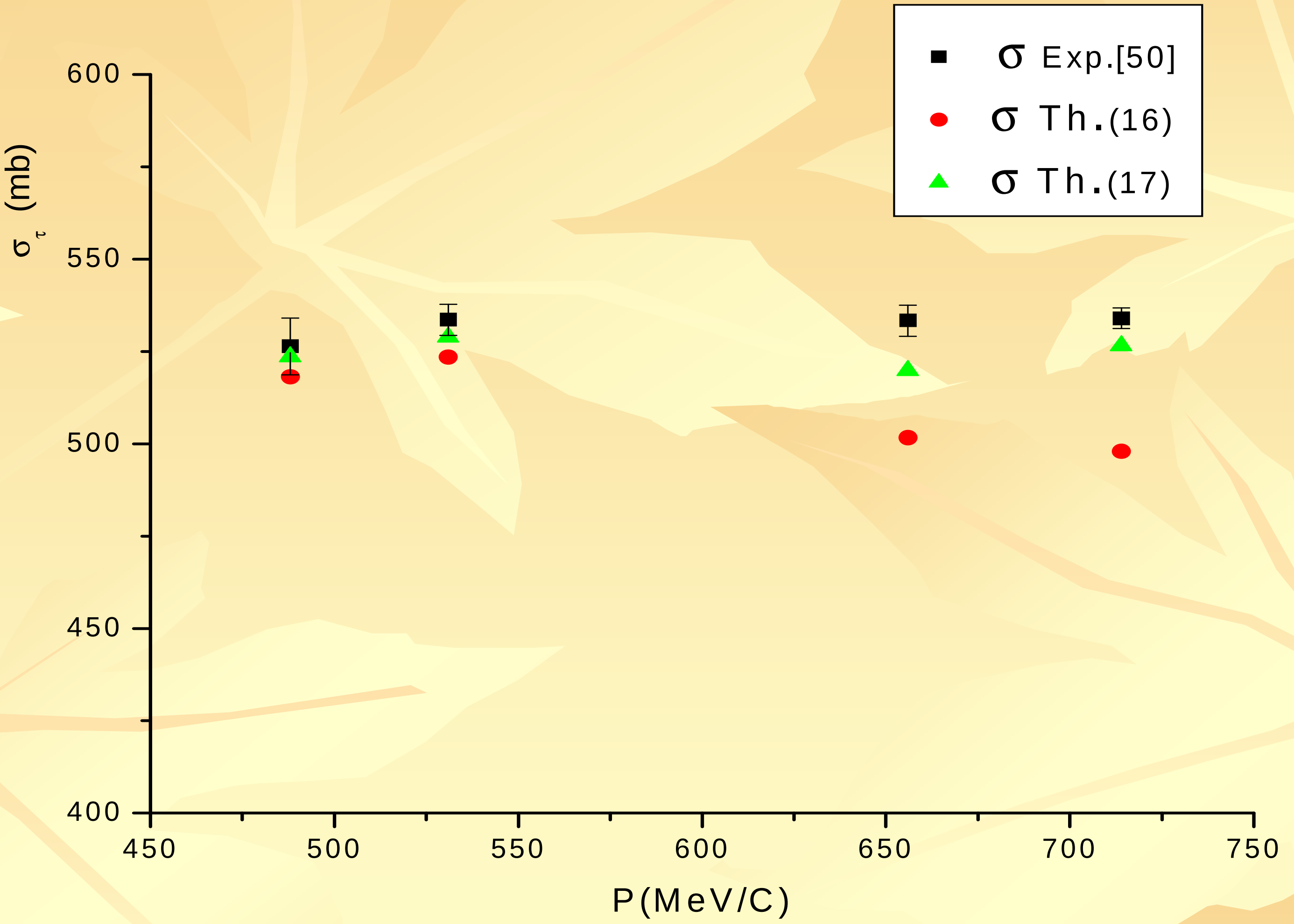


Fig.(B): **The total elastic cross section has been plotted versus the momentum energy reaction ,** P_L**=(488~715) MeV/c.**

Conclusion

we summarize our conclusions as follows

1) The derived $(K^+ - N)$ potential there are two features required by the experimental data i.e. the repulsive and short range characters.

2) The above two features help us to apply the impulse approximation in construction of $V_{(k^+,N)}(r)$ to use in $V_{(k^+,A)}(r)$

3) For interaction of K^+ with nucleus ^{40}Ca

We have some important properties which are associated with the nuclear medium effects and charge dependence.

4) The spin- orbit force were incorporated in our calculations without any further adjustable constants.

5) Good agreement with the experimental data are obtained for the total elastic cross sections.

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Thank you