Homogeneous Balitsky-Kovchegov hierarchy and Reggeon Field Theory

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Dipole picture

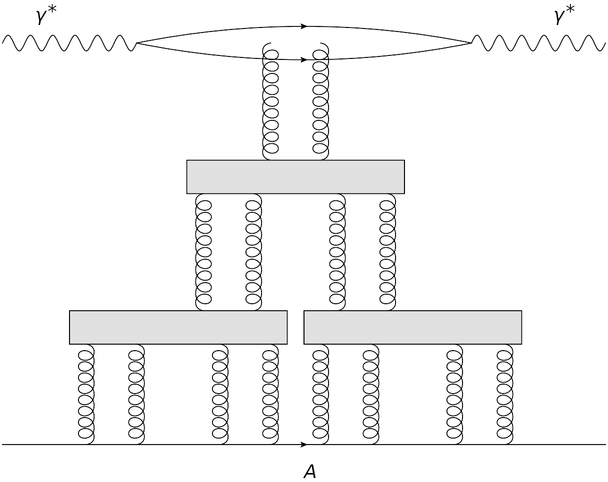

Figure: Dipole scattering amplitude $\mathcal{N}(x_1, x_2, y)$

Balitsky-Kovchegov equation

Balitsky-Kovchegov [Bal96, Bal99, Kov99] equation describes dipole scattering amplitude $\mathcal{N}(x_1, x_2, y)$:

$$\partial_y \mathcal{N}(x_1, x_2, y) = \overline{\alpha}_s \int d^2 x_3 \frac{x_{12}^2}{x_{13}^2 x_{32}^2} \left(\mathcal{N}(x_1, x_3, y) + \mathcal{N}(x_3, x_2, y) - \mathcal{N}(x_1, x_2, y) + \mathcal{N}_2(x_1, x_3, x_3, x_2, y)\right) \quad (1)$$

Contains 2-dipole correlator $\mathcal{N}_2(x_1, x_3, x_3, x_2, y)$. Similar equation for 2-dipole correlator contains 3-dipole correlator etc. Results in infinite hierarchy of equation. To solve

$$\mathcal{N}_2(x_1, x_3, x_3, x_2, y) = \mathcal{N}(x_1, x_3, y)\mathcal{N}(x_3, x_2, y) \quad (2)$$

Balitsky-Kovchegov equation

For spatially homogeneous target (big nucleus) one could remove impact parameter dependence (hence homogeneous) [Kov00] and to get

$$\partial_y \mathcal{N}(L, y) = \overline{\alpha}_s \chi(-\partial_L) \mathcal{N}(L, y) - \overline{\alpha}_s \mathcal{N}^2(L, y) \tag{3}$$

with

$$\mathcal{N}(L = \ln(k^2/k_0^2), y) = \int \frac{d^2 x_{12}}{2\pi x_{12}^2} e^{-ikx_{12}} \mathcal{N}(x_1, x_2, y) \tag{4}$$

and $\chi(-\partial_L)$ is defined by the characteristic BFKL function [KLF77, BL78]:

$$\chi(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1-\gamma), \tag{5}$$

Stochastic Balitsky-Kovchegov equation

Balitsky-Kovchegov hierarchy can be obtained from particular stochastic differential equation.

$$\partial_t u(L,t) = \chi(-\partial_L)u(L,t) - u^2(L,t) + \sqrt{2P(u(L,t))}\eta(L,t) \tag{6}$$

where $t = \overline{\alpha}_s Y$, $u(L,t)$ is stochastic variables which after averaging give correlators $\mathcal{N}_i$ and $\eta(L,t)$ is Gaussian noise with properties:

$$\langle \eta(L,t) \rangle = 0 \tag{7}$$

$$\langle \eta(L_1,t_1)\eta(L_2,t_2) \rangle = \delta(L_1 - L_2)\delta(t_1 - t_2) \tag{8}$$

Particular form of $P(u(L,t))$ on how dipole merging is accounted. Two forms used:

$$P(u(L,t)) = 0 \tag{9}$$

$$P(u(L,t)) = u(L,t) \tag{10}$$

corresponding to absence and presence of dipole merging.

Stochastic Balitsky-Kovchegov equation

Mostly interested in generational functional $Z[j]$:

$$Z[j] = \int e^{\int judtdL} \delta\left(\partial_t u - \chi(-\partial_L)u + u^2 - \sqrt{2P(u)}\eta\right) DuD\eta \qquad (11)$$

which can be rewritten with Martin-Siggia-Rose [MSR73] auxiliary fields $u^\dagger(L,t)$:

$$Z[j] = \int e^{\int judtdL} e^{\int u^\dagger\left(\partial_t u - \chi(-\partial_L)u + u^2 - \sqrt{2P(u)}\eta\right)dtdL} DuDiu^\dagger D\eta \qquad (12)$$

integrate over noise

$$Z[j] = \int e^{\int (u^\dagger(\partial_t u - \chi(-\partial_L)u) + u^\dagger u^2 + u^{\dagger 2}P(u) + ju)dtdL} DuDiu^\dagger \qquad (13)$$

with initial condition $j^\dagger(L,t) = u_0(L)\delta(t)$:

$$Z[j,j^\dagger] = \int e^{\int (u^\dagger(\partial_t u - \chi(-\partial_L)u) + u^\dagger u^2 + u^{\dagger 2}P(u) + ju + j^\dagger u^\dagger)dtdL} DuDiu^\dagger \qquad (14)$$

Does in corresponds to BFKL?

Proposed in [Bra05]

$$Z[J,J^\dagger] = \int D\Phi D\Phi^\dagger e^{S_0+S_I+S_E} \quad (15)$$

terms S_0, S_I, S_E correspond to propagation, merging/splitting of BFKL pomerons, interaction with the target/projectile

$$S_0 = \int dy d^2x_1 d^2x_2 \Phi^\dagger(x_1,x_2,y)\nabla_1^2\nabla_2^2\left[\partial_y + H\right]\Phi(x_1,x_2,y) \quad (16)$$

$$S_I = \frac{2\pi\overline{\alpha}_s^2}{N_c}\int dy \frac{d^2x_1 d^2x_2 d^2x_3}{x_{12}^2 x_{23}^2 x_{31}^2}\Big[(L_{12}\Phi^\dagger(x_1,x_2,y))\Phi(x_2,x_3,y)\Phi(x_3,x_1,y)+$$
$$(L_{12}\Phi(x_1,x_2,y))\Phi^\dagger(x_2,x_3,y)\Phi^\dagger(x_3,x_1,y)\Big] \quad (17)$$

$$S_E = -\int dy d^2x_1 d^2x_2 \Phi\left(x_1,x_2,y\right)\tau_A\left(x_1,x_2,y\right) + \Phi^\dagger\left(x_1,x_2,y\right)\tau_B\left(x_1,x_2,y\right) \quad (18)$$

$$L_{12} = x_{12}^4\nabla_1^2\nabla_2^2 \quad (19)$$

BFKL Field Theory is too complicated. Not evident it can be reduced. Two attempts of dimensional reduction was made [Bon07] and even more complicated [CLM12]. [Bon07] is fine but one interaction term have different structure:

$$S_\phi = \frac{2\alpha_s^2 N_c}{\pi} \int dy d^2k (\nabla_k^2 k^4 \nabla_k^2 \phi(k,y)) \phi^\dagger(k,y) \phi^\dagger(k,y) \tag{20}$$

BFKL can not be reduced to constructed RFT. Why? Because structure of Pomeron fusion vertex.

$$\frac{1}{x_{12}^2 x_{23}^2 x_{31}^2} L_{12} \Phi(x_1, x_2, y)) \Phi^\dagger(x_2, x_3, y) \Phi^\dagger(x_3, x_1, y) \tag{21}$$

Choose different Pomeron fusion vertex. Above one was proposed in [Bra05] to provide symmetry between target and projectile in theory

$$\frac{1}{x_{12}^2 x_{23}^2 x_{31}^2} \Phi(x_1, x_2, y)) P_{23} \Phi^\dagger(x_2, x_3, y) P_{31} \Phi^\dagger(x_3, x_1, y) \tag{22}$$

$$P_{23} = \frac{1}{x_{23}} L_{23}^{-1} x_{23}^2$$

Also gives symmetric theory by $\Phi^\dagger$ redefinition. Same high k^2 behaviour.

Classical solutions

We have the following classical field equation

$$\begin{aligned} \partial_t u &= \chi(-\partial_L)u - u^2 - 2u^\dagger u \\ -\partial_t u^\dagger &= \chi(\partial_L)u^\dagger - u^{\dagger 2} - 2u^\dagger u \end{aligned} \quad (23)$$

Without cross-term $uu^\dagger$

$$\begin{aligned} \partial_t u &= \chi(-\partial_L)u - u^2 \\ -\partial_t u^\dagger &= \chi(\partial_L)u^\dagger - u^{\dagger 2} \end{aligned} \quad (24)$$

and using diffusive approximation with substitution which represents traveling of wave

$$\chi_0 = \gamma_0 \chi_0' \, , \, t = \frac{2\tau}{\gamma_0^2 \chi_0''} \, , \, L = \sqrt{\frac{\chi_0''}{\gamma_0^2 \chi_0''}} x - 2 \frac{\gamma_0 \chi_0'' - \chi_0'}{\gamma_0^2 \chi_0''} \tau \quad (25)$$

Indeed gives two non linear diffusion equations (diffusive scaling)

$$\begin{aligned} \partial_\tau \phi &= \partial_x^2 \phi + \phi - \phi^2 \\ -\partial_\tau \phi^\dagger &= \partial_x^2 \phi^\dagger + \phi^\dagger - \phi^{\dagger 2} \end{aligned} \quad (26)$$

Classical solutions

If we leave cross-term $uu^\dagger$ then in the same diffusive approximation

$$\begin{aligned} \partial_\tau \phi &= \partial_x^2 \phi + \phi - \phi^2 - 2\phi\phi^\dagger \\ -\partial_\tau \phi^\dagger &= \partial_x^2 \phi^\dagger + \phi^\dagger - \phi^{\dagger 2} - 2\phi^\dagger \phi \end{aligned} \tag{27}$$

Traveling waves? Solutions exist but unphysical (raises with momentum). Example [AMCP76]

$$\begin{aligned} \{\phi, \phi^\dagger\} &= \frac{1}{2}\left(1 \pm tanh(z) - \frac{1}{2cosh^2(z)}\right) \\ z &= \frac{x}{2\sqrt{2}} - \frac{\tau}{4} \end{aligned} \tag{28}$$

$$\begin{aligned} \{\phi, \phi^\dagger\} &= \frac{1}{2}\left(1 \pm tanh(z) - \frac{3}{2cosh^2(z)}\right) \\ z &= \frac{3x}{2\sqrt{2}} - \frac{\tau}{4} \end{aligned} \tag{29}$$

Traveling waves only if $\phi = 0$ or $\phi^\dagger = 0$. Scaling destroyed.

Can define gluon production cross-section (up to normalization)

$$\frac{d\sigma}{d^2q\ dy} = \frac{1}{q^2}\int d^2k u(t_1, L_q)u^\dagger(t_2, L_{q-k}) \tag{30}$$

$$t_1 = ln(q) - \frac{Y}{2} - y\,,\; t_2 = ln(q) - \frac{Y}{2} + y \tag{31}$$

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