CONDENSATE CONFORMAL SYMMETRY BREAKING

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OUTLINE

FROM QUARKS TO UNIVERSE

1. CONCEPTS:
   - conformal action,
   - frame of reference, and
   - normal ordering (Casimir energy & condensate)
2. LOW-ENERGY QCD
3. MINIMAL STANDARD MODEL
4. COSMOLOGY and GENERAL RELATIVITY

Symmetry of S-Matrix elements

The S-matrix elements

as the evolution operator expectation values between in- and out- states

$$\underbrace{\mathcal{M}_{\text{in,out}}}_{P-inv,G-inv} = \underbrace{\langle\text{out}|}_{P-variant} \underbrace{\hat{S}[\hat{\ell}]}_{P-variant,G-inv} \overbrace{\underbrace{|\text{in}\rangle}_{P-variant}}^{\prod_P |\Phi_P\rangle},$$

are gauge-invariant ("G-inv"), and Poincaré-invariant ("P-inv")
See [A. Cherny at el. arXiv:1112.5856 [hep-th]]

The evolution operator $\hat{S}[\hat{\ell}]$ and $\langle\text{out}|$, $\hat{\ell}_\mu|\text{in}\rangle = \ell_\mu|\text{in}\rangle$states

are gauge-invariant and Poincaré-variant; $\ell \cdot \ell = 1$ is a frame.

The gauge-invariant S-matrix elements in arbitrary frame of reference?

It was Heisenberg and Pauli's question W. Heisenberg and W. Pauli, Z. Phys. 56 (1929) 1; Z. Phys. 59(1930) 166 to von Neumann in 1930: "How to generalize the Dirac Hamiltonian approach to QED of 1927 P.A.M. Dirac, Proc. Roy. Soc. A 114, (1927) 243; Can. J. Phys. 33 (1955) 650 to any frame?" von Neumann reply was to go back to the initial Lorentz-invariant formulation and to choose the comoving frame

$$\ell^0_\mu = (1,0,0,0) \to \ell^{\text{comoving}}_\mu = \ell_\mu, \qquad \ell_\mu \ell^\mu = \ell \cdot \ell = 1$$

and to repeat the gauge-invariant Dirac scheme in this frame. The comoving frame for a bound state is $\ell_\mu \sim \mathcal{P}_\mu$
M.A.Markov, J. Phys. 3 (1940) 453, and H.Yukawa, Phys. Rev. 77 (1950) 219

NORMAL ORDERING

Oscillator

$$:\sum_n \frac{p_n^2 + \omega_n^2 q_n^2}{2} \eqqcolon \sum_n \omega_n \frac{a_n^+ a_n^- + a_n^- a_n^+}{2} \coloneqq \sum_n \omega_n \Big(a_n^+ a_n^- + \frac{1}{2} \Big)$$

Non-Abelian Interactions

The normal ordering of gluons leads to their condensate

$$g^2 f^{ba_1 d} f^{da_2 c} \langle A_i^{a_1 *} A_j^{a_2 *} \rangle = 2g^2 [N_c^2 - 1] \delta^{bc} \delta_{ij} C_{\text{gluon}} = M_g^2 \delta^{bc} \delta_{ij}$$

$$\nabla^{db}(A) A_0^b \nabla^{dc}(A) A_0^c =: \nabla^{db}(A) A_0^b \nabla^{dc}(A) A_0^c : + M_g^2 A_0^d A_0^d$$

Coulomb(QED) $\to$ Yukawa(QCD)

$$V(\mathbf{k}) = \frac{4}{3} g^2 \frac{1}{\mathbf{k}^2 + M_g^2}$$

Low-energy QCD

Hadronization:

Ladder diagram sum of Coulomb Interaction in QED $\Rightarrow$
Schrödinger Equation [Bethe, Salpeter Phys. Rev. 84 (1951) 1232.]
Ladder diagram sum of Yukawa Interaction in QCD $\Rightarrow$
Salpeter Equation and Schwinger-Dyson Equation
Yu.Kalinovsky, L.Kaschluhn, V.Pervushin,Phys.Lett. B 231 (1989) pp. 288–292.; [arXiv:1112.5856.]

In Dirac approach, confinement means

complete destructive interference of Non-abelian Phase factors:

$$e^{ipx} \to \lim_{L\to\infty} \sum_{n=-L}^{L} v^{(n)}(x) \underbrace{e^{ipx}}_{parton} = 0, \text{ if } x \neq 0$$

and leads to Quark-Hadron Duality for $\hat{S} = 1 + i\hat{T}$
$\sum_h < *|\hat{T}|h >< h|\hat{T}|* >= 2Im < *|\hat{T}_{\rm Pert.Th.}|* >$
V.Pervushin, Riv. del Nuovo Cimento 8 (1985) N 10, pp. 1–48; [arXiv:1112.5856.]

Quark spectrum

Schwinger-Dyson equation

$$\Sigma(k) = m^0 + \frac{i}{2}\int \frac{dq_0 d^3 q}{(2\pi)^4} V_{\rm Yu}(k^\perp - q^\perp) \not{\ell} G_\Sigma(q) \not{\ell}$$

where $G_\Sigma(q) = (\not{q} - \Sigma(q))^{-1}$, $k_\mu^\perp = k_\mu - \ell_\mu (k\cdot\ell)$ and $\ell^2 = 1$.
In the reference frame $\ell^0 = (1,0,0,0)$, $q^\perp = (0, \mathbf{q})$, and we can put

$$\Sigma_{\rm a}(q) = E_{\rm a}(\mathbf{q}) \cos 2\upsilon_{\rm a}(\mathbf{q}) \equiv M_{\rm a}(\mathbf{q}).$$

Solution: A. Cherny, et al. arXiv: 1112.5856 [hep-th]

$|\mathbf{q}| \ll M_g \quad \cos 2\upsilon_{\rm a} \simeq 1; \qquad \alpha_s = g^2/(4\pi)$

$|\mathbf{q}| \gg M_g \quad \cos 2\upsilon_{\rm a} \simeq (M_g/|\mathbf{q}|)^{1+\beta}$, where $\alpha_s \frac{\cot(\beta\pi/2)}{1-\beta} = \frac{3}{2}$.

$$\cos 2\upsilon_{\rm a}(p) \simeq \theta(X_{\rm input}\cdot M_g - p) \sim step - \ function$$

Bethe-Salpeter equation: meson spectrum

BS Equation

In the limit of zero current masses $m_u \simeq m_d$ and pion mass M_π
S.-D. Eq.$\sim$ S. Eq.:

$$\frac{1}{F_\pi} \otimes \Big| m_d = M_d(p) - \frac{1}{2}\int \frac{d^3q}{(2\pi)^3} V(\mathrm{p}-\mathrm{q}) \cos 2\upsilon_u(q),$$

$$\frac{M_\pi L^\pi_{(2)}(\mathrm{p})}{2} = \sqrt{p^2 + M_d^2(p)}\ \ L^\pi_{(1)}(\mathrm{p}) - \frac{1}{2}\int \frac{d^3q}{(2\pi)^3} V(\mathrm{p}-\mathrm{q}) L^\pi_1(\mathrm{q}),$$

$$L^\pi_{(1)}(p) = \frac{M_d(p)}{F_\pi\sqrt{p^2 + M_d^2(p)}} = \frac{\cos 2\upsilon_d(p)}{F_\pi}$$

$$L^\pi_{(2)}(\mathrm{p}) = \frac{2m_d}{F\pi \cdot M_\pi}$$

are wave functions, $M_d(p)$ is the constituent quark mass,
$F_\pi = 93$ GeVis the weak-decay coupling constant.

Definition of Quark Condensate

Normaliz. condition $4N_c \int \frac{d^3q}{(2\pi)^3} L_1^\pi(q) L_2^\pi(q) = M_\pi$ yields

Gell-Mann-Oakes-Renner relation: Phys. Rev. 175 (1968) 2195

$$M_\pi^2 F_\pi^2 = 2m_d < d\bar{d} > .$$

where

$$< d\bar{d} >= \sum_{n=1}^{N_c} \langle q_n(t, \mathbf{x}) \overline{q}_n(t, \mathbf{x}) \rangle = 4N_c \int \frac{d^3q}{(2\pi)^3} \frac{1}{2} \cos 2\upsilon_u(q),$$

is the light-quark condensate.

Input parameter

$m_u \simeq m_d \simeq 4$ MeV , $M_u \simeq M_d \simeq 330$ MeV ,$M_s = 400 MeV$, $M_\pi \simeq 135$ MeV. $M_K = 494 MeV$ and $M_\rho = 770 MeV$.

Numerical results

$M_\sigma = 660 MeV$, $M_{a_1} = (1132 - i654) MeV$, $M_{K^\star} = 896 MeV$, $M_\Phi = 970 MeV$.

[Yu.Kalinovsky, L.Kaschluhn, V.Pervushin, A new QCD inspired version of the Nambu-Jona-Lasinio model, Phys.Lett. B 231 (1989) pp. 288–292] used θ-function approximation.

GMOR: $$\frac{< d\bar{d} >}{M_d^3} = \frac{M_\pi^2 F_\pi^2}{2 m_d M_d^3} \simeq 0.41 \pm 0.08$$

CONCEPTS: CONFORMAL SYMMETRY AND NORMAL ORDERING

THEORIES

1. LOW-ENERGY QCD: GMOR - relation!
2. CONFORMAL invariant STANDARD MODEL (CSM)?
3. CONFORMAL version of GENERAL RELATIVITY (CGR)?

INPUT: the largest mass t-quark condensate and GMOR-rel.

$$\boxed{\mathcal{H}_{\rm int}^{\rm CSM} = \lambda\varphi^4/4 + \varphi g_t \bar{t}t} \to V_{\rm eff}(v) = \lambda v^4/4 - v g_t < t\bar{t} >$$

$$\bar{t}t =: \bar{t}t : - < t\bar{t} >, \text{ where } \frac{< t\bar{t} >}{M_t^3} = \frac{< d\bar{d} >}{M_d^3} \simeq 0.41 \pm 0.08,$$

$M_t = g_t v_c = 174$ GeV, $g_t = 1/\sqrt{2}$, and

$v = 246$ GeV is the constant part of the Higgs field $\varphi = v + \eta$

OUTPUT: λ and e-w boson masses

$$V'_{\rm eff}(v) = 0 \to \lambda v^3 = g_t < t\bar{t} > \;\; \to \;\; \lambda = \frac{< t\bar{t} >}{4M_t^3} = \frac{0.41 \pm 0.08}{4},$$

$$m_{\rm higgs}^{tree} = \sqrt{3 g_t < t\bar{t} > /v} = 131 \pm 8 \text{ GeV}$$

Other condensate contributions $< \eta\eta >$, $< W^+W^- >$, $< ZZ >$:

$$m_{\rm higgs} \simeq m_{\rm higgs}^{\rm tree}\left[1 + \frac{\Delta m_{\rm higgs}^2}{m_{\rm higgs}^2}\right]^{1/2} = m_{\rm higgs}^{\rm tree}[1 + 0.014]$$

CONCEPTS: CONFORMAL SYMMETRY AND NORMAL ORDERING

THEORIES

1. LOW-ENERGY QCD!
2. CONFORMAL invariant STANDARD MODEL (CSM)!
3. CONFORMAL version of GENERAL RELATIVITY (CGR)?

$W_{\rm H-E} = -(1/6)\int d^4x\sqrt{-g}R^{(4)}; \qquad ds^2 = g_{\mu\nu}dx^\mu dx^\nu$

NATURAL UNITS: $M_{\rm Pl}\sqrt{3/(8\pi)} = c = \hbar = 1.$

CONFORMAL-INVARIANT ACTION

$$W_{\rm CGR} = -\int d^4x\left[\frac{\sqrt{-\widetilde{g}}}{6}R^{(4)}(\widetilde{g})e^{-2D} - e^{-D}\partial_\mu\left(\sqrt{-\widetilde{g}}\widetilde{g}^{\mu\nu}\partial_\nu e^{-D}\right)\right],$$

$$\widetilde{ds}^2 = \widetilde{g}_{\mu\nu}dx^\mu dx^\nu = \widetilde{\omega}^{\rm Fock}_{(\alpha)} \otimes \widetilde{\omega}^{\rm Fock}_{(\beta)}\eta^{(\alpha)(\beta)}$$

Deser S., Scale invariance and gravitational coupling Annals Phys. 59 (1970) 248; Dirac P.A.M, Long range forces and broken symmetries Proc. Roy. Soc. Lond. A. 333 (1973) 403.

FRAME of REFERENCE

$$\widetilde{\omega}^{\rm Fock}_{(0)} = e^{-2D}Ndx^0, \qquad \widetilde{\omega}^{\rm Fock}_{(b)} = \mathbf{e}_{(b)i}dx^i + N_{(b)}dx^0,$$

$D(x^0,x^1,x^2,x^3) = \langle D\rangle(x^0) + \overline{D}(x^0,x^1,x^2,x^3)$ is DILATON

$W_{\mathrm{CGR}} = W_{\mathrm{Universe}} + W_{\mathrm{graviton}} + W_{\mathrm{potential}}$

UNIVERSE

$$W_{\mathrm{Universe}}[\langle D\rangle, N_0] = -V_0 \int\limits_{\tau_I}^{\tau_0} \underbrace{dx^0 N_0}_{=d\tau} \left[\left(\frac{d\langle D\rangle}{N_0 dx^0}\right)^2 + \rho_{\mathrm{Cas}}(\langle D\rangle)\right],$$

GRAVITON

$$W_{\mathrm{graviton}}^{\mathrm{Fock}} = \int d^4x \frac{N}{6}\left[v_{(a)(b)}v_{(a)(b)} - e^{-4D}R^{(3)}(\mathbf{e})\right],$$

Newtonian potentials

$$W_{\mathrm{potential}} = \int d^4xN\left[-v_{\overline{D}}^2 - \underbrace{\frac{4}{3}e^{-7D/2}\triangle^{(3)}e^{-D/2}}_{\text{Newtonian potentials}}\right],$$

$[v_{\overline{D}} = 0]$ IS COMOVING FRAME of reference

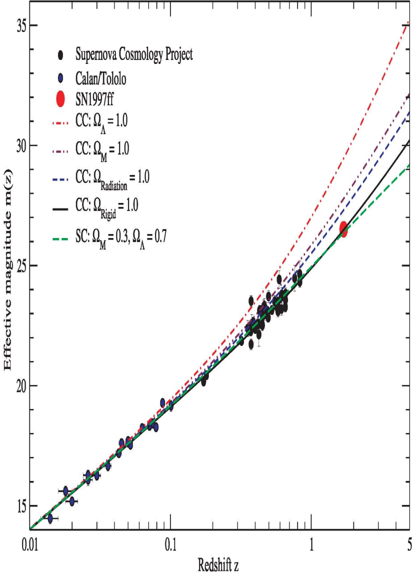
CONFORMAL SCENARIO: EMPTY UNIVERSE $W_{\text{graviton}} = W_{\text{potential}} = 0$

Equation of VOID SPACE:

$$d_{\text{Cas}}(a) = d_{\text{horison}}(a) = \int\limits_{a_I \to 0}^{a} d\overline{a}\ \ \rho_{\text{Cas}}^{-1/2}.$$

NORMAL ORDERING: $\rho_{\text{Cas}} = \sum_f \mathrm{H}_{\text{Cas}}^{(f)}/V_0 = \dfrac{H_0}{d_{\text{Cas}}(a)}.$

SOLUTION $d_{\text{hor}}(a) = \dfrac{a^2}{H_0}\ .$



DILATON long space interval

explains long Supernovae Distances $\uparrow$ R_{SNeIa} at z $\rightarrow$ via dominant Casimir energy

$$r_{\text{horizon}}(z) = H_0^{-1}(1+z)^{-2}$$

[SEE BLACK LINE];

ΛCDM model with

short space interval $R = ra$

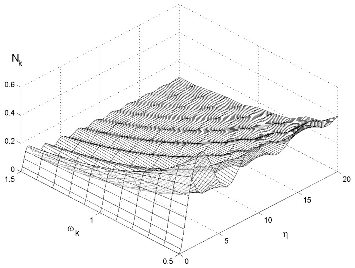
requires Inflation to explain long Supernovae Distances

$R_{\text{SNeIa}} = R_{\Omega_\Lambda=0.7,\Omega_M=0.3}$

[SEE GREEN LINE].

D. Behnke, D. Blaschke, V. Pervushin, and D. Proskurin, Description of Supernova Data in Conformal Cosmology without Cosmological Constant, Phys. Lett. B 530 (2002) 20;

A. Zakharov, V. Pervushin, Int. J. Mod. Phys. D19 (2010) No.9



$\boxed{\nearrow}$ is time-axis $\boxed{\uparrow}$ is number of bosons $N_{W,Z,h}$, $\boxed{\searrow}$ is their momentum, shows us creation of $N_h = 10^{90}$ Higgs particles at $1 + z_{\rm Pl} \sim 10^{15}$ during the first 10^{-12}sec.

$$\boxed{N_h(a) \simeq a_{\rm Pl}^{-4}\Omega_{\rm Cas}(a)} \to N_\gamma \simeq \alpha_W^2 10^{90} = 10^{87} \to T_{\rm CMB} \sim 3\text{K}.$$

A. Arbuzov, et al., Phys.Lett.B v. 691, p. 230 (2010); V.N. Pervushin, A.B. Arbuzov, B.M. Barbashov, R.G. Nazmitdinov, A. Borowiec, K.N. Pichugin, and A.F. Zakharov, Gen. Relativ. Gravit. (2012) DOI 10.1007/s10714-012-1423-7.

Planck Least Action Postulate

$$W_{\rm Universe} = \rho_{\rm cr} V^{(4)}_{\rm hor}(a_{\rm Pl}) = \frac{M^2_{\rm Pl}}{H^2_0} \frac{(1+z_{\rm Pl})^{-8}}{32} = 2\pi$$

$a^{-1}_{\rm Pl.} = (1+z_{\rm Pl.}) \simeq 0.62 \cdot 10^{15}$ E-W EPOCH!

Conformal Weights of Poincaré representations with respect to dilaton ENERGIES: $d\tau = d\eta/a^2 = dt/a^3$

$$\omega_\tau = a^2\sqrt{\mathbf{k}^2 + a^2 M^2_0} \quad \to \quad < \omega_{\rm n}(a) >= \frac{a^n}{a^n_{\rm Pl}} H_0,$$

give scales $< \omega_{\rm n}(a)_{\rm Pl} >= H_0 a^{-n}_{\rm Pl}$ for conformal weights n=0,1,2,3,4 in GeV:

n=0	n=1	n=2	n=3	n=4
$H_0 \sim 10^{-42}$	$R^{-1}_{\rm Cel.S.} \sim 10^{-27}$	$T_{\rm CMB} \sim 10^{-12}$	$M_{\rm EW} \sim 10^3$	$M_{\rm Pl} \sim 10^{19}$

Common origin of conformal symmetry breaking in both GR & SM

RESULTS: CONFORMAL SCENARIO

- EMPTY UNIVERSE $\to$ of SNe Ia DATA in CC
- Planck Least Action Postulate $\to$ Scale Hierarchy $H_0 a_{\rm Pl}^{-n}$

$$a_{\rm Pl}^{-1} = (1 + z_{\rm Pl}) \simeq \left[\frac{M_{\rm Pl}}{H_0}\right]^{1/4} \simeq 10^{15}$$

- Creation of Higgs particles $N_\phi \sim (1 + z_I)^6 \sim 10^{90}$
- Thermalization at $\frac{(1 + z_{\rm Term})}{(1 + z_{\rm Pl})} \sim \alpha_W^2 \qquad N_\phi \to N_\gamma = 10^{87}$
- Chemical evolution in CC: $a(\eta) \simeq \sqrt{\text{measurable time}}$

CONCEPTS: CONFORMAL SYMMETRY AND NORMAL ORDERING

THEORIES

1. LOW-ENERGY QCD GMOR relation
2. CONFORMAL SM Higgs particle mass
3. CONFORMAL GR SNeIa DATA

Thank you for your attention!