The nuclear first order phase transition at intermediate energies

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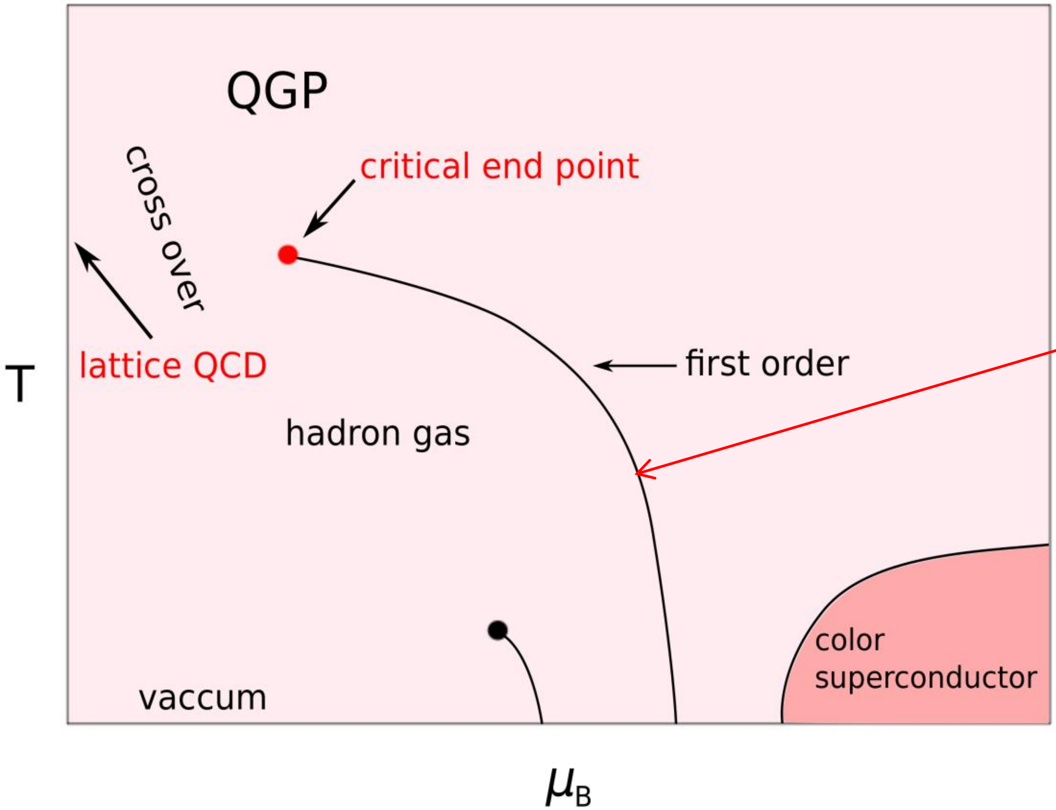
QCD phase diagram (page 1/2)
(T-μ plane)

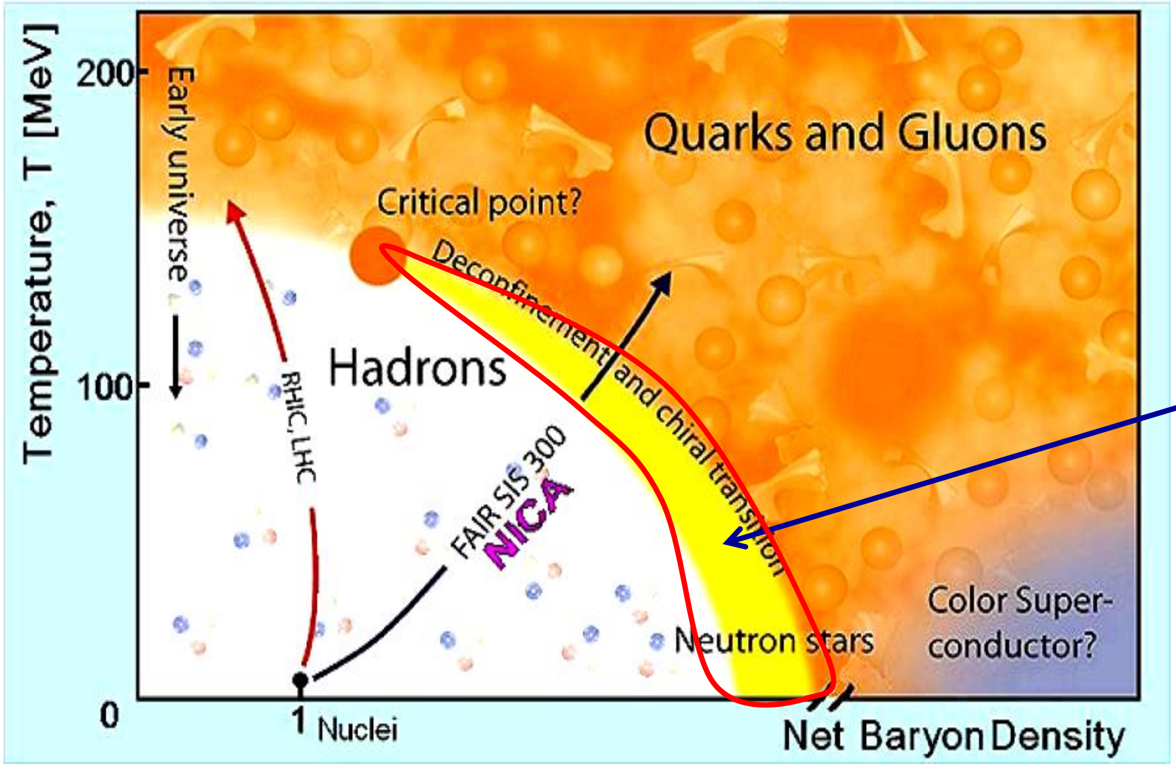


from [Li, arXiv:1002.4459]

- 1-st order hadron-QGP phase transition
- *a line in the phase diagram (T-μ)*
- *the liquid –gas phase transition*

QCD phase diagram (page 2/2)

(T-ρ plane)



- 1-st order hadron-QGP phase transition
- *an aria on the phase diagram (T-ρ)*
- *the liquid –gas phase transition*

First order phase transition (Ehrenfest definition)

(Thermodynamics and statistical mechanics)

Equilibrium statistical ensemble $(x_1, \ldots, x_m, x_{m+1}, \ldots, x_n)$

$x_1, \ldots, x_m$ -extensive, $x_{m+1}, \ldots, x_n$ -intensive

- Thermodynamic potential and its partial derivatives:

$$f(x) = f(x_1, \ldots, x_n),$$

$$\Delta f = df + \frac{1}{2!} d^2 f + \frac{1}{3!} d^3 f + \ldots,$$

$$df = \sum_{i=1}^{n} u_i dx_i, \qquad u_i = \frac{\partial f}{\partial x_i},$$

$$d^2 f = \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} dx_i dx_j, \quad a_{ij} = a_{ji} = \frac{\partial^2 f}{\partial x_i \partial x_j} = \frac{\partial u_j}{\partial x_i} = \frac{\partial u_i}{\partial x_j}.$$

- Homogeneous functions:

1. The homogeneous function of the first order (extensive)

$$f(\lambda x_1, \ldots, \lambda x_m, x_{m+1}, \ldots x_n) = \lambda f(x_1, \ldots, x_m, x_{m+1}, \ldots x_n),$$

$$\sum_{i=1}^{m} \frac{\partial f}{\partial x_i} = f. \qquad \textit{-Euler theorem}$$

2. The homogeneous function of the zero order (intensive)

$$\varphi(\lambda x_1, \ldots, \lambda x_m, x_{m+1}, \ldots x_n) = \varphi(x_1, \ldots, x_m, x_{m+1}, \ldots x_n),$$

$$\sum_{i=1}^{m} \frac{\partial \varphi}{\partial x_i} = 0. \qquad \textit{-Euler theorem}$$

First order phase transition (Ehrenfest definition)
(Properties of thermodynamic system)

- Thermodynamic systems:

1. Large number of particles: $N \approx N_A$

2. Zeroth law of thermodynamics

   State of a thermal equilibrium:

   1. dynamic state (chaotic motion, fluctuations)
   2. principle of transitivity
      (temperature T is a universal measure of equilibrium)

   $$T(1) = T(2) = T(1+2)$$

3. Principle of additivity $System(1+2) \rightarrow System(1) + System(2)$

   1. Thermodynamic limit: $E \rightarrow \infty, V \rightarrow \infty, N \rightarrow \infty$

      $$\varepsilon = E / N = const, v = V / N = const$$

   2. extensive variables: $E, S, V, N, \ldots$

      $$A(1+2) = A(1) + A(2)$$
      $$A(E, V, N) = Na(\varepsilon, v)$$

   3. intensive variables: $T, p, \mu, \ldots$

      $$\phi(1+2) = \phi(1) = \phi(2)$$
      $$\phi(E, V, N) = \phi(\varepsilon, v)$$

4. The I,II,III laws of thermodynamics

I law – energy conservation: $$dE = \delta Q - \delta W + dE_{mat},$$

II law – thermodynamic entropy:
(direction of heat transfer) $$dS_{th} = \frac{\delta Q}{T},$$

III law – absolute value of entropy:
(Planck rigid formulation) $$\lim_{T\to 0} S_{th} = 0.$$

5. Second part of II law (nonequilibrium states)

1. irreversible processes: $$dS_{th} \geq \frac{\delta Q}{T}$$
2. maximum entropy principle: $$(dS_{th})_{E,V,z,N} \geq 0, (TdS_{th} - dE)_{T,V,z,N} \geq 0$$

Main relations:

1. Fundamental equation of thermodynamics:
2. Euler theorem:
3. Gibbs-Duhem relation:

$$TdS_{th} = dE + pdV + Xdz - \mu dN,$$

$$TS_{th} = E + pV + Xz - \mu N,$$

$$S_{th}dT = Vdp + zdX - Nd\mu$$

Transitions between states

1. The quasistatic reversible proseses: $\Delta t >> \tau_{rel}$
2. The exchange of energy (heat, work) and matter

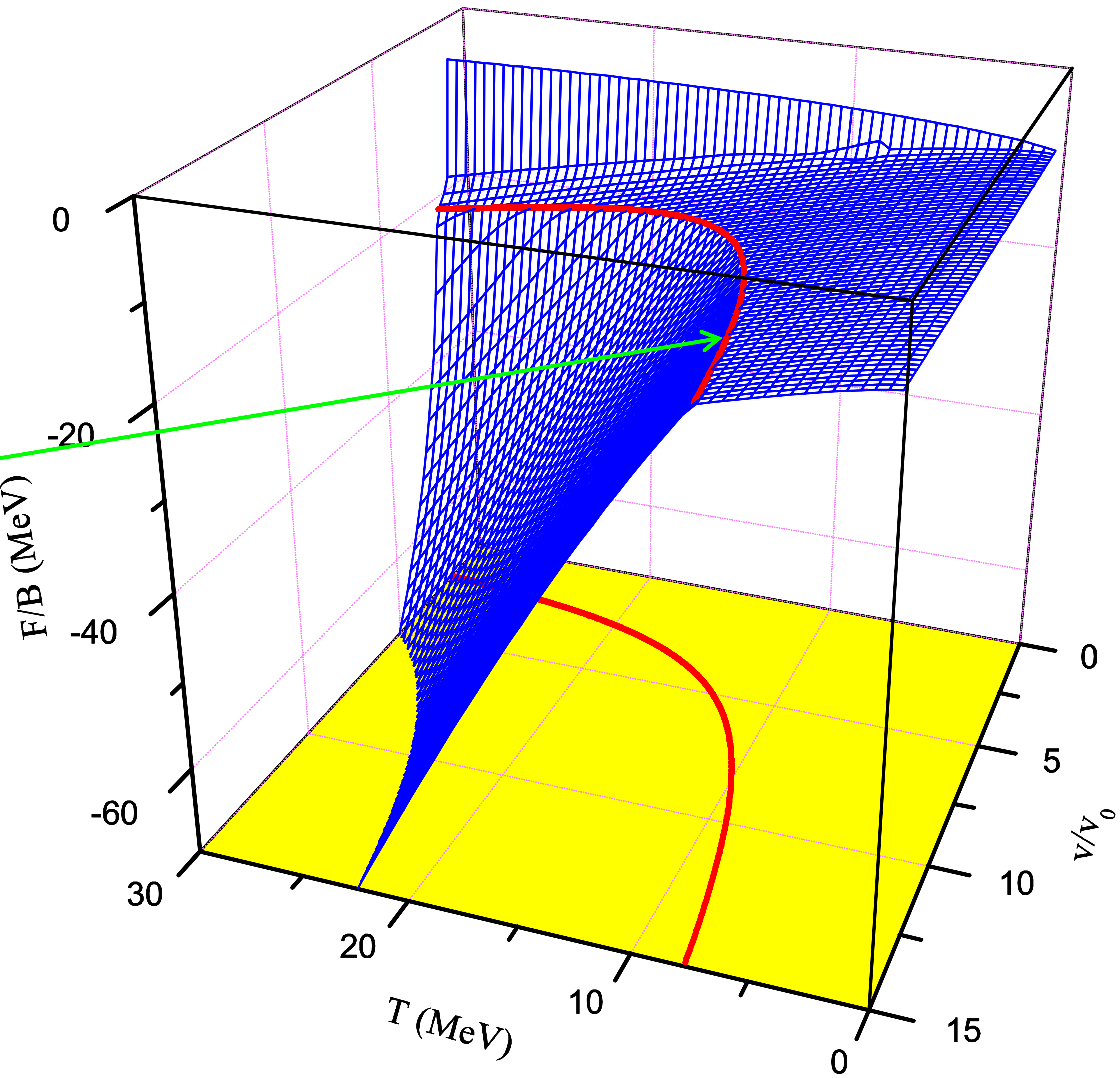
$$\delta Q = CdT, \quad \delta W = Xdz, \quad dE_{mat} = \mu dN$$

3. Caloric curve $$C = C(x_1, \ldots, x_n),$$
4. Equation of state $$X_i = X_i(x_1, \ldots, x_n).$$

First order phase transition (Ehrenfest definition)

Geometrical properties of Thermodynamical potential $f(x_1, x_2)$

- the first order phase transition
- the line of intersection of two surfaces



Ehrenfest definition of the first order phase transition for a given thermodynamic potential $f(x_1, x_2)$

1. The thermodynamic potential as a function of one variable of state at fixed values of its other variables of state have a cusp (sharp corner) at the points of phase transition.
2. The first order partial derivatives of the thermodynamic potential with respect to variables of state have the jump discontinuities at the points of phase transition.
3. The second order partial derivatives of the thermodynamic potential with respect to variables of state have the infinite maximum or minimum at the points of phase transition.

Two particular cases of the first order phase transition

1. First order phase transition associated with the Gibbs free energy (the liquid-gas phase transition)

- Thermodynamical potentials of the isobaric and grand canonical ensembles
- The relativistic mean-field hadronic model (Walecka model)

2. First order phase transition associated with the Helmholtz free energy

- The thermodynamical potential of the canonical ensemble
- The model of the decay of nuclei into nucleons

The relativistic mean-field hadronic model (page 1/5)

The quantum field theory B.D. Serot, J.D. Walecka, Adv. Nucl. Phys. 16 (1986) 1

- The Lagrangian density:

$$L=\bar{\psi}[i\gamma_\mu D^\mu-(m-g_s\phi)]\psi-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+\frac{1}{2}m_v^2A_\mu A^\mu+\frac{1}{2}(\partial_\mu\phi\partial^\mu\phi-m_s^2\phi^2),$$

$$F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu,\qquad D^\mu=\partial^\mu+ig_vA^\mu$$

-gauge invariant
-Lorentz invariant
-point like particles

- The Euler-Lagrange equations:

$$[i\gamma^\mu D_\mu-(m-g_s\phi)]\psi=0,$$

$$(\partial_\mu\partial^\mu+m_s^2)\phi=g_s\bar{\psi}\psi,$$

$$\partial_\mu F^{\mu\nu}+m_v^2A^\nu=g_v\bar{\psi}\gamma^\nu\psi.$$

- The conserved baryon current: (Noether theorem)

$$B^\mu=\bar{\psi}\gamma^\mu\psi$$

- The energy-momentum tensor:

$$T_{\mu\nu}=\frac{1}{2}\left(-\partial_\lambda\phi\partial^\lambda\phi+m_s^2\phi^2+\frac{1}{2}F_{\lambda\sigma}F^{\lambda\sigma}-m_v^2A_\lambda A^\lambda\right)g_{\mu\nu}+i\bar{\psi}\gamma_\mu\partial_\nu\psi+\partial_\mu\phi\partial_\nu\phi+\partial_\nu A^\lambda F_{\lambda\mu}$$

The relativistic mean-field hadronic model (page 2/5)

The mean-field approximation

$$\phi = \phi_0, \qquad A_\mu = \delta_{\mu 0} A_0$$

- The Lagrangian density:

$$L = \overline{\psi}[i\gamma_\mu \partial^\mu - g_v \gamma^0 A_0 - (m - g_s \phi_0)]\psi + \frac{1}{2}(m_v^2 A_0^2 - m_s^2 \phi_0^2),$$

- The equations of motion:

$$i\frac{\partial \psi}{\partial t} = \hat{H}_D \psi, \qquad \hat{H}_D = \vec{\alpha}\hat{\vec{p}} + \beta m^* + g_v A_0,$$

$$\phi_0 = \frac{g_s}{m_s^2}\overline{\psi}\psi,$$

$$A_0 = \frac{g_v}{m_v^2}\overline{\psi}\gamma^0\psi.$$

$$\vec{\alpha} = \gamma^0 \vec{\gamma}, \beta = \gamma^0,$$
$$\hat{\vec{p}} = -i\vec{\nabla},$$
$$m^* = m - g_s \phi_0$$

- The conserved baryon current:

$$B^\mu = \overline{\psi}\gamma^\mu \psi$$

- The energy-momentum tensor:

$$T_{\mu\nu} = i\overline{\psi}\gamma_\mu \partial_\nu \psi - \frac{1}{2}\left(m_v^2 A_0^2 - m_s^2 \phi_0^2\right) g_{\mu\nu}$$

The relativistic mean-field hadronic model (page 3/5)

The mean-field approximation. The second quantization method

- The plane wave solution for Dirac equation:

$$\psi(x)=\sum_{\vec{p},\sigma}\frac{1}{\sqrt{2E^{*}V}}\left(a_{\vec{p}\sigma}u_{p,\sigma}e^{-ip^{(+)}x}+b^{+}_{\vec{p}\sigma}u_{-p,-\sigma}e^{ip^{(-)}x}\right),$$

$$E^{*}=\sqrt{\vec{p}^{2}+m^{*2}},\qquad p^{\mu}=(E^{*},\vec{p}),\qquad p^{2}=m^{*2}$$

$$p^{(\pm)\mu}=(\varepsilon^{(\pm)},\vec{p}),\qquad \varepsilon^{(\pm)}=E^{*}\pm g_{v}A_{0}$$

- Anti-commutation relations:

$$\left\{a_{\vec{p}\sigma},a^{+}_{\vec{p}'\sigma'}\right\}=\delta_{\vec{p}\vec{p}'}\delta_{\sigma\sigma'},\qquad \left\{b_{\vec{p}\sigma},b^{+}_{\vec{p}'\sigma'}\right\}=\delta_{\vec{p}\vec{p}'}\delta_{\sigma\sigma'}$$

-create and annihilate particles

- The energy, baryon charge, scalar and vector fields:

$$\widehat{H}=\int_{V}d^{3}xT_{00}=\sum_{\vec{p},\sigma}\left[\varepsilon_{(+)}a^{+}_{\vec{p}\sigma}a_{\vec{p}\sigma}+\varepsilon_{(-)}b^{+}_{\vec{p}\sigma}b_{\vec{p}\sigma}\right]-\frac{1}{2}\left(m_{v}^{2}A_{0}^{2}-m_{s}^{2}\phi_{0}^{2}\right)V,$$

$$\widehat{B}=\int_{V}d^{3}xB^{0}=\sum_{\vec{p},\sigma}\left[a^{+}_{\vec{p}\sigma}a_{\vec{p}\sigma}-b^{+}_{\vec{p}\sigma}b_{\vec{p}\sigma}\right],$$

$$\phi_{0}=\frac{g_{s}}{m_{s}^{2}V}\int_{V}d^{3}x\overline{\psi}\psi=\frac{g_{s}}{m_{s}^{2}V}\sum_{\vec{p},\sigma}\frac{m^{*}}{E^{*}}\left[a^{+}_{\vec{p}\sigma}a_{\vec{p}\sigma}+b^{+}_{\vec{p}\sigma}b_{\vec{p}\sigma}\right],$$

$$A_{0}=\frac{g_{v}}{m_{v}^{2}V}\int_{V}d^{3}x\overline{\psi}\gamma^{0}\psi=\frac{g_{v}}{m_{v}^{2}V}\sum_{\vec{p},\sigma}\left[a^{+}_{\vec{p}\sigma}a_{\vec{p}\sigma}-b^{+}_{\vec{p}\sigma}b_{\vec{p}\sigma}\right]$$

-The condition of positive energies: Spin-statistics theorem (spin=1/2-fermi, Spin=0,1,... - bose)

The relativistic mean-field hadronic model (page 4/5)

Grand canonical ensemble (T,V,μ_B)

$$\hat{\rho} = \frac{1}{Z} e^{-\frac{\hat{H}-\mu\hat{B}}{T}}, \qquad Z = Tr\left(e^{-\frac{\hat{H}-\mu\hat{B}}{T}} \right), \qquad \langle O \rangle = Tr(\hat{\rho}\hat{O}).$$

$$\Omega = -T \ln Z = -T \sum_{\vec{p}\sigma} [\ln(1 + e^{-\frac{\varepsilon_{(+)}-\mu}{T}}) + \ln(1 + e^{-\frac{\varepsilon_{(-)}+\mu}{T}})] - \frac{1}{2}\left(a_v \rho_v^2 - a_s \rho_s^2\right)V,$$

$$\rho_v - \frac{1}{V} \sum_{\vec{p}\sigma} [n_{\vec{p}\sigma}^{(+)} - n_{\vec{p}\sigma}^{(-)}] = 0,$$

$$\rho_s - \frac{1}{V} \sum_{\vec{p}\sigma} \frac{m^*}{E^*} [n_{\vec{p}\sigma}^{(+)} + n_{\vec{p}\sigma}^{(-)}] = 0,$$

$$n_{\vec{p}\sigma}^{(+)} = \frac{1}{e^{\frac{\varepsilon_{(+)}-\mu}{T}} + 1}, \qquad n_{\vec{p}\sigma}^{(-)} = \frac{1}{e^{\frac{\varepsilon_{(-)}+\mu}{T}} + 1}$$

$$\frac{\partial \Omega}{\partial \rho_v} = 0, \qquad \frac{\partial \Omega}{\partial \rho_s} = 0,$$

$$\varepsilon_{(\pm)} = E^* \pm a_v \rho_v,$$

$$m^* = m - a_s \rho_s,$$

$$\rho_v = \frac{A_0 g_v}{a_v}, \qquad \rho_s = \frac{\phi_0 g_s}{a_s},$$

$$a_v = \frac{g_v^2}{m_v^2} = \frac{C_v^2}{m^2}, \qquad a_s = \frac{g_s^2}{m_s^2} = \frac{C_s^2}{m^2}$$

The relativistic mean-field hadronic model (page 5/5)

- **Thermodynamic quantities:**

$$\langle H\rangle = \sum_{\vec{p}\sigma} E^{*}[n_{\vec{p}\sigma}^{(+)} + n_{\vec{p}\sigma}^{(-)}] + \frac{1}{2}\left(a_v\rho_v^2 + a_s\rho_s^2\right)V,$$

$$\langle B\rangle = \sum_{\vec{p}\sigma}[n_{\vec{p}\sigma}^{(+)} - n_{\vec{p}\sigma}^{(-)}],$$

$$p = \frac{1}{3V}\sum_{\vec{p}\sigma}\frac{\vec{p}^{\,2}}{E^{*}}[n_{\vec{p}\sigma}^{(+)} + n_{\vec{p}\sigma}^{(-)}] + \frac{1}{2}\left(a_v\rho_v^2 - a_s\rho_s^2\right),$$

$$S = \frac{1}{T}(-\Omega + \langle H\rangle - \mu\langle B\rangle).$$

- **Constraints on parameters:**

$$\varepsilon_b(\rho_0) = \varepsilon_{b0} = -16 MeV, \qquad T = 0$$

$$\rho_v = \rho_0 = 0.16\, fm^{-3},$$

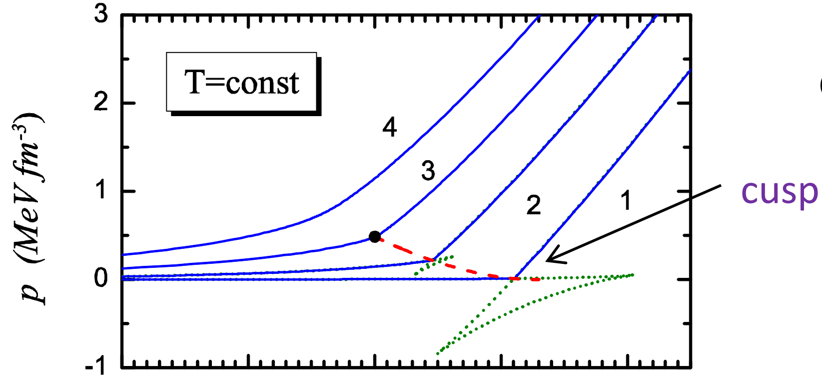
$$\varepsilon_b = \frac{\varepsilon}{\rho_v} - m, \qquad \varepsilon = \frac{\langle H\rangle}{V}$$

First order phase transition associated with the Gibbs free energy (of the liquid-gas type)

Grand canonical ensemble

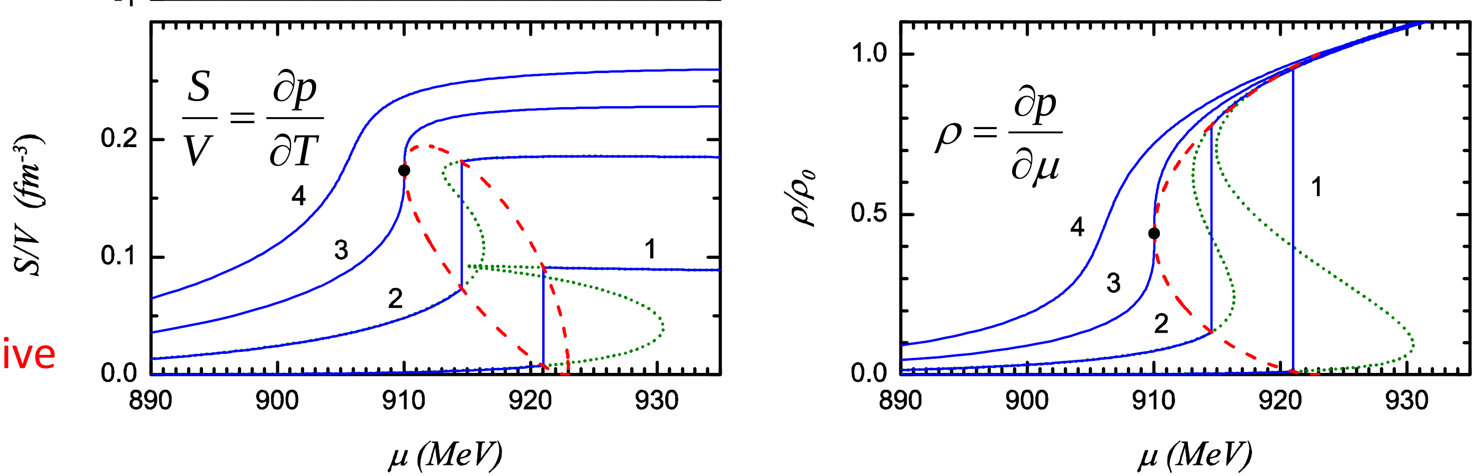
$$p = -\frac{\Omega}{V}$$ potential

A.P., NPA 887 (2012) 1



RMF
Grand Canonical Ensemble (T,μ)

first derivative



first derivative

First order phase transition associated with the Gibbs free energy (of the liquid-gas type)

Isobaric Ensemble

$\mu = \frac{G}{B}$ Potential (Gibbs free energy)

Legendre transform

RMF
Isobaric Ensemble (T,p)

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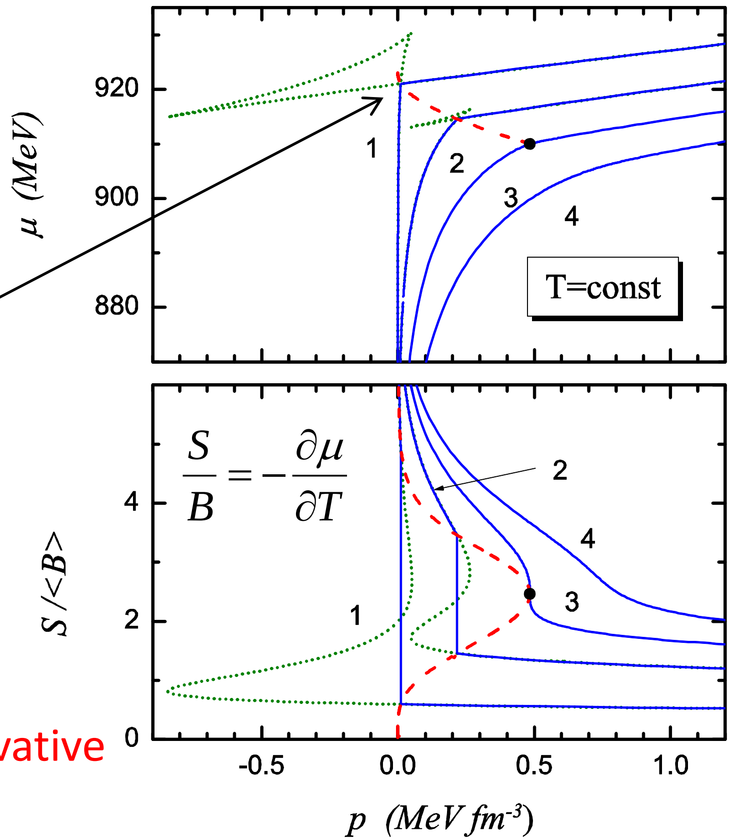
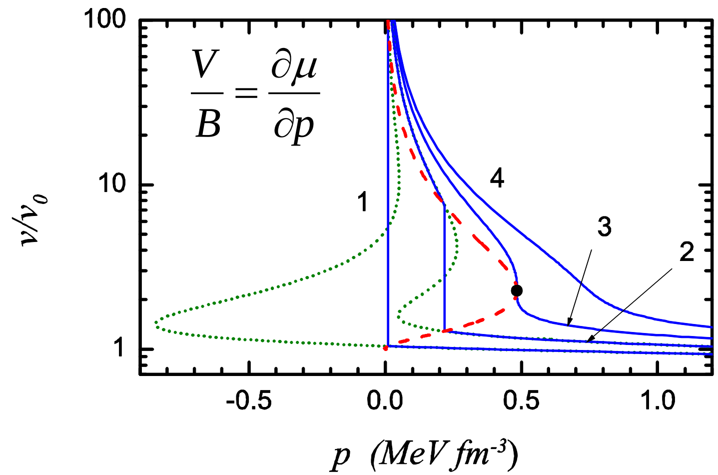
cusp



first derivative

first derivative



First order phase transition associated with the Gibbs free energy (of the liquid-gas type)

Canonical Ensemble

$$f = \frac{F}{B}$$ Potential (Helmholtz free energy)

Legendre transform

RMF
Canonical Ensemble (T,v)

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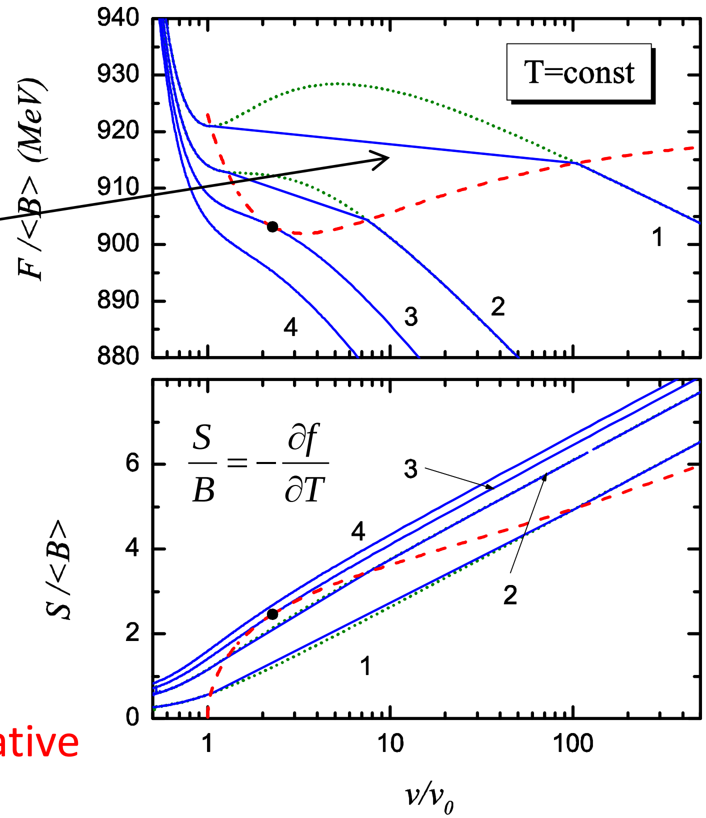
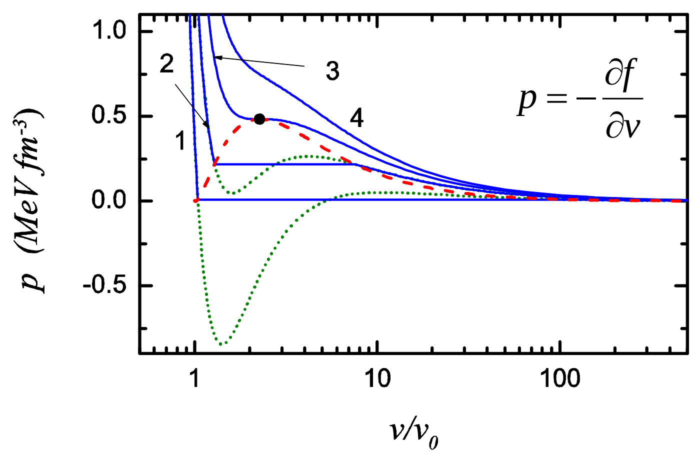
no cusp



first derivative

first derivative



First order phase transition associated with the Gibbs free energy (of the liquid-gas type)

Phase diagrams

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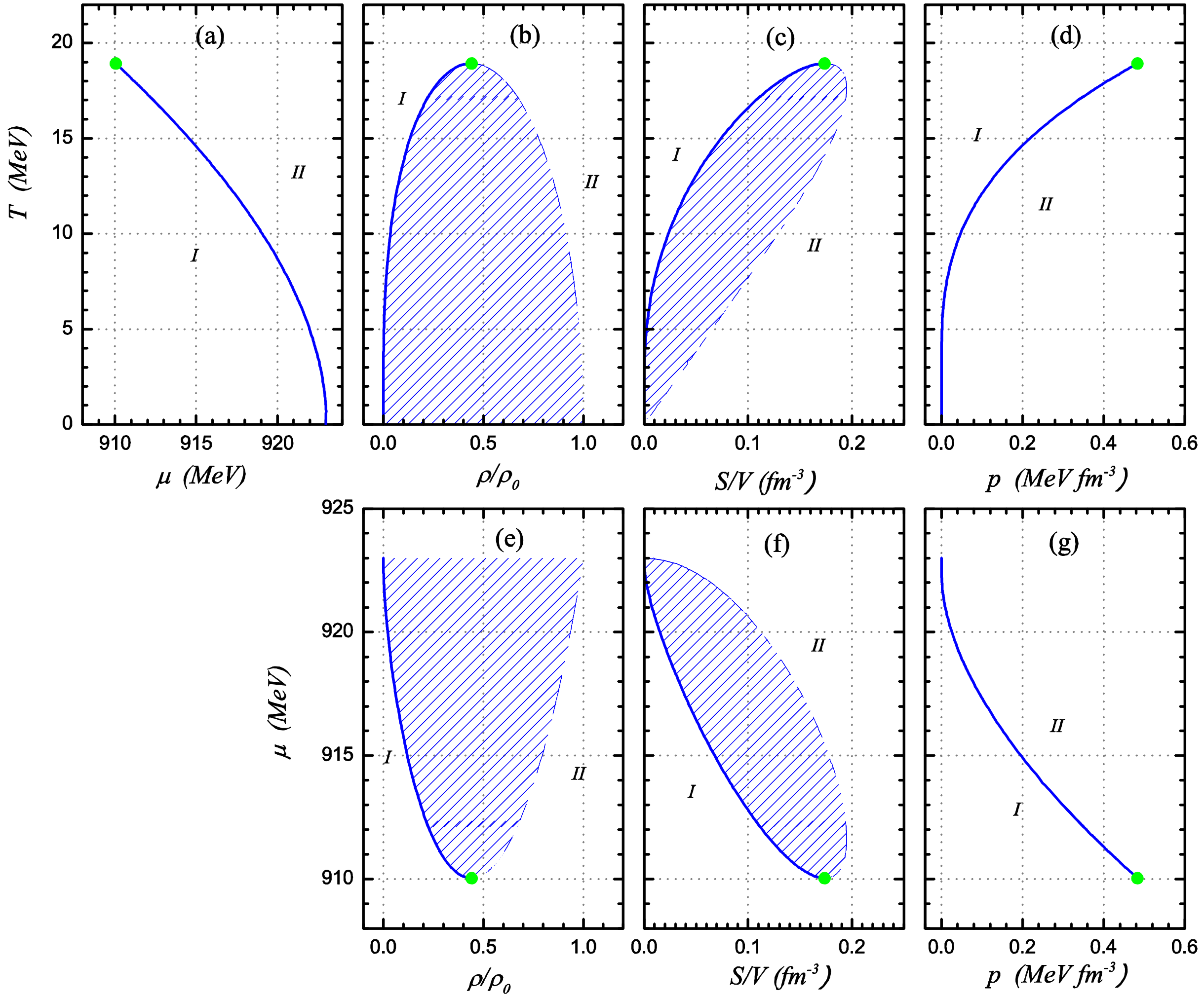

First order phase transition associated with the Gibbs free energy (of the liquid-gas type)

Caloric Curve and EoS

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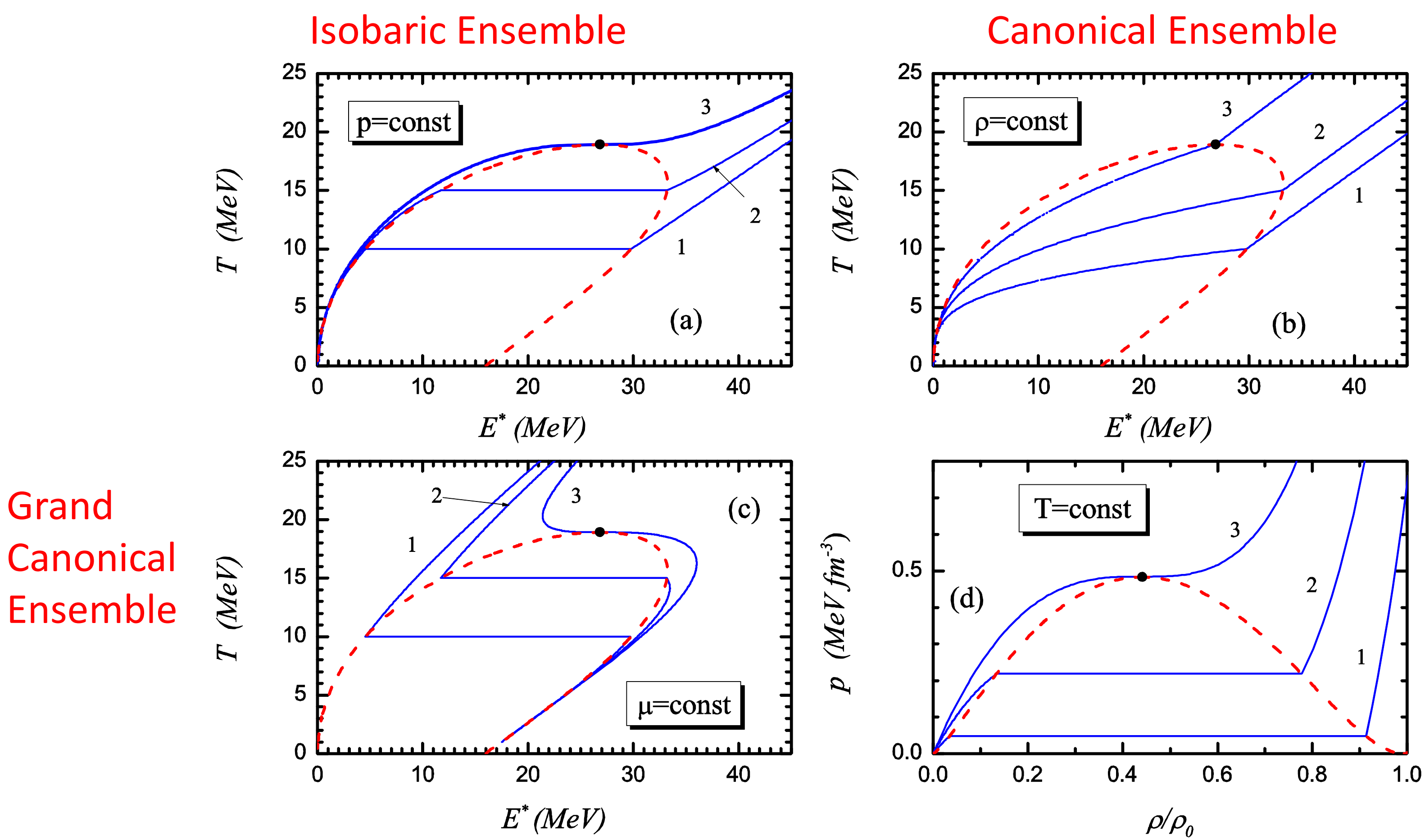


The model of decay of nuclei into nucleons

Canonical ensemble

- The statistical multifragmentation model (SMM):

$$Z_A = \sum_{\{n_k\}} \delta(\sum_{k=1}^{A} k n_k - A) \prod_{k=1}^{A} \frac{\omega_k^{n_k}}{n_k!} = \frac{1}{A} \sum_{k=1}^{A} k \omega_k Z_{A-k}, \qquad Z_0 = 1,$$

1-nucleus of A nucleons

$$\omega_k = g_k V_f \left(\frac{mkT}{2\pi} \right)^{3/2} e^{\frac{W_k}{T}}, \qquad V_f = V - v_0 A, \qquad v_0 = 1/\rho_0, \qquad W_1 = 0$$

- The model of decay of nuclei into nucleons (MDNN):

$$Z_B = \frac{\omega_1^B}{B!} + \frac{\omega_A^N}{N!}, \qquad V_f = V - v_0 B, \qquad B = NA,$$

N-nuclei of A nucleons

$$\omega_2 = \ldots = \omega_{A-1} = 0$$

$$\langle n_1 \rangle = \frac{B}{Z_B} \frac{\omega_1^B}{B!}, \qquad \langle n_A \rangle = \frac{N}{Z_B} \frac{\omega_A^N}{N!},$$

$$E = \frac{3}{2} T \langle m \rangle - \langle n_A \rangle W_A, \qquad \langle m \rangle = \langle n_1 \rangle + \langle n_A \rangle,$$

$$p = \frac{T}{V_f} \langle m \rangle,$$

$$\mu = p v_0 + \frac{T}{B} \{ 1 + [-\ln \omega_1 + \psi(B)] \langle n_1 \rangle + [-\ln \omega_A + \psi(N)] \langle n_A \rangle \}$$

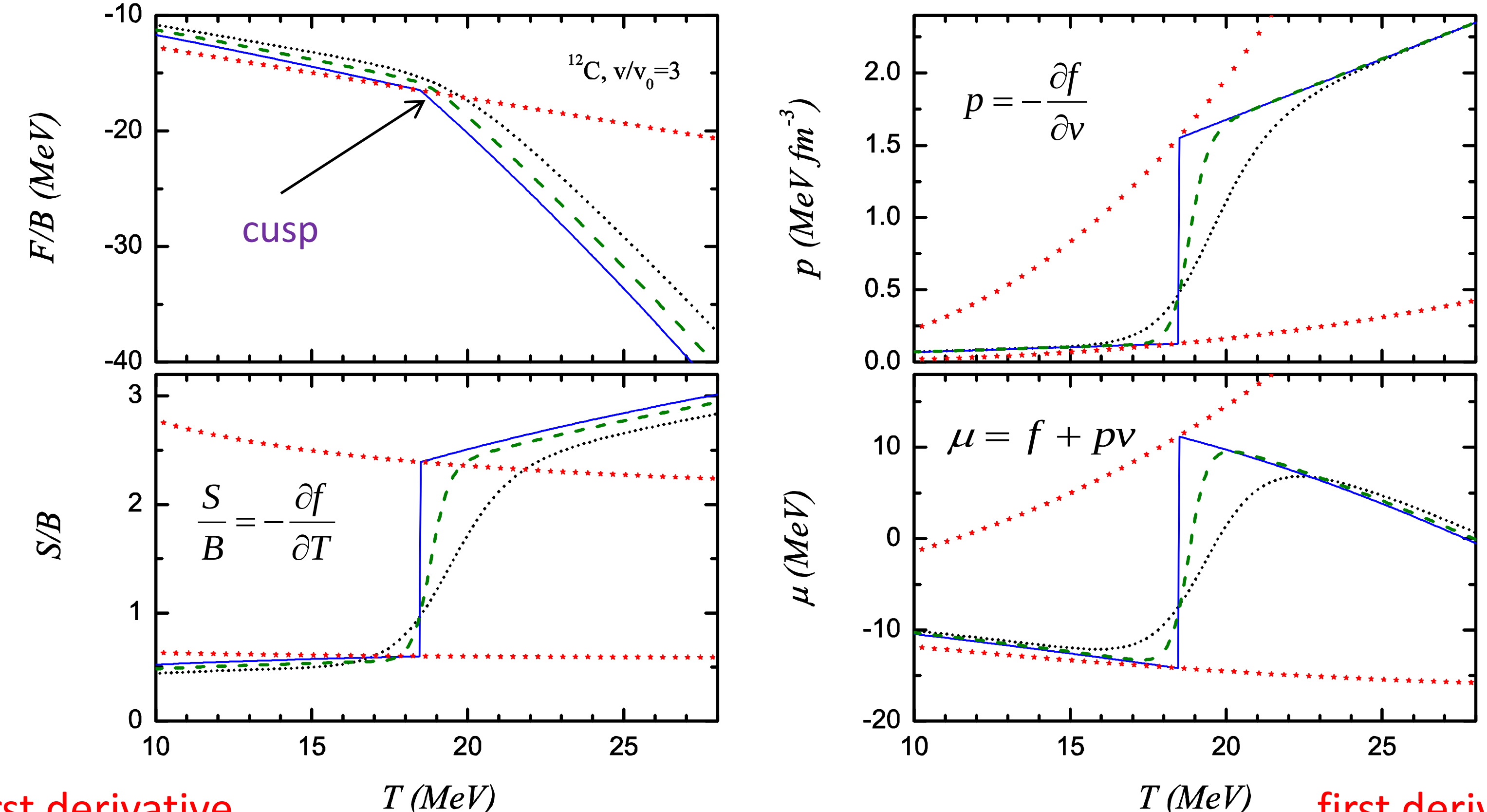
First order phase transition associated with Helmholtz free energy

Thermodynamical potential and its first derivatives

A.P., arXiv: 1206.6753

$f = \frac{F}{B}$ Potential (the Helmholtz free energy)

first derivative



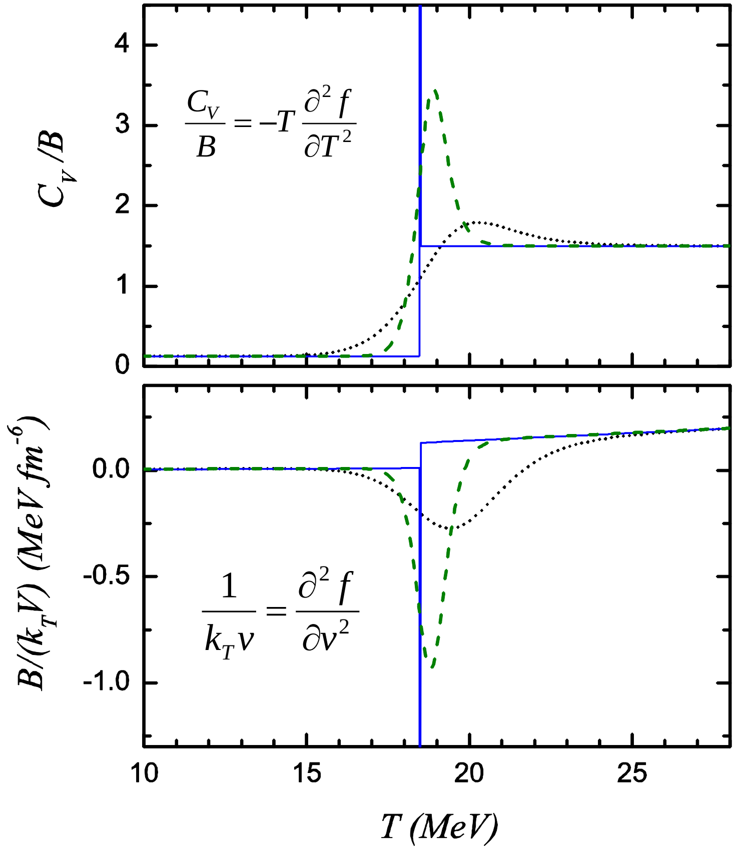
first derivative

first derivative

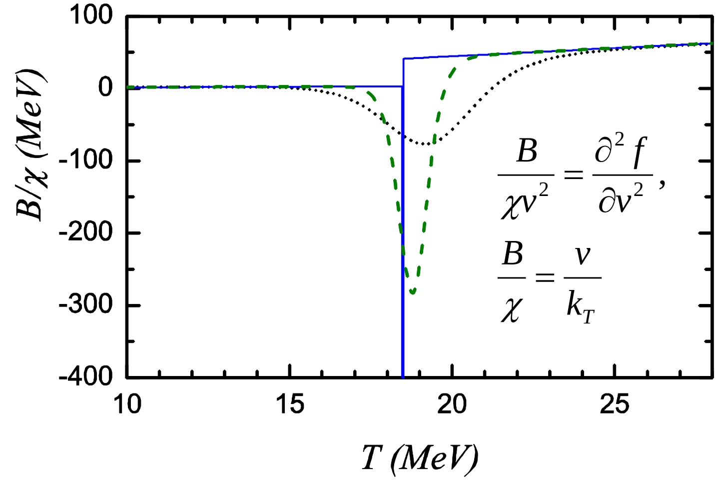
First order phase transition associated with Helmholtz free energy

Second derivatives

A.P., arXiv: 1206.6753



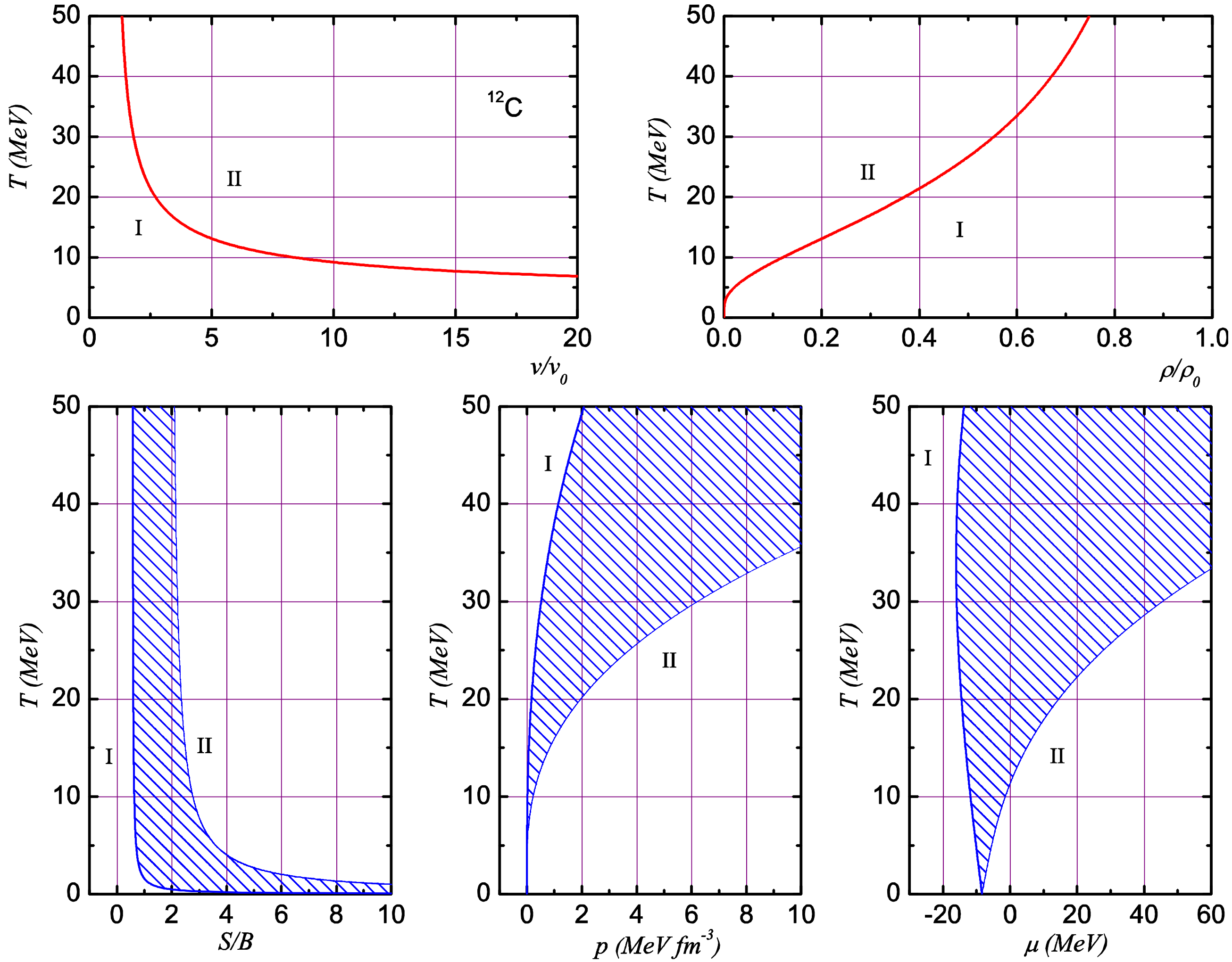
Infinite maximum or minimum



First order phase transition associated with Helmholtz free energy

A.P., arXiv: 1206.6753

Phase diagrams



Summary

1. The main thermodynamic properties of the nuclear liquid-gas phase transition associated with the Gibbs free energy were established by using the relativistic mean-field hadronic model.
2. The properties of the first order phase transition associated with the Helmholtz free energy were obtained on the basis of the model of the decay of nuclei into nucleons.
3. For the nuclear liquid-gas phase transition the phase diagram T-μ is represented by the coexistence line and the phase diagram T-ρ is represented by the coexistence area.
4. On the contrary, for the first order phase transition associated with the Helmholtz free energy the phase diagram T-μ is represented by the coexistence area and the phase diagram T-ρ is represented by the coexistence line.

Thank you for attention!