QCD in Infrared Region and Spontaneous Breaking of the Chiral Symmetry

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Outline

YM in the collaboration with K.Goeke, H-C.Kim, M.Siddikov,
Phys.Lett.**608**(2005)95,
Phys.Rev.D**76**(2007)076007,
Phys.Rev.D**76**(2007)0116007,
Mod.Phys.Lett.A**23**(2008)2360,
Phys.Rev.D **81**(2010)054029.
and
Y.M. talk given at 1st International Workshop on Theoretical Physics: Confinement and QCD vacuum, APCTP, Pohang, Korea, February 23 – 25, 2012.

The QCD coupling constant at small momentum

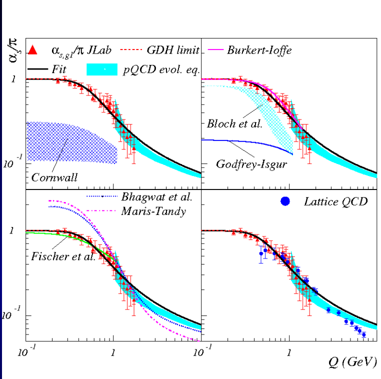


Figure: α_{s,g_1} from JLab data, Burkert-Ioffe model.α_s: Top left: S-D eqs. (Cornwall); Top right: S-D eqs. (Bloch), quark constituent model (Godfrey-Isgur); Bottom left: S-D eqs. (Maris-Tandy), (Fischer et al.), (Bhagwat et al.); Bottom right: Lattice QCD (Furui,Nakajima). *(A. Deur. The strong coupling constant at large distances. AIP Conf.Proc.1149:281-284,2009, arXiv:0901.2190v2 [hep-ph])*

The QCD coupling constant at small momentum

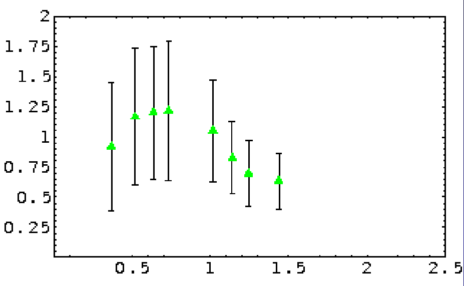


Figure: $\alpha_s(q)$ measured by using the gluon propagator and the ghost propagator has a maximum $\simeq 1$ at around $p = 0.5$ GeV and decreases as p approaches 0. *(Sadataka Furui, Hideo Nakajima. Infrared Features of the Landau Gauge, Phys.Rev. D69 (2004) 074505, arXiv:hep-lat/0305010v3).*

The QCD coupling constant at small momentum

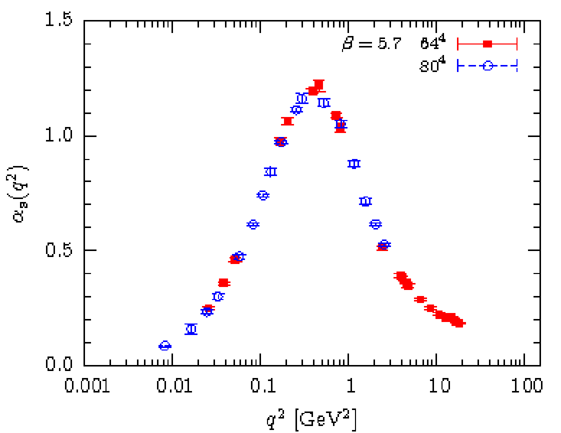


Figure: Running coupling $\alpha_s(q^2)$ versus q^2 for lattice sizes 64^4 and 80^4 at $\beta = 5.70$ *(I.Bogolubsky, E.Ilgenfritz, M.Muller-Preussker, A.Sternbeck. Lattice gluodynamics computation of Landau-gauge Green's functions in the deep infrared, Phys.Lett.B676:69-73,2009, arXiv:0901.0736v3 [hep-lat])*

The QCD coupling constant at small momentum

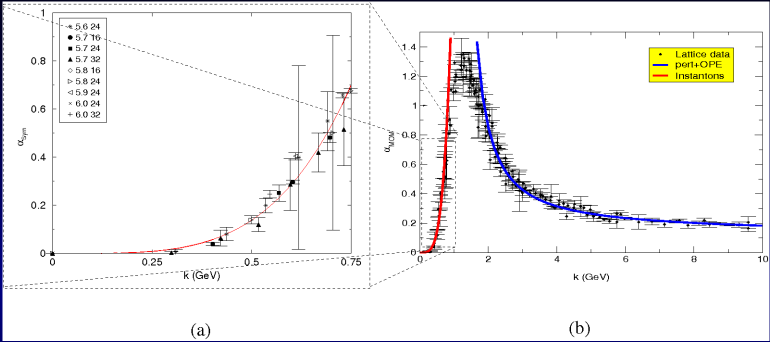


Figure: (b) $\alpha_s(p)$ measured by using the two and three gluon Green functions for different lattice settings. (a) Region of small momenta is compared with instanton vacuum model result $\alpha_s(p) = (18\pi)^{-1}\bar{R}^4 p^4$, $\bar{R} \approx 0.66\, fm$ *(Ph. Boucaud, J. P. Leroy, A. Le Yaouanc, J. Micheli, O. Pine, J. Rodriguez-Quintero. The Infrared Behavior of the Pure Yang-Mills Green Functions, arXiv:1109.1936v2 [hep-ph], JHEP, 0304:005, 2003).*

QCD vacuum on the lattice

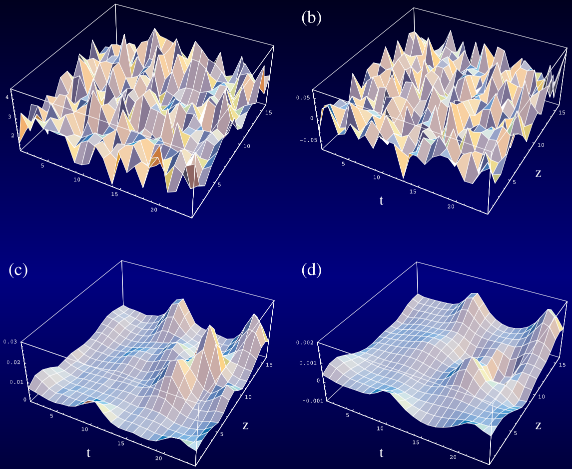


Figure: “Cooling” kill the normal zero-point oscillations but keep large topological objects like instantons and anti-instantons with random positions and sizes. The left column – the action density and the right column – the topological charge density *(J. Negele, Nucl. Phys. Proc. Suppl. 73 (1999) 92).*

The structure of QCD vacuum and dyons

Instantons are not the only possible large non-perturbative fluctuations of the gluon field: one can think also of merons, monopoles, vortices, dyons etc. QCD vacuum is a soup made all of these ingredients.
D. Diakonov, How to check that dyons are at work?, arXiv:1012.2296v2 [hep-ph].
The difference with the old instanton liquid model is:

- the Polyakov line is now nontrivial,
- the integration measure over collective coordinates is invariant under permutation of dyons belonging to different instantons, and allows instantons to overlap.

These circumstances are critical for obtaining confinement that was absent in former instanton models.

The structure of QCD vacuum and merons

Falk Zimmermann, Hilmar Forkel and Michael Muller-Preusker, Vacuum structure and string tension in Yang-Mills dimeron ensembles, arXiv:1202.4381v1 [hep-ph]

- Numerically simulated ensembles of SU(2) Yang-Mills dimeron solutions;
- Meron-induced quark confinement as triggered by dimeron dissociation;
- The density of coexisting, hardly dissociated and thus instanton-like dimerons remain large enough to reproduce successful non-confining instanton vacuum models;
- Dimeron ensembles provide an efficient basis for a rather complete description of the Yang-Mills vacuum.

Confinement and Spontaneous Breaking of Chiral Symmetry

Shinya Gongyo, Takumi Iritani, Hideo Suganuma, Gauge-Invariant Formalism with Dirac-mode Expansion for Confinement and Chiral Symmetry Breaking arXiv:1202.4130v1 [hep-lat].

- A direct analysis of the correlation between confinement and chiral symmetry breaking in lattice QCD Monte Carlo calculation;
- The confinement force is almost unchanged even after removing the low-lying Dirac modes, which are responsible to chiral symmetry breaking;
- No one-to-one correspondence between confinement and spontaneous chiral symmetry breaking in QCD.

Which component of QCD Vacuum most responsible for the SBCS?

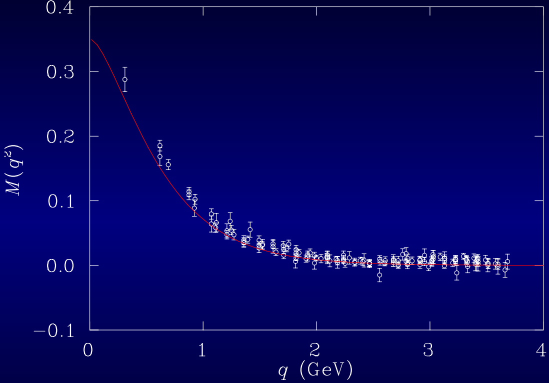


Figure: Momentum dependence of dynamical quark mass $M(q)$ in the chiral limit. Points: lattice result (P.Bowman *et. al.*,, 2004). Red line: Instanton Vacuum Model (Diakonov&Petrov86), **no fitting**.

Assumption: other components are not important for the SBCS.

QCD instantons

Instanton in singular gauge (Belavin *et.al.*, 1975):

$$A_\mu^{I,a}(x) = \frac{2\rho^2 \bar{\eta}^\nu_{\mu a}(x-z)_\nu}{(x-z)^2[\rho^2 + (x-z)^2]}\,.$$

For the antiinstanton just change the t'Hooft symbol $\bar{\eta} \to \eta$.

- Top. charge $Q_I = \frac{1}{32\pi^2}\int d^4x\, G^a_{\mu\nu}\tilde{G}^a_{\mu\nu} = +1$
- Action $S_I = \frac{8\pi^2}{g^2} \Rightarrow |\Delta N_W| = 1$ tunneling amplitude $\sim \exp\left(-S_I\right)$, $N_W = \frac{1}{24\pi^2}\int d^3x \epsilon_{ijk}\left\langle \left(U^\dagger \partial_i U\right)\left(U^\dagger \partial_j U\right)\left(U^\dagger \partial_k U\right)\right\rangle$.
- Instanton collective coordinates:

$$4\,(centre) \;+\; 1\,(size) \;+\; (4N_c{-}5)\,(orientations) \;=\; 4N_c$$

Dependence on N_{CS}

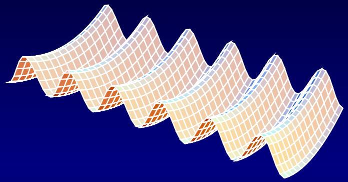


Figure: Vacuum gluon energy vs Chern-Simons number
$N_{CS} = \int d^3x\, K_0 = \frac{1}{16\pi^2}\int d^3x\, \epsilon^{ijk}\left(A_i^a \partial_j A_k^a + \frac{1}{3}\epsilon^{abc} A_i^a A_j^b A_k^c\right)$,
$N_{CS} \Rightarrow N_{CS} + N_W$.

Instanton Vacuum Model

Sum ansatz $A = \sum_I A^I + \sum_{\bar I} A^{\bar I}$. Parameters of the instanton vacuum:

- Instanton size distribution $D(\rho)$ and average size $\bar\rho$.
- Average interinstanton distance $\bar R$.
- Estimates:
  - Lattice estimate: $\bar R \approx 0.89\, fm$, $\bar\rho \approx 0.36\, fm$,
  - Phenomenological estimate: $\bar R \approx 1\, fm$, $\bar\rho \approx 0.33\, fm$,
  - Our estimate: $\bar R \approx 0.76\, fm$, $\bar\rho \approx 0.32\, fm$, correspond ChPT $F_{\pi,m=0} = 88 MeV$, $\langle \bar q q\rangle_{m=0} = -(255 MeV)^3$

  Thus within $10 - 15\%$ uncertainty different approaches give similar estimates
- Packing parameter $\underline{\pi^2 (\frac{\bar\rho}{\bar R})^4 \sim 0.1 - 0.3}$

  $\Rightarrow$ Independent averaging over instanton positions and orientations.

Instanton size distribution

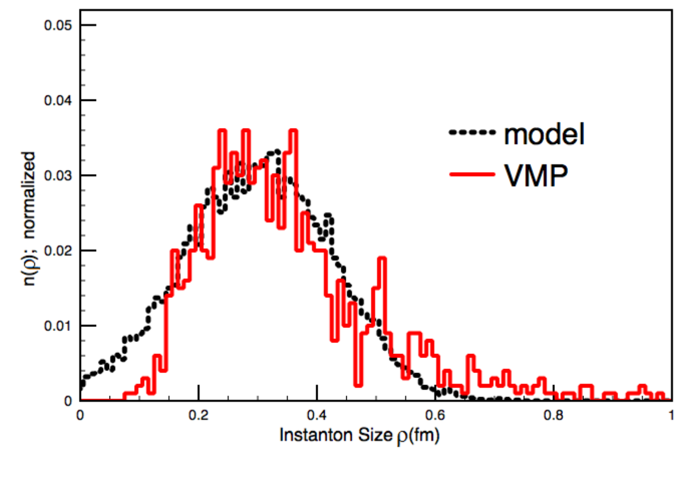


Figure: $n_{\text{model}}(\rho) \propto \exp\left[-\frac{(\rho-\bar{\rho})^2}{2\sigma^2}\right]$, with $\bar{\rho} = 0.3\ \text{fm}$, $\sigma = 0.13\ \text{fm}$, $\bar{R} \simeq 1.07\ \text{fm}$ and the VMP distribution from the corresponding lattice configurations *(R. Millo, P. Faccioli, Phys.Rev.D84:034504,2011)*.

Chiral Lagrangian at q^2 order

- To the lowest order in pion momenta q and external field $\hat{V} = s + p\gamma_5 + v_\mu\gamma_\mu + a_\mu\gamma_\mu\gamma_5$ it is

$$L_2 = \frac{F^2}{2}\left\langle D_\mu U^T D_\mu U\right\rangle + F^2\left\langle \chi^T U\right\rangle .$$

$(U = u_0 + i\vec{\tau}\vec{u},\ U^\dagger U = 1, D_\mu u_0 = \partial_\mu u_0 + a^i_\mu u_i, D_\mu u_i = \partial_\mu u_i - a^i_\mu u_0 + \epsilon_{ijk} v^j_\mu u_k, \chi = 2B(s,\vec{p})$, consider $N_f = 2$.)

- The simplest observables:
$\langle qq(m)\rangle = \frac{\delta \ln Z}{\delta s} \approx -F^2 B + \mathcal{O}(m)$,
$\int d^4x\, e^{-iq\cdot x}\left\langle j^{a,5}_\mu(x) j^{b,5}_\nu(0)\right\rangle = \int d^4x\, e^{-iq\cdot x}\frac{\delta^2 \ln Z}{\delta a^a_\mu \delta a^b_\nu} = F^2_\pi \delta^{ab}\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2+M^2_\pi}\right) + \mathcal{O}(q^2),\ M^2_\pi \approx 2B\, m + \mathcal{O}(m)$.
- The constants F, B define pion decay constant and quark condensate in the chiral limit.

(Gasser, Leutwyler, 1984).

Chiral Lagrangian at q^4 order

- $$L_4 = l_1(D_\mu U^T D_\mu U)^2 + l_2(D_\mu U^T D_\nu U)(D_\mu U^T D_\nu U) + l_3(\chi^T U)^2 + l_4(D_\mu \chi^T D_\mu U) + l_5(U^T F_{\mu\nu} F_{\mu\nu} U) + l_6(D_\mu \chi^T F_{\mu\nu} D_\nu U) + l_7(\tilde{\chi}^T U)^2 + h_1(\chi^T \chi) + h_2 \mathrm{tr}(F_{\mu\nu} F_{\mu\nu}) + h_3(\tilde{\chi}^T \tilde{\chi})$$
- We have 10 independent l_i, h_i *bare* constants.
- They are renormalized by pion loops to phenomenological low-energy constants $\bar{l}_i$ (LECs).
- Physical observables should be expressed in terms of $\bar{l}_i$.

LECs and observables in pion physics:

Universality of constants. Example of observables (Gasser, Leutwyler, 1984):

1. $\pi - \pi$ S-wave scattering length:

$$a_0^0 = \frac{7M_\pi^2}{32\pi F_\pi^2}\left[1 + \frac{5M_\pi^2}{84\pi F_\pi^2}(\bar{l}_1 + 2\bar{l}_2 - \frac{3}{8}\bar{l}_3 + \frac{21}{10}\bar{l}_4 + \frac{21}{8})\right]$$

2. Pion electromagnetic charge radius:

$$F_v(t) = 1 + \frac{1}{6}t\left\langle r_\pi^2\right\rangle_v + ...,\ \left\langle r_\pi^2\right\rangle_v = \frac{1}{16\pi F}(\bar{l}_6 - 1) + O(m_\pi^2)$$

3. $\pi \to e\nu\gamma$ decay amplitude has a part $\sim (\bar{l}_6 - \bar{l}_5)$.
4. Pion electromagnetic polarizabilities ($\gamma\pi \to \gamma\pi$ process) also are $\sim (\bar{l}_6 - \bar{l}_5)$.

Running and discontinued experiments

1. DIRAC@CERN $\Rightarrow$ lifetime of $\pi^+\pi^-, \pi K$ atoms. $\Rightarrow |a_0^0 - a_0^2|$ and $|a_0^{1/2} - a_0^{3/2}|$ in S-channel up to 5% (Gasser *et.al.*, 2001, J. Schweizer, 2004).
2. $K \to \pi\pi e\nu$ @BNL E865.$\Rightarrow a_0^0$.
3. $K^\pm \to \pi^\pm\pi^+\pi^-$ @NA48/2.$\Rightarrow |a_0^0 - a_0^2|$.
4. $\gamma p \to \gamma\pi^+ n$ reaction study at the Mainz Microtron MAMI to find pion electromagnetic polarizabilities.
5. (Discontinued) $\gamma\gamma \to \pi^+\pi^-$ experiments as PLUTO, DM1, DM2
6. Lattice evaluation of different constants (MILC, ETM, JLQCD, RBC/UKQCD, PACS-CS, *etc.*)

Light quarks in the instanton background - basic assumptions:

- Sum ansatz as background.
- Zero-mode approximation for single instanton quark propagator

$$S_i \approx \frac{|\Phi_{0i}\rangle\langle\Phi_{0i}|}{im} + \frac{1}{i\hat\partial}, \quad (i\hat\partial + g\hat A_i)\Phi_{0i} = 0.$$

- LO over N_c is kept.
- The width of the size distribution is neglected since suppressed as $1/N_c$.

These assumptions are working well at $m \Rightarrow 0$ (Diakonov *et.al.*, 1986-2006) but *wrong beyond the chiral limit.*

Beyond the chiral limit:

$$S_i = S_0 - S_0 \hat{p} \frac{|\Phi_{0i}\rangle\langle\Phi_{0i}|}{\langle\Phi_{0i}|\hat{p} S_0 \hat{p}|\Phi_{0i}\rangle} \hat{p} S_0, \;\; S_0 = \frac{1}{\hat{p} + im},$$

$$S_i|\Phi_{0i}\rangle = \frac{1}{im}|\Phi_{0i}\rangle, \;\; \langle\Phi_{0i}|S_i = \langle\Phi_{0i}|\frac{1}{im}.$$

$\Rightarrow$ most important $(m\rho)^2 \ln m\rho$-term of exact single-instanton quark effective action (Lee:2005),
$\Rightarrow$ full propagator (in the external fields $\hat{V} = s + p\gamma_5 + \hat{v} + \hat{a}\gamma_5$):

$$\tilde{S} - \tilde{S}_0 = -\tilde{S}_0 \sum_{i,j} \hat{p}|\phi_{0i}\rangle \left\langle \phi_{0i} \left| \left(\frac{1}{\hat{p}\tilde{S}_0\hat{p}} \right) \right| \phi_{0j} \right\rangle \langle\phi_{0j}|\hat{p}\tilde{S}_0$$

$$|\phi_0\rangle = \frac{1}{\hat{p}} L\hat{p}|\Phi_0\rangle, \;\; \tilde{S}_0 = \frac{1}{\hat{p} + \hat{V} + im}$$

$$L_i(x, z_i) = \mathrm{P}\exp\left(i \int_{z_i}^{x} dy_\mu (v_\mu(y) + a_\mu(y)\gamma_5) \right)$$

Steps to the partition function: $\tilde S \to \mathrm{Det} \to Z_N$

$\mathrm{Det} = \mathrm{Det}_{high} \cdot \mathrm{Det}_{low}$

$$\ln \tilde{\mathrm{Det}}_{low} = \mathrm{Tr} \int_m^{\bar M} dm' \, \tilde S(m') = \ln \det \langle \phi_{0,i} | \hat p \tilde S_0^{fg} \hat p | \phi_{0,j} \rangle,$$

Averaging of $\tilde{\mathrm{Det}}_{low}$ over instantons by means of fermionization $\to$ constituent quarks $\to$ partition function Z_N with t'Hooft-like nonlocal quark interaction term Y. Exponentiation in $Z_N \to$ dynamical coupling λ

$$Z_N = \int d\lambda_+ d\lambda_- D\bar\psi D\psi e^{-S}, \quad N_f = 2,$$

$$S = N_\pm \ln \frac{K}{\lambda_\pm} - N_\pm + \psi^\dagger (i\hat\partial + \hat V + im)\psi + \lambda_\pm Y_2^\pm,$$

$$Y_2^\pm = \int d\rho D(\rho) \left(\alpha^2 \det_f J^\pm + \beta^2 \det_f J^\pm_{\mu\nu} \right), \ \frac{\beta^2}{\alpha^2} = \frac{1}{8N_c - 4},$$

$$J^\pm_{fg} = \psi_f^\dagger \bar L \frac{1 \pm \gamma_5}{2} L \psi_g, \quad J^\pm_{\mu\nu,fg} = \psi_f^\dagger \bar L \sigma_{\mu\nu} \frac{1 \pm \gamma_5}{2} L \psi_g.$$

Bosonization $\Rightarrow$ mesons

$$Z_N = \int d\lambda_+ d\lambda_- D\bar\psi D\psi D\Phi^\pm D\Phi^\pm_{\mu\nu} e^{-S}$$

$$S = -N_\pm \ln\lambda_\pm + 2\left(\Phi_i^2 + \frac{1}{2}\Phi^2_{i,\mu\nu}\right) + \psi^\dagger\left[i\hat\partial + \hat V + im + \right.$$

$$\left. i\lambda^{0.5}\bar L F(p)\left(\alpha\Phi_i\Gamma_i + \frac{1}{2}\beta\Phi_{i,\mu\nu}\sigma_{\mu\nu}\Gamma_i\right)F(p)L^{-1}\right]\psi$$

$\Gamma_i = \{(1, i\vec\tau\gamma_5), (\gamma_5, i\vec\tau)\}$

Integrate out fermions:

$$S = -N_\pm \ln\lambda_\pm + 2\left(\Phi_i^2 + \frac{1}{2}\Phi^2_{i,\mu\nu}\right) - Tr\log\left[\hat p + \hat V + im + \right.$$

$$\left. i\lambda^{0.5}\bar L F(p)\left(\alpha\Phi_i\Gamma_i + \frac{1}{2}\beta\Phi_{i,\mu\nu}\sigma_{\mu\nu}\Gamma_i\right)F(p)L^{-1}\right]$$

Calculations – Double expansion ($1/N_c$, m):

- Mesons – chiral doublets: $(\sigma, \vec{\phi})$, $(\eta, \vec{\sigma})$ and $(\sigma_{\mu,\nu}, \vec{\phi}_{\mu\nu})$.
- Meson loops $\Rightarrow 1/N_c$ corrections.
- Regularization@$q \sim \rho^{-1}$ via nonlocality.
- Other sources of $1/N_c$-correction:
  - Finite width of size distribution.
  - Shift of the coupling λ.

Dynamical quark mass $M(m)$

$$M(m) = 0.36 - 2.36\, m - \frac{m}{N_c}(0.808 + 4.197 \ln m) + \mathcal{O}\left(m^2, \frac{1}{N_c}\right),$$

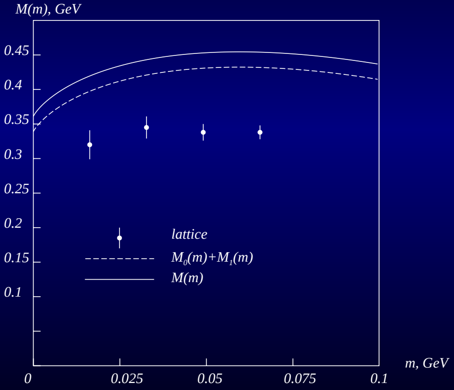


Figure: Dynamical quark mass $M(m)$. Comparison with lattice data (Bowman 2005).

Finite width correction

2-loop instanton size distribution (Diakonov ' 83, Vainshtein *et.al.*, ' 82)

$$D(\rho) \sim (\Lambda\rho)^{\frac{11N_c}{3}-5} \left(\ln(\Lambda\rho)\right)^{-N_c\left(\frac{5}{11}-\frac{255}{1331\ln(\Lambda\rho)}\right)}$$

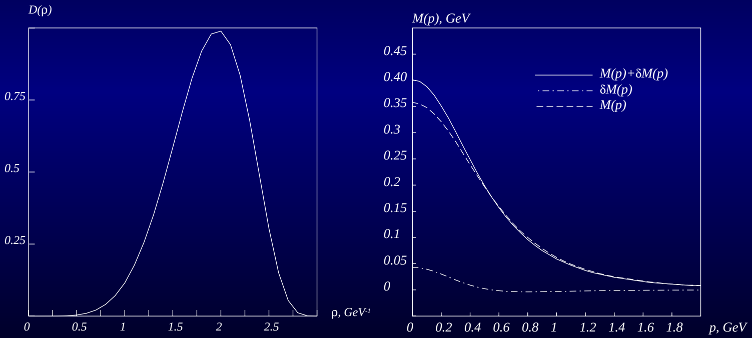


Figure: Left: Instanton size distribution. Right: change of the $M(p)$-dependence due to FWC.

Quark condensate

$$-\langle \bar{q}q\rangle(m) = ((0.00497 - 0.0343\, m)\; N_c +$$
$$(0.00168 - 0.0494\, m - 0.0580\, m \ln m)) + \mathcal{O}\left(m^2, \frac{1}{N_c^2}\right)$$

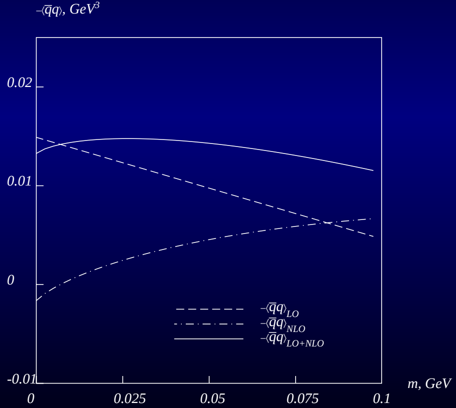


Figure: Quark condensate as a function of m.

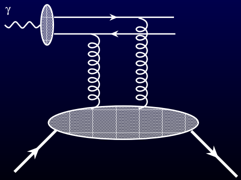
Magnetic susceptibility

$$\langle \psi_f^\dagger \sigma_{\mu\nu} \psi_f \rangle_F = e_f \, \chi_f \, \langle i \psi_f^\dagger \psi_f \rangle \, F_{\mu\nu}$$

- Sum rules estimate (Ioffe 1983, Belyaev 1984): $\chi_f \, \langle i \psi_f^\dagger \psi_f \rangle \sim 40 - 70 \, MeV$
- Measurable in jet production at $q_\perp \gg \Lambda_{QCD}$ (Braun *et.al*, 2002)

$$\gamma + N \to (\bar{q}q) + N$$

with *polarized* photon.



Magnetic susceptibility

Our result

$$\chi\langle\bar\psi\psi\rangle = N_c\left[0.015 + 5.3\times 10^{-4}\, m + \frac{m}{2\pi^2}\ln\, m\right] - [0.007 - 0.415\, m - 0.198\, m\ln\, m] + \mathcal{O}\left(m^2, \frac{1}{N_c^2}\right) = 0.038 - 0.413\, m - 0.0462\, m\ln\, m + \mathcal{O}\left(m^2, \frac{1}{N_c^2}\right).$$

Chiral log theorem:

$$\chi\langle\bar\psi\psi\rangle(m) = \chi\langle\bar\psi\psi\rangle(0)\left(1 - \frac{3m_\pi^2}{32\pi^2 F^2}\ln m_\pi^2\right)$$

Magnetic susceptibility

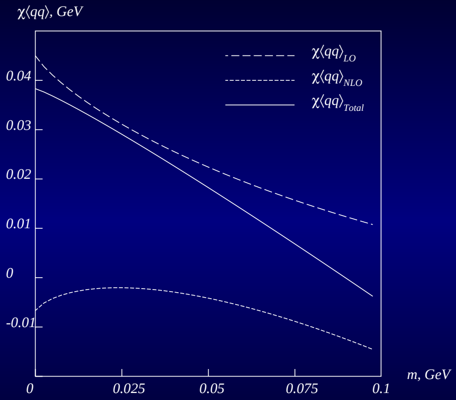


Figure: Magnetic susceptibility as a function of m.

$F_\pi,\ M_\pi$ from $\int d^4x\, e^{-iq\cdot x} \left\langle j_\mu^{a,5}(x) j_\nu^{b,5}(0)\right\rangle$

$$F_\pi^2 = N_c \left(\left(2.85 - \frac{0.869}{N_c}\right) - \left(3.51 + \frac{0.815}{N_c}\right) m - \right.$$

$$\left. - \frac{44.25}{N_c}\, m \ln m + \mathcal{O}(m^2) \right) \cdot 10^{-3}\ [GeV^2] =$$

$$(7.67 - 11.35\, m - 44.25\, m \ln m) \cdot 10^{-3}\ [GeV^2]$$

$$M_\pi^2 = m \left(\left(3.49 + \frac{1.63}{N_c}\right) + \right.$$

$$\left. m \left(15.5 + \frac{18.25}{N_c} + \frac{13.5577}{N_c} \ln m\right) + \mathcal{O}(m^2) \right) =$$

$$m\,(4.04 + 21.587\, m + 4.52\, m \ln m + \mathcal{O}(m^2))[GeV^2]$$

F_π, M_π

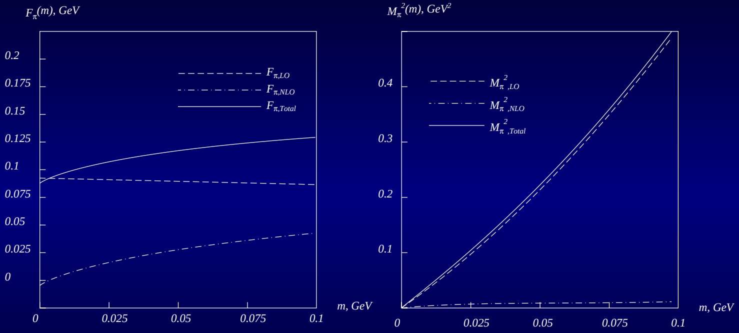


Figure: F_π and M_π^2 as a function of m.

F_π, M_π

- $F^2 = \left(2.85\, N_c - 0.87 + \mathcal{O}\left(\frac{1}{N_c}\right)\right) \times 10^{-3}[GeV^2]$
  $B = 1.75 + \frac{0.82}{N_c} + \mathcal{O}\left(\frac{1}{N_c^2}\right)[GeV]$
-

$$\bar{l}_3 = \frac{-1.14\, N_c \left(1 + \frac{0.872}{N_c} + \frac{0.875 \ln m}{N_c} + \mathcal{O}\left(\frac{1}{N_c^2}\right)\right)}{1 + \frac{0.94}{N_c} + \mathcal{O}\left(\frac{1}{N_c^2}\right)} =$$

$$-1.14\, N_c + 0.074 - \ln m + \mathcal{O}\left(\frac{1}{N_c}\right)$$

$$\bar{l}_4 = \frac{-0.079\, N_c \left(1 + \frac{0.232}{N_c} + \frac{12.6 \ln m}{N_c}\right)}{1 + \frac{0.47}{N_c} + \mathcal{O}\left(\frac{1}{N_c^2}\right)} =$$

$$-0.079\, N_c + 0.0187 - \ln m + \mathcal{O}\left(\frac{1}{N_c}\right)$$

The low-energy constant $\bar{l}_3$.

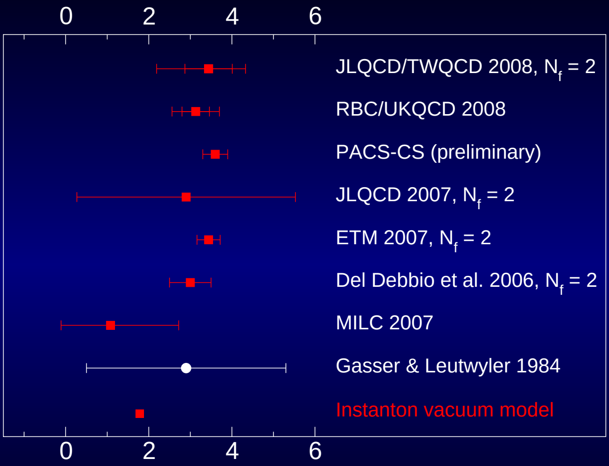


Figure: The low-energy constant $\bar{l}_3$: recent lattice results from different collaborations, phenomenological estimates from (Leutwyler 2008) and our result.

The low-energy constant $\bar{l}_4$.

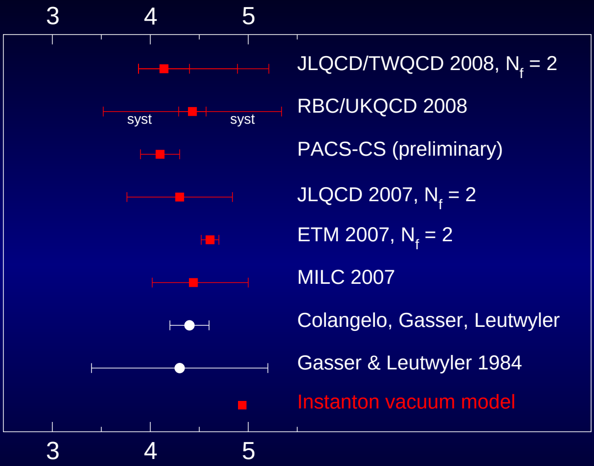


Figure: The low-energy constant $\bar{l}_4$: recent lattice results from different collaborations, phenomenological estimates from (Leutwyler 2008) and our result.

F_π, M_π

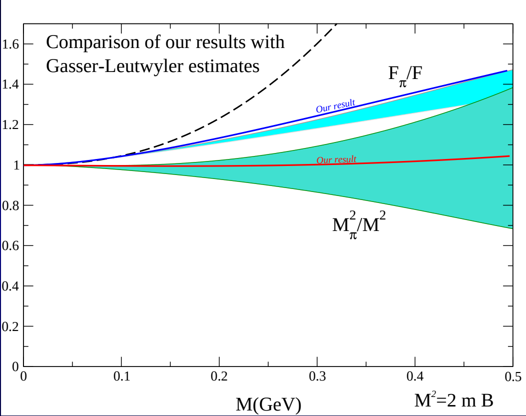


Figure: $m-$dependencies of F_π, M_π: comparison with phenomenological data from (Leutwyler 2001)

$m_u - m_d \neq 0$ and h_3, l_7

$$i\int dx \left\langle P^3(x)P^0(0)\right\rangle e^{iqx} \approx \frac{8B^3(m_u-m_d)l_7}{q^2-M_\pi^2} + O(q^2),$$

$$<\bar u u> - <\bar d d> = 4B^2(m_u-m_d)h_3.$$

Pion masses shift:

$$(M_{\pi_0}^2 - M_{\pi_\pm}^2)_{\delta m} = -(m_u-m_d)^2\frac{2B^2}{F^2}l_7 \approx -(m_u-m_d)^2 1.2\cdot 10^{-3} l_7.$$

Phenomenological estimate (GL, AP84) $l_7 \sim 5\cdot 10^{-3}$.

l_7 and h_3 in LO+NLO

$$l_7 \approx (6.6 \pm 2.4) \times 10^{-4}.$$

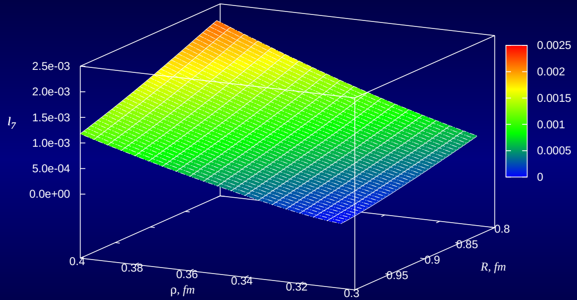


Figure: Dependence of the constant l_7 on the instanton vacuum parameters in the range $\rho \sim 0.32 - 0.35$ fm, $R \sim 0.8 - 1$ fm.

$$h_3 \approx 5.48 \times 10^{-3}.$$

QCD coupling $\alpha_s(Q)$ at the region $0 < Q < 1$ Gev

- Phenomenology and lattice measurements shows two options:
  1. $\alpha_s(Q)$ is scale invariant (conformal behavior), which is essential to apply a property of conformal field theories (CFT) to the study of hadrons: the Anti-de-Sitter space/Conformal Field Theory (AdS/CFT) correspondence.
  2. $\alpha_s(Q) \Rightarrow 0$ at $Q \Rightarrow 0$ and quasi-classical approximation to the QCD vacuum is applicable. This option are strongly supported by a number of lattice QCD studies.

Confinement and SBCS

- Quasi-classical QCD vacuum is a mixture of instantons and its constituents. Since the integration measure over collective coordinates is invariant under permutation of constituents belonging to different instantons, it allows instantons to overlap. This is a way for obtaining confinement.
- The momentum dependence of the dynamical quark mass measured at lattice perfectly coincide with the instanton vacuum model prediction with average instanton size $\bar{\rho} \sim 0.33$ fm and average inter-instanton distance $\bar{R} \sim 0.9$ fm. We conclude that such an instantons are responsible for the Spontaneous Breaking of Chiral Symmetry (SBCS).

ChPT low-energy constants

- We checked these assumptions by the evaluation the ChPT low-energy constants $\bar{l}_i$ *with account of all* $1/N_c$ *corrections*.
  - We evaluated the m-dependence of F_π, M_π and extracted the constants $\bar{l}_3$, $\bar{l}_4$. The found values are in reasonable agreement with lattice results and phenomenological estimates.
  - The calculated constant l_7, representing $m_u - m_d \neq 0$ effects, is a quite small and has a strong dependence on parameters of the model $\bar{R}, \bar{\rho}$. It will be quite important to make a lattice estimations of this quantity.