Fuzzy Topology, Quantization and Long-distance Gauge Fields

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Quantum space-time and novel geometries:

Discrete (lattice) space-time – Fundamental Plank length l_p
(Snyder, 1947; Markov,1948)

Noncommutative geometry (Connes, 1991)

Supergeometry (Rothstein,1986)

Hypothesis: Some nonstandard geometries can induce novel formalism of quantization (S.M.,2008)

Motivations of our theory :

Axioms of Set theory and Topology are the basis

of any particular geometry

Study of these fundamental structures can be important

for the construction of quantum space-time and related quantization

For simplicity only 1+1 geometry will be considered

Example: Set of all real numbers r is continuous ordered set R^1

R^1 is equivalent to fundamental set X of 1-dimensional Euclidian Geometry

Content

1. **Geometric properties induced by structure of fundamental set**

2. **Partial ordering of points, fuzzy ordering, fuzzy sets**

3. **Fuzzy manifolds and physical states**

4. **Derivation of Shroedinger dynamics for fuzzy states**

5. **Interactions on fuzzy manifold and gauge fields**

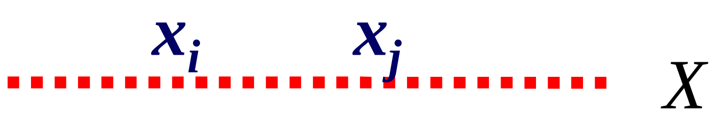
For simplicity only 1+1 geometry will be considered

Sets, Topology and Geometry

Example: Fundamental set of 1-dimensional Euclidian geometry is

(continious) ordered set of elements $X = \{ x_l \}$; x_l - points

$$\forall x_i, x_j \qquad x_i \le x_j . or . x_j \le x_i$$

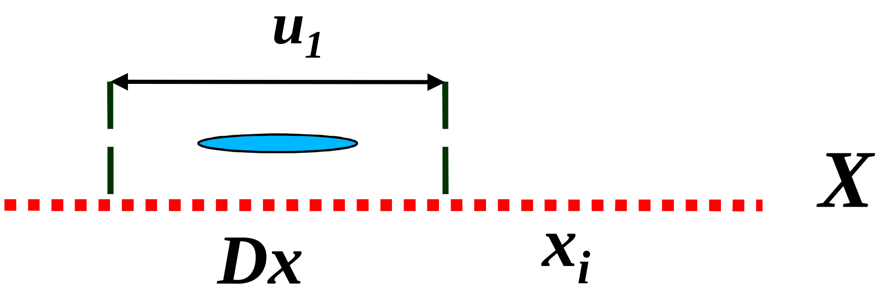


For illustration we consider first discrete set ***X***

In classical mechanics the particle *m* is ordered 'material' point *x(t)*

Partial ordered set - P

Beside $x_i \le x_j$ it can be also: $x_i \sim x_j$ - incomparabilty relation



S_T - total set; $S_T = X \cup P^u$

$X = \{ x_l \}$ - ordered subset; $P^u = \{ u_j \}$ - partial ordered subset

Example: let's consider interval - $Dx \in X$

$$u_1 \sim x_k , \quad \forall x_k \in Dx$$

u_1 , x_k are incomparable (equivalent) S_T elements

Fuzzy ordered set (Foset) - $F = \{ f_i \}$ *(Zeeman, 1964)*

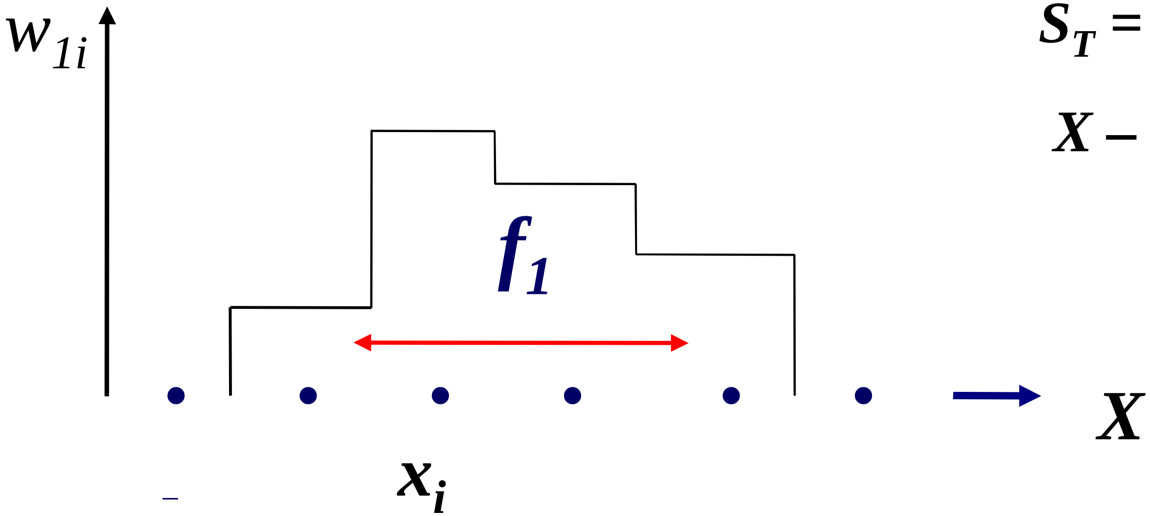
If f_j - fuzzy points, then: if $f_i \sim f_k$

$$\forall f_i , f_k , \quad \exists w_{i,k} > 0 , \quad F = \{ f_i \}$$ - set of fuzzy points; $\Sigma w_{jk} = 1$

Example: $\boldsymbol{S_T}$ - total set

$\boldsymbol{S_T = X}$ Y $\boldsymbol{F}$; $\boldsymbol{F = \{ f_1 \}}$

$\boldsymbol{X}$ – ordered subset



$$f_i \sim x_k , \quad \exists w_{i,k} > 0 , \qquad \Sigma w_{jk} = 1$$

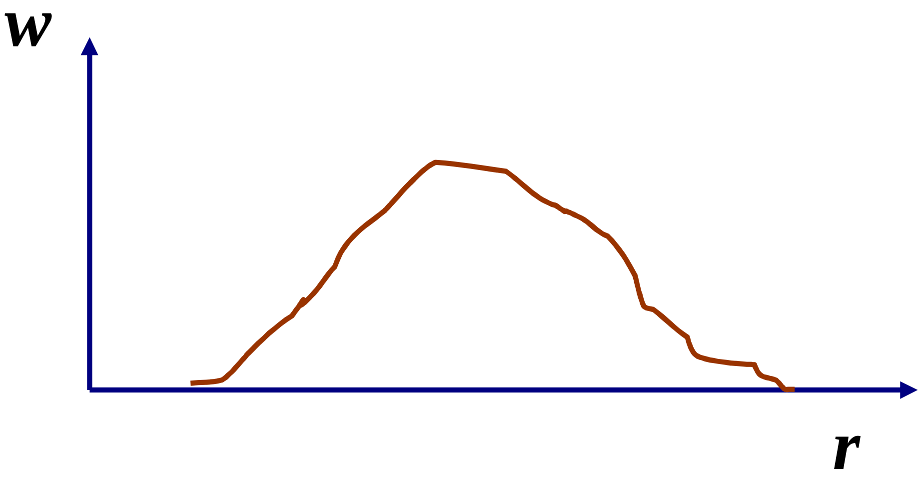
Continuous fuzzy sets

Example: S_T - total set ***;*** $S_T = R^1$ Y F;

$F = \{ f_1 \}$ – discrete subset of fuzzy points

X – continuous ordered subset; $X = \{ r_a \}$ $-\infty \leq r_a \leq \infty$

$$\forall\ f_i, r; \quad \exists\ w(r) > 0; \quad \int w(r)dr = 1$$



$\tilde{r}$ is fuzzy parameter

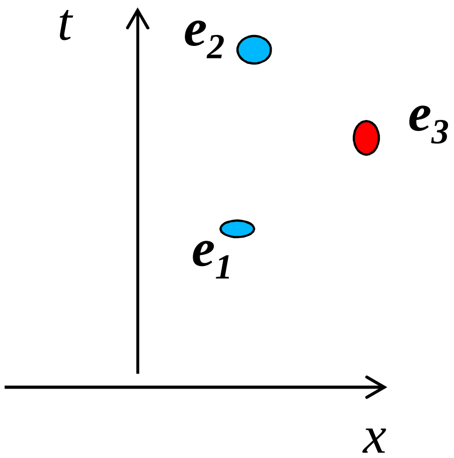
S_T is fuzzy space

w(r) isn't probability density, because S_T is topological structure ,

and not probabilistic one!

Partial ordering in classical mechanics

Beside geometric relations between events there is also causal relations



events - causal set: $\{ e_j \}$

Their time ordering corresponds to :

$$e_1 \leq e_3 \leq e_2$$

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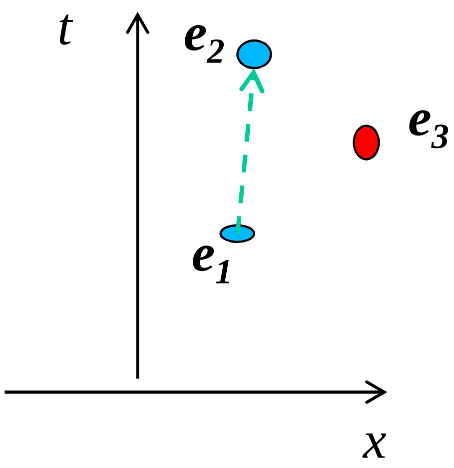


events - causal set: $\{e_j\}$

Their time ordering corresponds to:

$$e_1 \leq e_3 \leq e_2$$

now one event dynamically induce another :

$$e_1 \rightarrow e_2$$

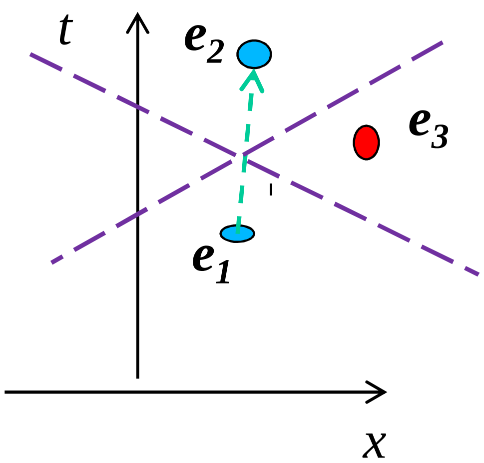
If to describe also causal relations between events:

$$e_1 \leq e_2 \;;$$
$$e_1 \sim e_3 \;;$$
$$e_2 \sim e_3$$

So e_3 is incomparable dynamically to e_1 and e_2

Partial ordering in special relativity

The causal relations acquire invariant, principal meaning due to light cone



event set: $\{e_j\}$

Their time ordering is :

$$e_1 \leq e_3 \leq e_2$$

Let’s describe also causal relations between events, then:

$$e_1 \leq e_2 \;;$$
$$e_1 \sim e_3 \;;$$
$$e_2 \sim e_3$$

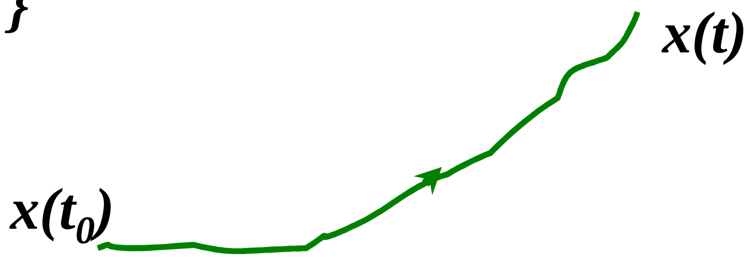
if e_3 is outside the light cone, then other relations between e_3 and $e_{1,2}$ can’t exist

Classical mechanics:

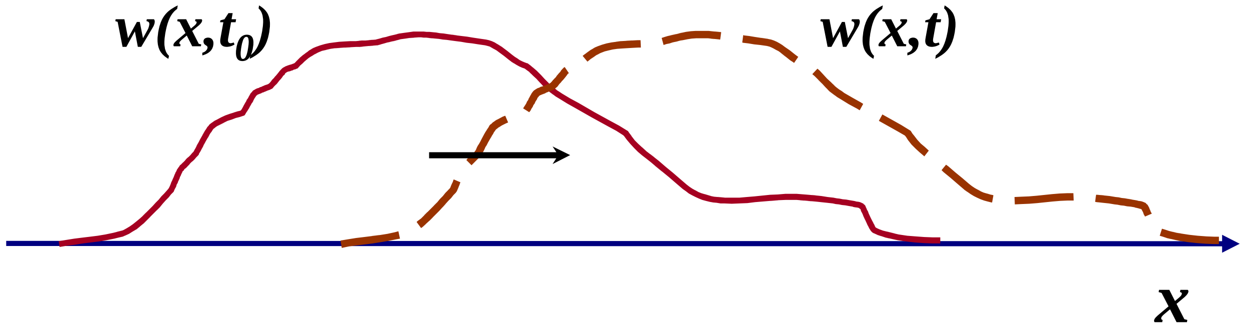
Massive particle *m* is ordered point ***x(t)***

its state $|\, g \,\} = \{(x(t),\ v_x(t),\ \ldots \}$

describes ***m*** trajectory



Fuzzy mechanics:

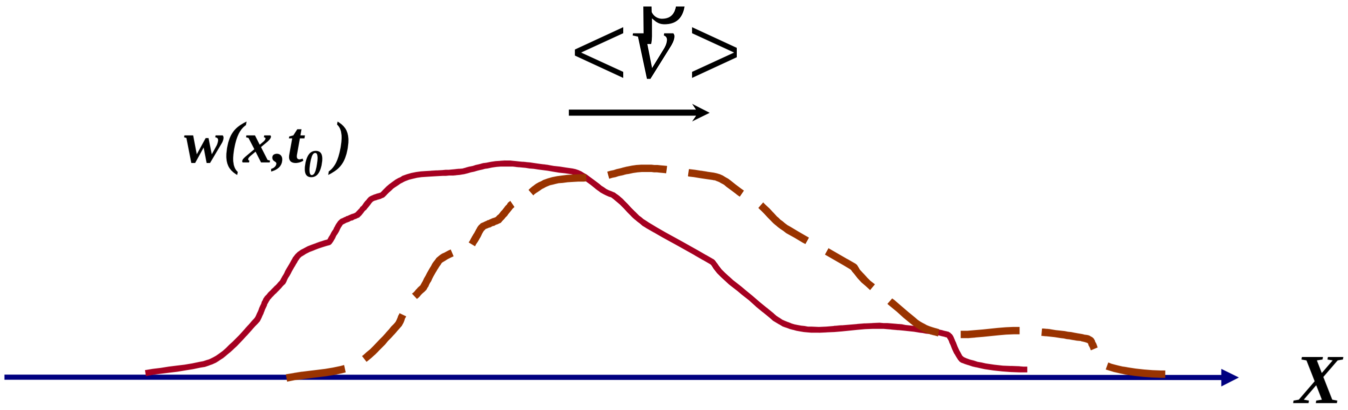


Massive particle is fuzzy point ***m(t),*** its coordinate $\tilde{x}(t)$ is fuzzy parameter

its state $|g\} = \{w(x,t), \ldots.\ \ ?\ \}$

Fuzzy motion

Fuzzy state $|g\}$: $|g\} = \{w(x), q_1, q_2 \ \dots\}$



Beside $w(x)$, another $|g\}$ parameter is $<\vec{v}(t)>$ - average *m* velocity

we shall look for another $|g\}$ parameters assuming they are

represented as functions $q(x)$

Fuzzy Evolution Equations

Fuzzy state $|g\} = \{w(x), q(x), \ldots\};$ Let's consider free ***w(x,t)*** evolution

Fuzzy (partial) ordering assumes that only short-distance correlations exist,

so ***w(x)*** evolution can be described as:

$$\frac{\partial w}{\partial t} = -f(x,t)$$

where f - arbitrary function

then from *w(x)* norm conservation:

$$\int_{-\infty}^{\infty} f(x)dx = \int_{-\infty}^{\infty} \frac{\partial w}{\partial t}(x,t)dx = \frac{\partial}{\partial t}\int_{-\infty}^{\infty} w(x,t)dx = 0$$

Fuzzy Evolution Equations

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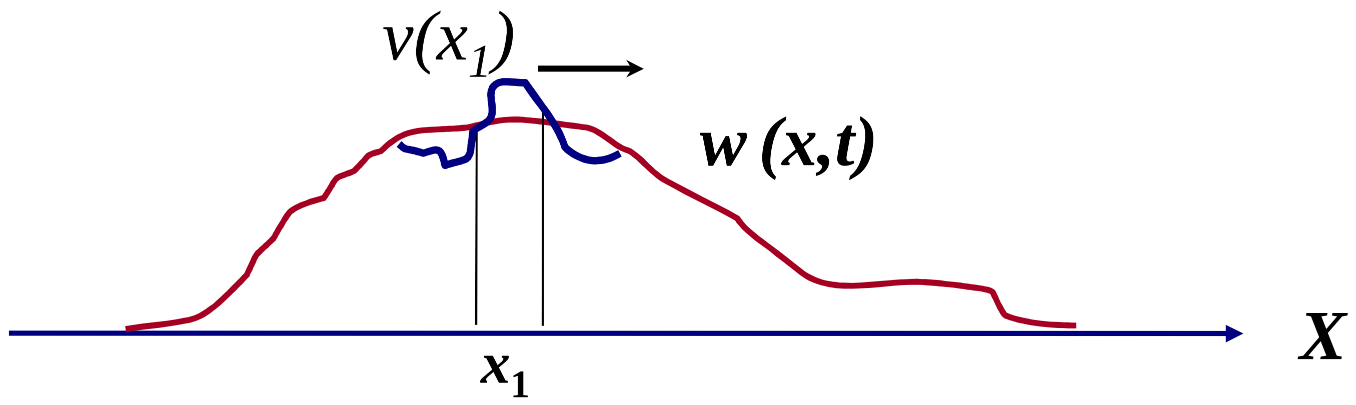
if to substitute : $\frac{\partial J}{\partial x} = f(x)$ then $J(\infty,t) - J(-\infty,t) = 0$

from ***X*** - reflection invariance: $J(\pm\infty,t) = 0$ then it results in:

$$\boxed{\frac{\partial w}{\partial t} = -\frac{\partial J}{\partial x}(x,t)}$$

- 1-dimensional flow continuity equation



in 1-dimension *m* flow can be written as: $J(x) = v(x)w(x)$

where *v(x)* is *w* flow velocity, it is supposed to be independent of *w(x)*

hence ***v(x)*** can be chosen as free parameter of ***m*** state ***g***

m state ***g(x) = { w(x), v(x)}*** **–** it's physical (fuzzy) ***g*** representation

Symmetric Representation of Fuzzy state g

if ***m*** state ***g = { w(x), v(x)}***

let's substitute $v(x)$ by: $\alpha(x) = k\int_{-\infty}^{x} v(\varsigma)d\varsigma$

here k is arbitrary constant

then average *m* velocity: $<V_x> = \frac{1}{k} < \frac{d\alpha}{dx} >$

hence $\alpha(x)$ is analogue of quantum phase

and so *m* state is: $g = \{ w(x), \alpha(x)\}$

It can be transformed to symmetric complex *g* representation:

$$g(x) = \sqrt{w(x)} \cdot e^{i\alpha(x)}$$

here $g(x) \rightarrow \{ w(x), v(x)\}$ is unambiguous map

Evolution Equation for Fuzzy state g

$$i\frac{\partial g}{\partial t} = \hat{H} g$$

We start with linear operator ***H*** for fuzzy dynamics

later the nonlinear operators will be considered also

For free m motion ***H*** should be invariant relative to space shifts so:

$$[\hat{H}, \frac{d}{dx}] = 0$$

It means that ***H*** can be only polinom:

$$\hat{H} = \sum_{i=2}^{n} c_i \frac{\partial^i}{\partial x^i}$$

Here ***i*** can be only even numbers, because noneven ***i*** contradict to ***X*** reflection invartiance

equation for operator *H*

From flow equation: $\frac{\partial w}{\partial t} = -\frac{\partial}{\partial x}(wv)$ and $v(x) = \frac{1}{k}\frac{\partial \alpha}{\partial x}$

it follows: $\frac{\partial \sqrt{w}}{\partial t} = -\frac{1}{k}\frac{\partial \alpha}{\partial x}\frac{\partial \sqrt{w}}{\partial x} - \frac{1}{2k}\sqrt{w}\frac{\partial^2 \alpha}{\partial x^2}$

$g(x) = \sqrt{w}e^{i\alpha}$ hence: $i\frac{\partial g}{\partial t} = (i\frac{\partial \sqrt{w}}{\partial t} - \sqrt{w}\frac{\partial \alpha}{\partial t})e^{i\alpha}$

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$$g(x) = \sqrt{w}e^{i\alpha}$$ hence: $$i\frac{\partial g}{\partial t} = \left(i\frac{\partial \sqrt{w}}{\partial t} - \sqrt{w}\frac{\partial \alpha}{\partial t}\right)e^{i\alpha}$$

and $$i\frac{\partial g}{\partial t} = \hat{H}g = \sum_{i=2}^{n} c_i \frac{\partial^i g}{\partial x^i} = \sum_{i=2}^{n} c_i \frac{\partial^i}{\partial x^i}(\sqrt{w}e^{i\alpha})$$

$$i\frac{\partial g}{\partial t} = \left(\sqrt{w}\frac{\partial \alpha}{\partial t} - \frac{i}{k}\frac{\partial \alpha}{\partial x}\frac{\partial \sqrt{w}}{\partial x} - \frac{i}{2k}\sqrt{w}\frac{\partial^2 \alpha}{\partial x^2}\right)e^{i\alpha} = \hat{H}\sqrt{w}e^{i\alpha} = \ldots + ic_n\sqrt{w}\frac{\partial^n \alpha}{\partial x^n})e^{i\alpha}$$

From comparison of highest α derivatives it follows *n=2;* $c_2 = -\frac{1}{2k}$

fuzzy state *g(x,t)* obeys to free Schroedinger equation with mass *m = k*

Nonlinear evolution operator

Bargman - Wigner theorem *(1964)* : $\hat{U}_t g(t_0) \to g'(t)$

If arbitrary pure state evolves only into the pure state and module of scalar product for all pure states $|<\psi|\varphi>|$ is conserved, then the evolution operator is linear.

Jordan (2006): If for any $\hat{U}_t\, g(t_0) \to g'(t)$

and U_t^{-1} is defined unambiguously, then U_t is linear operator

i.e. if pure states evolve only to pure ones

Fuzzy pure states $g(x,t)$ evolve only to pure ones, so their evolution should be linear

$g(x,t)$ are normalized complex functions,

we obtained free particle's quantum evolution on Hilbert space $\mathcal{H}$

Fuzzy space: $\mathcal{F} = \{ \tilde{\chi} \}$ induces Hilbert space $\mathcal{H}$ of normalized complex functions $g(x,t)$

Observables Q are supposed to be linear, self-adjoint operators on $\mathcal{H}$

In particular, particle’s momentum: $p_x = i\frac{d}{dx}$

hence $[p_x, x] = i$ commutation relations are obtained from geometrical and Set-theoretical arguments, I.e. in this case fuzzy topology represents the underlying structure for such operator algebra

In this model Plank constant is just coefficient, which connects x and p_x scales, so Relativistic unit system gives its natural description

$$\eta = c = 1 \qquad e^2 = \frac{1}{137}$$

Fuzzy Correlations

We exploit fuzzy state: $g(x) = \sqrt{w(x)} \cdot e^{i\alpha(x)}$

Yet more correct expression is: $g = \{w(x), \alpha(x, x')\}$

here: $\alpha(x, x') = \alpha(x) - \alpha(x')$

is the state g correlation between points x, x'

it describes $w(x)$ flow velocity $v(x)$ and defines state g free evolution.

Such ansatz corresponds to quantum density matrix

More complicated correlations can appear in fuzzy mechanics.

We didn't suppose that our theory possesses Galilean invariance, in fact,

Galilean transformations can be derived from fuzzy dynamics,

if to suppose that any RF corresponds to physical object with $m \to \infty$

Dirac equation : $i\frac{\partial \psi}{\partial t} = (\vec{\alpha}\vec{p} + \beta m)\psi$

can be derived assuming relativistic invariance of fuzzy dynamics

Its fuzzy space: $F = \{ \tilde{r}^3 \otimes \tilde{s}_z \}$

We derived free particle's dynamics from some geometrical and set-theoretical axioms.

Is it possible to describe particle's interactions in the same framework ?

We exploited $\boldsymbol{m}$ states: $\boldsymbol{g} = \{ w(x), \alpha(x)\}$ **defined in**

flat Hilbert space $\mathcal{H}$

Hypothesys: By the analogy with general relativity

we suppose that for fuzzy topology the external field

induces curvature of $\mathcal{H}$ or $\mathcal{H} \otimes \mathcal{I}$ **fiber bundle**

Interactions in Fuzzy Space

Free Schroedinger equation: $i\frac{\partial g}{\partial t} = Hg$

is equivalent to system of two equations for real and imaginary parts of $i\frac{\partial g}{\partial t}$

I) $\frac{\partial \sqrt{w}}{\partial t} = -\frac{1}{m}\frac{\partial \alpha}{\partial x}\frac{\partial \sqrt{w}}{\partial x} - \frac{1}{2m}\sqrt{w}\frac{\partial^2 \alpha}{\partial x^2}$ - flow equation

II) $\frac{\partial \alpha}{\partial t} = \frac{1}{m\sqrt{w}}\frac{\partial^2 \sqrt{w}}{\partial x^2} - \frac{1}{2m}\left(\frac{\partial \alpha}{\partial x}\right)^2$

here **I)** is kinematical equation, it connects *w(x)* and flow velocity $v(x) = \frac{1}{m}\frac{\partial \alpha}{\partial x}$

Hence particle's interactions can be included only in **II)**

Interactions in Fuzzy Space

Free Schroedinger equation: $i\frac{\partial g}{\partial t} = \hat{H}g$

Is equivalent to system of two equations:

I) $$\frac{\partial \sqrt{w}}{\partial t} = -\frac{1}{m}\frac{\partial \alpha}{\partial x}\frac{\partial \sqrt{w}}{\partial x} - \frac{1}{2m}\sqrt{w}\frac{\partial^2 \alpha}{\partial x^2}$$ - flow equation

II) $$\frac{\partial \alpha}{\partial t} = \frac{1}{m\sqrt{w}}\frac{\partial^2 \sqrt{w}}{\partial x^2} - \frac{1}{2m}\left(\frac{\partial \alpha}{\partial x}\right)^2 + H_{int}$$

here **I)** is kinematical equation, it connects $w(x)$ and flow velocity $v(x) = \frac{1}{m}\frac{\partial \alpha}{\partial x}$

Hence particle's interactions can be included only in **II)**

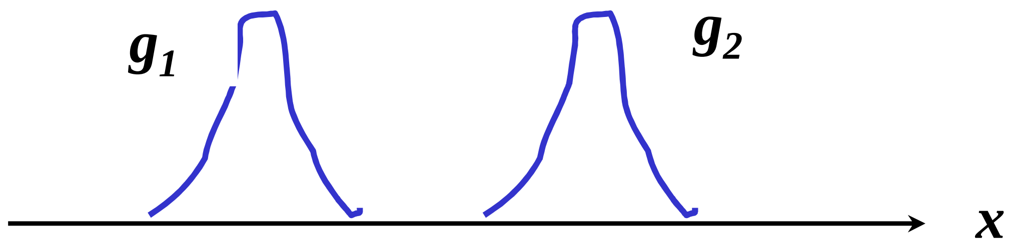
But it supposes their gauge invariance, because $\alpha(x,t)$ is quantum phase

Particle's Interactions in Fuzzy Space - Toy model

I) $$\frac{\partial\sqrt{w}}{\partial t} = -\frac{1}{m}\frac{\partial\alpha}{\partial x}\frac{\partial\sqrt{w}}{\partial x} - \frac{1}{2m}\sqrt{w}\frac{\partial^2\alpha}{\partial x^2}$$

II) $$\frac{\partial\alpha}{\partial t} = \frac{1}{m\sqrt{w}}\frac{\partial^2\sqrt{w}}{\partial x^2} - \frac{1}{2m}\left(\frac{\partial\alpha}{\partial x}\right)^2 + H_{int}$$

Let's consider interaction of two particles $\boldsymbol{m_1}$, $\boldsymbol{m_2}$, their states: $\boldsymbol{g_{1,2}(x,t)}$



In geometrical framework it's natural to assume that H_{int} is universal,

i. e. if $|\vec{p}_{1,2}| \to 0$, then $H_{\mathrm{int}} \neq 0$ It means that

there is term in $H_{int} \sim Q_1\, Q_2\, F(r_{12})$, where $\boldsymbol{Q_1}$,$\boldsymbol{Q_2}$ are particle's charges.

Particle's Interactions on fuzzy manifold

to restore space symmetry two identical particles $\boldsymbol{n_1}$, $\boldsymbol{n_2}$ should repulse each other ?

Deflections from $\boldsymbol{m_1}$ free motion induced by $\boldsymbol{m_2}$ presence at some distance,

even if $|\vec{p}_{1,2}| \to 0$,yet $H_{\text{int}} \neq 0$

It means that $H_{\text{int}} = Q_1 Q_2 F(r_{12}) = Q_1 A_0(r_{12})$ for $|\vec{p}_{1,2}| \to 0$

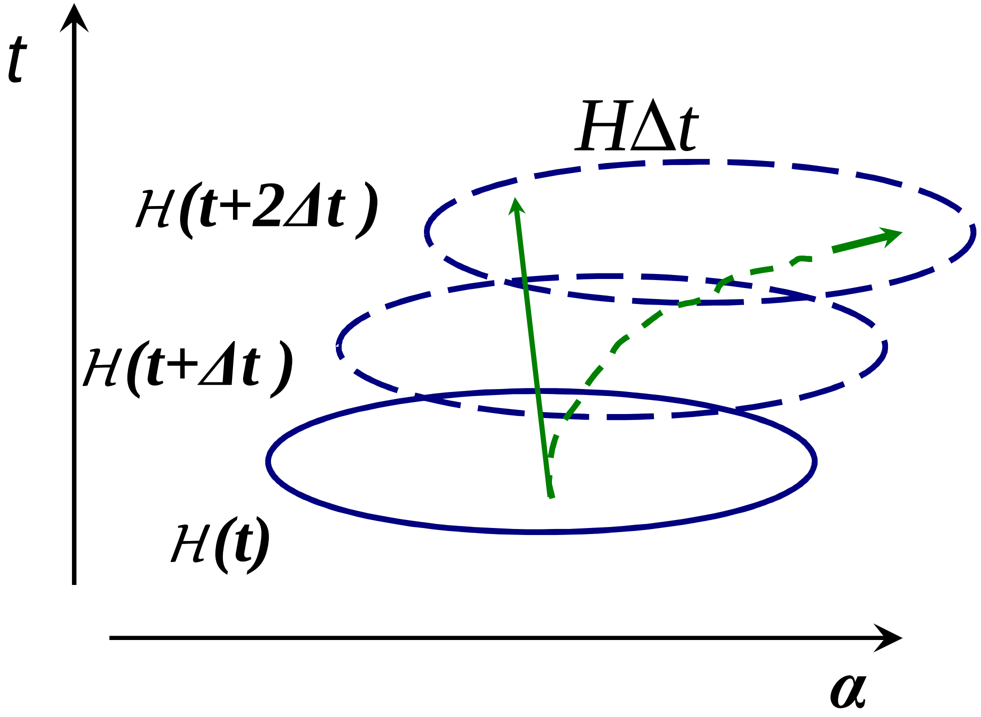
$$\frac{\partial \alpha}{\partial t} = \frac{1}{m\sqrt{w}} \frac{\partial^2 \sqrt{w}}{\partial x^2} - \frac{1}{2m} \left(\frac{\partial \alpha}{\partial x}\right)^2 + H_{int}$$

Hence: $$\frac{\partial \alpha}{\partial t} = \left(\frac{\partial \alpha}{\partial t}\right)_{free} + Q_1 A_0(\vec{r}_{12})$$

this is deformation of $\boldsymbol{m_1}$ fiber bundle $\mathcal{H} \otimes T$

$$\frac{\partial \alpha}{\partial t} = \left(\frac{\partial \alpha}{\partial t}\right)_{free} + Q_1 A_0(\vec{r}_{12})$$

Hence: $\Delta\alpha = \Delta\alpha_0 + Q_1 A_0(r_{12})\Delta t = \Delta\alpha_0 + H_{int}\Delta t$



Hence this is shift of $\alpha(x,t)$ subspace of $E_t = \mathcal{H} \otimes T$ - fiber bundle

So that each $\mathcal{H}(t_i)$ is flat but their connection $C(t_i, t_k)$ depends on H_{int}

Particle's Interactions in Fuzzy Space

suppose that m_1 is described by Klein-Gordon equation with $E>0$ and m_2 is classical particle, i.e. $m_2 \longrightarrow \infty$

in initial RF:
$$i\frac{\partial g}{\partial t} = \sqrt{\frac{\partial^2}{\partial \vec{r}^{\,2}} + m_1^2}\, g + Q_1 A_0(\vec{r})\, g$$

For Lorenz transform RF to RF' with velocity $\vec{v}$

$$i\frac{\partial g'}{\partial t} = \sqrt{\left[i\frac{\partial}{\partial \vec{r}\,'} + Q_1 \vec{A}'(\vec{r}\,')\right]^2 + m_1^2}\, g' + Q_1 A'_0(\vec{r}\,')\, g'$$

where : $\vec{A}' = A_0 \dfrac{\vec{v}}{\sqrt{1-\vec{v}^{\,2}}}$ and $A'_0 = A_0 \dfrac{1}{\sqrt{1-\vec{v}^{\,2}}}$

only such vector field $\vec{A}$ makes $g \rightarrow g'$ transformation consistent

So this toy-model is equivalent to Electrodynamics if A_μ is massless field

Fuzzy Mechanics and Gauge Invariance

In QFT local gauge invariance is postulated.

We shall argue that in Fuzzy mechanics it can be derived

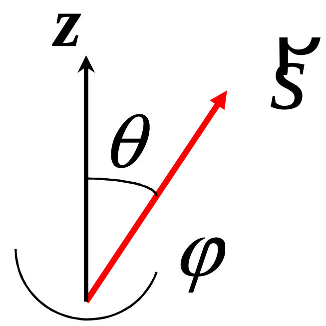
from global isotopic invariance

Fuzzy state for fermion (iso)doublet

Consider fermion SU_2 (iso)spin doublet of mass m_0

Fuzzy parameters: $\tilde{x}, \tilde{y}, \tilde{z}, \tilde{s}_z$; particle's state : $g(\vec{r}, s_z)$

Fuzzy representation: $g = w(\vec{r}), \alpha(\vec{r}), \theta(\vec{r}), \varphi(\vec{r})$



$$g \to \psi_s(\vec{r}) = \begin{Bmatrix} \psi_1 \\ \psi_2 \end{Bmatrix}$$ - linear representation

$\vec{\xi}$ - (iso)spin vector

$$i\frac{\partial \psi_s}{\partial t} = H\psi_s$$

from Jordan theorem ψ_s evolution should be linear

Nonabelian case: **Yang-Mills Fields**

Consider interaction of two femions ***m₁ , m₂*** of SU_2 doublet and suppose

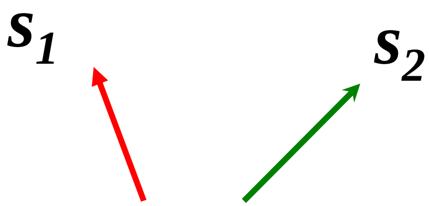
that their interaction is universal, so that at $\left|\vec{p}_{1,2}\right| \to 0$ it gives $H_{\text{int}} \neq 0$

$$\psi_s(\vec{r}) = \begin{Bmatrix} \psi_1 \\ \psi_2 \end{Bmatrix}$$

$\vec{S}_{1,2}$ - isospin vectors

To agree with global isotopic symmetry, two isospin vectors in nonrelativistic limit

can interact with each other only via this ansatz: $H_{\text{int}} = k\vec{s}_1\vec{s}_2 f(r_{1,2})$



Hence: $\vec{A}_0(r_{12}) = \vec{s}_2 f(r_{12})$

and so : $H_{\text{int}} = \lambda\vec{\sigma}_1\vec{A}_0(r_{1,2})$ - Yang-Mills

For nonrelativistic QCD limit it gives: $f(r) \approx \dfrac{g}{r^2}$

But it supposes that local gauge invariance follows from
global isotopic SU_2 symmetry :

Yang-Mills Interactions

Suppose that $\boldsymbol{m_1}$ is described by Klein-Gordon equation with ***E>0***

and $\boldsymbol{m_2}$ is classical particle, i.e. $\boldsymbol{m_2} \longrightarrow \infty$***;*** and $\left|\vec{p}_{1,2}\right| \to 0$

so $\boldsymbol{m_2}$ generates classical SU_2 Yang-Mills field $\vec{A}_0(r)$

in initial RF: $$i\frac{\partial g}{\partial t} = \sqrt{\frac{\partial^2}{\partial r_j^2} + m_1^2}\, g + Q_1 \vec{A}_0(r_j)\, g$$

For Lorenz transform RF to RF' with velocity $\vec{v}$

$$i\frac{\partial g'}{\partial t} = \sqrt{\left[i\frac{\partial}{\partial r_j'} + Q_1 \vec{A}_j'(r_j')\right]^2 + m_1^2}\, g' + Q_1 \vec{A}'_0(r_j')\, g'$$

where : $\vec{A}_j' = \vec{A}_0 \dfrac{v_j}{\sqrt{1-\vec{v}^2}}$ and $\vec{A}'_0 = \vec{A}_0 \dfrac{1}{\sqrt{1-\vec{v}^2}}$

only such vector field $\vec{A}_j$ makes $g \to g'$ transformation consistent

So we obtain standard Yang-Mills Hamiltonian for particle-field interaction

(Non)locality of interactions on fuzzy space

We supposed that our model interactions are local, yet this is extra assumption, two particles with fuzzy coordinates $\tilde{x}_{12}$ can interact nonlocally, or fermion with fuzzy coordinate $\tilde{x}$ can nonlocally interact with boson field.

Fuzzy Space and Noncommutative Theories

$[p_{x,}\, x] = i$ commutation relations were obtained from geometrical and Set-theoretical arguments, I.e. in this case fuzzy space represents the underlying structure for such operator algebra

What about $[y_{,}\, x]$ and $[p_x, p_y]$?

Modified fuzzy space F supposedly can be underlying structure for them

Fuzzy Approach to Quantization

Consider an arbitrary system ***S***

How to perform its quantization ?

I) To define main ***S*** coordinates q_i and to assume that they are fuzzy values with distributions ***w(q)***.

ii) To analyze ***S*** free evolution and to define the state correlations α between ***q, q'*** points

Example: Fock quantized fields $\{ n_k \}$

n_k - particle numbers can be regarded as fuzzy values

Conclusions

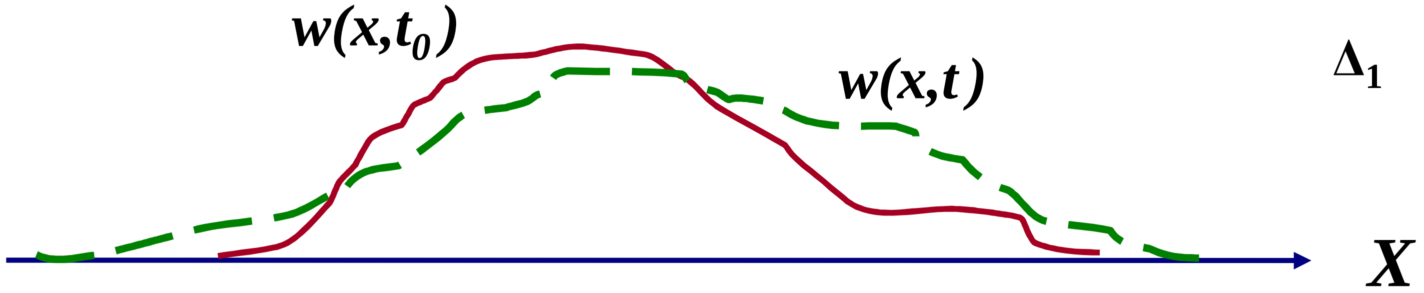
1. Fuzzy topology is simple and natural formalism for introduction
   of quantization into physical theory
2. Shroedinger equation is obtained from simple assumptions,
   Galilean invariance follows from it.
3. Gauge invariance of fields corresponds to dynamics on fuzzy manifold
4. Interactions in fuzzy space are generically nonlocal

Fuzzy evolution and linearity

We don't assume that ***w(x,t)*** is linear function of ***w(x',t₀)***

which is standard assumption of classical kinetics

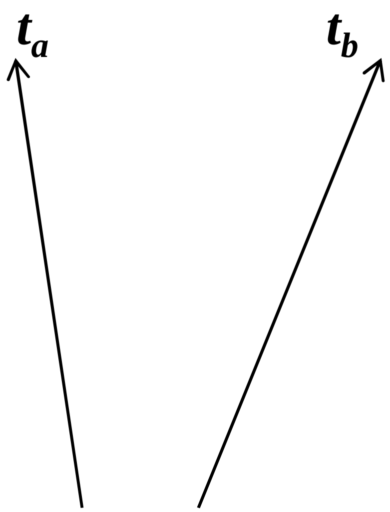
$$w(x,t) = \int G(x - x', t - t_0) w(x', t_0) dx'$$



Example : Boltzman diffusion

Clocks, proper times and partial ordering

In special relativity each dynamical system has its own proper time, there is no universal time, so proper time connected with clock dynamics of given reference frame (***RF)***



$$\frac{\partial w}{\partial t} = -div\,\vec{j}$$

Constraint: each event time t_1 in one RF is mapped to the point on time axe in other RF t'_1

$e_1 \sim e_3$

m particle's state $|g\}$ **= { *w(x)*, *α(x)* }**

The problem : to find dynamical $|g\}$ representation $g(x,t)$:

$$i\frac{\partial g}{\partial t} = \hat{H}g$$

$\hat{H}$ is dynamical operator (Hamiltonian)

We have two degrees of freedom: $g = \{ w(x), \alpha(x) \}$,

it can be transferred to: $g(x) = g_1(x) + ig_2(x)$

such that $g_1(x), g_2(x) = 0$, if $w(x) = 0$

This is symmetric ***g*** representation

example: $g(x) = \sqrt{w(x)} \cdot e^{i\alpha(x)}$

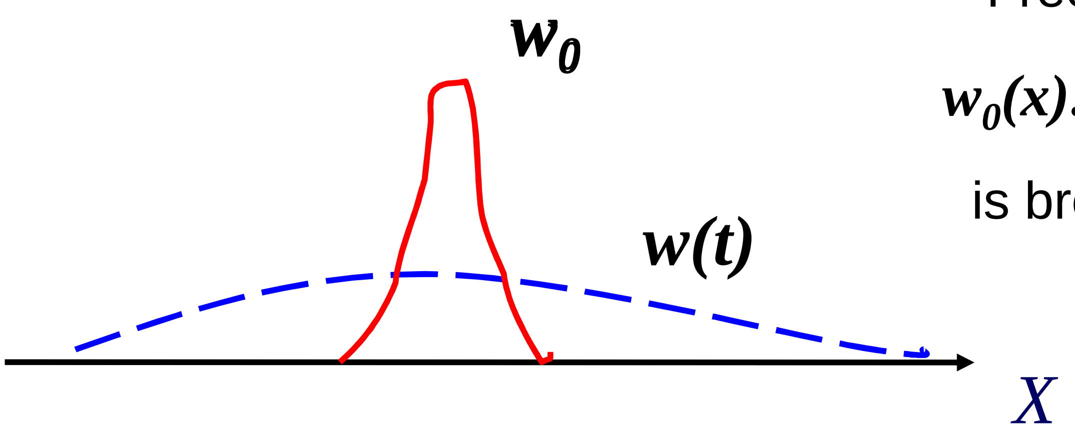
Generalization of event time mapping

one can substitute points by δ-functions

Fuzzy dynamics – space symmetry restoration

as the possible fundamental low of dynamics

Global symmetries are important in quantum physics



Free particle ***m:***

$w_0(x)$: space shift symmetry

is broken

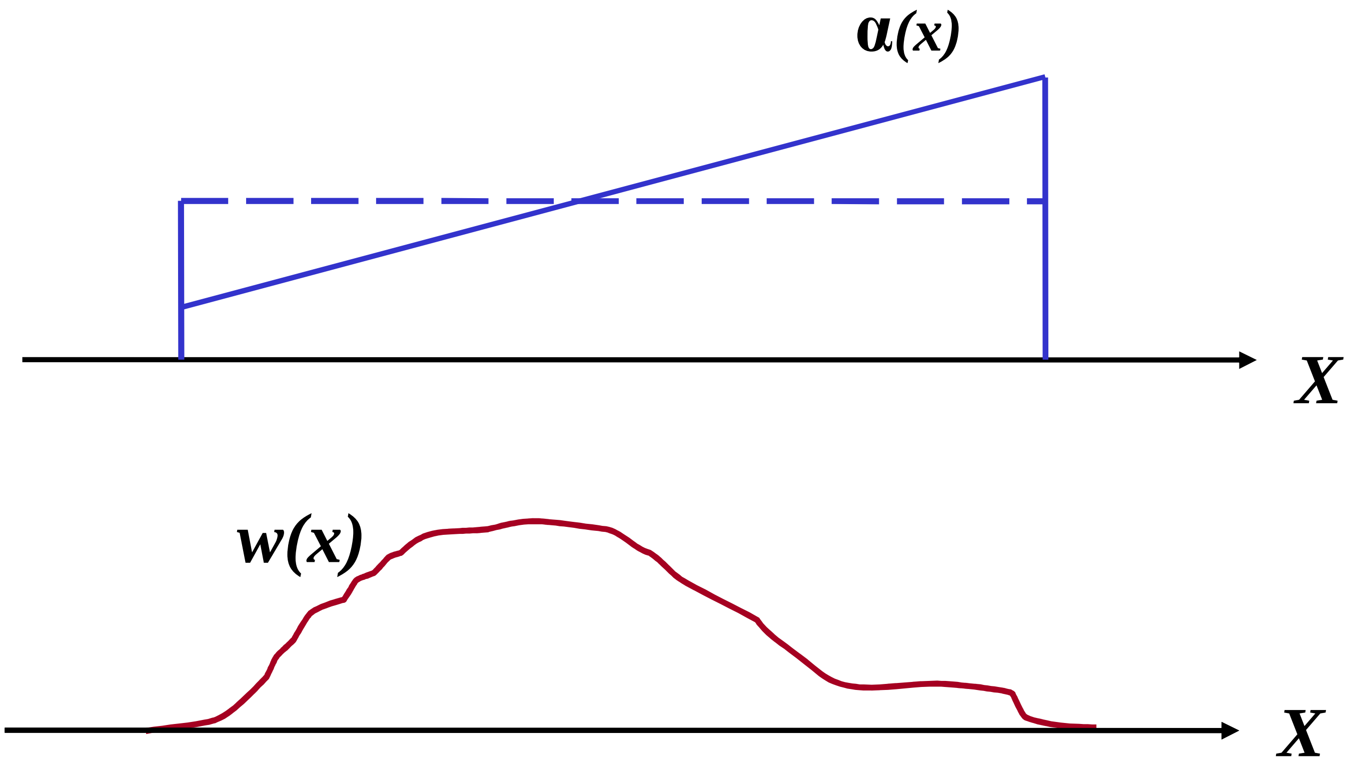
Low of fuzzy evolution :

space symmetry of *m* state |*g*} is restored as fast as possible :

So free *m* evolution *U(t) :* $w_0(x) \rightarrow const$ at very large *t*

Interactions: to restore space symmetry two identical particles e_1 , e_2 should repulse each other

m motion can be described as the space symmetry violation for |*g*}

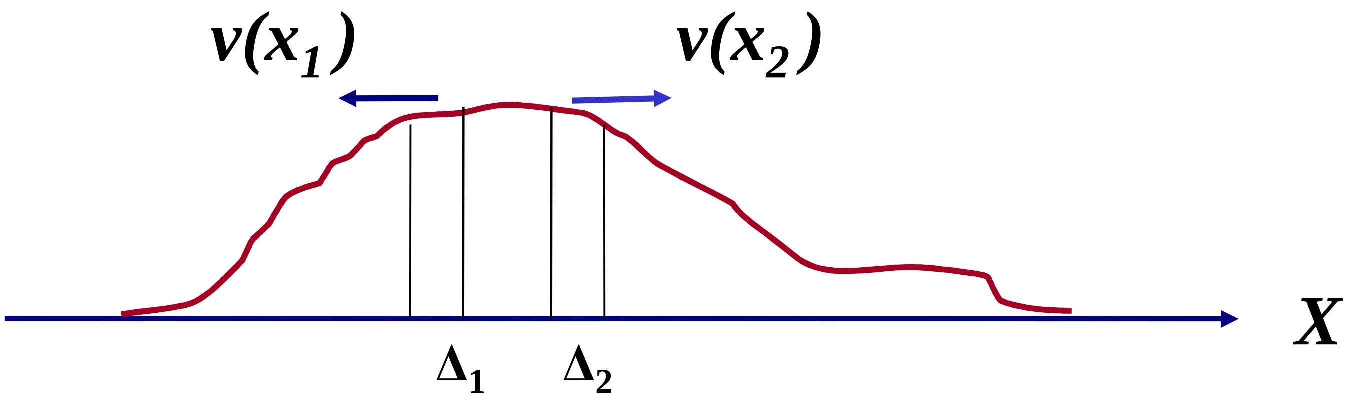


$$m \text{ velocity: } \langle V_x \rangle = \left\langle \frac{d\alpha}{dx} \right\rangle$$

α(*x*) is analog of quantum phase

hence w flow equation can be written as:

$$\frac{\partial w}{\partial t} = -\frac{\partial j}{\partial x} = -v\frac{dw}{dx} - w\frac{dv}{dx}$$

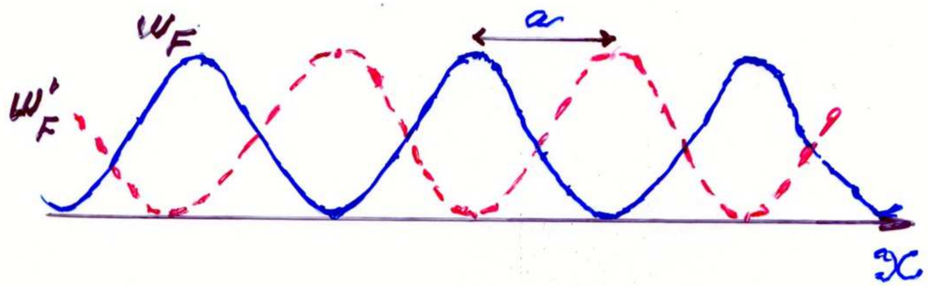


$$\frac{\partial w}{\partial t} = -div\,\vec{j}$$

$$v(x) = -\frac{1}{w(x)}\int_{-\infty}^{x}\frac{\partial w}{\partial t}(\varsigma)\,d\varsigma$$

$|g_0\} \sim w_0(x) = \frac{1}{2}\delta(x - x_1) + \frac{1}{2}\delta(x - x_2)$

Does $|g_0\}$ include other parameters g_i?



for $|g_F\}$ $\langle x \rangle$ is undefined

hence if $w_F(x,t)$ is solution

then $w_F'(x,t) = w_F(x+a,t)$ is solution

$\forall a$; $-\infty \leq a \leq \infty$

so, $|g_0\}$ has a-parameter
in addition to $w_0(x)$

Fuzzy mechanics – space symmetry restoration

as the general low of free motion