

# Baryon Structure in AdS/QCD

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PRD 80 (2009) 055014  
PRD 82 (2010) 074022  
PRD 83 (2011) 036001  
PRD 85 (2012) 076003  
PRD 86 (2012) 036007

ISHEPP-XXI, September 10-15, 2012, JINR, Dubna

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# Introduction

- **AdS/QCD  $\equiv$  Holographic QCD (HQCD)** – approximation to QCD: attempt to model Hadronic Physics in terms of fields/strings living in extra dimensions – **anti-de Sitter (AdS) space**
- **HQCD models** reproduce main features of QCD at low and high energies: chiral symmetry, confinement, power scaling of hadron form factors
- **Motivation:** **AdS/CFT correspondence** 1998 (Maldacena, Polyakov, Witten et al)  
  
Dynamics of the superstring theory in **AdS<sub>5</sub>** background is encoded in 4-dim conformal field theory living on the AdS boundary.  
  
**More general:** extra-dimensional (ED) theories including gravity are holographically equivalent to gauge theories living on boundary of ED space
- **Symmetry arguments:** Conformal group acting in boundary theory isomorphic to **SO(4, 2)** – the isometry group of **AdS<sub>5</sub>** space

# Introduction

- AdS metric  $ds^2 = \frac{R^2}{z^2} (dx_\mu dx^\mu - dz^2)$  Poincaré form  
 $z$  is extra dimensional (holographic) coordinate;  $z = 0$  is UV boundary  
Light-Front Holography (Brodsky-Teramond)  $z \rightarrow \zeta$  with  $\zeta^2 = \mathbf{b}_\perp^2 x(1-x)$
- Fields in AdS<sub>5</sub> classified by representation  $D(\Delta, J_1, J_2)$  of  $SO(4, 2)$   
 $\Delta$  – conformal dimension,  $J_1$  and  $J_2$  – spins related to  $SU(2)$
- Example: Scalar fields  $\Phi(x, z)$  according  $D(3, 0, 0)$  with  $\Delta = 3$  and  $J_1 = J_2 = 0$   
Holographic analogue of scalar mesons with  $J = 0$
- Action:  $S = \frac{1}{2} \int d^4x dz \sqrt{g} \left[ \partial_M \Phi(x, z) \partial^M \Phi(x, z) - m^2 \Phi^2(x, z) \right]$   
where  $m$  – 5D mass,  $m^2 R^2 = \Delta(\Delta - 4)$ .
- Kaluza-Klein (KK) expansion  $\Phi(x, z) = \sum_n \phi_n(x) \Phi_n(z)$
- Tower of KK modes  $\phi_n(x)$  dual to 4-dimensional fields describing hadrons
- Bulk profiles  $\Phi_n(z)$  dual to hadronic wave functions.  
Constrained dynamics: breaking conformal symmetry and introducing confinement due to background fields (dilaton) or infrared cut of  $z$ -coordinate

# Introduction

- Bound state problem:  
Derive Schrödinger-type EOM for  $\Phi_n(z)$ , solve it on mass-shell  $p^2 = M_H^2$ .  
Solutions: hadronic w.f. and mass spectra.
- Scattering problem for AdS field (without KK expansion) gives information about propagation of external field from  $z$  to the boundary  $z = 0$  — bulk-to-boundary propagator  $V(q, z)$  — Fourier-transform of AdS field  $V(x, z) = \int \frac{d^4 q}{(2\pi)^4} e^{-iqx} V(q, z)$   
On UV boundary  $V(q, z = 0) = 1$ .
- Hadron properties  
$$F_n(Q^2) = \int_0^\infty dz V(Q, z) \Phi_n^2(z), \quad Q^2 = -q^2$$
  
$$F_n(0) \equiv 1 \quad \text{due to } V(0, z) = 1 \text{ and } \int_0^\infty dz \Phi_n^2(z) = 1$$
- Nonlocality of strong interactions implemented by the 5-th (holographic) coordinate and hidden in the bulk profiles  $\Phi_n(z)$  and bulk propagators  $V(Q, z)$
- No scale or frame dependence, all gauge and discrete symmetries of 4-dim theory are preserved

# Introduction

- **Top-down approaches** Low-energy approximation of string theory trying to find a gravitational background with features similar to QCD (e.g. Sakai-Sugimoto model)
- **Bottom-up approaches** More phenomenological use the features of QCD to construct 5D dual theory including gravity on AdS space
- **Hard-wall:**  
AdS geometry is cutted by two branes **UV** ( $z = 0$ ) and **IR** ( $z = z_{\text{IR}}$ )  
Analogue of quark bag model, linear dependence on  $J(L)$  of hadron masses
- **Soft-wall:**  
Soft cutoff of AdS space by dilaton field  $e^{-\varphi(z)}$   
Analytical solution of EOM, Regge behavior  $M^2 \sim J(L)$

# Introduction

- **Scalar and Pseudoscalar fields**  $X(x, z) = S(x, z) e^{\vec{\pi}(x, z)\vec{\tau}/F_\pi}$
- **Pseudoscalar mesons** are dual to the mixing of  $\pi_i(x, z)$  and  $\phi_i(x, z)$  – longitudinal component of axial field  $A_\mu^i(x, z) = A_\mu^{i,\perp}(x, z) + \partial_\mu \phi^i(x, z)$
- Action

$$S = \int d^5x e^{-\varphi(z)} \sqrt{g} \left\{ -\frac{1}{4g_5^2} (F_L^2 + F_R^2) - |DX|^2 + 3|X|^2 \right\}$$

where

$$\varphi(z) = \kappa^2 z^2, \quad g_5^2 = 12\pi^2/N_c,$$

$$V = \frac{1}{2}(A_L + A_R), \quad A = \frac{1}{2}(A_L - A_R),$$

$$F_{L,R}^{MN} = \partial^M A_{L,R}^N - \partial^N A_{L,R}^M - i[A_{L,R}^M, A_{L,R}^N],$$

$$D^M X = \partial^M X - iA_L^M X + iX A_R^M$$

# Introduction

- **How it works:** Mass spectrum and WFs of vector mesons

- Apply axial gauge  $V_z = 0$  and derive EOM for the 4D-transverse component  $V_\mu^\perp$

$$e^{\varphi(z)} \partial_z \left[ \frac{e^{-\varphi(z)}}{z} \partial_z V_\mu^\perp(x, z) \right] - \partial^2 V_\mu^\perp(x, z) = 0$$

- KK expansion  $V_\mu^\perp(x, z) = \sum_n V_{\mu, n}^\perp(x) v_n(z)$

$$e^{\varphi(z)} \partial_z \left[ \frac{e^{-\varphi(z)}}{z} \partial_z v_n(z) \right] + M_{V_n}^2 v_n(z) = 0$$

- After the substitution  $v_n(z) = e^{\varphi(z)/2} \sqrt{z} \psi_n(z)$  derive **Schrödinger EOM**

$$-\partial_z^2 \psi_n(z) + U(z) \psi_n(z) = M_{V_n}^2 \psi_n(z)$$

where  $U(z) = \kappa^4 z^2 + \frac{3}{4z^2}$  **dilaton potential**

- Analytical solutions

$$M_{V_n}^2 = 4\kappa^2(n+1), \quad \psi_n(z) = \sqrt{\frac{2n!}{(n+1)!}} \kappa^2 z^{3/2} e^{-\kappa^2 z^2/2} L_n^1(\kappa^2 z^2)$$



# Introduction

## Main results in meson sector

- Reproduces consequences of chiral symmetry:

Gell-Mann – Oakes – Renner relation  $M_\pi^2 F_\pi^2 = 2\hat{m} \langle \bar{q}q \rangle$

- Properties of correlators of vector and axial current:

$$\Pi_V(Q^2) \rightarrow -\frac{N_c}{24\pi^2} \log Q^2 \quad \text{at } Q^2 \rightarrow \infty$$

$$\Pi_A(Q^2) \rightarrow \frac{F_\pi^2}{Q^2} \quad \text{at } Q^2 \rightarrow 0$$

VV (AA) Correlators are expanded as a sum over vector (axial) meson resonances

- Mass spectrum (Regge-like behavior)
- Form factors with correct power scaling, distribution functions, etc.

# Baryons in soft-wall model

- Baryons in soft-wall model: Forkel–Beyer–Frederico, Brodsky–Teramond, Abidin–Carlson, Gutsche–Lyubovitskij–Schmidt–Vega, ...
- **SW holographic approach** for baryons with inclusion of high Fock states dual to bulk fermion fields of higher dimension.
- **Objective:** Application to nucleon form factors, GPDs, nucleon resonances (Roper)

# Baryons in soft-wall model

- Bulk fermion fields  
 $\Psi_+(x, z)$  and  $\Psi_-(x, z)$  dual to  $\mathcal{O}_R = (p_R, n_R)$  and  $\mathcal{O}_L = (p_L, n_L)$
- Bulk fermion mass  $\pm m = \pm (\Delta - 3/2)$ , where  $\Delta$  - scaling dimension
- Scaling dimension  $\equiv$  Twist-dimension  $\tau = N + L$ ,  
 $N$  - number of partons,  $L = \max |L_z|$
- Action for the fermion field of twist  $\tau$

$$S_\tau = \int d^4x dz \sqrt{g} e^{-\varphi(z)} \sum_{i=+,-} \bar{\Psi}_{i,\tau}(x, z) \hat{\mathcal{D}}_i(z) \Psi_{i,\tau}(x, z),$$

$$\hat{\mathcal{D}}_\pm(z) = \frac{i}{2} \Gamma^M \overleftrightarrow{\partial}_M \mp \frac{m + \varphi(z)}{R}, \quad \Gamma^M = (\gamma^\mu, -i\gamma^5)$$

- dilaton  $\varphi(z) = \kappa^2 z^2$  (Regge behavior of hadron masses)
- metric  $g_{MN}(z) = \epsilon_M^a(z) \epsilon_N^b(z) \eta_{ab}$ ,  $g = |\det g_{MN}|$
- vielbein  $\epsilon_M^a(z) = e^{A(z)} \delta_M^a$ ,  $A(z) = \log(R/z)$  (conformal)
- interval  $ds^2 = g_{MN} dx^M dx^N = e^{2A(z)} (g_{\mu\nu} dx^\mu dx^\nu - dz^2)$

# Baryons in soft-wall model

- P-transformations

$$U_P^{-1} \Psi_{\tau, \pm}(t, \vec{x}, z) U_P = \pm \gamma^0 \gamma^5 \Psi_{\tau, \mp}(t, -\vec{x}, z)$$

$$U_P^{-1} \bar{\Psi}_{\tau, \pm}(t, \vec{x}, z) U_P = \pm \bar{\Psi}_{\tau, \mp}(t, -\vec{x}, z) \gamma^0 \gamma^5$$

$$\pm U_P^{-1} \bar{\Psi}_{\pm, \tau}(t, \vec{x}, z) \Psi_{\pm, \tau}(t, \vec{x}, z) U_P = \mp \bar{\Psi}_{\mp, \tau}(t, -\vec{x}, z) \Psi_{\mp, \tau}(t, \vec{x}, z)$$

$$U_P^{-1} S_{\tau}^{\pm} U_P = S_{\tau}^{\mp}$$

- C-transformations

$$U_C^{-1} \Psi_{\pm}(x, z) U_C = \mp C \gamma^5 \bar{\Psi}_{\mp}^T(x, z)$$

$$U_C^{-1} \bar{\Psi}_{\pm}(x, z) U_C = \pm \Psi_{\mp}^T(x, z) \gamma^5 C$$

$$\pm U_C^{-1} \bar{\Psi}_{\pm}(x, z) \Psi_{\pm}(x, z) U_C = \mp \bar{\Psi}_{\mp}(x, z) \Psi_{\mp}(x, z)$$

$$U_C^{-1} S_{\tau}^{\pm} U_C = S_{\tau}^{\mp}$$

# Baryons in soft-wall model

- **Redefinition**  $\Psi_{i,\tau}(x, z) = e^{\varphi(z)/2 - 2A(z)} \psi_{i,\tau}(x, z)$
- **Expansion on left- and right-chirality components (eigenstates of  $\gamma^5$ )**  
 $\psi_{i,\tau}(x, z) = \psi_{i,\tau}^L(x, z) + \psi_{i,\tau}^R(x, z)$
- **Kaluza-Klein expansion**

$$\psi_{i,\tau}^{L/R}(x, z) = \frac{1}{\sqrt{2}} \sum_n \psi_n^{L/R}(x) f_{i,\tau,n}^{L/R}(z),$$

- **Relations between bulk profiles**

$$\begin{aligned} f_{\tau,n}^R(z) &\equiv f_{+,\tau,n}^R(z) = -f_{-,\tau,n}^L(z), \\ f_{\tau,n}^L(z) &\equiv f_{+,\tau,n}^L(z) = f_{-,\tau,n}^R(z). \end{aligned}$$

- **EOM**

$$\left[ -\partial_z^2 + \kappa^4 z^2 + 2\kappa^2 \left( m \mp \frac{1}{2} \right) + \frac{m(m \pm 1)}{z^2} \right] f_{\tau,n}^{L/R}(z) = M_{n\tau}^2 f_{\tau,n}^{L/R}(z),$$

# Baryons in soft-wall model

- Solutions

$$S_z = +\frac{1}{2}, L_z = 0 : f_{\tau,n}^R(z) = \sqrt{\frac{2\Gamma(n+1)}{\Gamma(n+\tau-1)}} \kappa^{\tau-1} z^{\tau-3/2} e^{-\kappa^2 z^2/2} L_n^{\tau-2}(\kappa^2 z^2),$$

$$S_z = -\frac{1}{2}, L_z = 1 : f_{\tau,n}^L(z) = \sqrt{\frac{2\Gamma(n+1)}{\Gamma(n+\tau)}} \kappa^\tau z^{\tau-1/2} e^{-\kappa^2 z^2/2} L_n^{\tau-1}(\kappa^2 z^2)$$

and

$$M_{n\tau}^2 = 4\kappa^2 (n + \tau - 1)$$

with

$$\int_0^\infty dz f_{\tau,n_1}^{L/R}(z) f_{\tau,n_2}^{L/R}(z) = \delta_{n_1 n_2}$$

# Baryons in soft-wall model

- Inclusion of high Fock states

$$S = \sum_{\tau} c_{\tau} S_{\tau}$$

$c_{\tau}$  - set of free parameters

- Integration over  $z$  using normalization condition for  $f^{L/R}$

$$S = \int d^4x \bar{\psi}_n(x) \left[ \underbrace{\sum_{\tau} c_{\tau} i \not{\partial}}_{=1} - \underbrace{\sum_{\tau} c_{\tau} M_{n\tau}}_{=M_n} \right] \psi_n(x).$$

Correct normalization of kinetic term of 4D spinor field

$$\sum_{\tau} c_{\tau} = 1, \quad \sum_{\tau} c_{\tau} M_{n\tau} = M_n \quad (\text{baryon mass})$$

# Baryons in soft-wall model

- **Large  $N_c$  expansion**  $M_n = \sum_{\tau} c_{\tau} M_{n\tau} \sim \sum_{\tau} c_{\tau} \cdot \underbrace{\kappa}_{\sim \sqrt{N_c}} \cdot \underbrace{\sqrt{n + \tau - 1}}_{\sim \sqrt{N_c}} \sim N_c$

- $\kappa \sim \sqrt{N_c}$  consistent with LFH (Brodsky-Teramond)

$$F_{\pi} = \frac{\sqrt{3}}{8} \kappa = 83 \text{ MeV} \sim \sqrt{N_c}$$

- Holographic variable  $z$  (as conjugate to  $\kappa$ ) scales as  $z \sim \frac{1}{\sqrt{N_c}}$

- Physical interpretation:

- Limit  $N_c \rightarrow \infty$  means  $z = 0$  or approaching to UV boundary

baryons are bound states of infinitely number of  $N_c$  quarks

- Limit  $N_c \rightarrow 0$  means  $z \rightarrow \infty$

no bound states of quarks due to confinement imposed by the dilaton

see also [Brodsky, Huet PLB 417 \(1998\) 145](#)



# Electromagnetic structure of nucleons

- **Abidin-Carlson:** First application of SW model (3q configurations)
- **Coupling of bulk vector and fermion fields**

$$\mathcal{L}_{\text{int}}(x, z) = \sum_{i=+,-} \sum_{\tau} c_{\tau} \bar{\Psi}_{i,\tau}(x, z) \hat{V}_i(x, z) \Psi_{i,\tau}(x, z)$$

$$\hat{V}_{\pm}(x, z) = \underbrace{Q_N \Gamma^M V_M(x, z)}_{\text{min. coupling}} \pm \underbrace{\frac{i}{4} \eta_V [\Gamma^M, \Gamma^N] V_{MN}(x, z) \pm g_V \tau_3 \Gamma^M i\Gamma^z V_M(x, z)}_{\text{nonmin. coupling}}$$

$$\langle p' | J^{\mu}(0) | p \rangle = \bar{u}(p') \left[ \gamma^{\mu} F_1^N(t) + \frac{i}{2m_N} \sigma^{\mu\nu} q_{\nu} F_2^N(t) \right] u(p)$$

$$F_1^p(Q^2) = C_1(Q^2) + g_V C_2(Q^2) + \eta_V^p C_3(Q^2)$$

$$F_2^p(Q^2) = \eta_V^p C_4(Q^2)$$

$$F_1^n(Q^2) = -g_V C_2(Q^2) + \eta_V^n C_3(Q^2)$$

$$F_2^n(Q^2) = \eta_V^n C_4(Q^2)$$

# Electromagnetic structure of nucleons

Matveev-Muradyan-Tavkhelidze-Brodsky-Farrar quark-counting rules at large  $Q^2$

- $$C_1(Q^2) = \frac{1}{2} \int_0^\infty dz V(Q, z) \sum_\tau c_\tau \left( [f_\tau^L(z)]^2 + [f_\tau^R(z)]^2 \right)$$

$$= \sum_\tau c_\tau B(a+1, \tau) \left( \tau + \frac{a}{2} \right) \sim \sum_\tau \frac{c_\tau}{a^{\tau-1}}$$

- $$C_2(Q^2) = \frac{1}{2} \int_0^\infty dz V(Q, z) \sum_\tau c_\tau \left( [f_\tau^R(z)]^2 - [f_\tau^L(z)]^2 \right)$$

$$= \frac{a}{2} \sum_\tau c_\tau B(a+1, \tau) \sim \sum_\tau \frac{c_\tau}{a^{\tau-1}}$$

- $$C_3(Q^2) = \frac{1}{2} \int_0^\infty dz z \partial_z V(Q, z) \sum_\tau c_\tau \left( [f_\tau^L(z)]^2 - [f_\tau^R(z)]^2 \right)$$

$$= a \sum_\tau c_\tau B(a+1, \tau+1) \frac{a(\tau-1)-1}{\tau} \sim \sum_\tau \frac{c_\tau}{a^{\tau-1}}$$

- $$C_4(Q^2) = 2m_N \int_0^\infty dz z V(Q, z) \sum_\tau c_\tau f_\tau^L(z) f_\tau^R(z)$$

$$= \frac{2m_N}{\kappa} \sum_\tau c_\tau (a+1+\tau) B(a+1, \tau+1) \sqrt{\tau-1} \sim \sum_\tau \frac{c_\tau}{a^\tau}$$

- $$a = \frac{Q^2}{4\kappa^2}, \quad B(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)} \text{ is the beta-function.}$$

# Electromagnetic structure of nucleons

- $V(Q, z)$  – propagator of trans. massless vector field (analogue of EM field)

$$\begin{aligned} V(Q, z) &= \Gamma\left(1 + \frac{Q^2}{4\kappa^2}\right) U\left(\frac{Q^2}{4\kappa^2}, 0, \kappa^2 z^2\right) \\ &= \kappa^2 z^2 \int_0^1 \frac{dx}{(1-x)^2} x^{\frac{Q^2}{4\kappa^2}} e^{-\frac{\kappa^2 z^2 x}{1-x}} \end{aligned}$$

Radyushkin-Grigoryan (integral repr.)

Brodsky-Teramond (identify  $x$  with LC momentum fraction)

$$V(0, z) = 1, \quad V(Q, 0) = 1, \quad V(Q, \infty) = 0.$$

At  $Q^2 = 0$  functions  $C_i(Q^2)$  are normalized as

$$C_1(0) = 1, \quad C_2(0) = C_3(0) = 0, \quad C_4(0) = \frac{2m_N}{\kappa} \sum_{\tau} c_{\tau} \sqrt{\tau - 1}$$

# Electromagnetic structure of nucleons

## Choice of free parameters

- $\kappa = 383 \text{ MeV}$ ,  $c_3 = 1.25$ ,  $c_4 = 0.16$ ,  $g_V = 0.3$
- $c_5$  is expressed through  $c_3$  and  $c_4$ :  $c_5 = 1 - c_3 - c_4 = -0.41$
- $c_3, c_4$  are constrained by the nucleon mass
- $\kappa$  is fixed by the nucleon mass and nucleon electromagnetic radii
- $g_V$  is fixed by fine tuning of the neutron electromagnetic radii
- Nonminimal couplings  $\eta_V^{p,n}$  from nucleon magnetic moments

$$\eta_V^p = \frac{\kappa (\mu_p - 1)}{2m_N C_0} = 0.30, \quad \eta_V^n = \frac{\kappa \mu_n}{2m_N C_0} = -0.32, \quad C_0 = \sqrt{2} c_3 + \sqrt{3} c_4 + 2c_5$$

- Power scaling

$$\text{Dirac FF } F_1(Q^2) \sim \left( \frac{\Lambda^2}{Q^2} \right)^2$$

$$\text{Pauli FF } F_2(Q^2) \sim \left( \frac{\Lambda^2}{Q^2} \right)^3$$

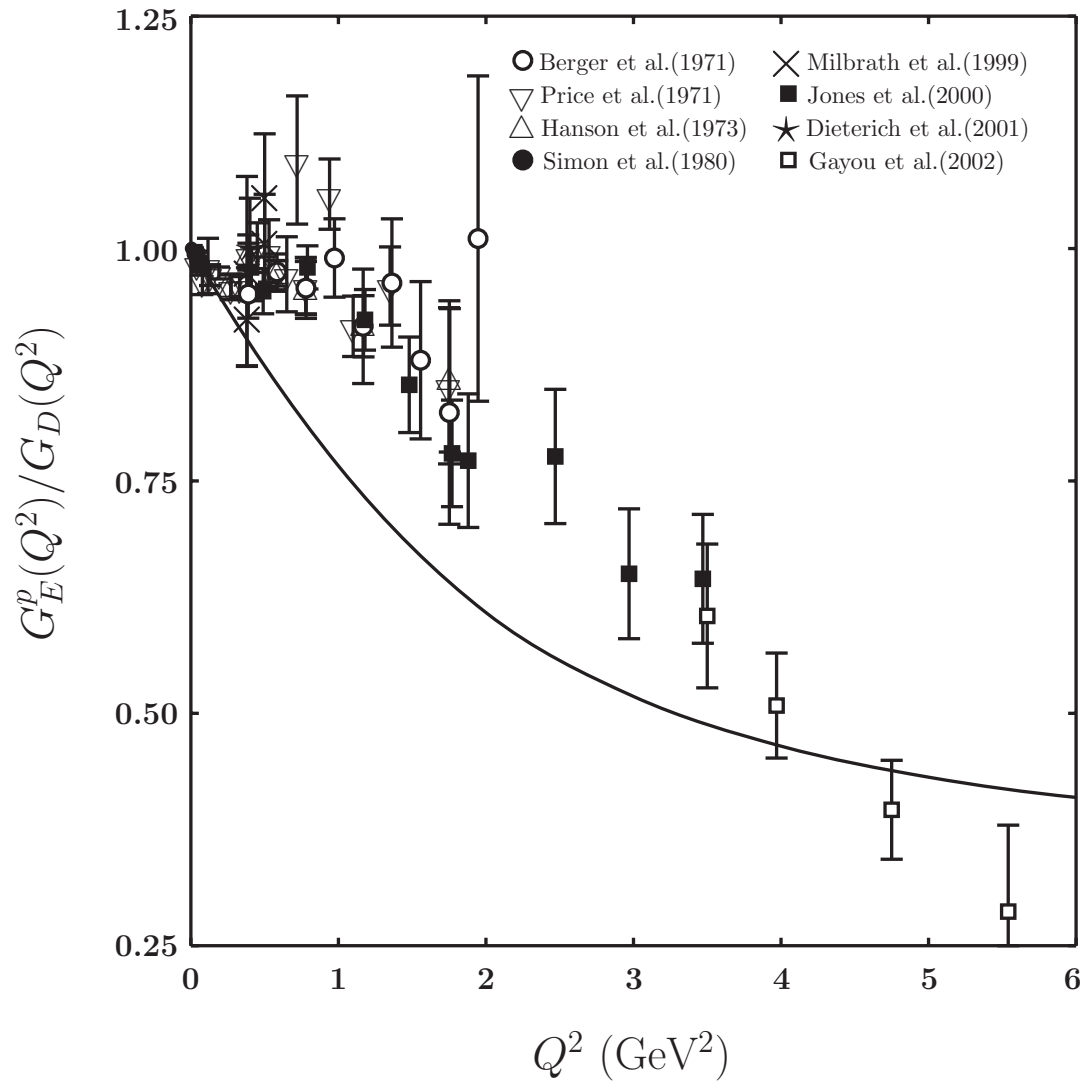
where  $\Lambda = 2\kappa = 0.77 \text{ GeV} \simeq \Lambda_{\text{dipole}} = 0.84 \text{ GeV}$

# Electromagnetic structure of nucleons

## Mass and electromagnetic properties of nucleons

Quantity	Our results	Data
$m_p$ (GeV)	0.93827	0.93827
$\mu_p$ (in n.m.)	2.793	2.793
$\mu_n$ (in n.m.)	-1.913	-1.913
$r_E^p$ (fm)	0.840	$0.8768 \pm 0.0069$
$\langle r_E^2 \rangle^n$ (fm <sup>2</sup> )	-0.117	$-0.1161 \pm 0.0022$
$r_M^p$ (fm)	0.785	$0.777 \pm 0.013 \pm 0.010$
$r_M^n$ (fm)	0.792	$0.862^{+0.009}_{-0.008}$
$r_A$ (fm)	0.667	$0.67 \pm 0.01$

# Electromagnetic structure of nucleons

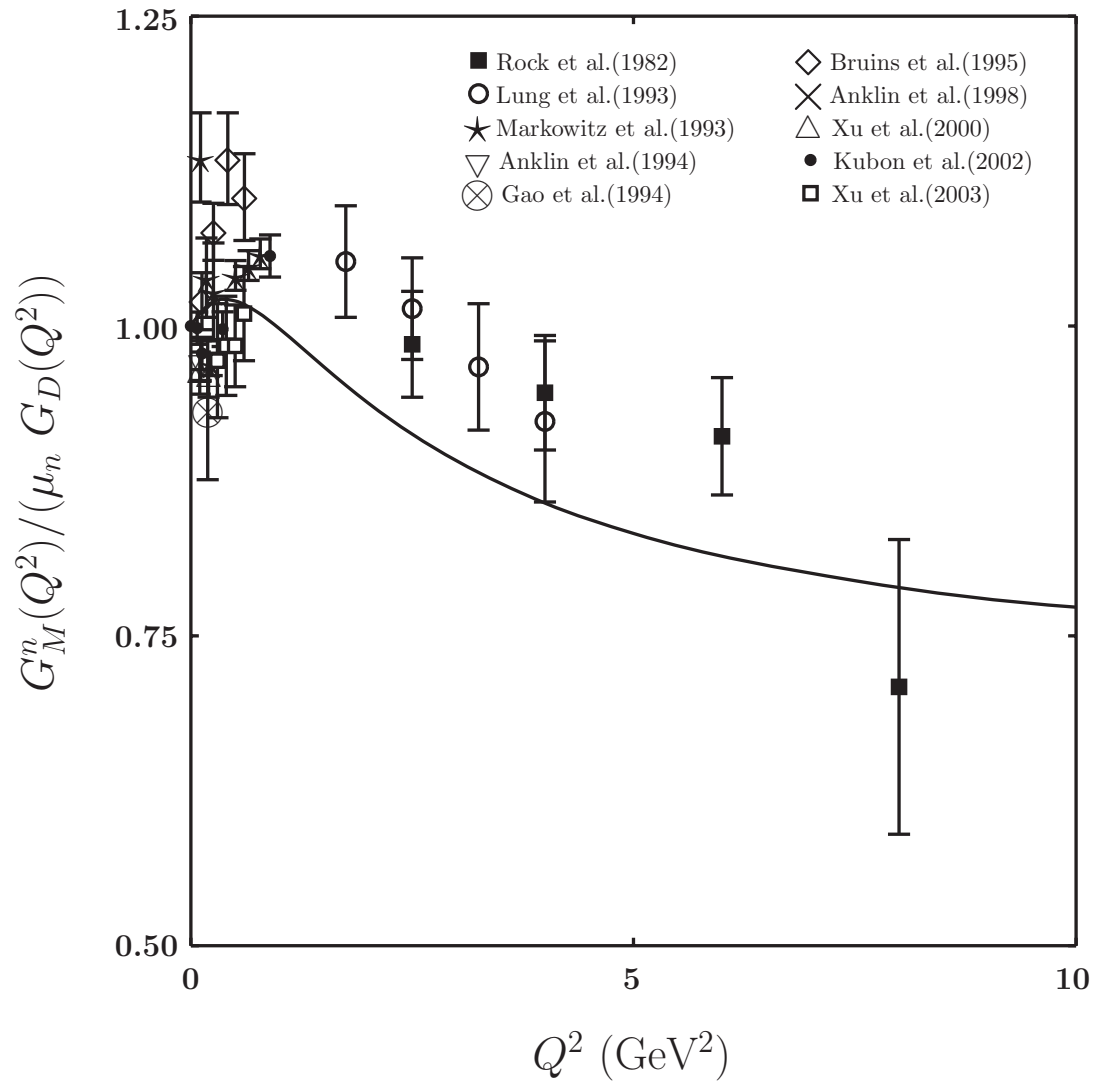




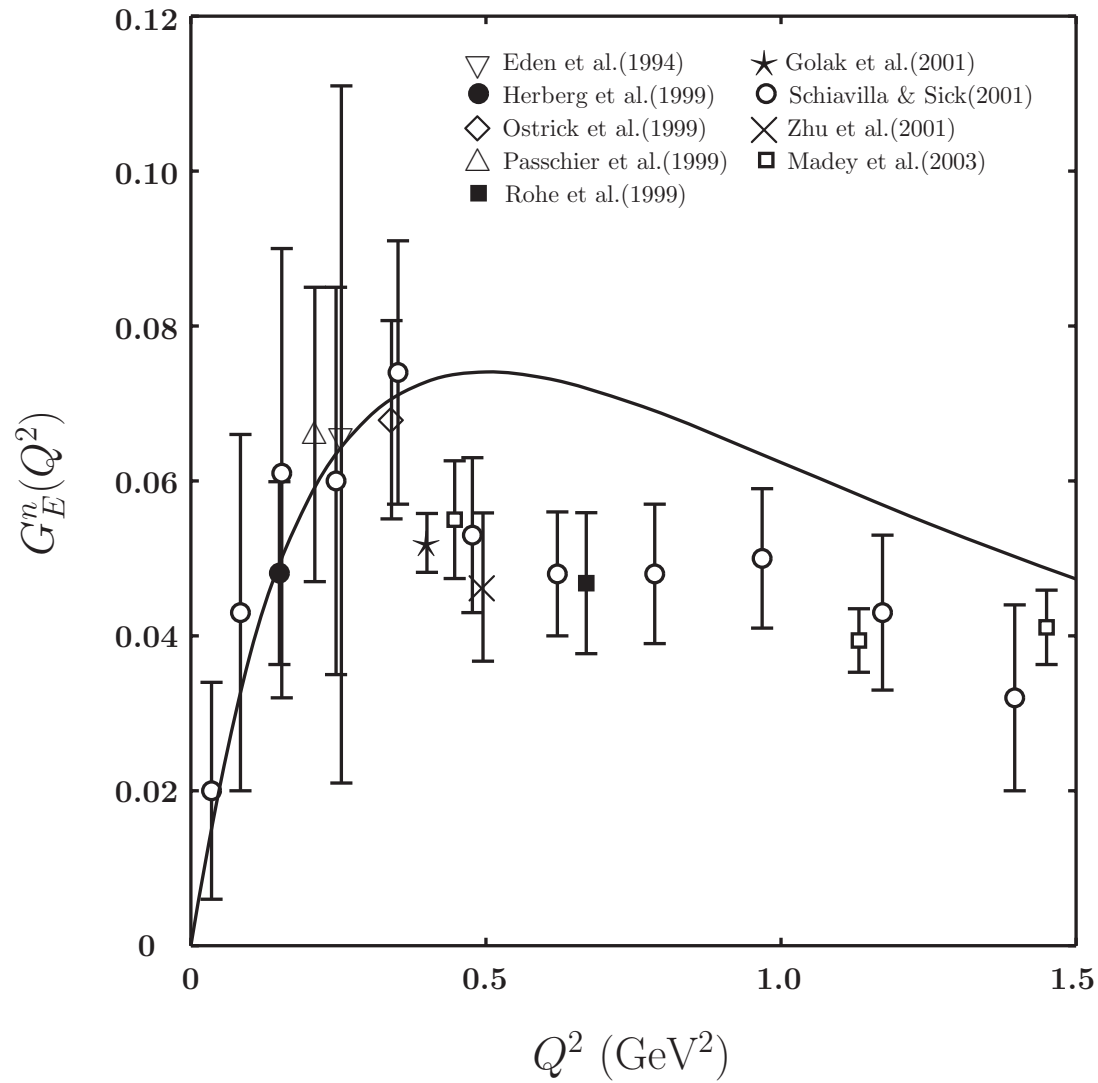




# Electromagnetic structure of nucleons

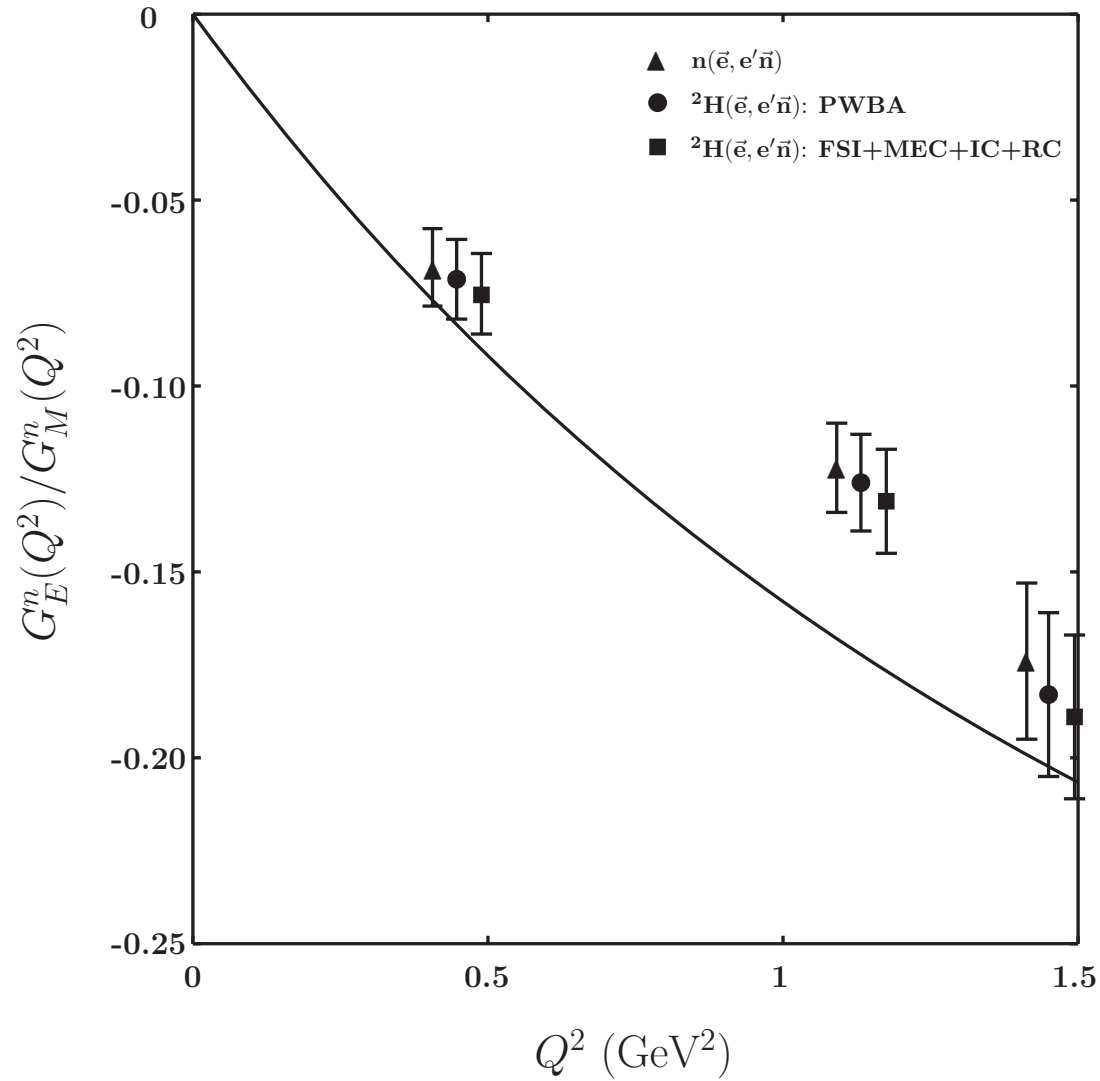


# Electromagnetic structure of nucleons

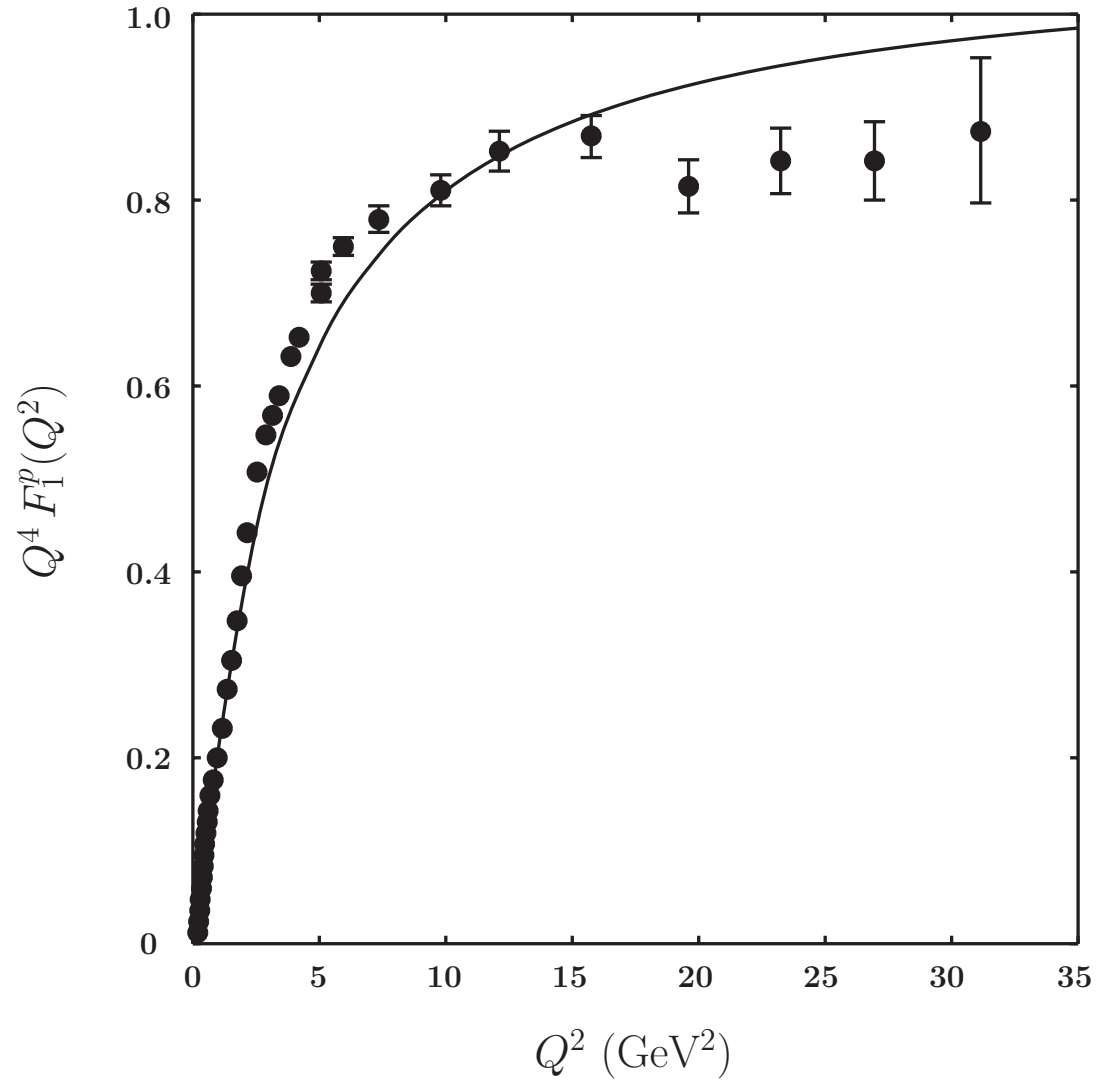




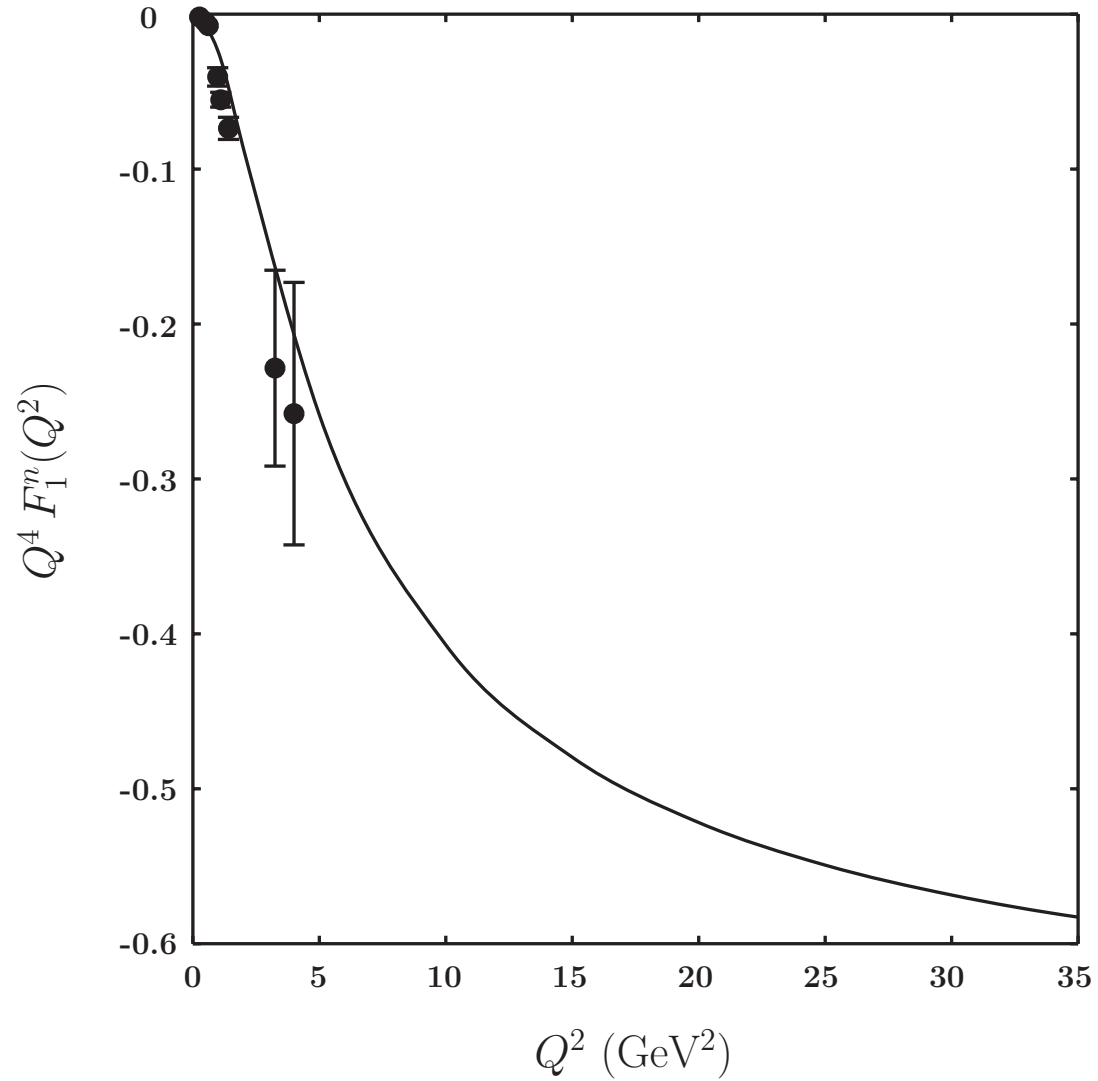
# Electromagnetic structure of nucleons



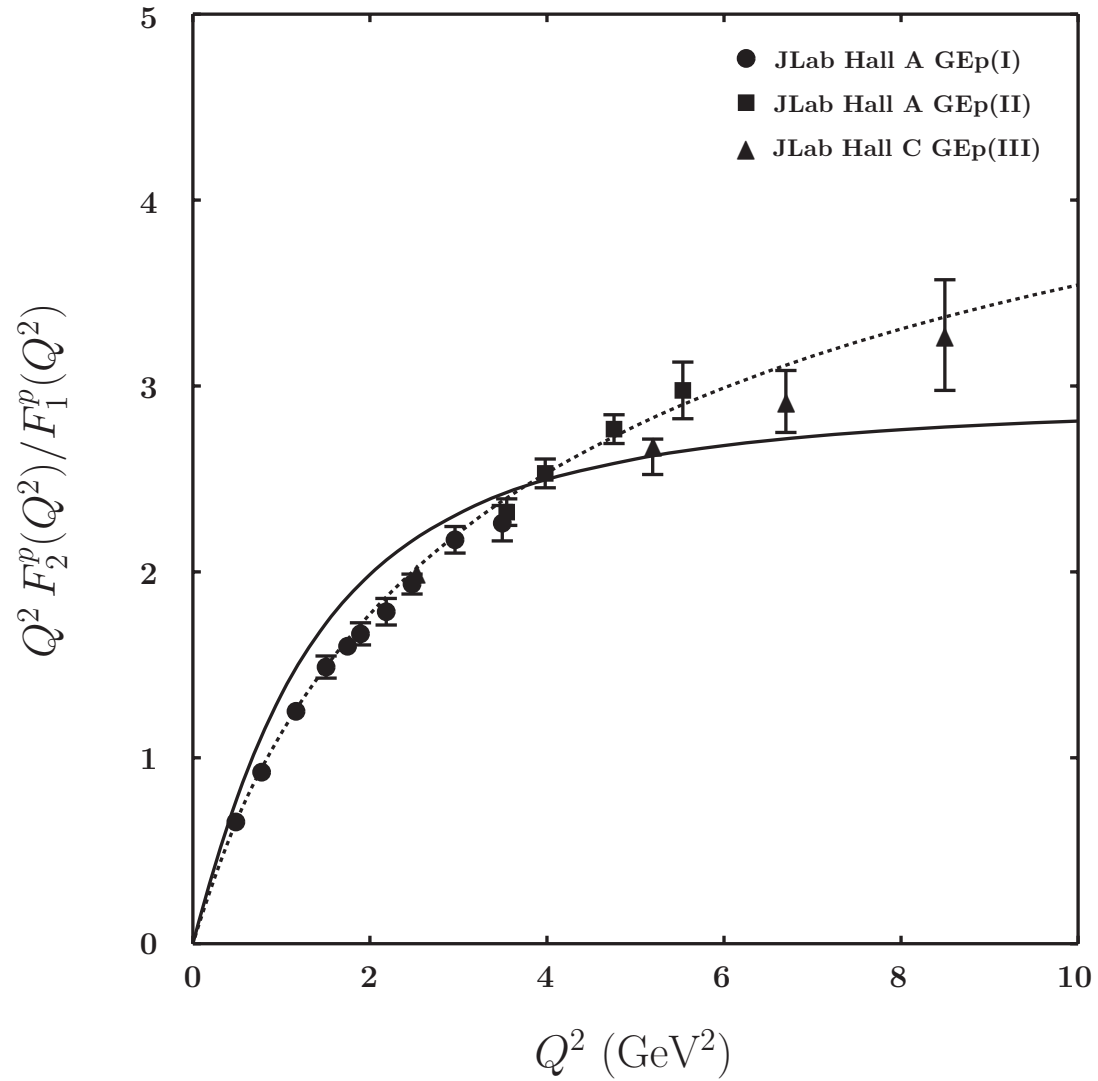
# Electromagnetic structure of nucleons



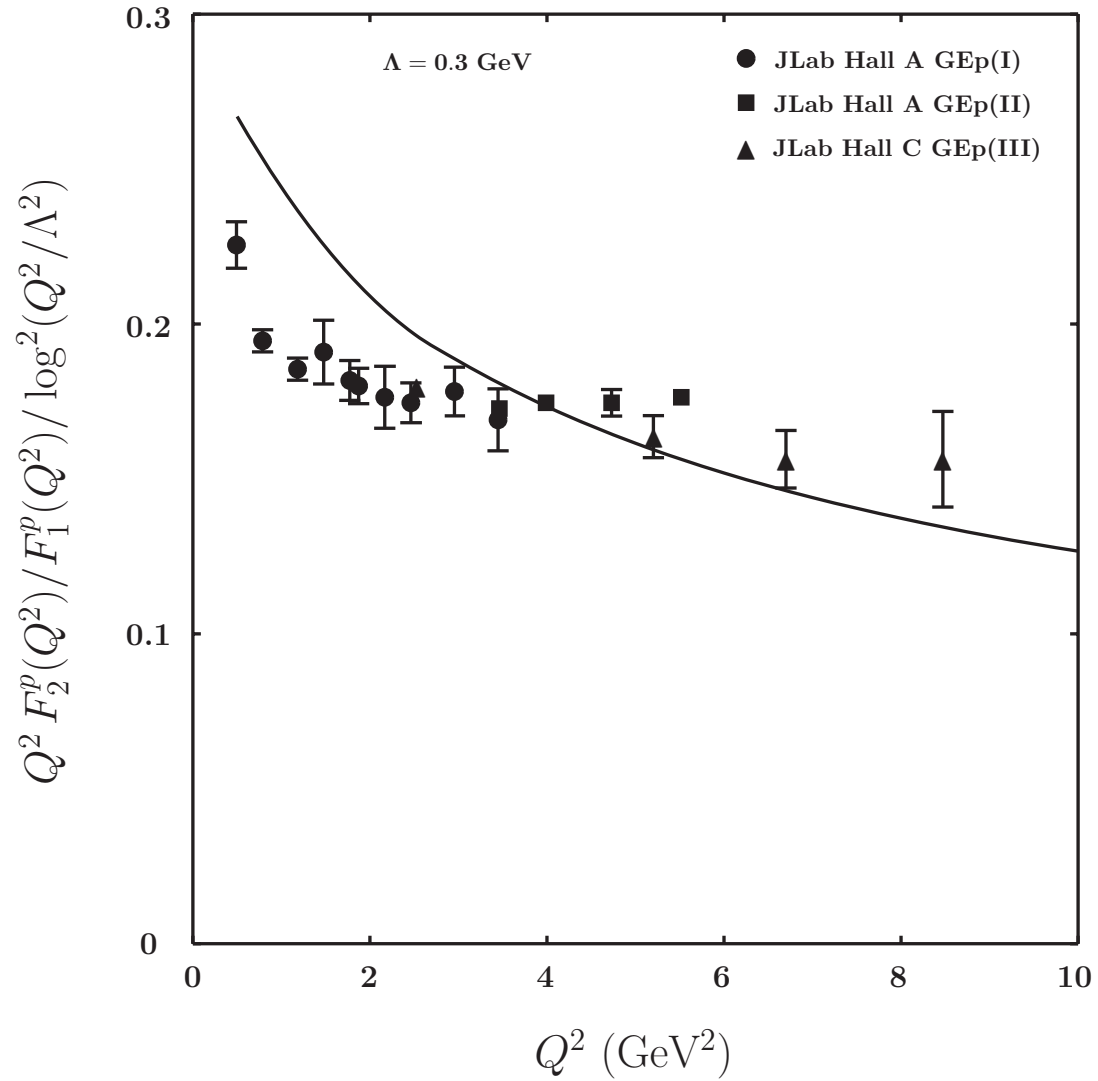
# Electromagnetic structure of nucleons



# Electromagnetic structure of nucleons



# Electromagnetic structure of nucleons





# Nucleon GPDs

- Sum rules relating EM FF and GPDs Ji, Radyushkin

$$F_1^p(t) = \int_0^1 dx \left( \frac{2}{3} H_v^u(x, t) - \frac{1}{3} H_v^d(x, t) \right)$$

$$F_1^n(t) = \int_0^1 dx \left( \frac{2}{3} H_v^d(x, t) - \frac{1}{3} H_v^u(x, t) \right)$$

$$F_2^p(t) = \int_0^1 dx \left( \frac{2}{3} E_v^u(x, t) - \frac{1}{3} E_v^d(x, t) \right)$$

$$F_2^n(t) = \int_0^1 dx \left( \frac{2}{3} E_v^d(x, t) - \frac{1}{3} E_v^u(x, t) \right)$$

- Grigoryan-Radyushkin integral representation for bulk-to-boundary propagator

$$V(Q, z) = \kappa^2 z^2 \int_0^1 \frac{dx}{(1-x)^2} x^{\frac{Q^2}{4\kappa^2}} e^{-\frac{x}{1-x} \kappa^2 z^2}$$

- LF mapping (Brodsky-Teramond):  $x$  is equivalent to LC momentum fraction

# Nucleon GPDs

- **GPDs**  $H_v^q(x, Q^2) = q(x) x \frac{Q^2}{4\kappa^2}$ ,  $E_v^q(x, Q^2) = e^q(x) x \frac{Q^2}{4\kappa^2}$
- **Distribution functions**  $q(x)$  and  $e^q(x)$

$$q(x) = \alpha^q \gamma_1(x) + \beta^q \gamma_2(x), \quad e^q(x) = \beta^q \gamma_3(x)$$

**Flavor couplings**  $\alpha^q, \beta^q$  and **functions**  $\gamma_i(x)$  are written as

$$\alpha^u = 2, \quad \alpha^d = 1, \quad \beta^u = 2\eta_p + \eta_n, \quad \beta^d = \eta_p + 2\eta_n$$

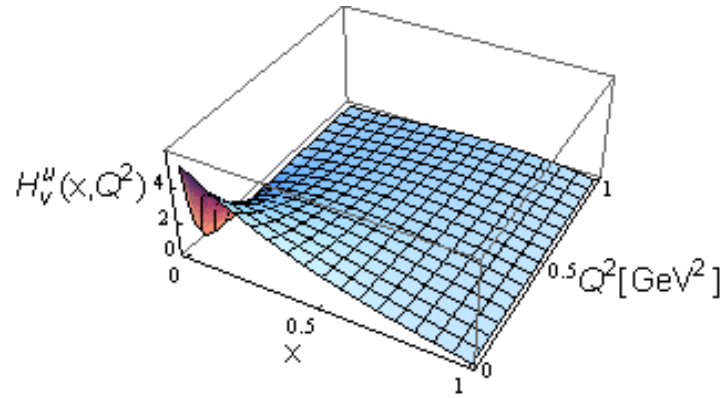
and

$$\gamma_1(x) = \frac{1}{2}(5 - 8x + 3x^2)$$

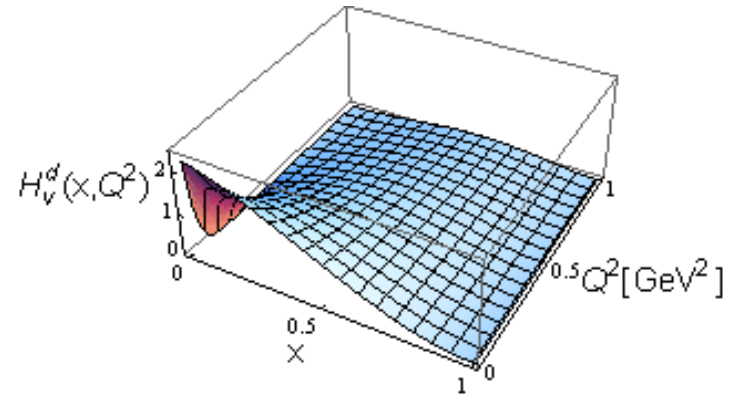
$$\gamma_2(x) = 1 - 10x + 21x^2 - 12x^3$$

$$\gamma_3(x) = 24(1 - x)^2$$

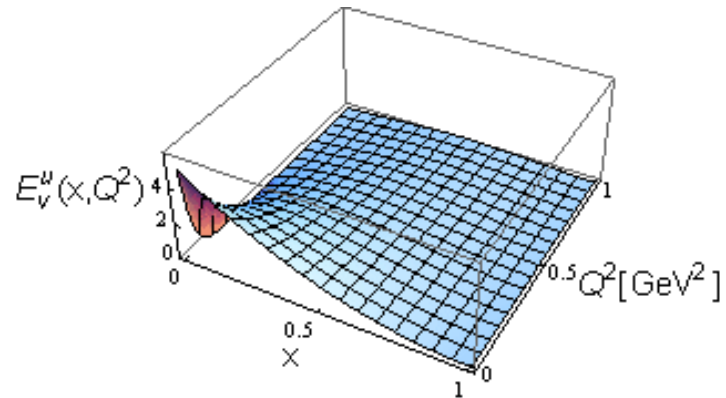
# Nucleon GPDs



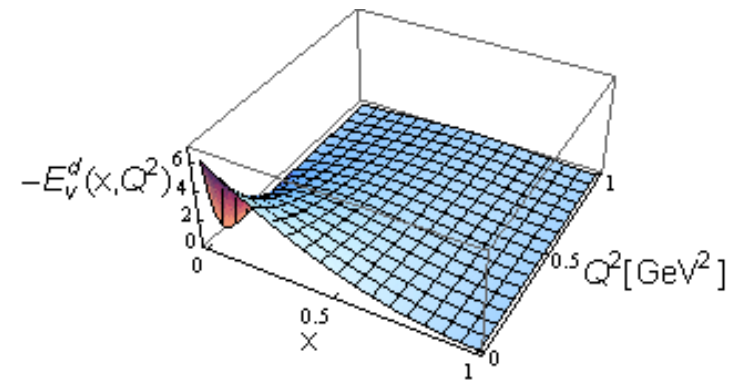
$H_v^u(x, Q^2)$



$H_v^d(x, Q^2)$



$E_v^u(x, Q^2)$



$E_v^d(x, Q^2)$

# Nucleon GPDs

- Nucleon GPDs in impact space Burkardt, Miller, Diehl, Kroll et al

$$q(x, \mathbf{b}_\perp) = \int \frac{d^2 \mathbf{k}_\perp}{(2\pi)^2} H_q(x, \mathbf{k}_\perp^2) e^{-i \mathbf{b}_\perp \cdot \mathbf{k}_\perp} = q(x) \frac{\kappa^2}{\pi \log(1/x)} e^{-\frac{\mathbf{b}_\perp^2 \kappa^2}{\log(1/x)}}$$
$$e^q(x, \mathbf{b}_\perp) = \int \frac{d^2 \mathbf{k}_\perp}{(2\pi)^2} E_q(x, \mathbf{k}_\perp^2) e^{-i \mathbf{b}_\perp \cdot \mathbf{k}_\perp} = e^q(x) \frac{\kappa^2}{\pi \log(1/x)} e^{-\frac{\mathbf{b}_\perp^2 \kappa^2}{\log(1/x)}}$$

- Parton charge and magnetization densities in transverse impact space

$$\rho_E^N(\mathbf{b}_\perp) = \sum_q e_q^N \int_0^1 dx q(x, \mathbf{b}_\perp) = \frac{\kappa^2}{\pi} \sum_q e_q^N \int_0^1 \frac{dx}{\log(1/x)} q(x) e^{-\frac{\mathbf{b}_\perp^2 \kappa^2}{\log(1/x)}}$$
$$\rho_M^N(\mathbf{b}_\perp) = \sum_q e_q^N \int_0^1 dx e^q(x, \mathbf{b}_\perp) = \frac{\kappa^2}{\pi} \sum_q e_q^N \int_0^1 \frac{dx}{\log(1/x)} e^q(x) e^{-\frac{\mathbf{b}_\perp^2 \kappa^2}{\log(1/x)}}$$

where  $e_u^p = e_d^n = 2/3$  and  $e_u^n = e_d^p = -1/3$

# Nucleon GPDs

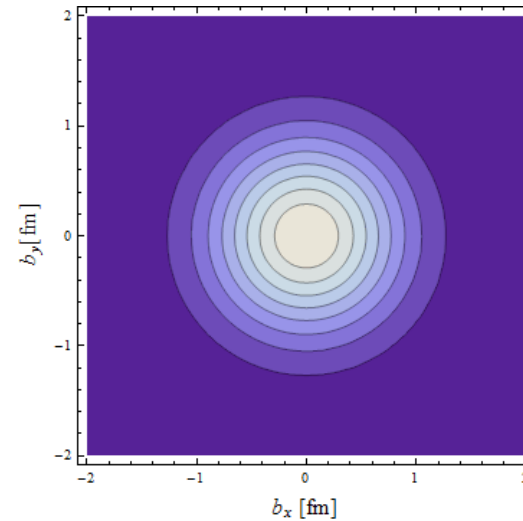
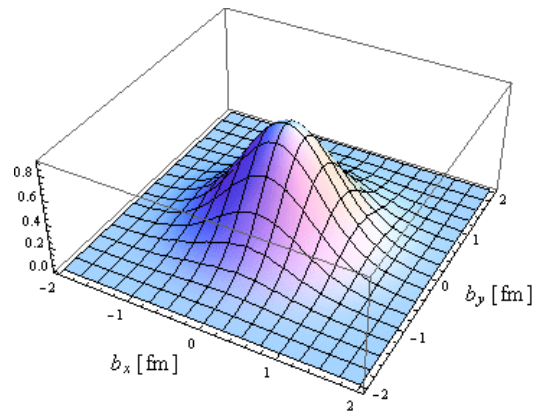
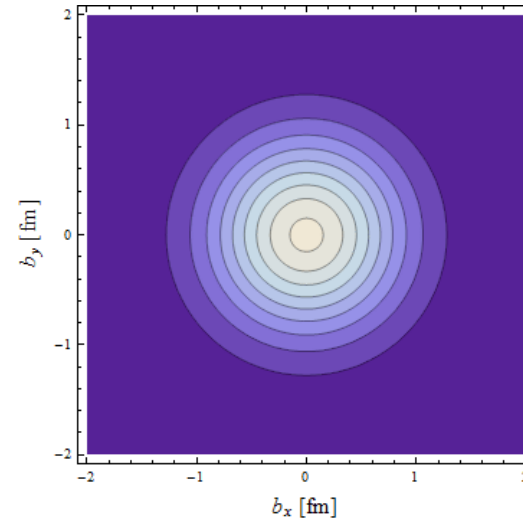
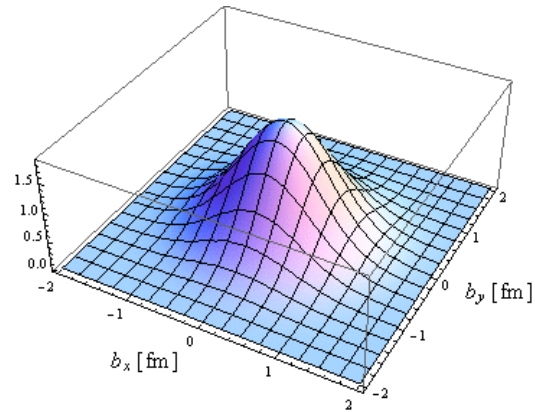
- Transverse width of impact parameter dependent GPD

$$\langle R_{\perp}^2(x) \rangle_q = \frac{\int d^2 \mathbf{b}_{\perp} \mathbf{b}_{\perp}^2 q(x, \mathbf{b}_{\perp})}{\int d^2 \mathbf{b}_{\perp} q(x, \mathbf{b}_{\perp})} = -4 \frac{\partial \log H_v^q(x, Q^2)}{\partial Q^2} \Big|_{Q^2=0} = \frac{\log(1/x)}{\kappa^2}$$

- Transverse rms radius

$$\langle R_{\perp}^2 \rangle_q = \frac{\int d^2 \mathbf{b}_{\perp} \mathbf{b}_{\perp}^2 \int_0^1 dx q(x, \mathbf{b}_{\perp})}{\int d^2 \mathbf{b}_{\perp} \int_0^1 dx q(x, \mathbf{b}_{\perp})} = \frac{1}{\kappa^2} \left( \frac{5}{3} + \frac{\beta^q}{12\alpha^q} \right) \simeq 0.527 \text{ fm}^2$$

# Nucleon GPDs



Plots for  $q(x, \mathbf{b}_\perp)$  for  $x = 0.1$ :  $u(x, \mathbf{b}_\perp)$  - upper pannels,  $d(x, \mathbf{b}_\perp)$  - lower pannels

# Roper resonance $N(1440)$

- Put  $n = 1$  and get solutions dual to Roper:

$$M_{\mathcal{R}} \simeq 1440 \text{ MeV}$$

- $N \rightarrow R + \gamma$  transition

$$M^\mu = \bar{u}_{\mathcal{R}} \left[ \gamma_{\perp}^{\mu} F_1(q^2) + i\sigma^{\mu\nu} \frac{q_{\nu}}{M_{\mathcal{R}}} F_2(q^2) \right] u_N, \quad \gamma_{\perp}^{\mu} = \gamma^{\mu} - q^{\mu} \frac{\not{q}}{q^2}$$

- Helicity amplitudes

$$H_{\pm\frac{1}{2}0} = \sqrt{\frac{Q_-}{Q^2}} \left( F_1 M_+ - F_2 \frac{Q^2}{M_{\mathcal{R}}} \right)$$
$$H_{\pm\frac{1}{2}\pm 1} = -\sqrt{2Q_-} \left( F_1 + F_2 \frac{M_+}{M_{\mathcal{R}}} \right)$$

- Alternative set [Weber, Capstick, Copley et al]

$$A_{1/2} = -b H_{\frac{1}{2}1}, \quad S_{1/2} = b \frac{|\mathbf{p}|}{\sqrt{Q^2}} H_{\frac{1}{2}0},$$

$$Q_{\pm} = M_{\pm}^2 + Q^2, \quad M_{\pm} = M_{\mathcal{R}} \pm M_N, \quad b = \sqrt{\frac{\pi\alpha}{2EM_{\mathcal{R}}M_N}}$$

# Roper resonance $N(1440)$

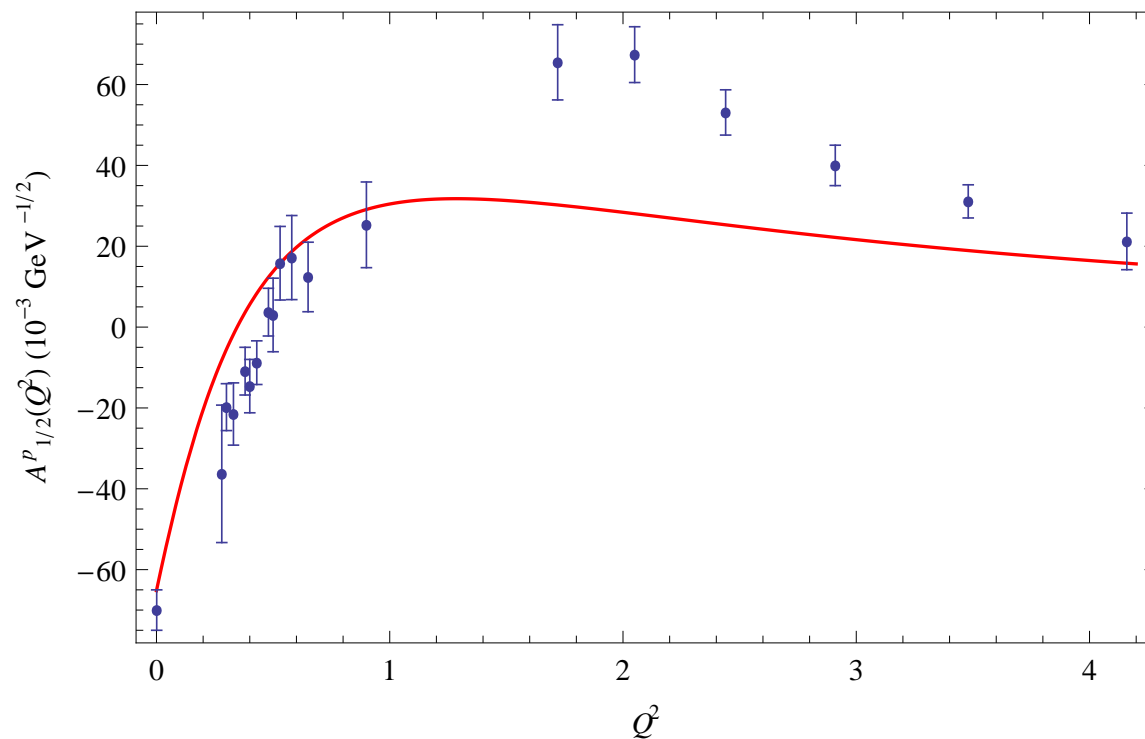
Helicity amplitudes  $A_{1/2}^N(0)$ ,  $S_{1/2}^N(0)$

Quantity	Our results	Data
$A_{1/2}^p(0)$ ( $\text{GeV}^{-1/2}$ )	-0.065	$-0.065 \pm 0.004$
$A_{1/2}^n(0)$ ( $\text{GeV}^{-1/2}$ )	0.040	$0.040 \pm 0.010$
$S_{1/2}^p(0)$ ( $\text{GeV}^{-1/2}$ )	0.040	
$S_{1/2}^n(0)$ ( $\text{GeV}^{-1/2}$ )	-0.040	



# Roper resonance $N(1440)$

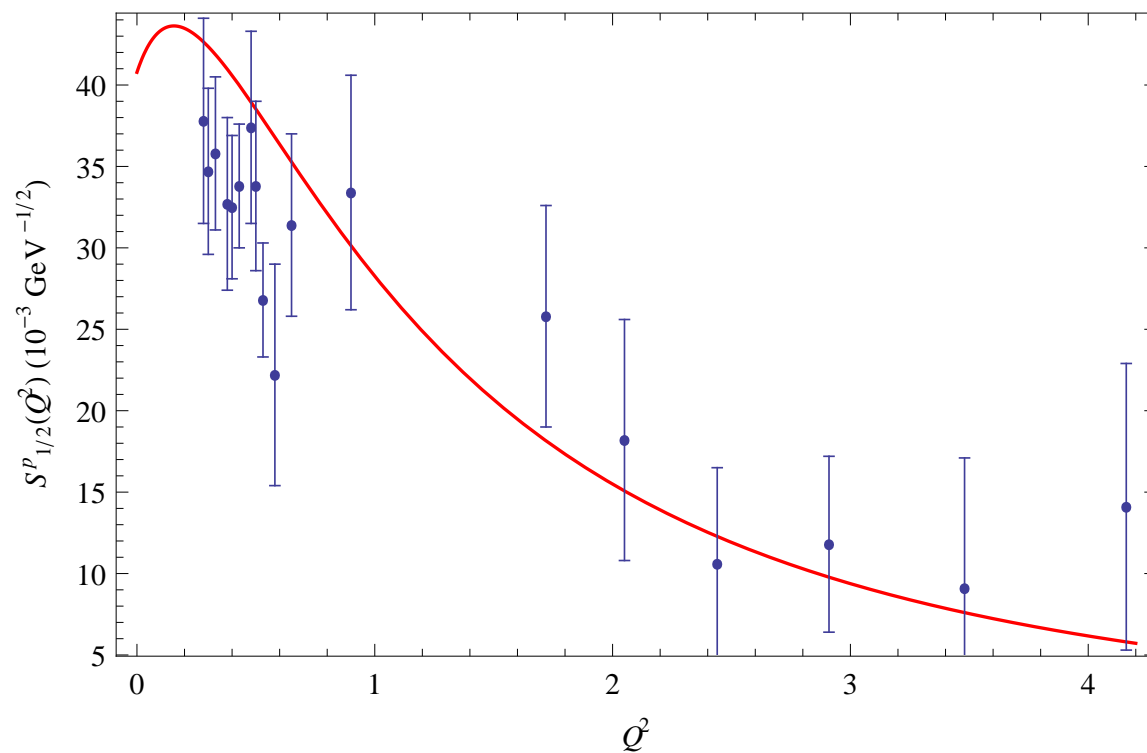
Helicity amplitude  $A_{1/2}^p(Q^2)$



Data: CLAS Coll at JLab, Moiseev et al, 1205.3948 [nucl-ex]

# Roper resonance $N(1440)$

Helicity amplitude  $S_{1/2}^p(Q^2)$



Data: CLAS Coll at JLab, Mokeev et al, 1205.3948 [nucl-ex]

# Summary

- AdS/QCD  $\equiv$  Holographic QCD (HQCD) – approximation to QCD: attempt to model Hadronic Physics in terms of fields/strings living in extra dimensions – anti-de Sitter (AdS) space
- HQCD models reproduce main features of QCD at low and high energies
- Soft-wall holographic approach – covariant and analytic model for hadron structure with confinement at large distances and conformal behavior at short distances
- Mesons, baryons, exotics from unified point view and including high Fock states
- Future work: nucleon TMDs, DVCS